Light-flavor sea-quark distributions in the nucleon in the SU(3) chiral quark soliton model. II. Theoretical formalism

M. Wakamatsu*

Department of Physics, Faculty of Science, Osaka University, Toyonaka, Osaka 560, Japan

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The path integral formulation is given to obtain quark and antiquark distribution functions in the nucleon within the flavor SU(3) version of the chiral quark soliton model. The basic model action is a straightforward generalization of the corresponding SU(2) one, except for one distinguishable feature, i.e., the presence of the SU(3) symmetry breaking term arising from the sizably large mass difference Δm_s between the strange and nonstrange quarks. We treat this SU(3) symmetry breaking effect by relying upon the first-order perturbation theory in the mass parameter Δm_s. We also address the problem of the ordering ambiguity of the relevant collective space operators, which arises in the evaluation of the parton distribution functions at the subleading order of the $1/N_c$ expansion.

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I. INTRODUCTION

In the preceding paper [26], which is referred to as I, we have shown that the flavor SU(3) version of the chiral quark soliton model (CQSM) can give reasonable predictions for the hidden strange-quark distributions in the nucleon, while preserving the success of the SU(2) CQSM. The detailed theoretical formulation of the model was left out, however, in consideration of its quite elaborate nature. The purpose of the present paper is to make up for this point.

The generalization of the CQSM to the case of flavor SU(3) was already done many years ago independently by two groups [1,2]. The basic dynamical assumption of the SU(3) CQSM is very similar to that of the SU(3) Skyrme model [3,4]. It is the embedding of the SU(3) hedgehog mean field into the SU(3) matrix followed by the quantization of the collective rotational motion in the full SU(3) collective coordinate space. The physical octet and decuplet baryons including the nucleon with good spin and flavor quantum numbers are obtained through this quantization process. For the usual low energy observables of baryons such as the magnetic moments or the axial-vector couplings, the theory can be formulated by using the standard cranking procedure which is familiar in the nuclear theory of collective rotation. However, what we want to investigate here is not the usual low energy observables of baryons but the quark and antiquark distributions in the nucleon, which are fully relativistic objects. For obtaining these quantities, we must evaluate nucleon matrix elements of quark bilinear operators containing two space-time coordinates with light-cone separation. The most convenient method for investigating such quantities is the path integral formalism, which was already used in the formulation of the similar observables in the SU(2) version of the CQSM [5–11].

The standard mean-field approximation in the nuclear theory corresponds to the stationary-phase approximation in the path integral formalism [9]. The rotational motion of the symmetry breaking mean-field configuration, which appears as a zero-energy mode, is treated by using the first-order perturbation theory in the collective rotational velocity Ω of the soliton. This is justified since the velocity of this collective rotational motion is expected to be much slower than the velocity of intrinsic quark motion in the hedgehog mean field. According to this theoretical structure of the model, any baryon observables including parton distribution functions (PDF) are given as a sum of the $O(\Omega^0)$ contributions and the $O(\Omega^1)$ one [9,10].

A completely new feature of the SU(3) CQSM, which is not shared by the SU(2) model, is the existence of SU(3) symmetry breaking term due to the appreciable mass difference between the strange and nonstrange quarks. We believe that this mass difference (or the mass of the strange quark itself) of the order 100 MeV is still much smaller than the typical energy scale of hadron physics of the order 1 GeV, and it can be treated by relying upon the perturbation theory.

Now, in the next section, we start to explain the detailed path integral formulation of the SU(3) CQSM for evaluating PDF. After explaining the general theoretical structure of the model, we shall discuss the $O(\Omega^0)$ contributions to the PDF, the $O(\Omega^1)$ contributions, and the first-order corrections in Δm_s in three separate subsections. Finally, in Sec. IV, we briefly summarize our achievement as well as what still remains to be clarified in future studies.

