Pion and kaon parton distribution functions in a meson-cloud model

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In a model of hadrons in which the hadron wave function at low Q^2 is represented by a hadron-like Fock state expansion, we calculate the π and K parton distribution functions by comparing with the experimental data. We show that the model gives an accurate description of available data and, in addition, gives a clear prediction of the initial flavors to be considered at the low Q^2 scale where perturbative evolution starts.

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The parton content of mesons is not well known due to the scarce experimental information as compared to the rich and accurate data existing for the proton. The meson structure is measured in dilepton Drell-Yan (DY) production in meson-nucleon interactions. So far, few experiments [1,2] have measured the valence quark distribution in charged pions, while for kaons, the only experimental information today available is on the $\bar{u}_k$ distribution in the K^-, which has been obtained by the NA3 collaboration [3] in the 1980s.

In DY dilepton production in the reaction $H^A + H^B \rightarrow l^+ l^-$, the biggest contribution comes from the subprocesses $\bar{q}+q \rightarrow l^+ l^-$, where $\bar{q},q$ are quarks in the initial hadrons H^A, H^B. Then the partonic structure of one of the initial hadrons can be extracted if the structure of the other one is known. Thus, the structure of charged pions can be measured in $\pi^{\pm} N$ interactions. Furthermore, some authors have claimed that adequate linear combinations of $\pi^{\pm} p$ and $\pi^{\pm} n$ DY cross sections can allow the measurement of the valence quark distributions in charged pions independently of their sea quark and gluon distributions [4]. In a similar way, the $\bar{u}$ valence quark distribution in the K^- can be measured. However, the valence s quark distribution cannot be easily determined in kaons since it has to annihilate with a $\bar{s}$ sea quark in the nucleon to produce the $l^+ l^-$ pair, giving a negligible contribution to the DY cross section in the experimentally accessible kinematic region.

Then, to know the kaon structure, which is desirable since, for instance, kaons are frequently used as beam particles in experiments, one has to rely on theoretical models to complement the experimental information. On this respect, several models have been proposed. Among them we can mention the model of Ref. [5], where the kaon structure is derived in a constituent quark model; the model of Ref. [6], where some quark and gluon sea is considered in addition to the constituent quarks at the scale where evolution starts; and most recently, the model presented in Ref. [7], where an expansion of the pion and kaon wave functions in terms of hadronlike Fock states is done at the input Q^2 scale. One interesting feature of the last one is that the model predicts the parton structure of the pion and kaon, at the low Q^2 scale where evolution starts, up to a few parameters which must be fixed from experimental data. This is a remarkable property since it leaves the model free of the arbitrariness of the other two, where by no means both the form and initial parton distribution functions (PDF) are justified.

In what follows, after a short revision of the model of Ref. [7], we shall use it to extract information on the pion and kaon parton content from simultaneous fits to the available experimental data.

Following Ref. [7], the π^- and K^- wave functions can be written as

$$|\pi^-\rangle = a_0^{\pi}|\pi^-\rangle + a_1^{\pi}|\pi^- g\rangle + a_2^{\pi}|K^0 K^-\rangle + \cdots, \tag{1}$$

$$|K^-\rangle = a_0^{K}|K^-\rangle + a_1^{K}|K^- g\rangle + a_2^{K}|\overline{K^0} \pi^-\rangle + \cdots, \tag{2}$$

at some low Q_v^2 scale, where we have neglected higher order contributions involving heavier mesons and fluctuations to Fock states containing more than two mesons. These fluctuations should be far off-shell and they can be safely ignored at this point. The first terms in Eqs. (1), (2) are the bare meson states, which are formed only by valons [5]. The following terms are fluctuations, whose origin can be traced back to the processes $v \rightarrow v + g$ and $g \rightarrow \bar{q} + q$ followed by the recombination of the perturbative $q\bar{q}$ pair with the valons v to form the hadronlike structure. As discussed in Ref. [8], we assume that interactions between a quark or antiquark and a valon of the same flavor do not form a neutral, unflavored, virtual meson structure but annihilates nonperturbatively to a gluon. Additionally, in-meson hadrons are assumed to be formed only by valons.

Thus, these fluctuations are an effective representation of the nonperturbative, *intrinsic* sea of quarks and gluons which should provide the necessary binding among constituent quarks to form hadrons [8,9]. Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution to higher Q^2 generates the perturbative, *extrinsic* sea of $q\bar{q}$ pairs and gluons.

