

Hadronic D decays involving scalar mesons

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The nonleptonic weak decays of charmed mesons into a scalar meson and a pseudoscalar meson are studied. The scalar mesons under consideration are σ [or $f_0(600)$], κ , $f_0(980)$, $a_0(980)$ and $K_0^*(1430)$. A consistent picture provided by the data suggests that the light scalars below or near 1 GeV form an SU(3) flavor nonet and are predominately the $q^2\bar{q}^2$ states, while the scalar mesons above 1 GeV can be described as a $q\bar{q}$ nonet. Hence, we designate $q^2\bar{q}^2$ to σ , κ , $a_0(980)$, $f_0(980)$ and $q\bar{q}$ to K_0^* . Sizable weak annihilation contributions induced from final-state interactions are essential for understanding the data. Except for the Cabibbo doubly suppressed channel $D^+ \rightarrow f_0 K^+$, the data of $D \rightarrow \sigma\pi$, $f_0\pi$, f_0K , $K_0^*\pi$ can be accommodated in the generalized factorization approach. However, the predicted rates for $D \rightarrow a_0\pi$, a_0K are too small by one to two orders of magnitude when compared with the preliminary measurements. Whether or not one can differentiate between the two-quark and four-quark pictures for the $f_0(980)$ produced in the hadronic charm decays depends on the isoscalar f_0 - σ mixing angle in the $q\bar{q}$ model.

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I. INTRODUCTION

There are two essential ingredients for understanding the hadronic decays of charmed mesons. First, the nonfactorizable contributions to the internal W -emission amplitude, which is naively expected to be color suppressed, are very sizable. Second, final-state interactions (FSIs) play an essential role. The nonfactorizable corrections to nonleptonic charm decays will compensate the Fierz terms appearing in the factorization approach to render the naively color-suppressed modes no longer color suppressed. The weak annihilation (W -exchange or W -annihilation) amplitude receives long-distance contributions via inelastic final-state interactions from the leading tree or color-suppressed amplitude. As a consequence, weak annihilation has a sizable magnitude comparable to the color-suppressed internal W -emission with a large phase relative to the tree amplitude. A well known example is the decay $D^0 \rightarrow \bar{K}^0 \phi$ which proceeds only through the W -exchange process. Even in the absence of the short-distance W -exchange contribution, rescattering effects required by unitarity can produce this reaction [1]. Then it was shown in [2] that this rescattering diagram belongs to the generic W -exchange topology.

There exist several different forms of FSIs: elastic scattering and inelastic scattering such as quark exchange, resonance formation, . . . , etc. The resonance formation of FSIs via $q\bar{q}$ resonances is probably the most important one to hadronic charm decays owing to the existence of an abundant spectrum of resonances known to exist at energies close to the mass of the charmed meson. Since FSIs are nonperturbative in nature, in principle it is notoriously difficult to calculate their effects. Nevertheless, most of the properties of resonances follow from unitarity alone, without regard to the dynamical mechanism that produces the resonance [3,4]. Consequently, the effect of resonance-induced FSIs can be described in a model-independent manner in terms of the masses and decay widths of the nearby resonances (for details, see e.g. [5]).

It has been established sometime ago that a least model-independent analysis of heavy meson decays can be carried out in the so-called quark-diagram approach [2,6,7]. Based on SU(3) flavor symmetry, this model-independent analysis enables us to extract the topological quark-graph amplitudes for $D \rightarrow PP, VP$ decays and see the relative importance of different underlying decay mechanisms. From this analysis one can learn the importance of the weak annihilation amplitude and a nontrivial phase between tree and color-suppressed amplitudes [8].

In the present paper we shall study the nonleptonic decays of charmed mesons into a pseudoscalar meson and a scalar meson. Light scalar mesons are traditionally studied in low energy S -wave $\pi\pi$, $K\pi$ and $K\bar{K}$ scattering experiments and in $p\bar{p}$ and $Z\bar{N}$ annihilations. Thanks to the powerful Dalitz plot analysis technique, many scalar meson production measurements in charm decays are now available from the dedicated experiments conducted at CLEO, E791, FOCUS, and BaBar. Hence the study of three-body decays of charmed mesons opens a new avenue to the understanding of the light scalar meson spectroscopy. Specifically, the decays $D \rightarrow f_0\pi(K)$, $D \rightarrow a_0\pi(K)$, $D \rightarrow \bar{K}_0^*\pi$ and $D^+ \rightarrow \sigma\pi^+$ have been observed. Moreover, in some three-body decays of charmed mesons, the intermediate scalar meson accounts for the main contribution to the total decay rate. For example, $D_s^+ \rightarrow f_0(980)\pi^+$ and $D_s^+ \rightarrow f_0(1370)\pi^+$ account for almost 90% of the $D_s^+ \rightarrow \pi^+\pi^+\pi^-$ rate [9], while about half of the total decay rate of $D^+ \rightarrow \pi^+\pi^+\pi^-$ comes from $D^+ \rightarrow \sigma\pi^+$ [9].

The study of $D \rightarrow SP$ is very similar to $D \rightarrow PP$ except for the fact that the quark structure of the scalar mesons, especially $f_0(980)$ and $a_0(980)$, is still not clear. A consistent picture provided by the data implies that light scalar mesons below or near 1 GeV can be described by the $q^2\bar{q}^2$ states, while scalars above 1 GeV will form a conventional $q\bar{q}$ nonet. Another salient feature is that the decay constant of the scalar meson is either zero or very small. We shall see

TABLE I. Experimental branching ratios of various $D \rightarrow SP$ decays measured by ARGUS, E687, E691, E791, CLEO, FOCUS and BaBar, where use of Eqs. (2.1) and (2.2) for the branching fractions of $f_0(980)$ and $a_0(980)$ has been made. For simplicity and convenience, we have dropped the mass identification for $f_0(980)$, $a_0(980)$ and $K_0^*(1430)$.

Collaboration	$\mathcal{B}(D \rightarrow SP) \times \mathcal{B}(S \rightarrow P_1 P_2)$	$\mathcal{B}(D \rightarrow SP)$
E791	$\mathcal{B}(D^+ \rightarrow f_0 \pi^+) \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) = (1.9 \pm 0.5) \times 10^{-4}$	$\mathcal{B}(D^+ \rightarrow f_0 \pi^+) = (4.3 \pm 1.1) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \rightarrow f_0 K^+) \mathcal{B}(f_0 \rightarrow K^+ K^-) = (3.84 \pm 0.92) \times 10^{-5}$	$\mathcal{B}(D^+ \rightarrow f_0 K^+) = (2.4 \pm 0.6) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \rightarrow f_0 K^+) \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) = (6.12 \pm 3.65) \times 10^{-5}$	$\mathcal{B}(D^+ \rightarrow f_0 K^+) = (1.4 \pm 0.8) \times 10^{-4}$
FOCUS	$\mathcal{B}(D^+ \rightarrow a_0^0 \pi^+) \mathcal{B}(a_0^0 \rightarrow K^+ K^-) = (2.38 \pm 0.47) \times 10^{-3}$	$\mathcal{B}(D^+ \rightarrow a_0^0 \pi^+) = (3.2 \pm 0.6)\%$
E791	$\mathcal{B}(D^+ \rightarrow \sigma \pi^+) \mathcal{B}(\sigma \rightarrow \pi^+ \pi^-) = (1.4 \pm 0.3) \times 10^{-3}$	$\mathcal{B}(D^+ \rightarrow \sigma \pi^+) = (2.1 \pm 0.5) \times 10^{-3}$
E791	$\mathcal{B}(D^+ \rightarrow \kappa \pi^+) \mathcal{B}(\kappa \rightarrow K^- \pi^+) = (4.4 \pm 1.2)\%$	$\mathcal{B}(D^+ \rightarrow \kappa \pi^+) = (6.5 \pm 1.9)\%$
E691, E687	$\mathcal{B}(D^+ \rightarrow \bar{K}_0^{*0} \pi^+) \mathcal{B}(\bar{K}_0^{*0} \rightarrow K^- \pi^+) = (2.3 \pm 0.3)\%$	$\mathcal{B}(D^+ \rightarrow \bar{K}_0^{*0} \pi^+) = (3.7 \pm 0.4)\%$
E791	$\mathcal{B}(D^+ \rightarrow \bar{K}_0^{*0} \pi^+) \mathcal{B}(\bar{K}_0^{*0} \rightarrow K^- \pi^+) = (1.14 \pm 0.16)\%$	$\mathcal{B}(D^+ \rightarrow \bar{K}_0^{*0} \pi^+) = (1.8 \pm 0.3)\%$
ARGUS, E687	$\mathcal{B}(D^0 \rightarrow f_0 \bar{K}^0) \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) = (3.2 \pm 0.9) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow f_0 \bar{K}^0) = (7.2 \pm 2.0) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \rightarrow f_0 \bar{K}^0) \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) = (2.5_{-0.5}^{+0.8}) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow f_0 \bar{K}^0) = (5.7_{-1.1}^{+1.8}) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \rightarrow f_0 \bar{K}^0) \mathcal{B}(f_0 \rightarrow K^+ K^-) = (1.2 \pm 0.9) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow f_0 \bar{K}^0) = (7.4 \pm 5.5) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \rightarrow a_0^+ K^-) \mathcal{B}(a_0^+ \rightarrow K^+ \bar{K}^0) = (3.3 \pm 0.8) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow a_0^+ K^-) = (2.2 \pm 0.5)\%$
BaBar	$\mathcal{B}(D^0 \rightarrow a_0^- K^+) \mathcal{B}(a_0^- \rightarrow K^- \bar{K}^0) = (3.1 \pm 1.9) \times 10^{-4}$	$\mathcal{B}(D^0 \rightarrow a_0^- K^+) = (2.1 \pm 1.3) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \rightarrow a_0^0 \bar{K}^0) \mathcal{B}(a_0^0 \rightarrow K^+ K^-) = (5.9 \pm 1.3) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow a_0^0 \bar{K}^0) = (7.9 \pm 1.7)\%$
BaBar	$\mathcal{B}(D^0 \rightarrow a_0^+ \pi^-) \mathcal{B}(a_0^+ \rightarrow K^+ \bar{K}^0) = (5.1 \pm 4.2) \times 10^{-4}$	$\mathcal{B}(D^0 \rightarrow a_0^+ \pi^-) = (3.4 \pm 2.8) \times 10^{-3}$
BaBar	$\mathcal{B}(D^0 \rightarrow a_0^- \pi^+) \mathcal{B}(a_0^- \rightarrow K^- \bar{K}^0) = (1.43 \pm 1.19) \times 10^{-4}$	$\mathcal{B}(D^0 \rightarrow a_0^- \pi^+) = (9.5 \pm 7.9) \times 10^{-4}$
ARGUS, E687	$\mathcal{B}(D^0 \rightarrow K_0^{*-} \pi^+) \mathcal{B}(K_0^{*-} \rightarrow \bar{K}^0 \pi^-) = (7.3 \pm 1.6) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow K_0^{*-} \pi^+) = (1.18 \pm 0.25)\%$
CLEO	$\mathcal{B}(D^0 \rightarrow K_0^{*-} \pi^+) \mathcal{B}(K_0^{*-} \rightarrow \bar{K}^0 \pi^-) = (4.3_{-0.8}^{+1.9}) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow K_0^{*-} \pi^+) = (7.0_{-1.3}^{+3.1}) \times 10^{-3}$
CLEO	$\mathcal{B}(D^0 \rightarrow K_0^{*-} \pi^+) \mathcal{B}(K_0^{*-} \rightarrow K^- \pi^0) = (3.6 \pm 0.8) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow K_0^{*-} \pi^+) = (1.17 \pm 0.26)\%$
CLEO	$\mathcal{B}(D^0 \rightarrow \bar{K}_0^{*0} \pi^0) \mathcal{B}(\bar{K}_0^{*0} \rightarrow K^- \pi^+) = (5.3_{-1.4}^{+4.2}) \times 10^{-3}$	$\mathcal{B}(D^0 \rightarrow \bar{K}_0^{*0} \pi^0) = (8.6_{-2.3}^{+6.8}) \times 10^{-3}$
E687	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) \mathcal{B}(f_0 \rightarrow K^+ K^-) = (4.9 \pm 2.3) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) = (3.1 \pm 1.4)\%$
E791	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) = (5.7 \pm 1.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) = (1.3 \pm 0.4)\%$
FOCUS	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) = (9.5 \pm 2.7) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) = (2.1 \pm 0.6)\%$
FOCUS	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) \mathcal{B}(f_0 \rightarrow K^+ K^-) = (7.0 \pm 1.9) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) = (4.4 \pm 1.2)\%$
FOCUS	$\mathcal{B}(D_s^+ \rightarrow f_0 K^+) \mathcal{B}(f_0 \rightarrow K^+ K^-) = (2.8 \pm 1.3) \times 10^{-4}$	$\mathcal{B}(D_s^+ \rightarrow f_0 K^+) = (1.8 \pm 0.8) \times 10^{-3}$
E687	$\mathcal{B}(D_s^+ \rightarrow \bar{K}_0^{*0} K^+) \mathcal{B}(\bar{K}_0^{*0} \rightarrow K^- \pi^+) = (4.3 \pm 2.5) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow \bar{K}_0^{*0} K^+) = (7 \pm 4) \times 10^{-3}$
FOCUS	$\mathcal{B}(D_s^+ \rightarrow K_0^{*0} \pi^+) \mathcal{B}(K_0^{*0} \rightarrow K^+ \pi^-) = (1.4 \pm 0.8) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow K_0^{*0} \pi^+) = (2.3 \pm 1.3) \times 10^{-3}$

