Studying $K\pi$ S-wave scattering in the K-matrix formalism

Long Li*
Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China

Bing-song Zou†
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China,
Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China,
and Institute of Theoretical Physics, Beijing 100080, China

Guang-lie Li‡
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China,
Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China,
and Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China
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We generalize our previous work on $\pi\pi$ scattering to $K\pi$ scattering, and reanalyze the experiment data for $K\pi$ scattering below 1.6 GeV. Without any free parameter, we explain the $K\pi$ $I=3/2$ S-wave phase shift very well by using t-channel ρ and u-channel K^* meson exchange. With the t-channel and u-channel meson exchange fixed as the background term, we fit the $K\pi$ $I=1/2$ S-wave data of the LASS experiment quite well by introducing one or two s-channel resonances. It is found that there is only one s-channel resonance between the $K\pi$ threshold and 1.6 GeV, i.e., $K_0^*(1430)$ with a mass around 1438–1486 MeV and a width of about 346 MeV, while the t-channel ρ exchange gives a pole at $(450-480i)$ MeV for the amplitude, rather uncertain due to the limitations of the approach used.

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I. INTRODUCTION

The assignment of the scalar mesons has been a long standing problem. Recently, the existence of the low-lying $\pi\pi$ scalar state σ has been well established, i.e., $f_0(400\text{–}1200)$ as listed by the Particle Data Group (PDG) [1]. Now the PDG lists five well-established isoscalar 0^{++} mesons: $f_0(400\text{–}1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, which are obviously too many for a standard $q\bar{q}$ nonet. Given the existence of two isovector scalars $a_0(980)$ and $a_0(1450)$, two scalar nonets have been suggested [2,3]: an unconventional one composed of σ, κ, $a_0(980)$, and $f_0(980)$ and the conventional $q\bar{q}$ nonet composed of $f_0(1370)$, $K_0^*(1430)$, $a_0(1450)$, and $f_0(1500)$ or $f_0(1710)$.

However, the existence of the κ is still in controversy. Evidence for this resonance has been claimed within certain models [4–8], while other studies dispute this [9,10]. Recently, a less model-dependent analysis of the LASS $K\pi$ scattering data between 825 MeV and 2 GeV by Cherry and Pennington [11] concluded that there is no $\kappa(900)$, but a very low-mass κ well below 825 MeV cannot be ruled out.

In fact the phase shifts of $K\pi$ S-wave scattering at low energies [12,13] look very similar to those of $\pi\pi$ S-wave scattering. In our previous study on $\pi\pi$ scattering in the K-matrix formalism [14], the negative phase shifts for the isotensor $\pi\pi$ S-wave were naturally explained by the t-channel ρ meson exchange while the broad $f_0(400\text{–}1200)$ structure in the isoscalar $\pi\pi$ S wave was decomposed into a t-channel ρ meson exchange part dominating at the low-energy end plus an additional s-channel wide resonance $f_0(1670)$. Considering the similarity between $K\pi$ scattering and $\pi\pi$ scattering, it is natural to extend our previous work on $\pi\pi$ scattering to $K\pi$ scattering. We find that the negative phase shifts of the $K\pi$ $I=3/2$ S wave can be very well reproduced by t-channel ρ and u-channel K^* meson exchange without any free parameter. With the t-channel and u-channel meson exchange fixed as the background term, the positive smoothly rising phase shifts for the $K\pi$ $I=1/2$ S wave can be well fitted by introducing one or two additional s-channel resonances. It is found that there is only one s-channel resonance between the $K\pi$ threshold and 1.6 GeV, i.e., $K_0^*(1430)$ with a mass around 1438–1486 MeV and a width about 346 MeV, while the t-channel ρ exchange gives a pole at $(450-480i)$ MeV for the amplitude.

II. FORMALISM

For the pseudoscalar-pseudoscalar-vector coupling, we use the SU(3)-symmetric Lagrangian [15]

$$\mathcal{L}_{PPV}=-\frac{1}{2}iG_V\mathrm{Tr}([P,\partial_\mu P]V^\mu), \tag{1}$$

where G_V is the coupling constant, P is the 3×3 matrix representation of the pseudoscalar meson octet, $P=\lambda^aP^a, a=1,\dots,8$, and λ^a are the 3×3 generators of SU(3). A similar definition of V^μ is used for the vector meson octet.