It is worth noting that, on very general basis, individual hadrons in the $|MM'\rangle$ Fock states are colored. The same is

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also true for the $|Mg\rangle$ fluctuations in the second term of Eqs. (1), (2) as far as the gluon is in a color octet state. However, the fluctuation itself is colorless. Notice that, if the two components of a generic fluctuation are in the **8** representation of color SU(3), and since $\mathbf{8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus \cdots}$, then there is a singlet (colorless) representation where the fluctuation can be accommodated.

Coefficients a_i^M; $M=\pi,K$; $i=1,2,3,\ldots$; in Eqs. (1), (2) are constrained by probability conservation: $\Sigma_i|a_i^M|^2=1$.

Since valon distributions in pions and kaons are related by

$$v_\pi(x) \equiv v_{\bar{u}/\pi^-}(x) = v_{d/\pi^-}(x) = v_{u/\pi^+}(x) = v_{\bar{d}/\pi^+}(x), \tag{3}$$

$$v_k(x) \equiv v_{\bar{u}/K^-}(x) = v_{d/K^\circ}(x) = v_{\bar{d}/\bar{K}^\circ}(x) = v_{u/K^+}(x), \tag{4}$$

$$v_{s/k} \equiv v_{s/K^-}(x) = v_{\bar{s}/K^+}(x) = v_{s/\bar{K}^\circ}(x) = v_{\bar{s}/K^\circ}(x) \tag{5}$$

due to isospin invariance, then at the Q_v^2 scale, parton distribution functions can be written as

$$\bar{u}_\pi(x) = d_\pi(x) = |a_0^\pi|^2 v_\pi(x) + |a_1^\pi|^2 P_{\pi g} \otimes v_\pi + |a_2^\pi|^2 P_{KK} \otimes v_k, \tag{6}$$

$$s_\pi(x) = \bar{s}_\pi(x) = |a_2^\pi|^2 P_{KK} \otimes v_{s/k}, \tag{7}$$

$$g_\pi(x) = |a_1^\pi|^2 P_{g\pi}(x) \tag{8}$$

for pions, and

$$\bar{u}_K(x) = |a_0^K|^2 v_k(x) + |a_1^K|^2 P_{Kg} \otimes v_k + |a_2^K|^2 P_{\pi K} \otimes v_\pi, \tag{9}$$

$$s_K(x) = |a_0^K|^2 v_{s/k}(x) + |a_1^K|^2 P_{Kg} \otimes v_{s/k} + |a_2^K|^2 P_{K\pi} \otimes v_{s/k}, \tag{10}$$

$$d_K(x) = |a_2^K|^2 P_{\pi K} \otimes v_\pi, \tag{11}$$

$$\bar{d}_K(x) = |a_2^K|^2 P_{K\pi} \otimes v_k, \tag{12}$$

$$g_K(x) = |a_1^K|^2 P_{gK}(x) \tag{13}$$

for kaons, where

$$P_{MM'} \otimes v_{q/M} \equiv \int_x^1 \frac{dy}{y} P_{MM'}(y) v_{q/M}\left(\frac{x}{y}\right) \tag{14}$$

is the probability density of the nonperturbative contribution to the parton distribution coming from the $|MM'\rangle$ fluctuation [8,10].

The meson probability density $P_{MM'}(x)$ in the $|MM'\rangle$ fluctuation has been calculated in Refs. [8,10]. It is given by

$$P_{MM'}(x) = \int_0^1 \frac{dy}{y} \int_0^1 \frac{dz}{z} F(y,z) R(x,y,z), \tag{15}$$

with

$$F(y,z) = \beta y v_q(y) z q'(z)(1-y-z)^a, \tag{16}$$

$$R(x,y,z) = \alpha \frac{yz}{x^2} \delta\left(1 - \frac{y+z}{x}\right). \tag{17}$$

In Eqs. (16), (17), v_q and q' are the valon and the quark or antiquark distributions which will form the meson M in the $|MM'\rangle$ fluctuation. The q' distribution is generated through the gluon emission from a valon followed by the $q'\bar{q}'$ pair creation. Thus, its probability density is [11]

$$q'(x) = \bar{q}'(x) = N \frac{\alpha_{st}^2(Q_v^2)}{(2\pi)^2} \int_x^1 \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) \int_y^1 \frac{dz}{z} P_{gq}\left(\frac{y}{z}\right) v_q(z), \tag{18}$$

where $P_{qg}(z)$ and $P_{gq}(z)$ are the Altarelli-Parisi splitting functions [12],

$$P_{gq}(z) = \frac{4}{3} \frac{1+(1-z)^2}{z}, \tag{19}$$

$$P_{qg}(z) = \frac{1}{2}[z^2 + (1-z)^2]. \tag{20}$$

The only scale dependence appearing in Eq. (18) arises through $\alpha_{st}(Q^2)$. Since the valon scale is typically of the order of $Q_v^2 \sim 0.64$ GeV^2 [5], then the $q'\bar{q}'$ pair creation can be safely evaluated perturbatively because $\alpha_{st}^2/(2\pi)^2$ is still sufficiently small. The normalization constants α, β, and N in Eqs. (16), (17), and (18) contribute to the global normalization coefficient of the corresponding Fock state fluctuation in the expansions of Eqs. (1), (2).