II. FORMULATION OF THE MODEL

We start with the familiar definition of the quark distribution function given as [12]

$$q(x) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dz_0 e^{ixM_N z_0} \times \langle N(\boldsymbol{P}=0) | \psi^\dagger(0) O_a \psi(z) \times | N(\boldsymbol{P}=0) \rangle |_{z_3=-z_0, z_\perp=0}. \qquad (1)$$

*Email address: wakamatu@miho.rcnp.osaka-u.ac.jp

Here O_a is to be taken as

$$O_a = \lambda_a (1+\gamma^0\gamma^3), \tag{2}$$

with $a=0,3$, and 8 for unpolarized distribution functions (note here we take that $\lambda_0 = 1$), while

$$O_a = \lambda_a (1+\gamma^0\gamma^3)\gamma_5, \tag{3}$$

for longitudinally polarized ones. We recall that the above definition of the quark distribution function can formally be extended to the negative x region. The function $q(x)$ with a negative argument should actually be interpreted as giving an antiquark distribution with a physical value of $x(>0)$ according to the rule

$$q(-x) = -\overline{q}(x) \quad (0<x<1), \tag{4}$$

for the unpolarized distributions, and

$$\Delta q(-x) = +\Delta\overline{q}(x) \quad (0<x<1), \tag{5}$$

for the longitudinally polarized distributions. Here, the sign difference between the two types of distributions arises from the different ways of their transformations under charge conjugation.

As was explained in the previous paper, the starting point of our theoretical analysis is the following path integral representation of a matrix element of a bilocal and bilinear quark operator between the nucleon state with definite momentum:

$$\begin{aligned}\langle N(\boldsymbol{P})|\psi^\dagger(0) O_a \psi(z)|N(\boldsymbol{P})\rangle &= \frac{1}{Z}\int d^3x\, d^3y\, e^{-i\boldsymbol{P}\cdot\boldsymbol{x}} e^{i\boldsymbol{P}\cdot\boldsymbol{y}} \int \mathcal{D}U \\ &\times \int \mathcal{D}\psi \mathcal{D}\psi^\dagger J_N\left(\frac{T}{2},\boldsymbol{x}\right) \psi^\dagger(0) O_a \psi(z) J_N^\dagger\left(-\frac{T}{2},\boldsymbol{y}\right) \\ &\times \exp\left[i\int d^4x \mathcal{L}(x)\right],\end{aligned} \tag{6}$$

where

$$\mathcal{L} = \overline{\psi}[i\partial\!\!\!/ - M U^{\gamma_5}(x) - \Delta m_s P_s]\psi, \tag{7}$$

with $U^{\gamma_5}(x) = \exp[i\gamma_5 \lambda_a \pi_a(x)/f_\pi]$ being the basic Lagrangian of the CQSM with three flavors [1,2]. Here, the mass difference Δm_s between the strange quark and nonstrange quarks is introduced with use of the projection operator

$$P_s = \frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

for the s-quark component. The quantity

$$J_N(x) = \frac{1}{N_c!}\epsilon^{\alpha_1\cdots\alpha_{N_c}} \Gamma^{\{f_1\cdots f_{N_c}\}}_{YTT_3;JJ_3} \psi_{\alpha_1 f_1}(x)\cdots\psi_{\alpha_{N_c} f_{N_c}}(x), \tag{9}$$

is a composite operator carrying the quantum numbers YTT_3, JJ_3 (hypercharge, isospin, and spin) of the baryon, where α_i the color index, while $\Gamma^{\{f_1\cdots f_{N_c}\}}_{YTT_3;JJ_3}$ is a symmetric matrix in spin-flavor indices f_i. A basic dynamical assumption of the SU(3) CQSM [which one may notice is similar to that of the SU(3) Skyrme model [3]] is the embedding of the SU(2) self-consistent mean-field solution of hedgehog shape into the SU(3) matrix as

$$U_0^{\gamma_5}(\boldsymbol{x}) = \begin{pmatrix} e^{i\gamma_5 \boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)} & 0 \\ 0 & 1 \end{pmatrix}. \tag{10}$$

