

Light gluino and the running of α_s

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A gluino in the mass range 12–16 GeV combined with a light (2–5.5 GeV) bottom squark, as has been proposed recently to explain an excess of b quark hadroproduction, would affect the momentum-scale dependence (“running”) of the strong coupling constant α_s in such a way as to raise its value at M_Z by about 0.014 ± 0.001 . If one combines sources of uncertainty at low (m_b) and high (M_Z) mass scales, one cannot exclude such a possibility. Prospects for improvement in this situation, which include better lattice QCD simulations and better measurements at M_Z , are discussed.

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I. INTRODUCTION

The production of b quarks in hadronic and electromagnetic reactions appears enhanced with respect to expectations based on perturbative QCD [1]. While questions have been raised about the magnitude or interpretation of this effect [2], the discrepancy has led to the suggestion of an additional mechanism of b quark production through the production of relatively light (12–16 GeV) gluinos, followed by the decay of gluinos to b quarks and their lighter (2–5.5 GeV) superpartners \tilde{b} [3]. The orthogonal mixture \tilde{b}' is assumed to be sufficiently heavy that it would not yet have been observed. The \tilde{b} squarks are assumed to be a mixture of the superpartners of b_L and b_R such that the decay $Z \rightarrow \tilde{b}\tilde{b}^*$ is suppressed [4]. Here we follow [3,5] in defining

$$\begin{pmatrix} \tilde{b} \\ \tilde{b}' \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{b}} \\ -\sin \theta_{\tilde{b}} & \cos \theta_{\tilde{b}} \end{pmatrix} \begin{pmatrix} \tilde{b}_R \\ \tilde{b}_L \end{pmatrix}. \quad (1)$$

We will see later that the light bottom squark \tilde{b} is dominantly right handed in order to have a sufficiently weak coupling with the Z boson.

A light gluino has been proposed before [6–8]. Clavelli [9] noted that the value of α_s as extracted from quarkonia (see, e.g., Ref. [10]), when extrapolated using the standard beta function of QCD to M_Z , led to a slightly lower value than measured directly at the Z . The running effect can be slowed down through the introduction of new fermionic and/or scalar particles with mass below M_Z . Recent analyses do not exclude or favor the possibility of a light gluino in the mass range of interest to us [11–13]. (A light gluino with a mass of the order of a few GeV, however, has been experimentally excluded [14].) Reference [15] further shows that

the inclusion of a light bottom squark only changes the running slightly and is still compatible with the current experimental data. However, the α_s extractions in these analyses do not take into account the contributions of the new particles. It is our purpose here to include such effects using available results and identify the improvements in data and calculations needed for a definite conclusion about the effect of a light gluino on the running of α_s between scales below 10 GeV and the Z mass. This question is of interest because of foreseen improved determinations at lower mass scales using quarkonium data and lattice gauge theories [16–18], and at M_Z using future linear colliders. We show that a distinction between the behavior of the QCD beta function with and without a 12–16 GeV gluino is not possible at present, but will be so with anticipated improvements in the low-energy determination of α_s and with reduction of errors in $\Gamma(Z \rightarrow b\bar{b})$ and $\Gamma(Z \rightarrow \text{hadrons})$.

Section II treats two-loop formulas for the scale dependence (“running”) of α_s in the standard model (SM) and in the presence of a light gluino and bottom squark. Typical effects range from $\delta\alpha_s(M_Z) \equiv \alpha_s^{MSSM}(M_Z) - \alpha_s^{SM}(M_Z) \approx 0.015$ at $m_{\tilde{g}} = 12$ GeV to $\delta\alpha_s(M_Z) \approx 0.009$ at $m_{\tilde{g}} = 30$ GeV, with $\delta\alpha_s(M_Z) \approx 0.002$ due to the bottom squark. These effects are somewhat larger than those found in Ref. [3] based upon one-loop running, but errors on α_s at low mass scales (Sec. III), at M_Z (Sec. IV), and above M_Z (Sec. V) still are large enough that no distinction is possible between the standard model and the light-gluino or bottom squark scenario in the minimal supersymmetric standard model (MSSM). We collect results and discuss the prospects for improved measurements in Sec. VI, summarizing briefly in Sec. VII.