Momentum conservation also requires

$$P_{MM'}(x) = P_{M'M}(1-x), \tag{21}$$

a condition which relates the in-meson M and M' probability densities. Additionally, hadrons in the $|MM'\rangle$ fluctuation must be correlated in velocity in order to form a bound state. This implies that

$$\frac{\langle x P_{MM'}(x)\rangle}{m_M} = \frac{\langle x P_{M'M}(x)\rangle}{m'_M}, \tag{22}$$

fixing the exponent a in Eq. (16). Notice also that P_{gM} is calculated from the "recombination" of an antiquark with a valon of the same flavor [8]. Then, formally, the P_{gM} corresponds to the would be π^0 distribution in a hypothetical $|M\pi^0\rangle$ fluctuation.

With the π and K PDF given in Eqs. (6)–(8) and (9)–(13), we proceed to fit simultaneously the experimental data on the $\bar{u}_\pi$ and $\bar{u}_K/\bar{u}_\pi$ by the E615 [2] and NA3 [3] collaborations, respectively. To that end we parametrize [5]

TABLE I. Coefficients of the pion and kaon PDF obtained from simultaneous fits to experimental data from the E615 [2] and NA3 [3] experiments. The χ^2 per degree of freedom of the fit is 0.882. Note that coefficients $|a_2^\pi|^2$ and $|a_2^K|^2$ are not independent but fixed by probability conservation.

$\lvert a_0^\pi\rvert^2$	0.689	± 0.041
$\lvert a_1^\pi\rvert^2$	0.310	± 0.130
$\lvert a_2^\pi\rvert^2$	0.001	± 0.136
$\lvert a_0^K\rvert^2$	0.411	± 0.067
$\lvert a_1^K\rvert^2$	0.229	± 0.097
$\lvert a_2^K\rvert^2$	0.360	± 0.118
a_π	0.044	± 0.036
b_π	0.372	± 0.025
a_k	0.917	± 0.898
b_k	0.743	± 0.240

$$v_\pi(x)=\frac{1}{\beta(a_\pi+1,b_\pi+1)}x^{a_\pi}(1-x)^{b_\pi}, \tag{23}$$

$$v_k(x)=\frac{1}{\beta(a_k+1,b_k+1)}x^{a_k}(1-x)^{b_k}, \tag{24}$$

$$v_{s/k}(x)=\frac{1}{\beta(a_k+1,b_k+1)}x^{b_k}(1-x)^{a_k}, \tag{25}$$

where the last two are related to one another by momentum conservation; and construct a χ^2 function as

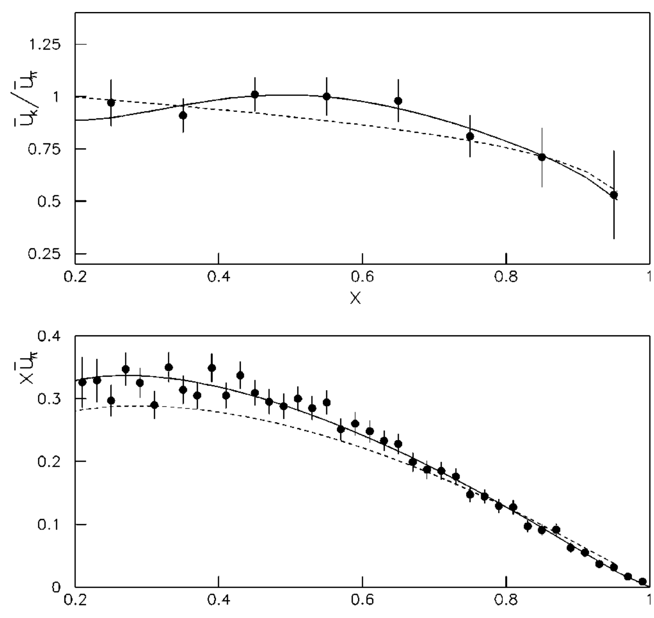

FIG. 1. Fit to experimental data from the E615 [2] and NA3 [3] experiments. Upper: the $\overline{u}_k/\overline{u}_\pi$ ratio as a function of x as measured by NA3. Full line: our fit. Dashed line: $\overline{u}_k/\overline{u}_\pi$ from Ref. [6]. Lower: the $\overline{u}_\pi$ distribution as a function of x as measured by the E615 collaboration. Full line: our fit. Dashed line: $x\overline{u}_\pi(x)$ from Ref. [6].