That this would give the lowest energy classical configuration can be deduced from a simple variational argument [13]. In fact, an arbitrary small variation of the (3,3) component of $U_0^{\gamma_5}(\boldsymbol{x})$ would induce a change of the strange-quark single-particle spectra in such a way that weak bound states appear from the positive energy Dirac continuum as well as from the negative energy one in a charge-conjugation symmetric way. Since only the negative energy continuum is originally occupied, this necessarily increases the total energy of the baryon-number-one system. Because of energy degeneracy of all the configurations attainable from the above configuration under the spatial rotation or the rotation in the flavor SU(3) internal space, a spontaneous zero-energy rotational mode necessarily arises. We also notice the existence of another important zero mode corresponding to the translational motion of the soliton center. As in the previous paper [5–8], the translational zero mode is treated by using an approximate momentum projection procedure (of the nucleon state), which amounts to integrating over all the shift $\boldsymbol{R}$ of the soliton center-of-mass coordinates,

$$\begin{aligned}&\langle N(\boldsymbol{P})|\psi^\dagger(0) O_a \psi(z)|N(\boldsymbol{P})\rangle \\ &\quad\to \int d^3R \langle N(\boldsymbol{P})|\psi^\dagger(0,-\boldsymbol{R}) O_a \psi(z_0, \boldsymbol{z}-\boldsymbol{R})|N(\boldsymbol{P})\rangle.\end{aligned} \tag{11}$$

On the other hand, the rotational zero modes can be treated by introducing a rotating meson field of the form

$$U^{\gamma_5}(\boldsymbol{x},t) = A(t) U_0^{\gamma_5}(\boldsymbol{x}) A^\dagger(t), \tag{12}$$

where $A(t)$ is a time-dependent SU(3) matrix in flavor space. A key identity in the following manipulation is as follows:

$$\overline{\psi}[i\partial\!\!\!/ - M U^{\gamma_5}(x) - \Delta m_s P_s]\psi = \psi_A^\dagger (i\partial_t - H - \Delta H - \Omega)\psi_A, \tag{13}$$

where

$$\psi_A = A^\dagger(t)\psi, \tag{14}$$

$$H = \frac{\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}}{i} + M\beta U_0^{\gamma_5}(\boldsymbol{x}), \tag{15}$$

$$\Delta H = \Delta m_s \gamma^0 A^\dagger(t)\left(\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8\right)A(t), \tag{16}$$

$$\Omega = -iA^\dagger(t)\dot{A}(t). \tag{17}$$

Here H is a static Dirac Hamiltonian with the background-pion field $U_0^{\gamma_5}(\boldsymbol{x})$, playing the role of mean-field potential for quarks, whereas ΔH is the SU(3) symmetry breaking correction to H. The quantity Ω is the SU(3)-valued angular velocity matrix later to be quantized in an appropriate way. At this stage, it is convenient to introduce a change of quark field variable $\psi \to \psi_A$, which amounts to getting on a body-fixed rotating frame of a soliton. Denoting ψ_A anew ψ for notational simplicity, the nucleon matrix element (8) can then be written as

$$\langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle = \frac{1}{Z}\Gamma^{\{f\}}\Gamma^{\{g\}*}\int d^3x\, d^3y\, e^{-i\boldsymbol{P}\cdot\boldsymbol{x}}e^{i\boldsymbol{P}\cdot\boldsymbol{y}}\int d^3R\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\psi^\dagger \exp\left[i\int d^4x\,\psi^\dagger(i\partial_t - H - \Delta H - \Omega)\psi\right]\prod_{i=1}^{N_c}\left[A\left(\frac{T}{2}\right)\psi_{f_i}\left(\frac{T}{2},\boldsymbol{x}\right)\right]\psi^\dagger(0,-\boldsymbol{R})A^\dagger(0)O_aA(z_0)\psi(z_0,\boldsymbol{z}-\boldsymbol{R})\prod_{j=1}^{N_c}\times\left[\psi^\dagger_{g_j}\left(-\frac{T}{2},\boldsymbol{y}\right)A^\dagger\left(-\frac{T}{2}\right)\right]. \tag{18}$$