II. TWO-LOOP RUNNING

The two-loop evolution of the strong coupling constant is governed by the β function

$$\beta(\alpha_s) = \mu \frac{d\alpha_s}{d\mu} = -\frac{\alpha_s^2}{2\pi} \left(b_1 + b_2 \frac{\alpha_s}{4\pi} \right). \quad (2)$$

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In a minimally extended SUSY QCD model, the one- and two-loop coefficients are given by [19–21]

$$b_1 = \left(\frac{11}{3} - \frac{2}{3}n_g^- \right) C_A - \left(\frac{4}{3}n_q + \frac{1}{3}n_{\tilde{q}}^- \right) T_F, \quad (3)$$

$$b_2 = \left(\frac{34}{3} - \frac{16}{3}n_g^- \right) C_A^2 - (4n_q + 4n_{\tilde{q}}^- - 2n_g^-n_{\tilde{q}}^-) C_F T_F - \left(\frac{20}{3}n_q + \frac{2}{3}n_{\tilde{q}}^- - 2n_g^-n_{\tilde{q}}^- \right) C_A T_F, \quad (4)$$

where n_q is the number of quark flavors, $n_{\tilde{q}}^-$ the number of squarks, and n_g^- the number of gluinos, $T_F=1/2$ is the Dynkin index of the fundamental representation, and $C_A=3$ and $C_F=4/3$ are the Casimir invariants in the adjoint and fundamental representations, respectively. In the scheme with only one light bottom squark and one light gluino with masses less than M_Z ($n_{\tilde{q}}^- = n_g^- = 1$), the changes in the β function due to these new particles are

$$\delta b_1^{\tilde{g}} = -2, \quad \delta b_1^{\tilde{b}} = -\frac{1}{6}, \quad (5)$$

$$\delta b_2^{\tilde{g}} = -48, \quad \delta b_2^{\tilde{b}} = -\frac{11}{3}, \quad \delta b_2^{\tilde{g}\tilde{q}} = \frac{13}{3}. \quad (6)$$

Up to two loops, the decoupling relation between $\alpha_s^{(n_f)}(\mu)$ in the n_f -flavor theory and $\alpha_s^{(n_f-1)}(\mu)$ in the (n_f-1) -flavor theory is trivial when they are matched at the heavy flavor threshold; for example, $\alpha_s^{(n_f)}(m_b) = \alpha_s^{(n_f-1)}(m_b)$ for the modified minimal subtraction ($\overline{\text{MS}}$) scheme mass $m_b = m_b^{(n_f)}(m_b)$. Finite corrections start to come in when one considers three-loop running [22].

Starting from α_s at a low-energy scale, one can obtain its value at M_Z by solving the integral equation

$$\log\left(\frac{M_Z^2}{\mu_0^2}\right) = \int_{\alpha_s(\mu_0)}^{\alpha_s(M_Z)} \frac{2d\alpha}{\beta(\alpha)}. \quad (7)$$

Evolving the strong coupling constant in the SM and MSSM with initial values given in Ref. [18] at $m_b = 4.1$ GeV, $\alpha_s^{(n_f=5)}(m_b) = 0.239_{-0.010}^{+0.012}$, we take $m_{\tilde{b}} = 4$ GeV and $m_{\tilde{g}} = 15$ GeV as an example and obtain

$$\alpha_s^{\text{SM}}(M_Z) = 0.1216 \pm 0.0027, \\ \alpha_s^{\text{MSSM}}(M_Z) = 0.1352 \pm 0.0035. \quad (8)$$

It should be mentioned that the minor difference of the evolution within the SM of this paper from that given in Ref. [18] is because we restrict ourselves to two-loop running while they use the three-loop running result.

We find that the light gluino dominates over the light bottom squark in the evolution over a wide range of its mass. In Fig. 1, we plot the difference $\delta\alpha_s(M_Z)$ as a function of the gluino mass $m_{\tilde{g}}$. The solid curve gives the result with both the light bottom squark and gluino taken into account,