later that final-state interactions are essential for understanding the $D \rightarrow SP$ data. It is hoped that through the study of $D \rightarrow SP$, old puzzles related to the internal structure and related parameters, e.g. the masses and widths, of light scalar mesons can receive new understanding.

This work is organized as follows. In Sec. II we summarize the experimental measurements of $D \rightarrow SP$ decays and emphasize that many results are still preliminary. We then discuss the various properties of the scalar mesons in Sec. III, for example, the quark structure, the decay constants and the form factors. Section IV is devoted to the quark-diagram scheme and its implication for final-state interactions. We analyze the $D \rightarrow SP$ data in Sec. V based on the generalized factorization approach in conjunction with FSIs. Conclusions are presented in Sec. VI.

II. EXPERIMENTAL STATUS

It is known that three-body decays of heavy mesons provide a rich laboratory for studying the intermediate state resonances. The Dalitz plot analysis is a very useful tech-

nique for this purpose. We are interested in $D \rightarrow SP$ (S : scalar meson, P : pseudoscalar meson) decays extracted from the three-body decays of charmed mesons. Some recent results (many being preliminary) are available from E791 [9], CLEO [10], FOCUS [11] and BaBar [12]. The 0^+ scalar mesons that have been studied in charm decays are $\sigma(500)$ [or $f_0(600)$], $f_0(980)$, $f_0(1370)$, $a_0(980)$, $a_0(1450)$, κ and $K_0^*(1430)$. The results of various experiments are summarized in Table I where the product of $\mathcal{B}(D \rightarrow SP)$ and $\mathcal{B}(S \rightarrow P_1 P_2)$ is listed. In order to extract the branching ratios for $D \rightarrow f_0 P$, we use the value of $\Gamma(f_0 \rightarrow \pi \pi) / [\Gamma(f_0 \rightarrow \pi \pi) + \Gamma(f_0 \rightarrow K \bar{K})] = 0.68$ [13]. Therefore,

$$\mathcal{B}[f_0(980) \rightarrow K^+ K^-] = 0.16,$$

$$\mathcal{B}[f_0(980) \rightarrow \pi^+ \pi^-] = 0.45. \quad (2.1)$$

For $D \rightarrow a_0 P$, we apply the PDG (Particle Data Group) average, $\Gamma(a_0 \rightarrow K \bar{K}) / \Gamma(a_0 \rightarrow \pi \eta) = 0.177 \pm 0.024$ [14], to obtain

TABLE II. The masses and widths of the 1^3P_0 scalar mesons (except for κ , see the mini-review in [18]) quoted in [14].

	σ	κ	$f_0(980)$	$a_0(980)$	$K_0^*(1430)$
mass	400–1200 MeV	700–900 MeV	980 ± 10 MeV	984.7 ± 1.2 MeV	1412 ± 6 MeV
width	600–1000 MeV	400–600 MeV	40–100 MeV	50–100 MeV	294 ± 23 MeV

$$\mathcal{B}[a_0^+(980) \rightarrow K^+ \bar{K}^0] = \mathcal{B}[a_0^-(980) \rightarrow K^- K^0] = 0.15 \pm 0.02,$$

$$\mathcal{B}[a_0^0(980) \rightarrow K^+ K^-] = 0.075 \pm 0.010. \quad (2.2)$$

Needless to say, it is of great importance to have more precise measurements of the branching fractions of f_0 and a_0 .

Several remarks are in order.

(1) The Cabibbo doubly suppressed mode $D^+ \rightarrow K^+ K^+ K^-$ has been recently observed by FOCUS [11]. The Cabibbo-allowed $D^0 \rightarrow a_0^+ K^-$ and doubly Cabibbo-suppressed mode $D^0 \rightarrow a_0^- K^+$ have been extracted from the three-body decay $D^0 \rightarrow K^+ K^- \bar{K}^0$ by BaBar [12].

(2) The decay $D^+ \rightarrow K^- \pi^+ \pi^+$ has been measured by E691 [15] and E687 [16] and the combined branching ratio for $D^+ \rightarrow K_0^{*0} \pi^+$ is quoted to be $(3.7 \pm 0.4)\%$ by PDG [14] (see also Table I). A highly unusual feature is that this three-body decay is dominated by the nonresonant contribution at 90% level, whereas it is known that nonresonant effects account for at most 10% in other three-body decay modes of charmed mesons [5,14]. A recent Dalitz plot analysis by E791 [9] reveals that a best fit to the data is obtained if the presence of an additional scalar resonance called κ is included. As a consequence, the nonresonant decay fraction drops from 90% to $(13 \pm 6)\%$, whereas $\kappa \pi^+$ accounts for $(48 \pm 12)\%$ of the total rate. Therefore, the branching ratio of $D^+ \rightarrow \bar{K}_0^{*0} \pi^+$ is dropped from $(3.7 \pm 0.4)\%$ to $(1.8 \pm 0.3)\%$. We shall see in Sec. V that the form factor for $D \rightarrow K_0^*$ transition extracted from the E791 experiment is closer to the theoretical expectation than that inferred from E691 and E687.

(3) The new CLEO and BaBar results on the Cabibbo-allowed decay $D^0 \rightarrow f_0 \bar{K}^0$ are consistent with the early measurements by ARGUS [17] and E687 [16] quoted in Table I from PDG. The Cabibbo doubly suppressed mode $D^+ \rightarrow f_0 K^+$ was first measured by FOCUS recently.

(4) There are four measurements of $D^0 \rightarrow K_0^{*-} \pi^+$: three from $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ by ARGUS [17], E687 [16], CLEO [10], and one from $D^0 \rightarrow K^- \pi^+ \pi^0$ by CLEO. The CLEO result $(1.17 \pm 0.26)\%$ for the branching ratio of $D^0 \rightarrow K_0^{*-} \pi^+$ extracted from $D^0 \rightarrow K^- \pi^+ \pi^0$ is in good agreement with ARGUS and E687 (see Table I), while the CLEO number $(7.0_{-1.3}^{+3.1}) \times 10^{-3}$ determined from $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ is slightly lower.

(5) As for $D_s^+ \rightarrow f_0 \pi^+$, four measured results by E687, E791 and FOCUS are shown in Table I. The old measurement by E687 and two new ones by FOCUS are larger than the E791 one. The preliminary FOCUS measurement indicates that the $f_0(980)$ resonance accounts for $(94.4 \pm 3.8)\%$

of the total $D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$ rate [11]. Later we shall use the average value $\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+) = (1.8 \pm 0.3)\%$ in Table IV.

(6) As stressed in the Introduction, there exist three-body decay modes that are dominated by the scalar resonances. Apart from the decays $D_s^+ \rightarrow f_0 \pi^+$ and $D^+ \rightarrow \sigma \pi^+$ as mentioned in the Introduction, some other examples are $D_s^+ \rightarrow f_0(980) K^+$ and $D^+ \rightarrow f_0(980) K^+$ which account for 72% and 44.5%, respectively, of the decays $D_s^+ \rightarrow K^+ K^+ K^-$ and $D^+ \rightarrow K^+ K^+ K^-$ [11].

(7) The production of the resonance $f_0(1370)$ in $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^- \rightarrow f_0(1370) \bar{K}^0$, $D^+ \rightarrow K^+ K^- \pi^+ \rightarrow f_0(1370) \pi^+$ and $D_s^+ \rightarrow \pi^+ \pi^+ \pi^- \rightarrow f_0(1370) \pi^+$ has been measured by ARGUS, E687, CLEO, by FOCUS and by E791, respectively. Since the branching fractions of $f_0(1370)$ into $\pi^+ \pi^-$, $K^+ K^-$ are unknown, we will not discuss it until Sec. V D.

(8) Some preliminary measurements of $D \rightarrow SP$ do not have yet enough statistical significance, for example, the decays $D^0 \rightarrow a_0^\pm \pi^\mp$, $a_0^- K^+$ and $D_s^+ \rightarrow \bar{K}_0^{*0} K^+$.

III. PHYSICAL PROPERTIES OF SCALAR MESONS

The masses and widths of the 0^+ scalar mesons relevant for our purposes are summarized in Table II. The σ meson observed in $D^+ \rightarrow \pi^+ \pi^+ \pi^-$ decay by E791 [9] has a mass of $478_{-23}^{+24} \pm 17$ MeV and a width of $324_{-40}^{+42} \pm 21$ MeV. Recently, the decay $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ has been analyzed by CLEO [10]. By replacing the nonresonant contribution with a $K_S^0 \sigma$ component, it is found that $m_\sigma = 513 \pm 32$ MeV and $\Gamma_\sigma = 335 \pm 67$ MeV [10], in accordance with E791. The isodoublet scalar resonance κ observed in the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ by E791 has a mass of $797 \pm 19 \pm 43$ MeV and a width of $410 \pm 43 \pm 87$ MeV [9]. However, the signal of κ is much less evident than σ . Indeed, this resonance is not confirmed by CLEO in the Dalitz analysis of the decay $D^0 \rightarrow K^- \pi^+ \pi^0$ [10]. The well established scalars $f_0(980)$ and $a_0(980)$ are narrow, while σ and κ are very broad.