Performing the path integral over the quark fields, we obtain

$$\begin{aligned}
&\langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle \\
&= \frac{1}{Z}\tilde{\Gamma}^{\{f\}}\tilde{\Gamma}^{\{g\}\dagger}N_c\int d^3x\, d^3y\, e^{-i\boldsymbol{P}\cdot\boldsymbol{x}}e^{i\boldsymbol{P}\cdot\boldsymbol{y}}\int d^3R\int \mathcal{D}A\left\{ {}_{f_1}\left\langle \frac{T}{2},\boldsymbol{x}\left|\frac{i}{i\partial_t - H - \Delta H - \Omega}\right|0,-\boldsymbol{R}\right\rangle_\gamma [A^\dagger(0)O_aA(z_0)]_{\gamma\delta}\right. \\
&\quad\times {}_\delta\left\langle z_0,\boldsymbol{z}-\boldsymbol{R}\left|\frac{i}{i\partial_t - H - \Delta H - \Omega}\right|-\frac{T}{2},\boldsymbol{y}\right\rangle_{g_1} - \mathrm{Tr}\left(\left\langle z_0,\boldsymbol{z}-\boldsymbol{R}\left|\frac{i}{i\partial_t - H - \Delta H - \Omega}\right|0,-\boldsymbol{R}\right\rangle A^\dagger(0)O_aA(z_0)\right) \\
&\quad\left.\times {}_{f_1}\left\langle \frac{T}{2},\boldsymbol{x}\left|\frac{i}{i\partial_t - H - \Delta H - \Omega}\right|-\frac{T}{2},\boldsymbol{y}\right\rangle_{g_1}\right\}\prod_{j=2}^{N_c}\left[{}_{f_j}\left\langle \frac{T}{2},\boldsymbol{x}\left|\frac{i}{i\partial_t - H - \Delta H - \Omega}\right|-\frac{T}{2},\boldsymbol{y}\right\rangle_{g_j}\right] \\
&\quad\times \exp[N_c \mathrm{Sp}\log(i\partial_t - H - \Delta H\Omega)],
\end{aligned} \tag{19}$$

with $\tilde{\Gamma}^{\{f\}} = \Gamma^{\{f\}}[A(T/2)]^{N_c}$, etc. Here Tr is to be taken over spin-flavor indices. Now the strategy of the following manipulation is in order. As in all the previous works, we assume that the collective rotational velocity of the soliton is much slower than the velocity of internal quark motion, which provides us with a theoretical support to a perturbative treatment in Ω. Since Ω is known to be an $O(1/N_c)$ quantity, this perturbative expansion in Ω can also be taken as a $1/N_c$ expansion. We shall retain terms up to the first order in Ω. We also use the perturbative expansion in Δm_s, which is believed to be a small parameter as compared with the typical energy scale of low energy QCD ($\sim$1 GeV).

Applying this expansion to Eq. (19), we obtain

$$\begin{aligned}
\langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle &= \langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle^{\Omega^0} \\
&\quad + \langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle^{\Omega^1} \\
&\quad + \langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle^{\Delta m_s} + \cdots.
\end{aligned} \tag{20}$$