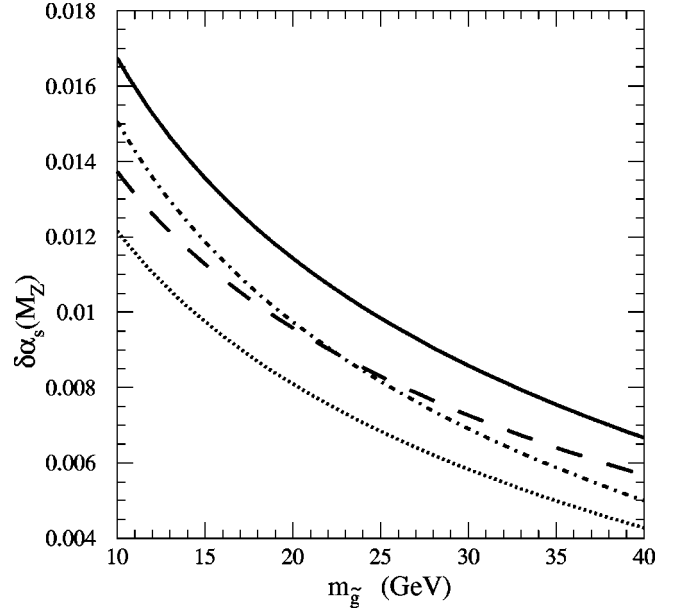


FIG. 1. Difference of $\alpha_s(M_Z)$ between the MSSM and SM starting from $\alpha_s(m_b) = 0.239$ as a function of the gluino mass $m_{\tilde{g}}$. The solid and dot-dashed curves give the two-loop results with and without a light bottom squark, respectively. The corresponding one-loop results are shown by the long-dashed curve (gluino plus light bottom squark) and dotted curve (gluino only).

whereas the dashed curve gives the result due to the light gluino contribution alone. The corresponding one-loop running results are indicated by the long-dashed curve (gluino and bottom squark) and dotted curve (gluino only). For the range of gluino mass of interest, 12 to 16 GeV, we find $\delta\alpha_s$ ranges from 0.015 to 0.013, so we shall quote it as $\delta\alpha_s = 0.014 \pm 0.001$ in what follows. We now ask whether present data favor or disfavor such an effect.

III. LOW-ENERGY INFORMATION ON α_s

In this section we review the main sources of low-energy information on α_s , concentrating on those with the smallest claimed errors and describing those errors critically. All results are quoted as $\alpha_s(M_Z)$ assuming SM running in this section. The corresponding values of $\alpha_s(M_Z)$ in the presence of a light gluino and a bottom squark can be obtained by adding $\delta\alpha_s (= 0.014 \pm 0.001)$.

A. τ decays

The lowest-energy determination of α_s , which appears in current reviews [11–13] comes from τ decays involving hadrons, with a *maximum* mass scale of m_{τ} . Impressive progress has been claimed in expressing the hadronic final state in $\tau \rightarrow \nu_{\tau} + X$, $X = \pi, \rho, a_1, \dots$, in terms of an effective quark-antiquark continuum describable via perturbative QCD, leading to values $\alpha_s(m_{\tau}) = 0.323 \pm 0.030$ [11,13] or 0.35 ± 0.03 [12]. Extrapolation via the renormalization group then leads to the values $\alpha_s(M_Z) = 0.1181 \pm 0.0031$ [11,13] or 0.121 ± 0.003 [12]. However, the assignment of errors to the contribution of nonperturbative effects in these analyses is

highly subjective, based on QCD sum rules for which independent tests of sufficient accuracy do not exist in our opinion. Remember that an error of 9% in $\alpha_s(m_\tau)$ corresponds to a change of less than 3% in the perturbative expression for $\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})$.

B. Deep inelastic scattering

The original and still one of the most powerful methods to measure α_s is deep inelastic lepton-hadron scattering. The most precise of several determinations, based on measurements of the structure function F_2 with electrons and muons, gives $\alpha_s(M_Z) = 0.1166 \pm 0.0022$ [13], from data points in the range $1.9 \text{ GeV} \leq Q \leq 15.2 \text{ GeV}$ [23]. However, the error is not based on an analysis by any experimental group. Other determinations are consistent with this value, but the smallest error quoted in any of them is ± 0.004 [11,13].

In principle the determination of α_s from deep inelastic scattering could be affected by a light \tilde{b} , since at the highest $Q^2 \gg m_{\tilde{b}}^2$ gluons can split into $\tilde{b}\tilde{b}^*$, affecting the evolution equations. However, such an analysis is beyond the scope of the present paper, and is more appropriately carried out by the experimental groups themselves.

C. Quarkonium

The measurement of $\alpha_s(m_b) = 0.22 \pm 0.02$ from the Y system (for $m_b = 4.75 \text{ GeV}$) implies $\alpha_s(M_Z) = 0.118 \pm 0.006$ in Bethke's review [11,13]. [A lower value $\alpha_s(m_b) = 0.185 \pm 0.01$, implying $\alpha_s(M_Z) = 0.109 \pm 0.004$, is quoted by Hinchliffe in the Particle Data Group review [12].] Neither value is competitive in its errors with the most precise one based on deep inelastic lepton-hadron scattering.