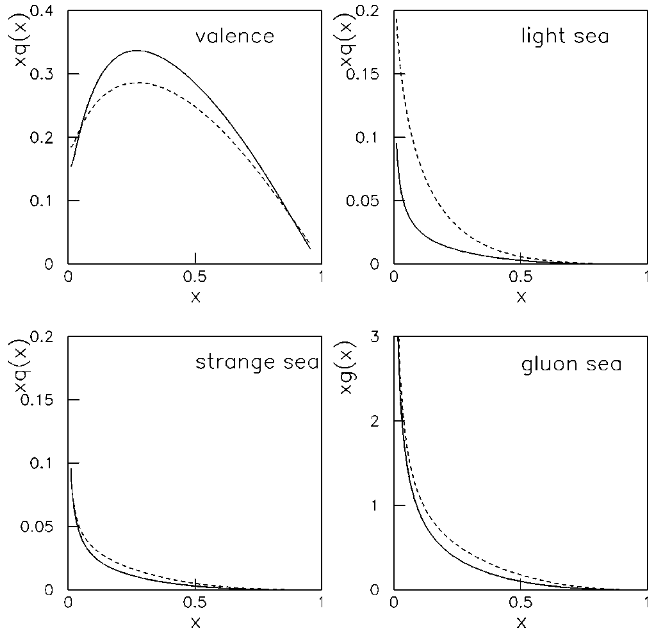

FIG. 2. Parton distribution functions in charged pions at 30 GeV^2. Full line: our pion PDF. Dashed line: pion PDF from Ref. [6]. Notice that our q-$\overline{q}$ sea is flavor SU(3) symmetric (see discussion in the text), while in the Gluck-Reya-Stratmann pion PDF, the flavor SU(3) symmetry is explicitly broken by the introduction of initial sea $u=d=\overline{u}=\overline{d}$ quark distributions. Sea quark and antiquark parton distributions are equal.

$$\begin{aligned}\chi^2&=\chi^2_{E615}+\chi^2_{NA3}\\&=\sum\frac{[y_i-x_i\overline{u}_\pi(x_i,Q^2_{E615})]^2}{\sigma^2_{y_i}}\\&\quad+\sum\frac{\left[z_i-\dfrac{\overline{u}_k}{\overline{u}_\pi}(x_i,Q^2_{NA3})\right]^2}{\sigma^2_{z_i}},\end{aligned} \tag{26}$$

to be minimized. $\beta(a,b)$ in Eqs. (23)–(25) is the beta function, which gives the correct normalization.

We have a total of eight free parameters to be fixed by the fit, namely, $|a_0^{\pi,K}|$, $|a_1^{\pi,K}|$ and the four exponents in the valon distributions of Eqs. (23)–(25). The fit procedure started from an arbitrary set of parameters, for which the low Q^2 π and K PDF were calculated and evolved to the Q^2 scale of the E615 and NA3 experiments, respectively. Then the χ^2 was calculated. This procedure was repeated until a minimun of χ^2 was found.

Results of the fit are shown in Table I and displayed in Fig. 1. As evidenced by the figure, the quality of the fit is quite good.

Once the parameters in the pion and kaon wave functions are known, we build the full set of PDF. They are shown for both, the π^- and the K^-, in Figs. 2 and 3, respectively at $Q^2=30$ GeV^2. To obtain the pion and kaon PDF, we have used the central values of parameters in Table I. Note that, since $|a_2^\pi|^2\sim 0$, then the quark sea of the pion is nearly SU(3)-flavor symmetric [13]. However, in the quark sea of