In the Gell-Mann representation, the Lagrangian can be expressed as

*Electronic address: lilong@mail.ihep.ac.cn
†Electronic address: zoubs@mail.ihep.ac.cn
‡Electronic address: ligl@mail.ihep.ac.cn

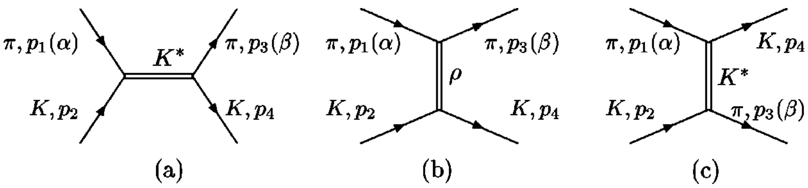

FIG. 1. The Born term of $K\pi$ scattering.

$$\mathcal{L}_{PPV}=2G_V f_{abc}P^a\partial_\mu P^b V^{c\mu}, \tag{2}$$

where f_{abc} are the antisymmetric structure constants of SU(3). For example,

$$\mathcal{L}_{\pi\pi\rho}=2G_V\epsilon_{ijk}\pi^i\partial_\mu\pi^j\rho^{k\mu} \tag{3}$$

and

$$\mathcal{L}_{\pi KK^*}=iG_V\{[(\partial_\mu\overline{K})\vec{\tau}K^{*\mu}-\overline{K^{*\mu}}\vec{\tau}(\partial_\mu K)]\cdot\vec{\pi} \\ -[\overline{K}\vec{\tau}K^{*\mu}-\overline{K^{*\mu}}\vec{\tau}K]\cdot(\partial_\mu\vec{\pi})\} \tag{4}$$

where

$$\vec{\pi}\equiv(\pi_1,\pi_2,\pi_3),\quad K^{*\mu}\equiv\begin{pmatrix}K^{*+\mu}\\ K^{*0\mu}\end{pmatrix},\quad K\equiv\begin{pmatrix}K^+\\ K^0\end{pmatrix},$$

$$\overline{K^{*\mu}}\equiv(K^{*-\mu},\overline{K^{*0\mu}}),\quad \overline{K}\equiv(K^-,\overline{K^0}),$$

and

$$\vec{\tau}=(\tau_1,\tau_2,\tau_3)$$

are the usual Pauli matrices acting on the kaon isospinors.

For $K\pi$ scattering, the amplitude T can be written in terms of two invariant amplitudes T^+ and T^- by [16]

$$T_{\beta\alpha}=\delta_{\beta\alpha}T^+ +\frac{1}{2}[\tau_\beta,\tau_\alpha]T^-, \tag{5}$$

where α,β are the isospin indices of the pions. Using isospin projection operators gives

$$3T^+=T^{1/2}(s,t,u)+2T^{3/2}(s,t,u),$$
$$3T^-=T^{1/2}(s,t,u)-T^{3/2}(s,t,u), \tag{6}$$

where s,t,u are the usual Mandelstam variables.

The partial-wave amplitudes are obtained from the full amplitude by the standard projection formula [14,16]

$$T_l(s)=\frac{1}{16\pi}\frac{1}{2}\int_{-1}^{+1}d(\cos\theta)P_l(\cos\theta)T(s,t,u) \\ =\frac{1}{16\pi}\frac{1}{4p^2}\int_{-4p^2}^{0}dtP_l\left[1+\frac{t}{2p^2}\right]T(s,t,u), \tag{7}$$

where $P_l(x)$ is the Legendre function and $p=\sqrt{[s-(m_\pi+m_K)^2][s-(m_\pi-m_K)^2]}/(2\sqrt{s})$. Our normalization is such that the unitarity relation for the partial-wave amplitude reads

$$\mathrm{Im}\,T_l(s)=\rho_1(s)|T_l(s)|^2,$$

with $\rho_1(s)=2p/\sqrt{s}$.