The presence of light \tilde{b} squarks affects the determination of α_s from certain quarkonium decays. For example, the total width of the Y is affected if the decay $Y \rightarrow \tilde{b}\tilde{b}^*$ is permitted. One has [5]

$$R_{\tilde{b}\tilde{b}^*}^Y = \frac{\Gamma_{\tilde{b}\tilde{b}^*}}{\Gamma_{\ell\bar{\ell}}} = \frac{1}{3} \left(\frac{\alpha_s(\mu)}{\alpha_{\text{em}}} \right)^2 \frac{m_Y(m_Y^2 - 4m_{\tilde{b}}^2)^{3/2}}{(t - m_{\tilde{g}}^2)^2}, \quad (9)$$

where $t = -(m_Y^2 - 4m_{\tilde{g}}^2)/4$. Typical effects on the bottom squark partial widths can exceed ten times the leptonic widths, thereby attaining values of tens of keV for different Y states and substantially affecting their expected total widths. Suppose the bottom squarks in the final state behave like usual hadronic jets within the detector. To compensate for the new open channel of $\tilde{b}\tilde{b}^*$, for a given *measured* hadronic width one must reduce the value of $\alpha_s(m_b)$. From Eq. (9) and $\Gamma_{\text{had}}^{Y(1S)} = 52.5 \pm 1.8 \text{ keV}$, we find that $R_{\tilde{b}\tilde{b}^*}^Y \approx 9$ for $m_{\tilde{b}} = 4 \text{ GeV}$, $m_{\tilde{g}} = 14 \text{ GeV}$ and α_s should be reduced by about 5%, consistent with the estimate in Ref. [5]. Such a change is well within the current error on the extracted $\alpha_s(m_b)$.

The decays of χ_{bJ} states to $\tilde{b}\tilde{b}^*$ occur with partial widths which can exceed 200 keV for $J=0$ and for $\sin\theta_{\tilde{b}} \cos\theta_{\tilde{b}} > 0$; for $J=1,2$ these partial widths are calculated to be much

smaller, and for the other sign of $\sin\theta_{\tilde{b}} \cos\theta_{\tilde{b}}$ they are small for all three values of J . One must then take into account these changes when extracting α_s from χ_{bJ} data, but their impact on the determination mentioned above is relatively modest [24].

D. Lattice

We shall argue that lattice calculations of α_s using the epsilon levels are relatively insensitive to the new physics introduced by light bottom squarks and gluinos. The inputs to the calculation of Ref. [16] are an overall mass scale (essentially adjustable through the choice of m_b) and either a $2S-1S$ or a $1P-1S$ level spacing. Both level spacings are used to obtain a value of α_s at a low mass scale characteristic of m_b . Consistency between the two values is used to argue in favor of an unquenched calculation with $n_f=3$ light quark flavors.

The new open $\tilde{b}\tilde{b}^*$ decay channels affect not only the decay widths, but also the masses of the $b\bar{b}$ bound states. In analogy with the neutral kaon system, in which the K_L-K_S mass splitting is of the same order as the K_S decay width to $\pi\pi$, we shall assume that the mass shifts in $b\bar{b}$ bound states due to the open $\tilde{b}\tilde{b}^*$ channels are of the *same order* as the contributions of $\tilde{b}\tilde{b}^*$ decay channels to their partial widths, i.e., tens of keV for the S-wave levels, at most a couple of hundred keV for the 3P_0 level, and unimportant for the other P-wave levels. A potential contribution from the heavy bottom squark \tilde{b}' is estimated to be unimportant because of the large mass suppression. The spin-weighted average $[5M(\chi_{b2}) + 3M(\chi_{b1}) + M(\chi_{b0})]/9 = \bar{M}(1P)$ is then affected by at most tens of keV. This is to be compared with the input level spacings $M(Y') - M(Y) = 563 \text{ MeV}$ and $M(\chi_b) - M(Y) = 440 \text{ MeV}$ [16]. We evaluate the effect of shifts in these quantities by tens of keV on α_s as follows.

A scale change r in the input mass splittings is reflected in a similar change in the scale at which α_s is evaluated, $\alpha_s(M) \rightarrow \alpha_s(Mr)$. Using this estimate, we find that the change in α_s due to a change by a factor of $r = 1 + \delta$ is $\Delta[\alpha_s^{-1}] \approx (b_1/2\pi)\delta$, where $b_1(n_f=5, \tilde{b}) = 7.5$ from Eq. (3) so $\Delta[\alpha_s^{-1}] \approx \delta \approx 10^{-4}$, $\Delta\alpha_s \approx 10^{-4}\alpha_s^2$. This is smaller by orders of magnitude than the effects which we consider to be important.