To be more explicit, they are given by

$$\begin{aligned}
&\langle N(\boldsymbol{P})|\psi^\dagger(0)O_a\psi(z)|N(\boldsymbol{P})\rangle^{\Omega^0} \\
&= \frac{1}{Z}\tilde{\Gamma}^{\{f\}}\tilde{\Gamma}^{\{g\}\dagger}N_c\int d^3x\, d^3y\, e^{-i\boldsymbol{P}\cdot\boldsymbol{x}}e^{i\boldsymbol{P}\cdot\boldsymbol{y}}\int d^3R\int \mathcal{D}A(\tilde{O}_a)_{\gamma\delta}\left[{}_{f_1}\left\langle \frac{T}{2},\boldsymbol{x}\left|\frac{i}{i\partial_t - H}\right|0,-\boldsymbol{R}\right\rangle_\gamma {}_\delta\left\langle z_0,\boldsymbol{z}-\boldsymbol{R}\left|\frac{i}{i\partial_t - H}\right|-\frac{T}{2},\boldsymbol{y}\right\rangle_{g_1}\right. \\
&\quad\left. - {}_\delta\left\langle z_0,\boldsymbol{z}-\boldsymbol{R}\left|\frac{i}{i\partial_t - H}\right|0,-\boldsymbol{R}\right\rangle_\gamma \cdot {}_{f_1}\left\langle \frac{T}{2},\boldsymbol{x}\left|\frac{i}{i\partial_t - H}\right|-\frac{T}{2},\boldsymbol{y}\right\rangle_{g_1}\right]\prod_{j=2}^{N_c}\left[{}_{f_j}\left\langle \frac{T}{2},\boldsymbol{x}\left|\frac{i}{i\partial_t - H}\right|-\frac{T}{2},\boldsymbol{y}\right\rangle_{g_j}\right] \\
&\quad\times \exp\left[N_c \mathrm{Sp}\log(i\partial_t - H) + i\frac{I}{2}\int \Omega_a^2 dt\right],
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\langle N(\boldsymbol{P})|\psi^{\dagger}(0)O_a\psi(z)|N(\boldsymbol{P})\rangle^{\Omega^1}\\
&=\frac{1}{Z}\tilde{\Gamma}^{\{f\}}\tilde{\Gamma}^{\{g\}\dagger}N_c\int d^3x\,d^3y\,e^{-i\boldsymbol{P}\cdot\boldsymbol{x}}e^{i\boldsymbol{P}\cdot\boldsymbol{y}}\int d^3R\int\mathcal{D}\mathcal{A}\Bigg\{\int d^3z'\,dz_0'\,i\Omega_{\alpha\beta}(z_0')[A^{\dagger}(0)O_aA(z_0)]_{\gamma\delta}\\
&\times\Bigg[{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|z_0',\boldsymbol{z}'\Big\rangle_{\alpha}\,{}_{\beta}\Big\langle z_0',\boldsymbol{z}'\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\,{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\\
&+{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\,{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|z_0',\boldsymbol{z}'\Big\rangle_{\alpha}\,{}_{\beta}\Big\langle z_0',\boldsymbol{z}'\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\\
&-{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\,{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|z_0',\boldsymbol{z}'\Big\rangle_{\alpha}\,{}_{\delta}\Big\langle z_0',\boldsymbol{z}'\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\Bigg]\\
&+iz_0\frac{1}{2}\{\Omega,\tilde{O}_a\}_{\gamma\delta}\Bigg[{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\,{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\,{}_{-\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\\
&\times{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\Bigg]\Bigg\}\prod_{j=2}^{N_c}\Bigg[{}_{f_j}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_j}\Bigg]\exp\Bigg[N_c\mathrm{Sp}\log(i\partial_t-H)+i\frac{I}{2}\int\Omega_a^2dt\Bigg],
\end{aligned}\tag{22}$$

and

$$\begin{aligned}
&\langle N(\boldsymbol{P})|\psi^{\dagger}(0)O_a\psi(z)|N(\boldsymbol{P})\rangle^{\Delta m_s}\\
&=\frac{1}{Z}\tilde{\Gamma}^{\{f\}}\tilde{\Gamma}^{\{g\}\dagger}N_c\int d^3x\,d^3y\,e^{-i\boldsymbol{P}\cdot\boldsymbol{x}}e^{i\boldsymbol{P}\cdot\boldsymbol{y}}\int d^3R\int\mathcal{D}\mathcal{A}\Bigg\{\int d^3z'\,dz_0'\,i\Delta H_{\alpha\beta}(z_0')[A^{\dagger}(0)O_aA(z_0)]_{\gamma\delta}\\
&\times\Bigg[{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|z_0',\boldsymbol{z}'\Big\rangle_{\alpha}\,{}_{\beta}\Big\langle z_0',\boldsymbol{z}'\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\,{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\\
&+{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\,{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|z_0',\boldsymbol{z}'\Big\rangle_{\alpha}\,{}_{\beta}\Big\langle z_0',\boldsymbol{z}'\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}-{}_{f_1}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_1}\\
&\times{}_{\delta}\Big\langle z_0,\boldsymbol{z}-\boldsymbol{R}\Big|\frac{i}{i\partial_t-H}\Big|z_0',\boldsymbol{z}'\Big\rangle_{\alpha}\,{}_{\beta}\Big\langle z_0',\boldsymbol{z}'\Big|\frac{i}{i\partial_t-H}\Big|0,-\boldsymbol{R}\Big\rangle_{\gamma}\Bigg]\Bigg\}\prod_{j=2}^{N_c}\Bigg[{}_{f_j}\Big\langle\frac{T}{2},\boldsymbol{x}\Big|\frac{i}{i\partial_t-H}\Big|-\frac{T}{2},\boldsymbol{y}\Big\rangle_{g_j}\Bigg]\\
&\times\exp\Bigg[N_c\mathrm{Sp}\log(i\partial_t-H)+i\frac{I}{2}\int\Omega_a^2dt\Bigg].
\end{aligned}\tag{23}$$

We shall treat these three contributions to the PDF in separate subsections below.