We start with the Born term of the $K\pi$ scattering amplitude by ρ meson and K^*meson exchange and follow the K-matrix formalism as in Refs. [14,17,18]. Figure 1 is the Feynman diagram of the $K\pi$ scattering Born term.

A. *s*-channel and *u*-channel K^* meson exchange amplitude

The Born term for the K^* meson exchange [(a) and (c) of Fig. 1] is

$$T^{1/2}(s,t,u)=g^2_{\pi KK^*}\left[\frac{3(t-u)}{m^2_{K^*}-s}+\frac{3(m^2_\pi-m^2_K)^2}{s(m^2_{K^*}-s)}+\frac{s-t}{m^2_{K^*}-u} -\frac{(m^2_\pi-m^2_K)^2}{m^2_{K^*}(m^2_{K^*}-u)}\right], \tag{8}$$

$$T^{3/2}(s,t,u)=-2g^2_{\pi KK^*}\left[\frac{s-t}{m^2_{K^*}-u}-\frac{(m^2_\pi-m^2_K)^2}{m^2_{K^*}(m^2_{K^*}-u)}\right], \tag{9}$$

where $g_{\pi KK^*}=G_V$ is the coupling constant. Their S-wave projections are

$$K^{1/2}_S(s)=-\frac{1}{2}K^{3/2}_S(s)=G_2\left\{-1+\frac{2(s-m^2_\pi-m^2_K)+m^2_{K^*}-(m^2_\pi-m^2_K)^2/m^2_{K^*}}{4p^2}\ln\frac{m^2_{K^*}+s-2(m^2_\pi+m^2_K)}{m^2_{K^*}+s-2(m^2_\pi+m^2_K)-4p^2}\right\}, \tag{10}$$

where $G_2=g^2_{\pi KK^*}/(16\pi)$. K-matrix unitarization is introduced by

$$T_S^{I=1/2}(s)=\frac{K_S^{I=1/2}(s)}{1-i\rho_1(s)K_S^{I=1/2}(s)}, \tag{11}$$

$$T_S^{I=3/2}(s)=\frac{K_S^{I=3/2}(s)}{1-i\rho_1(s)K_S^{I=3/2}(s)}. \tag{12}$$

Now we calculate the coupling constant G_2. Considering the $I=1/2$ P-wave amplitude,

$$T_P^{I=1/2}(s)=\frac{K_P^{I=1/2}(s)}{1-i\rho_1(s)K_P^{I=1/2}(s)}, \tag{13}$$

where $K_P^{I=1/2}$ is the $I=1/2$ P-wave Born amplitude,

$$K_P^{1/2}(s)=\frac{1}{4p^2}\int_{-4p^2}^{0}dt\left\{G_2\left[\frac{3(t-u)}{m_{K^*}^2-s}+\frac{3(m_\pi^2-m_K^2)^2}{s(m_{K^*}^2-s)}+\frac{s-t}{m_{K^*}^2-u}-\frac{(m_\pi^2-m_K^2)^2}{m_{K^*}^2(m_{K^*}^2-u)}\right]\times\left[1+\frac{t}{2p^2}\right]\right\}. \tag{14}$$

Near the K^* pole at $s\approx m_{K^*}^2$, we have

$$K_P^{1/2}(s)\approx\frac{G_2 4p^2}{m_{K^*}^2-s}, \tag{15}$$

and thus

$$T_P^{I=1/2}(s)=\frac{G_2 4p^2}{m_{K^*}^2-s-i\rho_1(s)G_2 4p^2}. \tag{16}$$

Comparing with the standard Breit-Wigner formula, we obtain

$$M_{K^*}\Gamma_{K^*}=\rho_1(s)4p^2G_2|_{s=M_{K^*}}, \tag{17}$$

which leads to $G_2=0.21$ with the K^* mass $M_{K^*}=891.66$ MeV and width $\Gamma_{K^*}=50.8$ MeV from Ref. [1].

The ratio of coupling constants is $g_{\rho\pi\pi}/g_{\pi KK^*}\simeq 1.9$ using the $g_{\rho\pi\pi}$ value of Refs. [14,17]: $g^2_{\rho\pi\pi}/(32\pi)=0.364$. It agrees well with the value from SU(3) symmetry: $g_{\rho\pi\pi}/g_{\pi KK^*}=2$.