The lattice calculation of α_s at scales of order m_b is thus not likely to be affected by the presence of a light gluino and bottom squark with the mass ranges considered here to more than ± 0.0001 , and possibly even greater accuracy.

The Particle Data Group review by Hinchliffe [12] quotes the average of several lattice determinations as implying $\alpha_s(M_Z) = 0.1134 \pm 0.003$, at a characteristic mass scale of m_b as in the case of quarkonium decays. Bethke [13] adopts only the latest lattice determination [18] and quotes $\alpha_s(M_Z) = 0.121 \pm 0.003$.

IV. INFORMATION ON α_s AT M_Z

A. Direct measurements: Standard model

Based on $\alpha_s(M_Z) = 0.1200 \pm 0.0028$ and the global best fit values of some other input parameters (e.g., M_Z , M_H , etc.),

the standard model predicts $\Gamma(Z \rightarrow \text{hadrons}) = 1.7429 \pm 0.0015$ GeV, to be compared with the experimental value 1.7444 ± 0.0020 GeV [25]. [The fact that the two numbers do not agree exactly is due to the existence of other inputs in the fit affected by $\alpha_s(M_Z)$.] The experimental error alone in $\Gamma(Z \rightarrow \text{hadrons})$ would imply an error in $\alpha_s(M_Z)$ of ± 0.0034 , consistent with the value quoted by Bethke [13]. Additional theoretical errors raise this to ± 0.005 . Fitting *only* $\Gamma(Z \rightarrow \text{hadrons})$, we find $\alpha_s(M_Z) = 0.123 \pm 0.005$. We shall adopt this more conservative error.

B. Effect of SUSY scenario on $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*)$

The light bottom squark \tilde{b} is assumed to be long lived at the collider scale or to decay promptly into light hadrons in this scenario [3]. In either case, it forms a hadronic jet within the detector due to its color charge. Therefore, the $Z \rightarrow \tilde{b}\tilde{b}^*$ decay mode will contribute to the total hadronic width of the Z boson.

The partial decay width $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*)$ can be expressed at the tree level as

$$\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*) \simeq \frac{G_F M_Z^3}{8\sqrt{2}\pi} [(g_V^b + g_A^b) \sin^2 \theta_{\tilde{b}} + (g_V^b - g_A^b) \cos^2 \theta_{\tilde{b}}]^2, \quad (10)$$

where we take the limit $m_{\tilde{b}} \approx 0$. (In our convention the $Zb\bar{b}$ vertex $\sim g_V^b - g_A^b \gamma^5$.) The $Z\tilde{b}\tilde{b}^*$ coupling must be small to agree with the electroweak precision measurements at the Z-pole. A vanishing tree-level $Z\tilde{b}\tilde{b}^*$ coupling is achieved if the mixing angle $\theta_{\tilde{b}}$ is chosen to satisfy $\sin \theta_{\tilde{b}} = \sqrt{2} \sin^2 \theta_W / 3 \approx 0.39$. However, a nonzero effective coupling, which can be obtained if $\sin \theta_{\tilde{b}} \neq 0.39$ and/or via loop corrections, may contribute to $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*)$. Carena *et al.* [4] calculated the $\tilde{b}\tilde{b}^*$ production cross section as a function of the effective $Z\tilde{b}\tilde{b}^*$ coupling. Their results indicate that $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*)$ is less than $\mathcal{O}(0.001)$ GeV for $0.30 \leq \sin \theta_{\tilde{b}} \leq 0.45$. For comparison, the tree-level formula (10) gives $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*) = (0-0.001)$ GeV in the same range of $\sin \theta_{\tilde{b}}$. For $\sin \theta_{\tilde{b}} \approx 0.39$, an upper bound can be obtained on the one-loop correction to $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*)$ by using an argument similar to that in Sec. IV D, which would also assert that $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*)$ is less than $\mathcal{O}(0.001)$ GeV.

C. Effect of SUSY scenario on $\Gamma(Z \rightarrow b\bar{b})$

The electroweak observables such as R_b have been considered to provide a stringent constraint on the allowed parameter space of the light gluino or bottom squark scenario [26–28]. For $\sin \theta_{\tilde{b}} = 0.39$, $m_{\tilde{b}} = 5$ GeV, $m_{\tilde{b}'} = 200$ GeV, and $m_{\tilde{g}} = 14$ GeV, Baek [28] calculated $\delta R_b \equiv R_b - R_b^{\text{SM}}$ as a function of the CP violating phases ϕ_b and ϕ_3 . The range of δR_b turns out to be $-(2.0-3.5) \times 10^{-3}$ for $\sin \theta_{\tilde{b}} = 0.39$ and $m_{\tilde{b}'} = 200$ GeV. This is unacceptably large. The observed value is $R_b^{\text{expt}} = 0.21664 \pm 0.00068$ [25], to be compared with the standard model prediction $R_b^{\text{SM}} = 0.21569 \pm 0.00016$. The