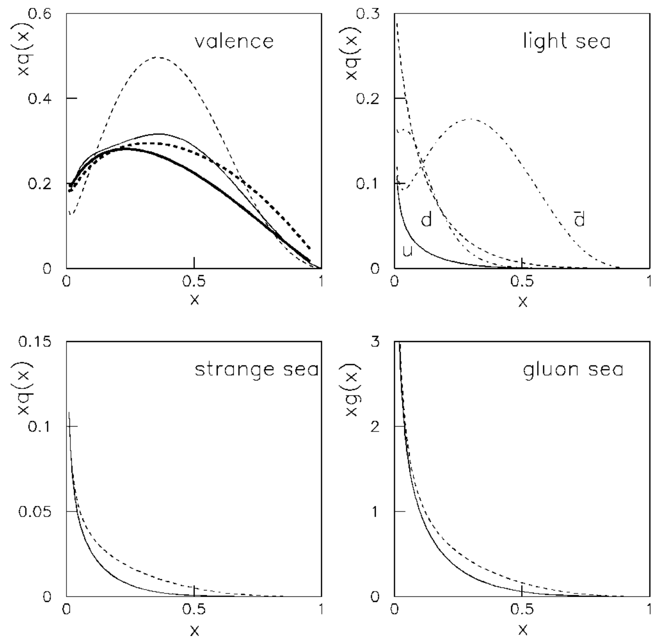

FIG. 3. Parton distribution functions in kaons at 30 GeV^2. Upper left corner: $x\overline{u}$ (full line) and xs (dashed line) parton distributions. Thin lines: our PDF. Fat lines: valence quark distributions from Ref. [6]. Upper right corner: our PDF (full and point dashed lines). Dashed line: sea quark distributions from Ref. [6]. Bottom line curves: Full lines: our PDF. Dashed lines: PDF from Ref. [6].

the kaon the SU(3) symmetry is largely broken due to the big contribution of fluctuations to the intrinsic sea. Another interesting feature of the kaon structure is the d-$\overline{d}$ asymmetry of the K^- sea. This is due to the fact that d is a valon in the π, while $\overline{d}$ is a valon in the K in the $|K\pi\rangle$ fluctuation. Isospin symmetry tell us that $(d-\overline{d})^{K^-}(x)=(\overline{d}-d)^{K^+}(x)=(u-\overline{u})^{\overline{K}^0}(x)=(\overline{u}-u)^{K^0}(x)$. The u and $\overline{s}$ sea are, however, equal in the K^-. Once again, via the isospin symmetry, we get $\overline{u}=s$ in the K^+ sea, $d=\overline{d}$ in the $\overline{K}^0$ sea and $\overline{d}=s$ in the K^0 sea [13].

The PDF of π^+, K^+, K^0, and $\overline{K^0}$ can be obtained just by using isospin symmetry. The π^0 structure, however, deserves separate considerations. In fact, the π^0 wave function at the low Q_v^2 scale can be written as [8]

$$|\pi^0\rangle=b_0|\pi^0\rangle+b_1|\pi^0 g\rangle+b_2|\pi^-\pi^+\rangle + \frac{b_3}{\sqrt{2}}[|K^- K^+\rangle-|K^0\overline{K^0}\rangle]+\cdots. \qquad (27)$$

Thus, contrarily to charged pions, where the intrinsic sea is formed only by gluons and eventually strange quarks, in the π^0 the intrinsic sea is formed also by $u\overline{u}$ and $d\overline{d}$ quarks due to the $|\pi^+\pi^-\rangle$ fluctuation. It should be noted also that, as the $|\pi^0 g\rangle$ and the $|\pi^+\pi^-\rangle$ fluctuations have the same origin, namely the splitting of a gluon to a $u\overline{u}$ or $d\overline{d}$ pair, then it can be assumed that $|b_1|^2\sim|b_2|^2$, thus possibly reducing the intrinsic gluon sea due to probability conservation. This indicates a remarkable difference between the structure of charged and neutral pions within the model.

We want to stress that we have determined the complete structure of charged pions and kaons from a minimal set of measurements of the π^- and K^- PDF just by extracting the three valon distributions v_π, v_k, and $v_{s/k}$ and the parameters $|a_0^{\pi,K}|^2$ and $|a_1^{\pi,K}|^2$ from experimental data. In this sense the model has an interesting predictive power. Note that, as a matter of fact, the experimental information one can get on the kaon structure is only on the light valence quark distribution. The measurement of the strange and even sea quark distributions in kaons is not possible due to practical reasons. Actually, strange and sea quarks only contribute to the total Drell-Yan dilepton cross section through valence-sea and sea-sea $q\overline{q}$ annihilation. Then their contributions are small and cannot be easily separated. A similar difficulty arises to measure the sea parton distributions in pions, where, once again, valence-sea and sea-sea contributions to the Drell-Yan cross section are smaller than the valence-valence ones.

Moreover, the model predicts the structure of pions and kaons at the low Q_v^2 scale, where perturbative QCD evolution starts. This gives a plausible solution to the long standing problem of the origin of the valencelike sea quark and gluon distributions needed, at the low Q^2 input scale for evolution, to describe experimental data on hadron structure.

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