In order to explain the $K\pi$ $I=3/2$ S-wave experimental data, a form factor is needed to take into account the off-shell behavior of the exchanged mesons. For t- and u-channel exchange, we use a form factor of conventional monopole type at each vertex:

$$F(q)=\frac{\Lambda^2-m^2}{\Lambda^2-q^2}, \tag{18}$$

where m and q are the mass and four-vector momentum of the exchanged mesons, and the cutoff parameter $\Lambda=1500$ MeV, the same value as for $\pi\pi$ scattering in Ref. [14].

After adding the form factor, $K_S^{I=1/2}(s)$ and $K_S^{I=3/2}(s)$ become

$$\begin{aligned}K_S^{1/2}(s)&=\frac{1}{4p^2}\int_{-4p^2}^{0}dt\left\{G_2\left[\frac{3(t-u)}{m_{K^*}^2-s}+\frac{3(m_\pi^2-m_K^2)^2}{s(m_{K^*}^2-s)}\right]+\left(\frac{\Lambda^2-m_{K^*}^2}{\Lambda^2-u}\right)^2\left[\frac{s-t}{m_{K^*}^2-u}-\frac{(m_\pi^2-m_K^2)^2}{m_{K^*}^2(m_{K^*}^2-u)}\right]\right\}\\&=G_2\left\{\frac{m_{K^*}^2-\Lambda^2}{A-4p^2}\times\left[1+\frac{s}{A}-\frac{(m_\pi^2-m_K^2)^2}{m_{K^*}^2A}\right]+\frac{s+B-(m_\pi^2-m_K^2)^2/m_{K^*}^2}{4p^2}\ln\frac{B(A-4p^2)}{A(B-4p^2)}\right\},\end{aligned} \tag{19}$$

$$\begin{aligned}K_S^{3/2}(s)&=\frac{1}{4p^2}\int_{-4p^2}^{0}dt\left\{-2G_2\left(\frac{\Lambda^2-m_{K^*}^2}{\Lambda^2-u}\right)^2\left[\frac{s-t}{m_{K^*}^2-u}-\frac{(m_\pi^2-m_K^2)^2}{m_{K^*}^2(m_{K^*}^2-u)}\right]\right\}\\&=-2G_2\left\{\frac{m_{K^*}^2-\Lambda^2}{A-4p^2}\times\left[1+\frac{s}{A}-\frac{(m_\pi^2-m_K^2)^2}{m_{K^*}^2A}\right]+\frac{s+B-(m_\pi^2-m_K^2)^2/m_{K^*}^2}{4p^2}\ln\frac{B(A-4p^2)}{A(B-4p^2)}\right\},\end{aligned} \tag{20}$$

where $A=\Lambda^2+s-2(m_\pi^2+m_K^2)$, $B=m_{K^*}^2+s-2(m_\pi^2+m_K^2)$.

B. t-channel ρ meson exchange amplitude

The Born term for the ρ meson exchange [see Fig. 1(b)] is

$$T^{Born}(I=1/2)=2g_{\pi\pi\rho}g_{\rho KK}\frac{s-u}{m_\rho^2-t}, \tag{21}$$

$$T^{Born}(I=3/2)=-g_{\pi\pi\rho}g_{\rho KK}\frac{s-u}{m_\rho^2-t}. \tag{22}$$

Their S-wave projections are

$$\begin{aligned}K_S^{1/2}(s)&=-2K_S^{3/2}(s)\\&=2G_1\left\{-1+\frac{2(s-m_\pi^2-m_K^2)+m_\rho^2}{4p^2}\ln\frac{m_\rho^2+4p^2}{m_\rho^2}\right\},\end{aligned} \tag{23}$$

where $G_1=g^2_{\pi\pi\rho}/(32\pi)=0.364$ [14,17]. Because we cannot obtain $g_{\rho KK}$ from experiment, the SU(3) symmetry $g_{\pi\pi\rho}=2g_{\rho KK}$ is used.