difference is $R_b^{\text{expt}} - R_b^{\text{SM}} = 0.00095 \pm 0.00070$, so that one must have $3.05 \times 10^{-3} > \delta R_b > -1.15 \times 10^{-3}$ to maintain agreement at the 3σ level. By suitable choice of phases Baek is able to reduce the predicted magnitude of δR_b by about a factor of 2, which would put it within reasonable limits. Dealing with the CP conserving MSSM, Cao *et al.* [26] obtained similar results for $m_{\tilde{b}} = 3.5$ GeV.

It is noted in Refs. [26,28] that the variation of $m_{\tilde{b}}$ does not change δR_b significantly. As will be seen later, the SUSY contribution to the decay channel $Z \rightarrow b\bar{b}$ is the dominant component in the change of the hadronic Z decay width. Therefore, we take $\delta\Gamma(Z \rightarrow \text{hadrons}) \simeq \delta\Gamma(Z \rightarrow b\bar{b})$, which is related to δR_b by

$$\begin{aligned} \delta R_b &\simeq \frac{\delta\Gamma(Z \rightarrow b\bar{b})}{\Gamma^{\text{SM}}(Z \rightarrow \text{hadrons})} - \frac{\Gamma^{\text{SM}}(Z \rightarrow b\bar{b}) \delta\Gamma(Z \rightarrow \text{hadrons})}{\Gamma^{\text{SM}}(Z \rightarrow \text{hadrons})^2} \\ &= (1 - R_b^{\text{SM}}) \frac{\delta\Gamma(Z \rightarrow b\bar{b})}{\Gamma^{\text{SM}}(Z \rightarrow \text{hadrons})}. \end{aligned} \quad (11)$$

In the following calculation, we will take the range $\delta R_b = -(1-2) \times 10^{-3}$ (which covers the most negative acceptable value if one takes the current 3σ bound seriously) for our estimation of changes in $\alpha_s(M_Z)$. Using $R_b^{\text{SM}} = 0.21569$ and $\Gamma^{\text{SM}}(Z \rightarrow \text{hadrons}) = 1.7429$ GeV, one finds that $\delta\Gamma(Z \rightarrow b\bar{b}) = -(0.0022-0.0044)$ GeV. Here we reiterate that the value of R_b predicted in the SUSY scenario remains a potentially dangerous feature of this scheme.

D. Effect of SUSY scenario on $\Gamma(Z \rightarrow \tilde{g}\tilde{g})$

With a light gluino in this scenario, the Z boson can decay into a pair of gluinos through loop-mediated processes. The gluinos then decay promptly to $b\bar{b}^*$ or $\tilde{b}\tilde{b}$, contributing to the total hadronic width of the Z. Previous analyses [29] indicate that the branching ratio of $Z \rightarrow \tilde{g}\tilde{g}$ falls in the range of 10^{-5} to 10^{-4} for a wide range of MSSM parameter space. This gives a partial width of less than $\mathcal{O}(1)$ MeV. Although the possibility of a light bottom squark is not considered in those analyses, it can be argued that any possible increase due to the light bottom squark should be comparatively small. The reason is that the effective coupling between the Z boson and the gluinos should be of the same order as the one-loop correction to the coupling between Z and bottom quarks, both of which are α_s and one-loop suppressed. The $Zb\bar{b}$ coupling receives an $\mathcal{O}(\alpha_s)$ correction coming from the interference between the SUSY contribution and the SM tree-level coupling, resulting in a decrease of at most 4.4 MeV in the total width of Z (see Sec. IV C). In the case of $Z\tilde{g}\tilde{g}$, however, there is no tree-level coupling; therefore, the amplitude for the process is further suppressed by $\mathcal{O}(\alpha_s)$. Using the result of $\delta\Gamma(Z \rightarrow b\bar{b})$ as given in Sec. IV C, it is easy to see that the partial width of $Z \rightarrow \tilde{g}\tilde{g}$ is indeed at most an MeV.

A lower bound can be obtained on $\Gamma(Z \rightarrow \tilde{g}\tilde{g})$ based on the unitarity of the S matrix ($S^\dagger S = 1$). We expect that this

TABLE I. Values of $\alpha_s(M_Z)$ based on determinations at different mass scales, in the standard model (1) and in the presence of a light gluino and bottom squark (2).