A. Quark structure of scalar mesons

It is known that the identification of scalar mesons is difficult experimentally and the underlying structure of scalar mesons is not well established theoretically (for a review, see e.g. [18–20]). It has been suggested that the light scalars—the isoscalars $\sigma(500)$, $f_0(980)$, the isodoublet κ and the isovector $a_0(980)$ —form an SU(3) flavor nonet. In the naive quark model, the flavor wave functions of these scalars read

$$\sigma = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad f_0 = s\bar{s},$$

$$a_0^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad a_0^+ = u\bar{d}, \quad a_0^- = d\bar{u},$$

$$\kappa^+ = u\bar{s}, \quad \kappa^0 = d\bar{s}, \quad \bar{\kappa}^0 = s\bar{d}, \quad \kappa^- = s\bar{u}. \quad (3.1)$$

However, this model immediately faces two difficulties:¹ (i) It is impossible to understand the mass degeneracy of $f_0(980)$ and $a_0(980)$. (ii) It is hard to explain why σ and κ are broader than $f_0(980)$ and $a_0(980)$. Recalling that $a_0 \rightarrow \pi\eta$, $\sigma \rightarrow \pi\pi$ and $\kappa \rightarrow K\pi$ are Okubo-Zweig-Iizuka(OZI) allowed (but not OZI superallowed) while $f_0 \rightarrow \pi\pi$ is OZI suppressed as it is mediated by the exchange of two gluons, it is thus expected that $m_\kappa \gg \Gamma_\kappa \sim \Gamma_\sigma \sim \Gamma_{a_0} > \Gamma_{f_0}$, a relation not borne out by experiment.

Although the data of $D_s^+ \rightarrow f_0(980)\pi^+$ and $\phi \rightarrow f_0(980)\gamma$ imply the copious $f_0(980)$ production via its $s\bar{s}$ component, there are some experimental evidences indicating that $f_0(980)$ is not purely an $s\bar{s}$ state. First, the measurements of $J/\psi \rightarrow f_0(980)\phi$ and $J/\psi \rightarrow f_0(980)\omega$

$$\begin{aligned} \mathcal{B}[J/\psi \rightarrow f_0(980)\phi] &= (3.2 \pm 0.9) \times 10^{-4}, \\ \mathcal{B}[J/\psi \rightarrow f_0(980)\omega] &= (1.4 \pm 0.5) \times 10^{-4}, \end{aligned} \quad (3.2)$$

clearly indicate the existence of the nonstrange and strange quark content in $f_0(980)$. Second, the fact that $f_0(980)$ and $a_0(980)$ have similar widths and that the f_0 width is dominated by $\pi\pi$ also suggests the composition of $u\bar{u}$ and $d\bar{d}$ pairs in $f_0(980)$; that is, $f_0(980) \rightarrow \pi\pi$ should not be OZI suppressed relative to $a_0(980) \rightarrow \pi\eta$. Therefore, isoscalars σ and f_0 must have a mixing

$$f_0 = s\bar{s} \cos \theta + n\bar{n} \sin \theta, \quad \sigma = -s\bar{s} \sin \theta + n\bar{n} \cos \theta, \quad (3.3)$$

with $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$.

The σ - $f_0(980)$ mixing angle can be inferred from the decays $J/\psi \rightarrow f_0(980)\phi$ and $J/\psi \rightarrow f_0(980)\omega$ [22]²

$$\frac{\mathcal{B}[J/\psi \rightarrow f_0(980)\omega]}{\mathcal{B}[J/\psi \rightarrow f_0(980)\phi]} = \frac{1}{\lambda} \tan^2 \theta, \quad (3.4)$$

¹However, for a different point of view of these difficulties in the $q\bar{q}$ picture, see e.g. [21].

²It has been shown by the $f_0(980)$ production data in Z^0 decays at OPAL [23] and DELPHI [24] that $f_0(980)$ is composed essentially of $u\bar{u}$ and $d\bar{d}$ pairs. This favors a mixing angle close to $\pi/2$. However, it is in contradiction to the experimental observation that the final state $f_0(980)\phi$ in hadronic J/ψ decays has a larger rate than $f_0(980)\omega$.

where the deviation of the parameter λ from unity characterizes the suppression of the $s\bar{s}$ pair production; that is, $\lambda = 1$ in the SU(3) limit. From the data (3.2) we obtain

$$\theta = (34 \pm 6)^\circ, \quad \text{or} \quad \theta = (146 \pm 6)^\circ \quad (3.5)$$

for $\lambda = 1$. Additional information on the mixing angle can be obtained from the $f_0(980)$ coupling to $\pi\pi$ and $K\bar{K}$ [22]:

$$R_g \equiv \frac{g_{f_0 K^+ K^-}^2}{g_{f_0 \pi^+ \pi^-}^2} = \frac{1}{4} (\lambda + \sqrt{2} \cot \theta)^2. \quad (3.6)$$

Using the average value of $R_g = 4.03 \pm 0.14$ obtained from the measurements: 4.00 ± 0.14 by KLOE [25], 4.6 ± 0.8 by SND [26] and 6.1 ± 2.0 from CMD-2 [27], we find

$$\theta = (25.1 \pm 0.5)^\circ, \quad \text{or} \quad \theta = (164.3 \pm 0.2)^\circ \quad (3.7)$$

for $\lambda = 1$. However, the WA102 experiment on $f_0(980)$ production in central pp collisions yields a result $R_g = 1.63 \pm 0.46$ [28], which differs from the aforementioned three measurements. This leads to

$$\theta = (42.3_{-5.5}^{+8.3})^\circ, \quad \text{or} \quad \theta = (158 \pm 2)^\circ, \quad (3.8)$$

again for $\lambda = 1$.

Recently, a phenomenological analysis of the radiative decays $\phi \rightarrow f_0(980)\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ yielded

$$\theta = (5 \pm 5)^\circ, \quad \text{or} \quad \theta = (138 \pm 6)^\circ \quad (3.9)$$

with the second solution being more preferable [29]. In this analysis, $\phi \rightarrow f_0(980)\gamma$ is calculated at the quark level by considering the $s\bar{s}$ quark loop coupled to both ϕ and $f_0(980)$.³ However, the experimental analysis and the theoretical study of this ϕ radiative decay are practically based on the chiral-loop picture, namely, $\phi \rightarrow K^+ K^- \rightarrow K^+ K^- \gamma \rightarrow f_0(980)\gamma$. It turns out that the predicted branching ratio in the $q\bar{q}$ picture is at most of order 5×10^{-5} (see e.g. [31]), while experimentally [14]

$$\mathcal{B}[\phi \rightarrow f_0(980)\gamma] = (3.3_{-0.5}^{+0.8}) \times 10^{-4}. \quad (3.10)$$

This is because the $f_0(980)$ coupling to $K^+ K^-$ is not strong enough as in the case of the four-quark model to be discussed shortly where $f_0 \rightarrow K^+ K^-$ is OZI superallowed.

In short, it is not clear if there exists a universal mixing angle θ which fits simultaneously to all the measurements from hadronic J/ψ decays, the $f_0(980)$ coupling to $\pi^+ \pi^-$

³It is pointed out in [30] that the mechanism $\phi \approx s\bar{s} \rightarrow s\bar{s}\gamma \rightarrow f_0(980)\gamma$ without creation of and annihilation of an additional $u\bar{u}$ pair cannot explain the $f_0(980)$ spectrum observed in the $\phi \rightarrow \gamma f_0(980) \rightarrow \gamma \pi^0 \pi^0$ process because it does not contain the $K^+ K^-$ intermediate state. For a criticism of [29], see also the remark in the footnote [28] in the first paper of [30].

and K^+K^- and the radiative decay $\phi \rightarrow f_0(980)\gamma$ followed⁴ by $f_0 \rightarrow \gamma\gamma$. Equations (3.5) and (3.9) indicate that $\theta \sim 140^\circ$ is preferred, while $\theta \sim 34^\circ$ is also allowed provided that R_g is of order 2. At any rate, the above two possible allowed angles imply the dominance of $s\bar{s}$ in the $f_0(980)$ wave function and $n\bar{n}$ in σ . However, as stressed before, the 2-quark picture for $f_0(980)$ has the difficulty of explaining the absolute $\phi \rightarrow f_0(980)\gamma$ rate.

As for $a_0(980)$, it appears at first sight that one needs an $s\bar{s}$ content in a_0 in order to explain the radiative decay $\phi \rightarrow a_0(980)\gamma$; otherwise, it is OZI suppressed. However, since a_0 is an isovector while $s\bar{s}$ is isoscalar, the mixing of $(u\bar{u} - d\bar{d})$ with $s\bar{s}$ is not allowed in the a_0 wave function within the 2-quark description.⁵ Nevertheless, $\phi \rightarrow a_0(980)\gamma$ can proceed through the process $\phi \rightarrow K^+K^- \rightarrow K^+K^-\gamma \rightarrow a_0(980)\gamma$ as both ϕ and $a_0(980)$ couple to K^+K^- . Indeed, it has been suggested that both $a_0(980)$ and $f_0(980)$ can be interpreted as a $K\bar{K}$ molecular bound state which is treated as an extended object. Since both $f_0(980)$ and $a_0(980)$ couple strongly to $K\bar{K}$ as they are just below the $K\bar{K}$ threshold, they can be imagined as $s\bar{s}$ and $n\bar{n}$ core states, respectively, surrounded by a virtual $K\bar{K}$ cloud [34]. In this $K\bar{K}$ molecular picture, one can explain the decay $f_0 \rightarrow \pi\pi$ without the light nonstrange quark content in $f_0(980)$ and the decay $\phi \rightarrow a_0(980)\gamma$ without the need of an intrinsic strange quark component in a_0 ; both decays are allowed by the OZI rule in the sense that only one gluon exchange is needed.

However, there are several difficulties with this $K\bar{K}$ molecular picture. First, the $K\bar{K}$ molecular width is less than its binding energy of order 20 MeV [34], while the measured widths of $f_0(980)$ and $a_0(980)$ lie in the range of 40 to 100 MeV [14]. Second, it is expected in this model that $\mathcal{B}[\phi \rightarrow f_0(980)\gamma]/\mathcal{B}[\phi \rightarrow a_0(980)\gamma] \approx 1$, while this ratio is measured to be 3.8 ± 1.0 [14]. (The most recent result is 6.1 ± 0.6 by KLOE [35].) Third, the predicted branching ratios

⁴The analysis of the three-body decays of $D^+ \rightarrow f_0(980)\pi^+$, $\bar{K}_0^{*0}(1430)\pi^+$ and $D_s^+ \rightarrow f_0(980)\pi^+$ gives $\theta = (42.14^{+5.8}_{-7.3})^\circ$ in [32]. However, in this analysis, the weak annihilation contribution has been neglected and SU(3) symmetry has been applied to relate $f_0\pi$ to $K_0^*\pi$, a procedure which is not justified since f_0 and K_0^* belong to different SU(3) flavor nonets (see the discussion in Sec. IV). If the mass parameters $m_{n\bar{n}}$ and $m_{s\bar{s}}$ are assigned to the $n\bar{n}$ and $s\bar{s}$ states, respectively, one will have the mass relations: $m_{n\bar{n}}^2 = m_\sigma^2 \cos^2 \theta + m_{f_0}^2 \sin^2 \theta$ and $m_{s\bar{s}}^2 = m_\sigma^2 \sin^2 \theta + m_{f_0}^2 \cos^2 \theta$ from Eq. (3.3). Inserting the dynamically generated Nambu–Jona-Lasinio–(NJL)-type masses for $m_{n\bar{n}}$ and $m_{s\bar{s}}$, it is found in [33] that $\theta = \pm(18 \pm 2)^\circ$ provided that $m_\sigma = 600$ MeV.