A. $O(\Omega^0)$ contribution to PDF

Although we do not need any essential change for the derivation of the $O(\Omega^0)$ contribution, we recall here some main ingredients, since it is useful for understanding the following manipulation. We first introduce the eigenstates $|m\rangle$ and the associated eigenenergies E_m of the static Dirac Hamiltonian H, satisfying

$$H|m\rangle=E_m|m\rangle. \tag{24}$$

The spectral representation of the single quark Green's function is then given as

$$
{}_{\alpha}\left\langle \boldsymbol{x},t\left|\frac{i}{i\partial_t-H}\right|\boldsymbol{x}',t'\right\rangle_{\beta}
=\theta(t-t')\sum_{m>0} e^{-iE_m(t-t')}{}_{\alpha}\langle \boldsymbol{x}|m\rangle\langle m|\boldsymbol{x}'\rangle_{\beta}
-\theta(t'-t)\sum_{m<0} e^{-iE_m(t-t')}{}_{\alpha}\langle \boldsymbol{x}|m\rangle\langle m|\boldsymbol{x}'\rangle_{\beta}. \tag{25}
$$

Using this equation together with the identity

$$
\langle \boldsymbol{z}-\boldsymbol{R}|=\langle -\boldsymbol{R}|e^{i\boldsymbol{p}\cdot\boldsymbol{z}}, \tag{26}
$$

with $\boldsymbol{p}$ being the momentum operator, we can perform the integration over $\boldsymbol{R}$ in Eq. (19). The resultant expression is then put into Eq. (18) to carry out the integration over z_0. This leads to the following expression for the quark distribution function:

$$
q(x;\Omega^0)=\int \Psi^{(n)*}_{YTT_3;JJ_3}[\xi_A]O^{(0)}[\xi_A]\Psi^{(n)}_{YTT_3;JJ_3}[\xi_A]d\xi_A. \tag{27}
$$

Here $O^{(0)}[\xi_A]$ is an $O(\Omega^0)$ effective operator given by

$$
O^{(0)}[\xi_A]=M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|\tilde{O}_a\delta(xM_N-E_n-p_3)|n\rangle. \tag{28}
$$

Note that it is still a functional of the collective coordinates ξ_A that specify the orientation of the hedgehog soliton in the collective coordinate space. The physical baryons are identified as rotational states of this collective motion and the corresponding wave functions are denoted as $\Psi^{(n)}_{YTT_3;JJ_3}[\xi_A]$, which belongs to a SU(3) representation of dimension n with relevant spin-flavor quantum numbers. Using the standard Wigner rotation matrix (or D function) of SU(3) group, they are represented as

$$
\Psi^{(n)}_{YTT_3;JJ_3}[\xi_a]=(-1)^{J+J_3}\sqrt{n}D^{(n)}_{\mu,\nu}(\xi_a) \tag{29}
$$

with $\mu=(YTT_3)$ and $\nu=(Y'=1,JJ_3)$. In the present study, we are interested in the quark distribution functions in the nucleon, so that we can set $Y=1$ and $T=J=1/2$.

The general formula can now be used to derive some more explicit form of the $O(\Omega^0)$ contribution to the quark distribution functions. We first consider the unpolarized distributions. For the flavor-singlet case, we take

$$
\tilde{O}_{a=0}=A^{\dagger}\lambda_0 A(1+\gamma^0\gamma^3)=1+\gamma^0\gamma^3, \tag{30}
$$

so that we find that

$$
O^{(0)}[\xi_A]=M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle, \tag{31}
$$

with the abbreviation $\delta_n=\delta(xM_N-E_n-p_3)$. This then gives

$$
q^{(0)}(x;\Omega^0)=\langle 1\rangle_p f(x), \tag{32}
$$

with the definition

$$
f(x)=M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle. \tag{33}
$$

Here and hereafter, $\langle O\rangle_B$ should be understood as an abbreviated notation of the matrix element of a collective operator O between a baryon state B (mostly, the spin-up proton state) with appropriate quantum numbers, i.e.,

$$
\langle O\rangle_B\equiv\int \Psi^{(n)*}_{YTT_3;JJ_3}[\xi_A]O[\xi_A]\Psi^{(n)}_{YTT_3;JJ_3}[\xi_A]d\xi_A. \tag{34}
$$