Source	Q (GeV)	(1)	(2)
τ	1.78	$0.118(0.121?) \pm (>)0.003$	a
DIS	~ 3	$0.1166 \pm (>)0.0022$	$0.130 \pm (>)0.003$
Lattice	~ 5	$0.121 \pm (>)0.003$	$0.135 \pm (>)0.003$
$\Gamma_h(Z)$	91.2	0.123 ± 0.005	$(0.123 - 0.131) \pm 0.005$
Event shapes	$> M_Z$	0.123 ± 0.006	Unknown

^aSee Sec. III A. Extrapolation from such a low Q is risky in our opinion.

bound is likely to provide a fairly good estimate of the actual $Z \rightarrow \tilde{g}\tilde{g}$ partial width as long as cancellations of loop contributions with high internal momenta are implemented, as in the calculations of Ref. [29]. The situation is analogous to the K_S-K_L mass difference and the decay $K_L \rightarrow \mu^+ \mu^-$. In each case the high-momentum components of the loop diagrams are suppressed (here, through the presence of the charmed quark [30]), leaving the low-mass on-shell states ($\pi\pi$ or $\gamma\gamma$, respectively) to provide a good estimate of the matrix element.

The imaginary part of the invariant matrix element $\mathcal{M}(Z \rightarrow \tilde{g}\tilde{g})$ can be written as

$$\text{Im}[\mathcal{M}(Z \rightarrow \tilde{g}\tilde{g})] = \frac{1}{2} \sum_f \int d\Pi_f \mathcal{M}(Z \rightarrow f) \mathcal{M}^*(\tilde{g}\tilde{g} \rightarrow f), \quad (12)$$

where the sum runs over all possible intermediate on-shell states f . Since \tilde{b} is the lightest supersymmetric particle in the scenario and all other supersymmetric particles (except \tilde{g}) are much heavier, we only need to consider the cases where f is $b\bar{b}$ and $\tilde{b}\tilde{b}^*$. The contribution of the latter can be neglected because we require that the tree-level $Z\tilde{b}\tilde{b}^*$ coupling be small. Furthermore, since the mass of the heavy bottom squark \tilde{b}' is very large, only $\tilde{g}\tilde{g} \rightarrow b\bar{b}$ via \tilde{b} exchange is considered to be significant. Based on the fact that $|\mathcal{M}(Z \rightarrow \tilde{g}\tilde{g})| \geq \text{Im}[\mathcal{M}(Z \rightarrow \tilde{g}\tilde{g})]$, our calculation indicates [31]

$$\begin{aligned} & \frac{\Gamma(Z \rightarrow \tilde{g}\tilde{g})}{\Gamma(Z \rightarrow b\bar{b})} \\ & \geq \frac{\alpha_s^2(M_Z) [(g_V^b + g_A^b) \sin^2 \theta_b - (g_V^b - g_A^b) \cos^2 \theta_b]^2}{24 [(g_V^b)^2 + (g_A^b)^2]}. \end{aligned} \quad (13)$$

Taking $\alpha_s(M_Z) = 0.123$ and $\sin \theta_b = 0.39$, we obtain $\Gamma(Z \rightarrow \tilde{g}\tilde{g}) \geq 0.02$ MeV. As mentioned above, it is likely that the actual partial width is not far above this lower bound. We will take the upper bound to be 1 MeV, as explained earlier.

In summary, we estimate $\Gamma(Z \rightarrow \tilde{b}\tilde{b}^*) = (0-1)$ MeV, $\delta\Gamma(Z \rightarrow b\bar{b}) = -(2.2-4.4)$ MeV and $\Gamma(Z \rightarrow \tilde{g}\tilde{g}) = (0.02-1)$ MeV. The total correction to the predicted hadronic width of Z is thus $-(0.2-4.4)$ MeV, which is equivalent to a change of $(0-0.008)$ in $\alpha_s(M_Z)$ with respect to the

SM value 0.123 ± 0.005 . We then have $\alpha_s(M_Z) = (0.123-0.131) \pm 0.005$ in the SUSY scenario.

V. INFORMATION ON α_s ABOVE THE Z

A number of determinations of α_s at the highest-available mass scales are based on event shapes in e^+e^- annihilations [13]. An example [32] of such determinations, based on data at center-of-mass energies up to 206 GeV, is $\alpha_s(M_Z) = 0.1227 \pm 0.0012 \pm 0.0058$. Since the dominant error is systematic, it will not be decreased substantially by combination with results of other experiments.