⁵Even in the presence of the hidden $s\bar{s}$ content in $a_0(980)$ within the 4-quark model, the direct radiative decay $\phi \rightarrow a_0(980)\gamma$ is prohibited owing to the opposite sign between the $u\bar{u}$ and $d\bar{d}$ components in $a_0(980)$. This means that it is necessary to consider the contribution from the K^+K^- intermediate states.

for both $\phi \rightarrow f_0(980)\gamma$ and $\phi \rightarrow a_0(980)\gamma$ are only of order 10^{-5} [31] which are too small compared to Eq. (3.10) and [36]

$$\mathcal{B}[\phi \rightarrow a_0(980)\gamma] = (0.88 \pm 0.17) \times 10^{-4}. \quad (3.11)$$

Alternatively, the aforementioned difficulties⁶ with a_0 and f_0 can be circumvented in the four-quark model in which one writes symbolically [39]

$$\begin{aligned} \sigma &= u\bar{d}\bar{u}\bar{d}, & f_0 &= (us\bar{u}\bar{s} + ds\bar{d}\bar{s})/\sqrt{2}, \\ a_0^0 &= (us\bar{u}\bar{s} - ds\bar{d}\bar{s})/\sqrt{2}, & a_0^+ &= us\bar{d}\bar{s}, & a_0^- &= ds\bar{u}\bar{s}, \\ \kappa^+ &= u\bar{d}\bar{d}\bar{s}, & \kappa^0 &= u\bar{d}\bar{u}\bar{s}, & \bar{\kappa}^0 &= us\bar{u}\bar{d}, & \kappa^- &= ds\bar{u}\bar{d}. \end{aligned} \quad (3.12)$$

This is supported by a recent lattice calculation [40]. This $q^2\bar{q}^2$ scenario for light scalars has several major advantages: (i) The mass degeneracy of $f_0(980)$ and $a_0(980)$ is natural and the mass hierarchy pattern of the SU(3) nonet is understandable. (ii) Why σ and κ are broader than f_0 and a_0 can be explained. The decays $\sigma \rightarrow \pi\pi$, $\kappa \rightarrow K\pi$ and $f_0, a_0 \rightarrow K\bar{K}$ are OZI superallowed without the need of any gluon exchange, while $f_0 \rightarrow \pi\pi$ and $a_0 \rightarrow \pi\eta$ are OZI allowed as it is mediated by one gluon exchange. Since $f_0(980)$ and $a_0(980)$ are very close to the $K\bar{K}$ threshold, the $f_0(980)$ width is dominated by the $\pi\pi$ state and a_0 governed by the $\pi\eta$ state. Consequently, their widths are narrower than σ and κ . (iii) It predicts the relation

$$\frac{\mathcal{B}[J/\psi \rightarrow f_0(980)\omega]}{\mathcal{B}[J/\psi \rightarrow f_0(980)\phi]} = \frac{1}{2}, \quad (3.13)$$

which is in good agreement with the experimental value of 0.44 ± 0.20 [14]. (iv) The coupling of f_0 and a_0 to $K\bar{K}$ is strong enough as the strong decays $f_0 \rightarrow K\bar{K}$ and $a_0 \rightarrow K\bar{K}$ are OZI superallowed. Consequently, the branching ratio of $\phi \rightarrow (f_0, a_0)\gamma$ can be as large as of order 10^{-4} [31].⁷ (v) It is concluded in [30] that production of $f_0(980)$ and $a_0(980)$ in

⁶Likewise, it has been argued in the literature that σ is not a $q\bar{q}$ state [37]. Furthermore, the QCD sum rule calculation also indicates that the lightest scalars are nearly decoupled from $q\bar{q}$, suggesting a non- $q\bar{q}$ structure [38]. In short, one always has some troubles when the light scalar mesons are identified as $q\bar{q}$ states.

⁷Just as in the $K\bar{K}$ molecular model, the ratio $r \equiv \mathcal{B}(\phi \rightarrow f_0\gamma)/\mathcal{B}(\phi \rightarrow a_0\gamma)$ is also an issue in the four-quark model in which $g_{f_0K^+K^-} = g_{a_0K^+K^-}$ and hence $\mathcal{B}(\phi \rightarrow f_0\gamma) = \mathcal{B}(\phi \rightarrow a_0\gamma)$ is predicted, in disagreement with the observed value of 3.8 ± 1.0 [14]. Close and Kirk [41] proposed that r can be explained by considering a large $a_0 - f_0$ mixing. However, as pointed out in [42], the isospin-violating $a_0 - f_0$ mixing is small, analogous to the smallness of $\pi^0 - \eta - \eta'$ mixing, and its correction to r amounts to at most a few percent. One possibility for a large r is that the superposition of the 4-quark state and $K\bar{K}$ has a different weight in $f_0(980)$ and $a_0(980)$.

the $\phi \rightarrow \gamma f_0(980) \rightarrow \gamma \pi^0 \pi^0$ and $\phi \rightarrow \gamma a_0(980) \rightarrow \gamma \pi^0 \eta$ decays is caused by the four-quark transitions, resulting in strong restrictions on the large- N_c expansions of the decay amplitudes. The analysis shows that these constraints give new evidences in favor of the four-quark picture of $f_0(980)$ and $a_0(980)$ mesons.

Therefore, it appears that the four-quark state in core with the $K\bar{K}$ in the outer regime gives a more realistic description of the light scalar mesons. If scalar mesons near and below 1 GeV are non- $q\bar{q}$ states, then the 0^+ mesons in the 1.3–1.7 GeV mass region may be more conventional. For example, it is natural to assume that $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ and $f_0(1500)/f_0(1710)$ are in the same SU(3) flavor nonet in the states $n\bar{n}$, $u\bar{d}$, $u\bar{s}$ and $s\bar{s}$, respectively [18]. In other words, they may have a simple $q\bar{q}$ interpretation. A global picture which emerged from the above discussions is as follows: The scalar meson states above 1 GeV form a $q\bar{q}$ nonet with some possible mixing with glueballs, whereas the light scalar mesons below or near 1 GeV form predominately a $qq\bar{q}\bar{q}$ nonet with a possible mixing with 0^+ $q\bar{q}$ and glueball states (see also [20]). This is understandable because in the $q\bar{q}$ quark model, the 0^+ meson has a unit of orbital angular momentum and hence it should have a higher mass above 1 GeV. On the contrary, four quarks $q^2\bar{q}^2$ can form a 0^+ meson without introducing a unit of orbital angular momentum. Moreover, color and spin dependent interactions favor a flavor nonet configuration with attraction between the qq and $\bar{q}\bar{q}$ pairs. Therefore, the 0^+ $qq\bar{q}\bar{q}$ nonet has a mass near or below 1 GeV.

It is conceivable that the two-quark and four-quark descriptions of light scalars, especially $f_0(980)$ and $a_0(980)$, may lead to some different implications for the hadronic weak decays of charmed mesons into the final state containing a scalar meson. This will be explored in Sec. V.

B. Decay constants

The scalar mesons under consideration are $\sigma(500)$, κ , $f_0(980)$, $a_0(980)$ and $K_0^*(1430)$. The decay constants of scalar and pseudoscalar mesons are defined by

$$\langle 0|A_\mu|P(q)\rangle = if_P q_\mu, \quad \langle 0|V_\mu|S(q)\rangle = f_S q_\mu. \quad (3.14)$$

For the neutral scalars σ , f_0 and a_0^0 , the decay constant must be zero owing to charge conjugation invariance or conservation of vector current:

$$f_\sigma = f_{f_0} = f_{a_0^0} = 0. \quad (3.15)$$

Applying the equation of motion, it is easily seen that the decay constant of K_0^{*+} (a_0^+) is proportional to the mass difference between the constituent s (d) and u quarks. Contrary to the case of pseudoscalar mesons, the decay constant of the scalar meson vanishes in the SU(3) limit or even in the isospin limit. Therefore, the decay constant of $K_0^*(1430)$ and the charged $a_0(980)$ is suppressed. We shall use the values

TABLE III. The $D \rightarrow S$ transition form factors $F_0^{DS}(0)$ at $q^2 = 0$ in various models. Except for the $D \rightarrow a_0$ form factor, the other form factors in this work are obtained by a fit to the data. The $D \rightarrow f_0$ form factor is obtained from the $D_s^+ \rightarrow f_0$ one via Eq. (3.20).

Transition	[48]	[49]	[50]	This work
$D \rightarrow \sigma$	0.74	0.57 ± 0.09		0.42 ± 0.05
$D \rightarrow f_0$				0.26 ± 0.02
$D_s^+ \rightarrow f_0$			$0.64^{+0.05}_{-0.03}$	0.52 ± 0.04
$D \rightarrow a_0^\pm$				0.77
$D \rightarrow \kappa$				0.85 ± 0.10
$D, D_s^+ \rightarrow K_0^*$				1.20 ± 0.07

$$f_{a_0^\pm} = 1.1 \text{ MeV}, \quad f_{K_0^*} = 42 \text{ MeV} \quad (3.16)$$

obtained from the finite-energy sum rules [43]. (A different calculation of the scalar meson decay constants based on the generalized NLJ model is given in [44].) Since they are derived using the $q\bar{q}$ quark model, it is not clear if the a_0^\pm decay constant remains the same in the $q^2\bar{q}^2$ picture, though it is generally expected that the decay constant is suppressed in the latter scenario because a four-quark state is larger than a two-quark state [43].

As for the decay constant of κ , we apply the equation of motion to Eq. (3.14) to obtain

$$m_{a_0}^2 f_{a_0} = i(m_d - m_u) \langle a_0^- | \bar{d}u | 0 \rangle,$$

$$m_\kappa^2 f_\kappa = i(m_s - m_u) \langle \kappa^- | \bar{s}u | 0 \rangle, \quad (3.17)$$

and assume $\langle \kappa^- | \bar{s}u | 0 \rangle \approx \langle a_0^- | \bar{d}u | 0 \rangle$. It follows that $f_\kappa \approx 65 \text{ MeV}$ for $m_u = 4.8 \text{ MeV}$, $m_d = 8.7 \text{ MeV}$, $m_s = 164 \text{ MeV}$ [45] and $m_\kappa = 800 \text{ MeV}$. In short, the decay constants of scalar mesons are either zero or very small.

C. Form factors

Form factors for $D \rightarrow P$ and $D \rightarrow S$ transitions are defined by [46]

$$\begin{aligned} \langle P(p) | V_\mu | D(p_D) \rangle &= \left(p_{D\mu} + p_\mu - \frac{m_D^2 - m_P^2}{q^2} q_\mu \right) F_1^{DP}(q^2) \\ &\quad + \frac{m_D^2 - m_P^2}{q^2} q_\mu F_0^{DP}(q^2), \\ \langle S(p) | A_\mu | D(p_D) \rangle &= i \left[\left(p_{D\mu} + p_\mu - \frac{m_D^2 - m_S^2}{q^2} q_\mu \right) F_1^{DS}(q^2) \right. \\ &\quad \left. + \frac{m_D^2 - m_S^2}{q^2} q_\mu F_0^{DS}(q^2) \right], \end{aligned} \quad (3.18)$$

where $q_\mu = (p_D - p)_\mu$. The form factors relevant for $D \rightarrow SP$ decays are $F_0^{DP}(q^2)$ and $F_0^{DS}(q^2)$. For $D \rightarrow P$ form

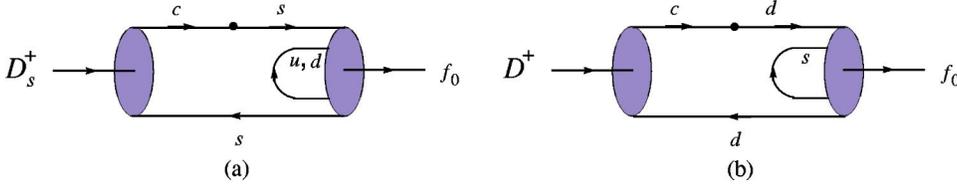


FIG. 1. The $D_s^+ \rightarrow f_0(980)$ and $D^+ \rightarrow f_0(980)$ transition form factors, where $f_0(980)$ is described by a $q^2\bar{q}^2$ state.

factors, we will use the Melikhov-Stech (MS) model [47] based on the constituent quark picture. Other form factor models give similar results.