After introducing the form factor,

$$K_S^{1/2}(s) = -2K_S^{3/2}(s)$$
$$= 2G_1\left\{\left[\frac{2(s-m_\pi^2-m_K^2)}{\Lambda^2}+1\right]\times\frac{m_\rho^2-\Lambda^2}{\Lambda^2+4p^2} - \frac{2(s-m_\pi^2-m_K^2)+m_\rho^2}{4p^2}\ln\frac{m_\rho^2(\Lambda^2+4p^2)}{\Lambda^2(m_\rho^2+4p^2)}\right\}. \tag{24}$$

C. Amplitude of *s*-channel *S*-wave resonances

The phase shift is known to be elastic below 1300 MeV. The threshold for the $K\eta'$ channel is at 1453 MeV and the $K\eta$ channel is only weakly coupled to the $K\pi$ channel [9,12]. Considering the $K\pi$ and $K\eta'$ channels, the explicit form is

$$T = \frac{M\Gamma_{K\pi}/\rho_1(M^2)}{M^2-s-i[M\Gamma_{K\pi}\rho_1(s)/\rho_1(M^2)+M\Gamma_{K\eta'}\rho_2(s)/\rho_2(M^2)]} \tag{25}$$

where $\rho_2(s)=\sqrt{[s-(m_{\eta'}+m_K)^2][s-(m_{\eta'}-m_K)^2]}/s$ is the phase space factor of $K\eta'$.

When fitting the experimental data, we first try introducing one *s*-channel resonance and then try introducing two such resonances.

III. NUMERICAL RESULTS AND DISCUSSION

As in Refs. [14,17], we use the Dalitz-Tuan method to combine the various components given in the last section to get the full partial-wave amplitudes and corresponding phase shifts.

For $K\pi$ $I=3/2$ *S*-wave scattering, the phase shift is negative, with magnitude slowly increasing as the center-of-mass energy increases as shown in Fig. 2. There is no *s*-channel quark-antiquark resonance contribution allowed for isospin $I=3/2$. So the only contributions here are the *t*-channel ρ and *u*-channel K^* meson exchanges. With the cutoff parameter $\Lambda=1.5$ GeV fixed to be the same as in $\pi\pi$ scattering [14], we get the prediction for the $K\pi$ $I=3/2$ *S*-wave phase shift as shown by the solid line in Fig. 2(b) without introducing any free parameters, which reproduces the data nicely. To show the effect of an off-shell form factor, the results without a form factor are shown in Fig. 2(a). The *t*-channel ρ exchange and *u*-channel K^* exchange give very similar contributions to the $K\pi$ $I=3/2$ *S*-wave phase shift as shown by the dotted line and dashed line, respectively, in Fig. 2.

Now we turn to $K\pi$ $I=1/2$ *S*-wave scattering. The data and our theoretical curves for the phase shift and amplitude magnitude are shown in Fig. 3. The *u*-channel K^* exchange and the *t*-channel ρ meson exchange with $\Lambda=1.5$ GeV give the contributions shown by the long-dashed line and dotted line, respectively. Here the *t*-channel ρ exchange gives a much larger contribution than the *u*-channel K^* meson exchange. The sum of these two contributions is shown by the dashed line and is obviously not enough to reproduce the experimental data. Some contribution from *s*-channel resonance(s) is definitely needed. By fixing the *t*-channel ρ exchange and the *u*-channel K^* exchange as the background contribution, we fitted the LASS data [12] first by introducing one *s*-channel resonance (dot-dashed line) and then by introducing two *s*-channel resonances (solid line). The fitting parameters for the *s*-channel resonance(s) and the corresponding χ^2 for the two cases are listed in Table I.