The determination of $\alpha_s(M_Z)$ from event shapes in high-energy e^+e^- annihilations will be affected in several ways by the light-gluino or light bottom squark scenario. Virtual bottom quarks will be able to radiate bottom squarks and gluinos; virtual gluons will be able to split into pairs of light bottom squarks and pairs of gluinos; and next-to-next-to-leading order (NNLO) perturbative expressions will be modified because of new loops in gluon and bottom quark propagators. Estimates of these effects are beyond the scope of the present paper, but are worth pursuing.

VI. RESULTS AND PROSPECTS FOR FURTHER IMPROVEMENTS

We show in Table I examples of the results for $\alpha_s(M_Z)$ based on the best determinations at various mass scales, both in the standard model and in the presence of a light gluino and bottom squark. As mentioned earlier, for a gluino in the 12 to 16 GeV mass range, values of $\delta\alpha_s(M_Z)$ due to this latter scenario range from ≈ 0.015 to ≈ 0.013 when extrapolating from m_b to M_Z , so we shall quote their effect as 0.014 ± 0.001 .

Table I presents a rather unsatisfactory situation at present, in our view. No clear-cut decision is possible in favor of either the standard model or the light gluino or bottom squark scenario. In Fig. 2 we show values of $\alpha_s(M_Z)$ extracted from determinations at various values of Q [13]. A straight line, corresponding to the standard model, clearly provides an excellent fit, while we have shown that the best-measured values of α_s are also compatible with the light gluino or bottom squark hypothesis.

We expect that some of the indeterminacy should be reduced when results of fully unquenched lattice calculations appear, reducing the error on the extrapolated coupling constant to $\Delta\alpha_s(M_Z) = \pm 0.002$ or less. However, further reduc-

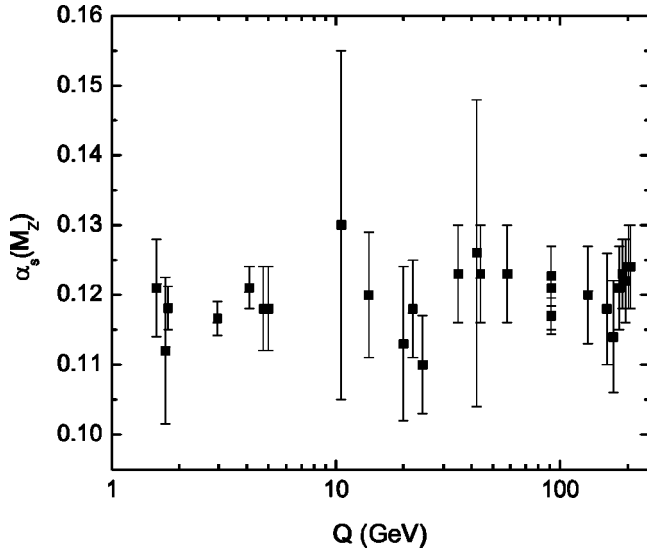


FIG. 2. Values of $\alpha_s(M_Z)$ as determined at various values of Q , based on standard model evolution. From Ref. [13].

tion of uncertainty will require improved determinations either at the Z mass [particularly of $\Gamma_{\text{tot}}(Z)$ and R_b] or above it (extrapolated down to M_Z). For the latter case, a calculation is needed for the effect of the light gluino or bottom squark proposal on hadronic event shapes.

VII. SUMMARY

We have outlined the current status of the scale dependence of the strong fine-structure constant α_s and the light it

can shed on the hypothesis of a light gluino and bottom squark. No conclusion is possible at present regarding this hypothesis with CP violating phases vis-à-vis the standard model. Improvements that will permit a more clear-cut test include refinement of lattice calculations, reduction of errors based in the hadronic and $b\bar{b}$ widths of the Z , and possibly more precise determinations based on event shapes in high-energy e^+e^- collisions. Of course, direct searches for light gluinos and bottom squarks will play a key role, but that is another story.

Note added. After this work was finished, we received a paper [33] considering the $Z \rightarrow b\bar{b}^* \tilde{g} + b^* \bar{b} \tilde{g}$ channel, whose partial width was estimated to be of order 10^{-3} GeV in the gluino mass range of interest to us. This positive contribution may partially cancel with the negative SUSY contribution to $\Gamma(Z \rightarrow b\bar{b})$ in both CP -conserving and CP -violating cases [26,28] and, therefore, brings down our estimate of α_s extracted at M_Z in Table I, where maximal CP violation is assumed.

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