As discussed in Sec. III A, the light scalar mesons σ , κ , $f_0(980)$ and $a_0(980)$ are predominately $q^2\bar{q}^2$, while K_0^* is described by the $q\bar{q}$ state. Nevertheless, it is useful to see what are the predictions of $D \rightarrow S$ form factors in the conventional quark model. There are some existing calculations of the form factor $F_0^{DS}(0)$ in the literature (see Table III). Paver and Riazuddin [48] obtained $F_0^{D\sigma}(0) = 0.74(f_D/200 \text{ MeV})$. Gatto *et al.* got $F_0^{D\sigma}(0) = 0.57 \pm 0.09$ [49] using the constituent quark model (CQM). Based on the same model, Deandrea *et al.* [50] obtained $F_0^{D_s^+ f_0}(0) = 0.64_{-0.03}^{+0.05}$ assuming a pure $s\bar{s}$ state for f_0 . A value of $F_0^{D_s^+ f_0}(m_\pi^2) = 0.36_{-0.08}^{+0.06}$ is obtained by Gourdin, Keum and Pham [51] based on a fit to the old data of $D_s^+ \rightarrow f_0(980)\pi^+$. The $B \rightarrow a_0$ form factor is estimated by Chernyak [52] to be $F_0^{Ba_0}(0) \sim 0.46$. Using the scaling law, it leads to $F_0^{Da_0}(0) \approx F_0^{Ba_0}(0) \sqrt{m_B/m_D} = 0.77$. Since the conventional quark model is not applicable to light scalars with four-quark content, we shall use the measured decay rates to extract the $D \rightarrow S$ form factors (except for $D \rightarrow a_0$) in Sec. V and the results are summarized in Table III.

In the $q\bar{q}$ description of $f_0(980)$, it follows from Eq. (3.3) that

$$F_0^{D^0 f_0} = \frac{1}{\sqrt{2}} \sin \theta F_0^{D^0 f_0^{u\bar{u}}}, \quad F_0^{D^+ f_0} = \frac{1}{\sqrt{2}} \sin \theta F_0^{D^+ f_0^{d\bar{d}}},$$

$$F_0^{D_s^+ f_0} = \cos \theta F_0^{D_s^+ f_0^{s\bar{s}}}, \quad (3.19)$$

where the superscript $q\bar{q}$ denotes the quark content of f_0 involved in the transition. In the limit of SU(3) symmetry, $F_0^{D^0 f_0^{u\bar{u}}} = F_0^{D^+ f_0^{d\bar{d}}} = F_0^{D_s^+ f_0^{s\bar{s}}}$ and hence

$$F_0^{D^0 f_0} = F_0^{D^+ f_0} = \frac{1}{\sqrt{2}} F_0^{D_s^+ f_0} \tan \theta. \quad (3.20)$$

In the four-quark picture, one has (see Fig. 1)

$$F_0^{D f_0}(0) = \frac{\lambda}{2} F_0^{D_s^+ f_0}(0), \quad (3.21)$$

where use of the $f_0(980)$ flavor function $s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ has been made. For $\theta \sim 34^\circ$, we see that the relation between $F_0^{D f_0}$ and $F_0^{D_s^+ f_0}$ is very similar in the $q\bar{q}$ and $q^2\bar{q}^2$ pictures,

but if $\theta \sim 140^\circ$ then $F_0^{D f_0}$ will have an opposite sign to $F_0^{D_s^+ f_0}$ in the former model. To compute the $D \rightarrow S$ form factors using the $q\bar{q}$ model, it is worth mentioning the Isgur-Scora-Grinstein-Wise (ISGW) model [53] and its improved version the ISGW2 model [54]. Contrary to the above-mentioned form-factor calculations shown in Table III, the form factors of interest in the ISGW model are F_1^{DS} evaluated at $q^2 = q_m^2 \equiv (m_D - m_S)^2$, the maximum momentum transfer, recalling that $F_1(0) = F_0(0)$. The reason for considering the form factor at zero recoil is that the form-factor q^2 dependence in the ISGW model is proportional to $\exp[-(q_m^2 - q^2)]$. Hence, the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. Consequently, the form factor is unreasonably small at $q^2 = 0$. This has been improved in the ISGW2 model in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$. However, we find that form factors in the ISGW2 model calculated even at zero recoil are already small compared to the other model calculations at $q^2 = 0$. For example, $F_1^{Da_0}$ and $F_1^{D_s K_0^*}$ at zero recoil are found to be 0.52 and 0.29, respectively, in the ISGW model and 0.12 as well as 0.09, respectively, in the ISGW2 model. These form factors become even much smaller at $q^2 = 0$. Therefore, the ISGW model and especially the ISGW2 version predict much smaller $D \rightarrow S$ form factors than other quark models.

For the q^2 dependence of the $D \rightarrow S$ form factors, we shall assume the pole dominance:

$$F_0^{DS}(q^2) = \frac{F_0^{DS}(0)}{1 - q^2/m_*^2}, \quad (3.22)$$

with m_* being the mass of the 0^- pole state with the same quark content of the current under consideration.

In the MS [47] or the Baver-Stech-Wirbel (BSW) [46] model, the typical $D \rightarrow P$ form factors have the values $F_0^{D\pi^\pm}(0) = 0.69$ and $F_0^{DK}(0) = 0.78$. In general, it is conceivable that the form factor for $D \rightarrow \sigma$ transition is comparable to the $D \rightarrow \pi^0$ one. The argument goes as follows. If the scalar meson is made from $q\bar{q}$, its distribution amplitude has the form [45]

$$\phi_S(x) = 6f_S x(1-x) \left[1 + \sum_{n=1}^{\infty} B_n C_n^{3/2}(2x-1) \right], \quad (3.23)$$

where f_S is the decay constant of the scalar meson S , B_n are constants, and $C_n^{3/2}$ is the Gegenbauer polynomial. For the isosinglet scalar mesons σ and f_0 , their decay constants vanish, but the combination $f_S B_n$ can be nonzero. For σ , f_0 and

a_0^0 , charge conjugation invariance implies that $\phi(x) = -\phi(1-x)$; that is, the distribution amplitude vanishes at $x = 1/2$. Using $|f_s B_1| \approx 75$ MeV obtained in [45], we have

$$\phi_\sigma(x) \approx 6\bar{B}_1 x(1-x)(3-6x), \quad (3.24)$$

where $\bar{B}_1 = -f_s B_1$. It is clear that the σ distribution amplitude peaks at $x=0.25$ and 0.75 . Now the D meson wave function is peaked at $x \sim \Lambda/m_D \sim 1/3$ [55]. Recall that the asymptotic pion distribution amplitude has the familiar expression

$$\phi_\pi(x) = 6f_\pi x(1-x), \quad (3.25)$$

which has a peak at $x = 1/2$. Though \bar{B}_1 is smaller than f_π , it is anticipated that the $D \rightarrow \sigma$ transition form factor is similar to that of $D \rightarrow \pi^0$ one because the peak of ϕ_σ is close to that of the D distribution amplitude. However, it is not clear if this argument still holds for the scalar mesons which are bound states of $q^2 \bar{q}^2$.

For $K_0^*(1430)$, the distribution amplitude reads

$$\phi_{K_0^*}(x) \approx 6f_{K_0^*} x(1-x)[1 + B_1(3-6x)]. \quad (3.26)$$

It is easy to check that $\phi_{K_0^*}$ has a large peak at $x=0.25$ and a small peak at $x=0.75$. Consequently, it is natural to have a large $D \rightarrow K_0^*$ transition form factor as shown in Table III. Another argument favoring a large $D \rightarrow K_0^*$ form factor is as follows (see [56]). Consider the decay $D^0 \rightarrow K_0^{*-} \pi^+$ and apply PCAC (partial conservation of axial vector current) to evaluate the matrix element $q^\mu \langle K_0^{*-} | \bar{s} \gamma_\mu (1 - \gamma_5) c | D^0 \rangle$. Assuming that this two-body matrix element is saturated by the D_s^+ pole, we find

$$\begin{aligned} & \langle \pi^+ | (\bar{u}d) | 0 \rangle \langle K_0^{*-} | (\bar{s}c) | D^0 \rangle \\ &= i\sqrt{2} f_\pi f_{D_s} g_{K_0^{*-} D^0 D_s^+} \frac{m_{D_s}^2}{m_{D_s}^2 - q^2}. \end{aligned} \quad (3.27)$$

Next apply the SU(4) symmetry to relate strong coupling of $K_0^{*-} D^0 D_s^+$ to $K_0^{*-} \pi^+ \bar{K}^0$

$$g_{K_0^{*-} D^0 D_s^+} = g_{K_0^{*-} \pi^+ \bar{K}^0}. \quad (3.28)$$

The coupling $g_{K_0^{*-} \pi^+ \bar{K}^0}$ can be determined from the measured decay rate of $K_0^{*-} \rightarrow \bar{K}^0 \pi^-$ via

$$\Gamma(K_0^{*-} \rightarrow \bar{K}^0 \pi^-) = g_{K_0^{*-} \pi^+ \bar{K}^0}^2 \frac{p_c}{8\pi m_{K_0^*}}, \quad (3.29)$$

where p_c is the c.m. momentum in the K_0^* rest frame. Using the K_0^* width given in Table II, we obtain $g_{K_0^{*-} \pi^+ \bar{K}^0} = 4.9$ GeV. Since

$$\langle \pi^+ | (\bar{u}d) | 0 \rangle \langle K_0^{*-} | (\bar{s}c) | D^0 \rangle = f_\pi (m_D^2 - m_{K_0^*}^2) F_0^{DK_0^*}(m_\pi^2), \quad (3.30)$$

it follows that

$$\begin{aligned} F_0^{DK_0^*}(m_\pi^2) &= \sqrt{2} \frac{g_{K_0^{*-} \pi^+ \bar{K}^0} f_{D_s^+}}{m_D^2 - m_{K_0^*}^2} \frac{m_{D_s}^2}{m_{D_s}^2 - m_\pi^2} \\ &= 1.23 \left(\frac{f_{D_s^+}}{270 \text{ MeV}} \right). \end{aligned} \quad (3.31)$$

This is consistent with the value of 1.20 ± 0.07 extracted directly from $D^+ \rightarrow \bar{K}_0^{*0} \pi^+$ (see Sec. V A).

IV. QUARK DIAGRAM SCHEME AND FINAL-STATE INTERACTIONS

It has been established sometime ago that a least model-independent analysis of heavy meson decays can be carried out in the so-called quark-diagram approach. In this diagrammatic scenario, all two-body nonleptonic weak decays of heavy mesons can be expressed in terms of six distinct quark diagrams [2,6,7]: T , the color-allowed external W -emission tree diagram; C , the color-suppressed internal W -emission diagram; E , the W -exchange diagram; A , the W -annihilation diagram; P , the horizontal W -loop diagram; and V , the vertical W -loop diagram. (The one-gluon exchange approximation of the P graph is the so-called ‘‘penguin diagram.’’) It should be stressed that these quark diagrams are classified according to the topologies of weak interactions with all strong interaction effects included and hence they are *not* Feynman graphs. All quark graphs used in this approach are topological and meant to have all the strong interactions included, i.e. gluon lines are included in all possible ways. Therefore, topological graphs can provide information on final-state interactions (FSIs).