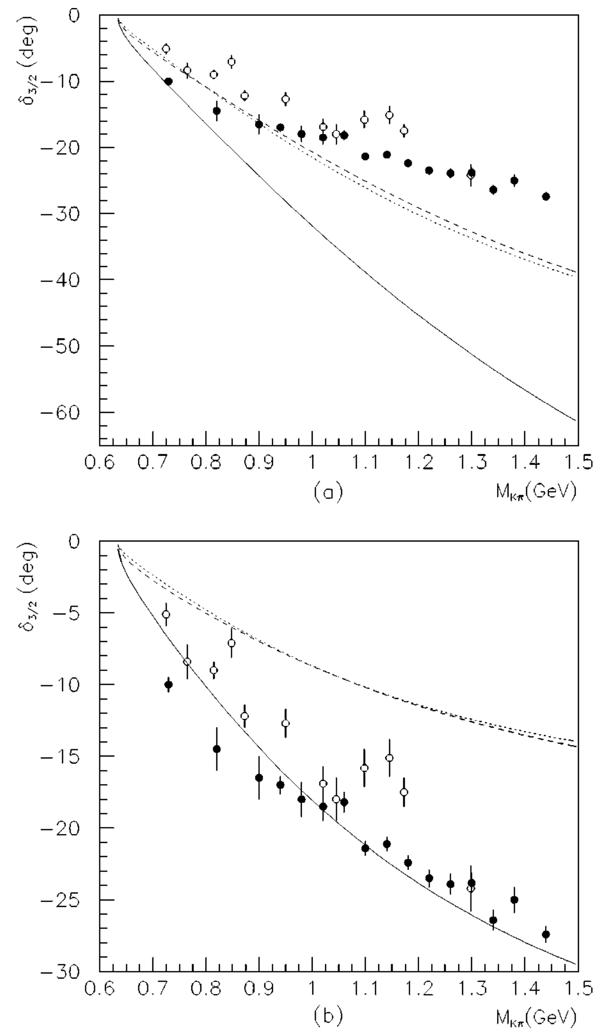

FIG. 2. $I=3/2$ $K\pi$ *S*-wave phase shift. Data are from Refs. [13] (dots) and [19] (circles). Theoretical curves are for *t*-channel ρ exchange (dotted line), *u*-channel K^* exchange (dashed line), and the sum (solid line); (a) without form factor and (b) with form factor and $\Lambda=1.5$ GeV.

It is natural that the fit with two *s*-channel resonances gives a smaller χ^2 value. But from Fig. 3 we see that both cases with one or two *s*-channel resonances give a quite good fit to the data. For the case of two resonances, the second resonance is very broad and has a mass above the upper energy limit (1.6 GeV) of the data, and could be an effective tail of resonances above 1.6 GeV. In both cases, there is only one *s*-channel resonance between the $K\pi$ threshold and 1.6 GeV, corresponding to the PDG well-established $K^*(1430)$ resonance. The fitted mass and width for $K^*(1430)$ depend