Based on the SU(3) flavor symmetry, this model-independent analysis enables us to extract the topological quark-graph amplitudes and see the relative importance of different underlying decay mechanisms. The quark-diagram scheme, in addition to being helpful in organizing the theoretical calculations, can be used to analyze the experimental data directly. When enough measurements are made with sufficient accuracy, we can find out the values of each quark-diagram amplitude from experiment and compare to theoretical results, especially checking whether there are any final-state interactions or whether the weak annihilations can be ignored as often asserted in the literature.

For charmed meson decays, the penguin contributions are negligible owing to the good approximation $V_{ud} V_{cd}^* \approx -V_{us} V_{cs}^*$ and the smallness of $V_{ub} V_{cb}^*$. Hence, for $D \rightarrow PP, VP, VV$ decays, only T, C, E and A contribute. The reduced quark-graph amplitudes T, C, E, A have been extracted from Cabibbo-allowed $D \rightarrow PP$ decays by Rosner [8,57] with the results:

TABLE IV. Quark-diagram amplitudes and branching ratios for various $D \rightarrow SP$ decays with and without the long-distance weak annihilation terms induced from final-state interactions. Light scalar mesons σ , κ , $a_0(980)$ and $f_0(980)$ are described by the $q^2\bar{q}^2$ states, while K_0^* is assigned by $q\bar{q}$. Experimental results are taken from Table I.

Decay	Amplitude	$\mathcal{B}_{\text{naive}}$	\mathcal{B}_{FSI}	$\mathcal{B}_{\text{expt}}$
$D^+ \rightarrow f_0 \pi^+$	$V_{cd}V_{ud}^*(T+C'+2A)/\sqrt{2} + V_{cs}V_{us}^*\sqrt{2}C'$	3.5×10^{-4}	see text	$(4.3 \pm 1.1) \times 10^{-4}$
$\rightarrow f_0 K^+$	$V_{cd}V_{us}^*(T+3A)/\sqrt{2}$	2.2×10^{-5}	2.2×10^{-5}	$(2.0 \pm 0.5) \times 10^{-4}$
$\rightarrow a_0^+ \bar{K}^0$	$V_{cs}V_{ud}^*(T'+C)$	1.7×10^{-2}	1.7×10^{-2}	
$\rightarrow a_0^0 \pi^+$	$V_{cd}V_{ud}^*(-T-C')/\sqrt{2}$	1.7×10^{-3}	1.7×10^{-3}	$(3.2 \pm 0.6)\%$
$\rightarrow \sigma \pi^+$	$V_{cd}V_{ud}^*(T+C'+2A)$	input		$(2.1 \pm 0.5) \times 10^{-3}$
$\rightarrow \kappa \pi^+$	$V_{cs}V_{ud}^*(T+C')$	input		$(6.5 \pm 1.9)\%$
$\rightarrow \bar{K}_0^{*0} \pi^+$	$V_{cs}V_{ud}^*(T+C')$	input		$(1.8 \pm 0.3)\%$
$D^0 \rightarrow f_0 \bar{K}^0$	$V_{cs}V_{ud}^*(C+3E)/\sqrt{2}$	8.2×10^{-4}	input for E	$(6.3 \pm 1.2) \times 10^{-3}$
$\rightarrow a_0^+ K^-$	$V_{cs}V_{ud}^*(T'+E)$	2.8×10^{-6}	1.1×10^{-3}	$(2.2 \pm 0.5)\%$
$\rightarrow a_0^0 \bar{K}^0$	$V_{cs}V_{ud}^*(C-E)/\sqrt{2}$	3.5×10^{-3}	3.6×10^{-3}	$(7.9 \pm 1.7)\%$
$\rightarrow a_0^- K^+$	$V_{cd}V_{us}^*(T+E)$	8.1×10^{-5}	7.9×10^{-5}	$(2.1 \pm 1.3) \times 10^{-3}$
$\rightarrow a_0^+ \pi^-$	$V_{cd}V_{ud}^*(T'+E)$	1.7×10^{-7}	6.5×10^{-5}	$(3.4 \pm 2.8) \times 10^{-3}$
$\rightarrow a_0^- \pi^+$	$V_{cd}V_{ud}^*(T+E)$	1.3×10^{-3}	1.3×10^{-3}	$(9.5 \pm 7.9) \times 10^{-4}$
$\rightarrow K_0^{*-} \pi^+$	$V_{cs}V_{ud}^*(T+E)$	1.3×10^{-2}	1.1×10^{-2}	$(1.18 \pm 0.25)\%$
$\rightarrow \bar{K}_0^{*0} \pi^0$	$V_{cs}V_{ud}^*(C'+E)/\sqrt{2}$	3.9×10^{-4}	3.7×10^{-3}	$(7.0_{-1.3}^{+3.1}) \times 10^{-3}$ $(8.6_{-2.3}^{+6.8}) \times 10^{-3}$
$D_s^+ \rightarrow f_0 \pi^+$	$V_{cs}V_{ud}^*(2T+2A)/\sqrt{2}$	input		$(1.8 \pm 0.3)\%$
$\rightarrow f_0 K^+$	$V_{cs}V_{us}^*(2T+3A)/\sqrt{2}$	1.2×10^{-3}	1.2×10^{-3}	$(1.8 \pm 0.8) \times 10^{-3}$
$\rightarrow \bar{K}_0^{*0} K^+$	$V_{cs}V_{ud}^*(C'+A)$	4.0×10^{-4}	1.5×10^{-3}	$(7 \pm 4) \times 10^{-3}$
$\rightarrow K_0^{*0} \pi^+$	$V_{cd}V_{ud}^*T + V_{cs}V_{us}^*A$	1.3×10^{-3}	1.1×10^{-3}	$(2.3 \pm 1.3) \times 10^{-3}$

$$\begin{aligned}
T &= (2.67 \pm 0.20) \times 10^{-6} \text{ GeV}, \\
C &= (2.03 \pm 0.15) \exp[-i(151 \pm 4)^\circ] \\
&\quad \times 10^{-6} \text{ GeV}, \\
E &= (1.67 \pm 0.13) \exp[i(115 \pm 5)^\circ] \\
&\quad \times 10^{-6} \text{ GeV}, \\
A &= (1.05 \pm 0.52) \exp[-i(65 \pm 30)^\circ] \\
&\quad \times 10^{-6} \text{ GeV}.
\end{aligned} \tag{4.1}$$

Hence, the weak annihilation (W -exchange E or W -annihilation A) amplitude has a sizable magnitude comparable to the color-suppressed internal W -emission amplitude C with a large phase relative to the tree amplitude T . As discussed in [5], it receives long-distance contributions from nearby resonance via inelastic final-state interactions from the leading tree or color-suppressed amplitude. The effects of resonance-induced FSI can be described in a model independent manner and are governed by the masses and decay widths of the nearby resonances. Weak annihilation topologies in $D \rightarrow PP$ decays are dominated by nearby scalar resonances via final-state rescattering. The relative phase between the tree and color-suppressed amplitudes arises from the final-state rescattering via quark exchange. This can be

evaluated by considering the t -channel chiral-loop contribution or by applying the Regge pole method (for details, see [5]).

For $D \rightarrow SP$ decays, there are several new features. First, one can have two different external W -emission and internal W -emission diagrams, depending on whether the emission particle is a scalar meson or a pseudoscalar one. We thus denote the prime amplitudes T' and C' for the case when the scalar meson is an emitted particle. The quark-diagram amplitudes for various $D \rightarrow SP$ decays are listed in Table IV. Second, because of the smallness of the decay constant of the scalar meson as discussed before, it is expected that $|T'| \ll |T|$ and $|C'| \ll |C|$. A noticeable example is the decay D^0

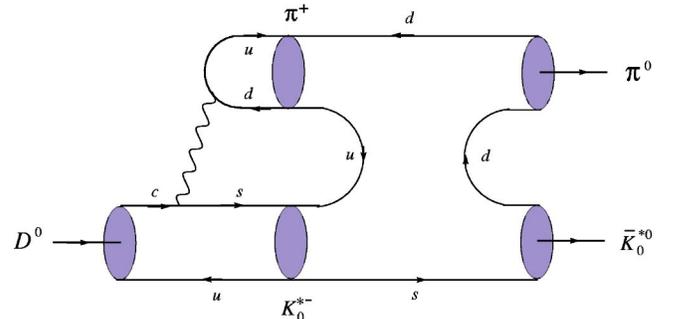


FIG. 2. Contributions to $D^0 \rightarrow \bar{K}_0^{*0} \pi^0$ from the color-allowed weak decay $D^0 \rightarrow K_0^{*-} \pi^+$ followed by a resonant-like rescattering. This has the same topology as the W -exchange graph.

$\rightarrow K^- a_0^+$. Its branching ratio is naively predicted to be of order 10^{-6} , which is strongly suppressed compared to the counterpart decay $D^0 \rightarrow K^- \pi^+$ in the PP sector. Experimentally, $K^- a_0^+$ has a branching ratio comparable to $K^- \pi^+$. This implies the importance of the W -exchange term in $D^0 \rightarrow K^- a_0^+$. Third, since K_0^* and the light scalars σ , κ , f_0 , a_0 fall into two different $SU(3)$ flavor nonets, one cannot apply $SU(3)$ symmetry to relate the topological amplitudes in $D^+ \rightarrow f_0 \pi^+$ to, for example, those in $D^+ \rightarrow \bar{K}_0^{*0} \pi^+$. Note that in the flavor $SU(3)$ limit, the primed amplitudes T' and C' diminish in the factorization approach due to the vanishing decay constants of scalar mesons.

Just as in $D \rightarrow PP$ decays, the topological weak annihilation amplitudes E and A , which are naively expected to be helicity suppressed, can receive large long-distance final-state interaction contributions. For example, there is a contribution to the W -exchange amplitude E of $D^0 \rightarrow \bar{K}_0^{*0} \pi^0$ from the color-allowed decay $D^0 \rightarrow K_0^{*-} \pi^+$ followed by a resonant-like rescattering. As discussed in [5], Fig. 2 manifested at the hadron level receives a s -channel resonant contribution from, for example, the 0^- resonance $K(1830)$ and a t -channel contribution with one-particle exchange. Likewise, the W -exchange term in $D^0 \rightarrow a_0^\pm \pi^\mp$ receives the $\pi(1800)$ resonance contribution.

V. GENERALIZED FACTORIZATION AND ANALYSIS

We will study $D \rightarrow SP$ decays within the framework of generalized factorization in which the hadronic decay amplitude is expressed in terms of factorizable contributions multiplied by the *universal* (i.e. process independent) effective parameters a_i that are renormalization scale and scheme independent. More precisely, the weak Hamiltonian has the form

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cq_1} V_{uq_2}^* [a_1 (\bar{u}q_2)(\bar{q}_1c) + a_2 (\bar{q}_1q_2)(\bar{u}c)] + \text{H.c.}, \quad (5.1)$$

with $(\bar{q}_1q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. For hadronic charm decays, we shall use $a_1 = 1.15$ and $a_2 = -0.55$. The parameters a_1 and a_2 are related to the Wilson coefficients via

$$a_1 = c_1(\mu) + c_2(\mu) \left(\frac{1}{N_c} + \chi_1(\mu) \right),$$

$$a_2 = c_2(\mu) + c_1(\mu) \left(\frac{1}{N_c} + \chi_2(\mu) \right), \quad (5.2)$$

where the nonfactorizable terms $\chi_i(\mu)$ will compensate the scale and scheme dependence of Wilson coefficients $c_i(\mu)$ to render a_i physical.

A. $D \rightarrow K_0^*(1430) \pi$

Among the four measured $D \rightarrow K_0^* \pi$ modes: $D^+ \rightarrow \bar{K}_0^{*0} \pi^+$, $D^0 \rightarrow K_0^{*-} \pi^+$, $\bar{K}_0^{*0} \pi^0$ and $D_s^+ \rightarrow K_0^* \pi^+$, only

the first one does not involve weak annihilation and hence it can be used to fix the form factor for $D \rightarrow K_0^*$ transition. More precisely,

$$A(D^+ \rightarrow \bar{K}_0^{*0} \pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [a_1 f_\pi (m_D^2 - m_{K_0^*}^2) F_0^{DK_0^*}(m_\pi^2) + a_2 f_{K_0^*} (m_D^2 - m_\pi^2) F_0^{D\pi}(m_{K_0^*}^2)]. \quad (5.3)$$

From a fit to the E791 result $\mathcal{B}(D^+ \rightarrow \bar{K}_0^{*0} \pi^+) = (1.8 \pm 0.3)\%$ (see Table I), we obtain

$$F_0^{DK_0^*}(0) = 1.20 \pm 0.07. \quad (5.4)$$

As explained in Sec. II, the E791 analysis for $D^+ \rightarrow K^- \pi^+ \pi^+$ has included the scalar contribution from the κ resonance and found a better improved fit. If the PDG value of $(3.7 \pm 0.4)\%$ for the branching ratio of $D^+ \rightarrow \bar{K}_0^{*0} \pi^+$ is employed [14], we will get a too large form factor of order 1.60.

Under the factorization approximation, the factorizable amplitudes of other $D \rightarrow K_0^* \pi$ decays read

$$A(D^0 \rightarrow K_0^{*-} \pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [a_1 f_\pi (m_D^2 - m_{K_0^*}^2) F_0^{DK_0^*}(m_\pi^2) + a_2 \langle K_0^{*-} \pi^+ | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle],$$

$$A(D^0 \rightarrow \bar{K}_0^{*0} \pi^0) = \frac{G_F}{2} V_{cs} V_{ud}^* a_2 [f_{K_0^*} (m_D^2 - m_\pi^2) F_0^{D\pi}(m_{K_0^*}^2) + \langle K_0^{*-} \pi^+ | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle],$$

$$A(D_s^+ \rightarrow K_0^{*0} \pi^+) = \frac{G_F}{\sqrt{2}} a_1 [V_{cd} V_{ud}^* f_\pi (m_D^2 - m_{K_0^*}^2) F_0^{DK_0^*}(m_\pi^2) + V_{cs} V_{us}^* \langle K_0^{*0} \pi^+ | (\bar{u}d) | 0 \rangle \times \langle 0 | (\bar{s}c) | D_s^+ \rangle]. \quad (5.5)$$

The factorizable (or short-distance) weak annihilation contribution is conventionally argued to be helicity suppressed.

We see from Table IV that the predictions for $D^0 \rightarrow K_0^{*-} \pi^+$ and $D_s^+ \rightarrow K_0^{*0} \pi^+$ are in agreement with experiment, whereas $D^0 \rightarrow \bar{K}_0^{*0} \pi^0$ and $D_s^+ \rightarrow \bar{K}_0^{*0} K^+$ are too small by an order of magnitude. The latter implies the importance of long-distance weak annihilation contributions induced from FSIs. If we assume that the relative phase and magnitude of the W -exchange amplitude E and the W -annihilation amplitude A relative to the external W -emission amplitude T are the same as in the case of $D \rightarrow PP$ decays, namely,

$$E/T \approx 0.63 e^{i115^\circ}, \quad A/T \approx 0.39 e^{-i65^\circ}, \quad (5.6)$$

we find that $D^0 \rightarrow \bar{K}_0^{*0} \pi^0$ and $D_s^+ \rightarrow \bar{K}_0^{*0} K^+$ are enhanced by an order of magnitude, while $D^0 \rightarrow K_0^{*-} \pi^+$ and $D_s^+ \rightarrow K_0^{*0} \pi^+$ are affected only slightly.

B. $D^+ \rightarrow \kappa \pi^+$

As noticed in passing, although the decay $D^+ \rightarrow \kappa \pi^+$ has very similar topological quark amplitudes as $D^+ \rightarrow \bar{K}_0^{*0} \pi^+$ (see Table IV), they cannot be related to each other via SU(3) symmetry as κ and K_0^* belong to two different SU(3) flavor nonets. Using the decay constant $f_\kappa = 65$ MeV as estimated in Sec. III B, we find that $F_0^{D\kappa}(0) = 0.85 \pm 0.10$ from the measured $D^+ \rightarrow \kappa \pi^+$ rate. If the decay constant is negligible, then the form factor will become smaller, $F_0^{D\kappa}(0) = 0.64 \pm 0.10$, due to the absence of a destructive contribution from C' .

C. $D^+ \rightarrow \sigma \pi^+$

Neglecting W -annihilation and taking $m_\sigma = 500$ MeV, the form factor $F_0^{D\sigma}$ extracted from $D^+ \rightarrow \sigma \pi^+$ reads

$$F_0^{D\sigma}(0) = 0.42 \pm 0.05. \quad (5.7)$$

This is quite different from the fit value 0.8 ± 0.2 obtained in [58] using the E791 data for $D^+ \rightarrow \pi^+ \pi^+ \pi^-$ [9] and the Breit-Wigner description of the σ resonance. Note that in the conventional quark model, the σ flavor wave function is given by $(u\bar{u} + d\bar{d})/\sqrt{2}$, while it is $u\bar{u}d\bar{d}$ in the four-quark picture. Therefore, in the SU(3) symmetry limit, the ratio of $|A(D^+ \rightarrow \sigma \pi^+)/A(D^+ \rightarrow \kappa \pi^+)|^2$ is $|V_{cd}/V_{cs}|^2$ if σ is made of four quarks, while it will be two times smaller if σ is a $q\bar{q}$ state.

D. $D \rightarrow f_0(980) \pi, f_0(980) K$

Since the W -annihilation contribution is smaller than the W -exchange one in $D \rightarrow PP$ decays [see Eq. (4.1)], we will neglect the W -annihilation amplitude A as a first approximation and determine the $D \rightarrow f_0$ form factor from experiment. We will choose $D_s^+ \rightarrow f_0 \pi^+$ or $D_s^+ \rightarrow f_0 K^+$ rather than $D^+ \rightarrow f_0 K^+$ to extract $F_0^{Df_0}(0)$. The reason is as follows. In the SU(3) limit, it is expected that (see Table IV)

$$\frac{\mathcal{B}(D_s^+ \rightarrow f_0 \pi^+)}{\mathcal{B}(D_s^+ \rightarrow f_0 K^+)} = \left| \frac{V_{ud}}{V_{us}} \right|^2. \quad (5.8)$$

It is easily seen that this relation is borne out by experiment. In contrast, the relation

$$\frac{\Gamma(D^+ \rightarrow f_0 K^+)}{\Gamma(D_s^+ \rightarrow f_0 K^+)} = \frac{1}{4} \left| \frac{V_{cd}}{V_{cs}} \right|^2 \quad (5.9)$$

is different from the experimental ratio which is close to $|V_{cd}/V_{cs}|^2$. This implies that the decay rate of $D^+ \rightarrow f_0 K^+$ inferred from FOCUS [11] is probably too large by a factor of 4. Indeed, since this mode is Cabibbo doubly suppressed, it is unlikely that its branching ratio is of the same order as the Cabibbo singly suppressed one $D^+ \rightarrow f_0 \pi^+$. At any rate, it is important to check this mode soon. From the decay $D_s^+ \rightarrow f_0 \pi^+$, we obtain

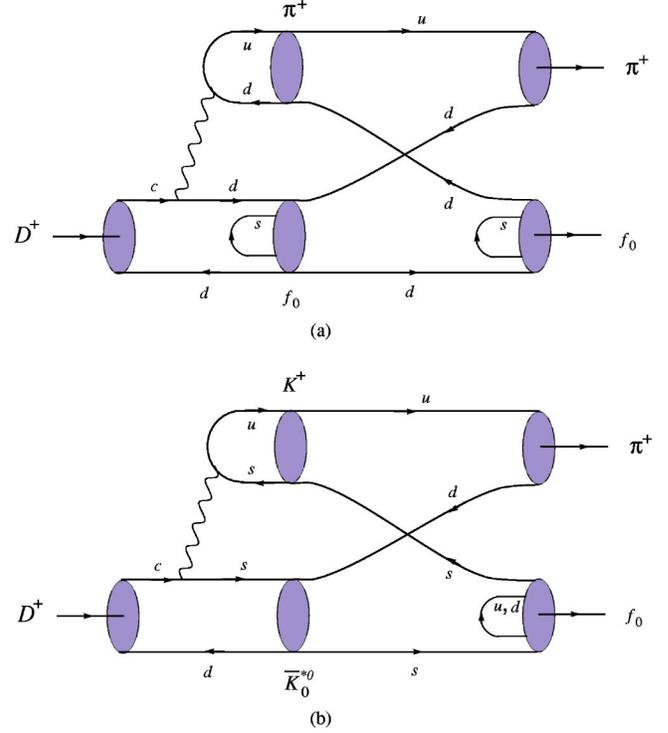


FIG. 3. (a) Long-distance contributions to the amplitude $V_{cd}V_{ud}^*C'$ of $D^+ \rightarrow f_0 \pi^+$ from the color-allowed weak decay $D^+ \rightarrow f_0 \pi^+$ followed by a rescattering via quark exchange, and (b) the long-distance contribution to the amplitude $V_{cs}V_{us}^*C'$ of $D^+ \rightarrow f_0 \pi^+$ from the color-allowed weak decay $D^+ \rightarrow K_0^{*+} \bar{K}_0, K^+ \bar{K}_0^{*0}$ followed by a rescattering via quark exchange.

$$F_0^{D_s^+ f_0}(0) = 0.52 \pm 0.04 \quad (5.10)$$

and hence $F_0^{Df_0}(0) = 0.26 \pm 0.02$ from Eq. (3.21).

Since we have neglected the W -annihilation contribution in the process of extracting the form factor $F_0^{D_s^+ f_0}(0)$, we will consistently ignore this contribution in all $(D, D_s^+) \rightarrow f_0 \pi, f_0 K$ decays listed in Table IV. It is clear from this table that one needs a sizable W -exchange to account for $D^0 \rightarrow f_0 \bar{K}^0$. One can utilize this mode to fix the amplitude E to be

$$E/T \approx 0.40 e^{i100^\circ}, \quad (5.11)$$

where we have assumed a phase of 100° of the W -exchange term relative to the tree amplitude.

As for the decay $D^+ \rightarrow f_0 \pi^+$, the factorizable internal W -emission amplitude is absent owing to a vanishing f_0 decay constant. Nevertheless, it does receive a long-distance contribution via final-state rescattering, see Fig. 3. Since $V_{cs}V_{us}^* \approx -V_{cd}V_{ud}^*$ and the amplitude C' is governed by a_2 , the amplitude $V_{cd}V_{ud}^*C'/\sqrt{2} + V_{cs}V_{us}^*\sqrt{2}C' \approx -V_{cd}V_{ud}^*C'/\sqrt{2}$ will give a constructive contribution and enhance slightly the decay rate of $D^+ \rightarrow f_0 \pi^+$. At any rate, the agreement between theory and experiment for this mode implies that the form factor for $D^+ \rightarrow f_0$ is indeed smaller than the one for $D_s^+ \rightarrow f_0$.