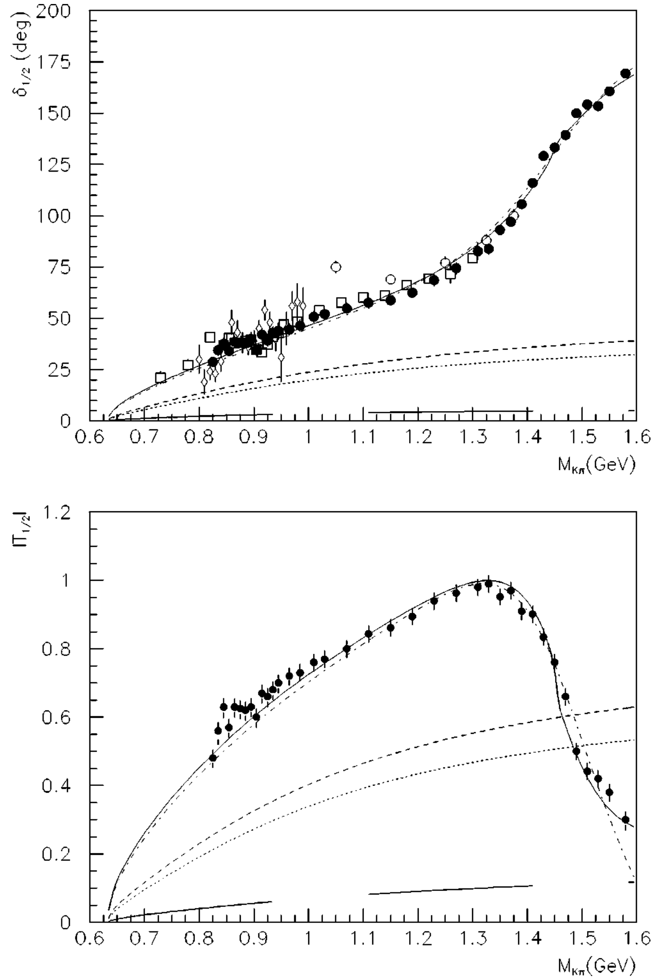

FIG. 3. The $I=1/2$ $K\pi$ *S*-wave phase shift and amplitude. The experimental data for $\delta_{1/2}$ and $T_{1/2}$ are from Ref. [12] (dots), Ref. [13] (boxes), Ref. [20] (circles), and Ref. [21] (diamonds). The long-dashed line is for K^* meson exchange, the dotted line is for ρ meson exchange, the dashed line is the sum of K^* and ρ meson exchange, the dot-dashed line includes one *s*-channel resonance, and the solid line includes two resonances.

on whether we introduce one or two *s*-channel resonances, with mass around 1438–1486 MeV and width about 346 MeV, which are very close to the values (1450, 350) MeV obtained by Tornqvist and Roos [22] with a different formalism. The need for a genuine nonet of scalar resonances around this energy, which has to be introduced as a pole driving term, was also established in the Refs. [7,8].

For the *t*-channel ρ meson exchange amplitude, we find a pole at 0.45–0.48 GeV. This is consistent with the conclusion by Cherry and Pennington that there is no $\kappa(900)$, but a very low-mass κ well below 825 MeV cannot be ruled out. However in the *K*-matrix approach, only the imaginary part

TABLE I. Fitting parameters for the *s*-channel resonances and the corresponding χ^2 for two cases: with one resonance (first line) and with two resonances (second line). Values for the mass and width are in units of GeV.

M_1	$\Gamma^{(1)}_{K\pi}$	$\Gamma^{(1)}_{K\eta'}$	M_2	$\Gamma^{(2)}_{K\pi}$	$\Gamma^{(2)}_{K\eta'}$	χ^2
1.438	0.345	0.001	—	—	—	86/45
1.486	0.346	0.000	1.668	0.150	0.491	57/45

of the loop function of two intermediate mesons is kept, while the model-dependent real part is neglected. This is an approximation. In the chiral unitary approach [7,8], this real part is included with some approximation and generates a pole around 780 MeV. So from our *K*-matrix approach results we cannot exclude the possible existence of $\kappa(900)$. But at least we can say that we can fit the data without a pole around 900 MeV in this approach.

Much work has been done recently on this issue within the context of chiral perturbation theory and its unitarized versions [7,8,23]. Since the chiral Lagrangians account for the basic dynamics of QCD at low energies, these methods should in principle be preferable. The coupling constants of the effective chiral Lagrangians will in general receive contributions from different sources, in particular from meson resonances. The exchange of crossed ρ and K^* is accounted for by the lowest-order chiral Lagrangians [7,8] and the *s*-channel K^* exchange would be accounted for by a higher-order Lagrangian [24]. However, our *K*-matrix approach gives a more intuitive picture about the contributions from each component. For example, for both $I=0$ $\pi\pi$ *S*-wave and $I=1/2$ $K\pi$ *S*-wave scattering at low energies, the *t*-channel ρ exchange dominates, while for $I=3/2$ $K\pi$ *S*-wave scattering, the *t*-channel ρ exchange and *u*-channel K^* exchange give almost equal contributions.

In summary, the $K\pi$ $I=3/2$ *S*-wave phase shift can be well reproduced by *t*-channel ρ and *u*-channel K^* meson exchange while the $K\pi$ $I=1/2$ *S*-wave phase shift is dominated by the *s*-channel $K_0^*(1430)$ resonance and the *t*-channel ρ exchange with a pole at $(450–480i)$ MeV. The $\kappa(450)$ found here, with the uncertainties mentioned above, has a similar nature to $\sigma(400)$ [14,17]: Both are produced by *t*-channel ρ exchange and are very broad with a width around 1 GeV.

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