If $f_0(980)$ is made from $q\bar{q}$, the ratio of $D^+ \rightarrow f_0 K^+$ to $D_s^+ \rightarrow f_0 K^+$ will be

$$\frac{\Gamma(D^+ \rightarrow f_0 K^+)}{\Gamma(D_s^+ \rightarrow f_0 K^+)} = \frac{1}{2} \tan^2 \theta \left| \frac{V_{cd}}{V_{cs}} \right|^2. \quad (5.12)$$

For $\theta \sim 34^\circ$ (see Sec. III A), it is easily seen that the two-quark relation (5.12) is similar to Eq. (5.9) for the four-quark case. However, for $\theta \sim 140^\circ$ as favored by hadronic J/ψ decays and the radiative decays $\phi \rightarrow f_0(980)\gamma$, $f_0(980) \rightarrow \gamma\gamma$, the $n\bar{n}$ and $s\bar{s}$ components in the $q\bar{q}$ wave function of $f_0(980)$ have opposite signs. This means that the interference between the tree amplitude T and the W -annihilation amplitude A in the decay, for example, $D_s^+ \rightarrow f_0(980)\pi^+$ is opposite in the 2-quark and 4-quark models. That is, if the interference is constructive in one of the quark models, it will be destructive in the other model, unless the relative phase between T and A is 90° . Therefore, whether or not one can distinguish between the $q\bar{q}$ and $q^2\bar{q}^2$ pictures for $f_0(980)$ via nonleptonic D decays depends on the f_0 - σ mixing angle and the magnitude and the phase of the W -annihilation term.

Finally we comment on the decays $D^0 \rightarrow f_0(1370)\bar{K}^0$, $D^+ \rightarrow f_0(1370)\pi^+$ and $D_s^+ \rightarrow f_0(1370)\pi^+$ which have been measured by ARGUS [17], E687 [16] and CLEO [10], by FOCUS [11] and by E791 [9], respectively, with the results

$$\begin{aligned} & \mathcal{B}[D^0 \rightarrow f_0(1370)\bar{K}^0] \mathcal{B}[f_0(1370) \rightarrow \pi^+ \pi^-] \\ &= \begin{cases} (4.7 \pm 1.4) \times 10^{-3} & \text{ARGUS, E687} \\ (5.9_{-2.7}^{+1.8}) \times 10^{-3}, & \text{CLEO} \end{cases} \\ & \mathcal{B}[D^+ \rightarrow f_0(1370)\pi^+] \mathcal{B}[f_0(1370) \rightarrow K^+ K^-] \\ &= (6.2 \pm 1.1) \times 10^{-4} \quad \text{FOCUS} \\ & \mathcal{B}[D_s^+ \rightarrow f_0(1370)\pi^+] \mathcal{B}[f_0(1370) \rightarrow \pi^+ \pi^-] \\ &= (3.3 \pm 1.2) \times 10^{-3} \quad \text{E791} \end{aligned} \quad (5.13)$$

Since the branching fractions of $f_0(1370) \rightarrow \pi^+ \pi^-$, $K^+ K^-$ are unknown, the individual branching ratio of D decays into $f_0(1370)$ cannot be determined at present. Nevertheless, if $f_0(1370)$ is a $n\bar{n}$ state in nature, the decay $D_s^+ \rightarrow f_0(1370)\pi^+$ can only proceed through the topological W -annihilation diagram. Hence, this will be the first direct evidence for a nonvanishing W -annihilation amplitude A in $D \rightarrow SP$ decays. The other modes $D^0 \rightarrow f_0(1370)\bar{K}^0$ and $D^+ \rightarrow f_0(1370)\pi^+$ proceed via the internal W -emission diagram C and the external W -emission diagram T , respectively. Taking into account the phase-space correction and noting that $D^+ \rightarrow f_0(1370)\pi^+$ is Cabibbo singly suppressed while the other two are Cabibbo allowed, it is obvious that $|T| > |C| > |A|$, as it should be.

E. $D \rightarrow a_0(980)\pi$, $a_0(980)K$

Because the primed amplitudes T' and C' are largely suppressed relative to the unprimed ones T and C owing to the smallness of the a_0 decay constant, it is interesting to

notice that the neutral state $a_0^0 \bar{K}^0$ is not color suppressed relative to the charged mode $a_0^+ K^-$, contrary to the case of $D^0 \rightarrow \pi^+ K^-$, $\pi^0 \bar{K}^0$. It is also anticipated that $a_0^- \pi^+ \gg a_0^+ \pi^-$, a relation that cannot be tested by the present preliminary data as they do not have enough statistical significance.

Just as the D decays to σ , κ , $f_0(980)$ and $K_0^*(1430)$, one may use the channel $D^+ \rightarrow a_0^0 \pi^+$ to fix the form factor for $D \rightarrow a_0$ transition. However, in the SU(3) limit, one has the relations (see Table IV)

$$\begin{aligned} R_1 &\equiv \frac{\mathcal{B}(D^+ \rightarrow a_0^0 \pi^+)}{\mathcal{B}(D^+ \rightarrow \kappa \pi^+)} = \frac{1}{2} \left| \frac{V_{cd}}{V_{cs}} \right|^2, \\ R_2 &\equiv \frac{\mathcal{B}(D^+ \rightarrow a_0^0 \pi^+)}{\mathcal{B}(D^+ \rightarrow \sigma \pi^+)} \approx \frac{1}{2}. \end{aligned} \quad (5.14)$$

Experimentally, $R_1 \sim \frac{1}{2}$ and $R_2 \sim 15$. This indicates that the preliminary data of $D^+ \rightarrow a_0^0 \pi^+$ is too large by at least an order of magnitude. Therefore, we will instead use the value of 0.77 for $F_0^{Da_0}(0)$ (see Sec. III C). For the W -exchange amplitude we can apply Eq. (5.11).

The results of calculations are shown in Table IV. Obviously all the predicted branching ratios for $D \rightarrow a_0 \pi$, $a_0 K$ (except for $D^0 \rightarrow a_0^- \pi^+$) are too small by one to two orders of magnitude when compared with experiment. Note that $D^0 \rightarrow a_0^- K^+$ is Cabibbo doubly suppressed and it appears to be very unlikely that it has a large branching ratio of order 10^{-3} . From Table IV we also see that $D^+ \rightarrow a_0^+ \bar{K}^0$ has the largest branching ratio among the two-body decays $D \rightarrow a_0 \pi(K)$.

If we fit the $D \rightarrow a_0$ form factor to $D^+ \rightarrow a_0^0 \pi^+$, we will get $F_0^{Da_0}(0) = 3.4$ and the large discrepancy between theory and experiment will be greatly improved. However, in the meantime we also predict that $\mathcal{B}(D^+ \rightarrow a_0^+ \bar{K}^0) = 0.35$ which is obviously too large. Moreover, it is impossible to achieve this abnormally large $D \rightarrow a_0$ form factor in the quark model.

It is possible that one has to apply the Breit-Wigner approximation for $a_0(980)$ to derive the branching ratios for $D \rightarrow a_0 \pi$, $a_0 K$ from the three-body decays of charmed mesons. Furthermore, the fraction of $a_0(980) \rightarrow K\bar{K}$ should be pinned down. It will be interesting to compare the experimental results with the predictions exhibited in Table IV.

VI. CONCLUSIONS

The nonleptonic weak decays of charmed mesons into a scalar meson and a pseudoscalar meson are studied. The scalar mesons under consideration are σ [or $f_0(600)$], κ , $f_0(980)$, $a_0(980)$ and $K_0^*(1430)$. The main conclusions are:

(1) Studies of the mass spectrum of scalar mesons and their strong as well as electromagnetic decays suggest that the light scalars below or near 1 GeV form an SU(3) flavor nonet and are predominately the $q^2\bar{q}^2$ states, while the scalar mesons above 1 GeV can be described as a $q\bar{q}$ nonet with a

possible mixing with $0^+ q\bar{q}$ and glueball states. Therefore, we designate $q^2\bar{q}^2$ to σ , κ , $a_0(980)$, $f_0(980)$ and $q\bar{q}$ to K_0^* .

(2) The topological quark-diagram scheme for $D \rightarrow SP$ decays is more complicated than the case of $D \rightarrow PP$. One can have two different external W -emission and internal W -emission diagrams, depending on whether the emission particle is a scalar meson or a pseudoscalar one. The quark-diagram amplitude for the case when the emitted particle is a scalar meson is largely suppressed relative to the one when the pseudoscalar meson is emitted. Moreover, the former amplitude vanishes in the limit of SU(3) symmetry.

(3) The charmed meson to κ and K_0^* transition form factors are extracted from the Cabibbo-allowed decays $D^+ \rightarrow \kappa\pi^+$, $\bar{K}_0^{*0}\pi^+$, respectively, while $D^+ \rightarrow \sigma$ and $D_s^+ \rightarrow f_0$ ones are inferred from $D^+ \rightarrow \sigma\pi^+$ and $D_s^+ \rightarrow f_0\pi^+$, respectively, based on the assumption of negligible W annihilation. We show that a large form factor for $D \rightarrow K_0^*$ is expected.

The relation $F_0^{D^+f_0} = F_0^{D_s^+f_0}/2$ obtained in the 4-quark picture for $f_0(980)$ leads to a prediction for $D^+ \rightarrow f_0\pi^+$ in agreement with experiment. Note that the value of the form factor $F_0^{D^+\sigma}(0) = 0.42 \pm 0.05$ obtained in this work is very different from the one, 0.8 ± 0.2 , quoted in the literature. It is pointed out that the ISGW model and its improved version the ISGW2 model predict too small $D \rightarrow S$ form factors even at zero recoil.

(4) Except for the Cabibbo doubly suppressed decay $D^+ \rightarrow f_0K^+$, the data of $D \rightarrow \sigma\pi$, $f_0\pi$, f_0K , $K_0^*\pi$ can be accommodated in the generalized factorization approach. Sizable weak annihilation contributions induced from final-state

interactions are crucial for understanding the data. For example, the importance of the W -exchange term is implied by the decays $D^0 \rightarrow f_0\bar{K}^0$, $\bar{K}_0^{*0}\pi^0$ and the W -annihilation one by $D_s^+ \rightarrow \bar{K}_0^{*0}K^+$. Without W -exchange or W -annihilation contributions, the decay rates of these modes will be too small by one order of magnitude. The branching ratio of $D^+ \rightarrow f_0K^+$ is predicted to be of order 10^{-5} and should be tested soon.

(5) The predicted $D \rightarrow a_0\pi$, a_0K rates are too small by one to two orders of magnitude when compared with the preliminary measurements. It is pointed out that $D^+ \rightarrow a_0^+\bar{K}^0$ should have the largest branching ratio among the decays $D \rightarrow a_0\pi$, a_0K .

(6) If $f_0(980)$ is a $q\bar{q}$ state in nature, it must contain not only $s\bar{s}$ but also $u\bar{u}$ and $d\bar{d}$ content. The f_0 - σ mixing angles inferred from the hadronic decays $J/\psi \rightarrow f_0\phi/\omega$, the radiative decay $\phi \rightarrow f_0\gamma$ followed by $f_0 \rightarrow \gamma\gamma$, and the strong coupling of f_0 to $K\bar{K}$ and $\pi\pi$ are not quite compatible with each other. If $\theta \sim 140^\circ$, then it will be possible to distinguish between the two-quark and four-quark pictures for $f_0(980)$ in the decay, for example, $D_s^+ \rightarrow f_0\pi^+$. In the SU(3) symmetry limit, the ratio of $|A(D^+ \rightarrow \sigma\pi^+)/A(D^+ \rightarrow \kappa\pi^+)|^2$ can be different by a factor of 2 in these two different pictures.

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