In the flavor-nonsinglet case ($a=3$ or 8),

$$
\tilde{O}_a=A^{\dagger}\lambda_a A(1+\gamma^0\gamma^3)=D_{ab}\lambda_b(1+\gamma^0\gamma^3), \tag{35}
$$

we have

$$
\begin{aligned}
O^{(a)}[\xi_A]&=D_{ab}M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|\lambda_a(1+\gamma^0\gamma^3)\delta_n|n\rangle\\
&=D_{a8}M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|\lambda_8(1+\gamma^0\gamma^3)\delta_n|n\rangle\\
&=\frac{D_{a8}}{\sqrt{3}}M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle.
\end{aligned} \tag{36}
$$

Here, we have used the generalized hedgehog symmetry of the classical configuration (10). This then gives, for $a=3$ or 8,

$$
q^{(a)}(x;\Omega^0)=\left\langle\frac{D_{a8}}{\sqrt{3}}\right\rangle_p f(x). \tag{37}
$$

Turning to the longitudinally polarized distribution, we take

$$
\tilde{O}_a=A^{\dagger}\lambda_0 A(1+\gamma^0\gamma^3)\gamma_5=\gamma_5+\Sigma_3, \tag{38}
$$

for the flavor-singlet case, so that we find

$$
\Delta q^{(0)}(x;\Omega^0)=0. \tag{39}
$$

On the other hand, for the flavor nonsinglet case we obtain

$$
\tilde{O}_a=A^{\dagger}\lambda_a A(1+\gamma^0\gamma^3)\gamma_5=D_{ab}\lambda_b(\gamma_5+\Sigma_3). \tag{40}
$$

This gives

$$
\begin{aligned}
O^{(a)}[\xi_A]&=M_N\frac{N_c}{2}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|D_{ab}\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle\\
&=D_{a3}M_N\frac{N_c}{3}\left(\sum_{n\le 0}-\sum_{n>0}\right)\langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle.
\end{aligned} \tag{41}
$$

We therefore have, for $a=3$ or 8,

$$\Delta q^{(a)}(x;\Omega^0) = \langle -D_{a3} \rangle_{p\uparrow} g(x), \tag{42}$$

with

$$g(x) = -M_N \frac{N_c}{2} \left(\sum_{n \leq 0} - \sum_{n > 0} \right) \langle n | \lambda_3 (\gamma_5 + \Sigma_3) \delta_n | n \rangle. \tag{43}$$

B. $O(\Omega^1)$ contribution to PDF

There is some controversy in the treatment of the $O(\Omega^1)$ term in the CQSM. The dispute began after our finding of the novel $1/N_c$ correction (or more explicitly the first-order rotational correction in the collective angular velocity Ω) to some isovector observables such as the isovector part of the nucleon axial-vector coupling constant $g_A^{(3)}$ or the isovector magnetic moment $\mu_{I=1}$ [14]. We showed that this new $1/N_c$ correction, which is entirely missing in the theoretical framework of the intimately connected effective meson theory, i.e., the Skyrme model, plays just a desirable role in solving the long-standing g_A problem inherent in the soliton model based on the hedgehog configuration [14,15]. According to Schechter and Weigel [16,17], however, this $O(\Omega^1)$ contribution originates from the ordering ambiguity of the collective operators and it breaks the *G*-parity symmetry of strong interactions. We agree that the operator ordering ambiguity is unavoidable when going from a classical theory to a quantum theory. A different choice of ordering would, in general, define a different quantum theory. It was shown, however, that the existence of this new $O(\Omega^1)$ contribution is a natural consequence of a physically reasonable choice of operator ordering that keeps the time order of the relevant operators and that this $O(\Omega^1)$ contribution to $g_A^{(3)}$ is nothing incompatible with any symmetry of strong interactions including the *G*-parity symmetry [18–20]. We also recall the fact that this time-order-keeping quantization procedure is nothing extraordinary in that it gives the same answer as the so-called cranking approach familiar in the nuclear many-body theory [19]. [Alkofer and Weigel also claimed that the new $O(\Omega^1)$ term breaks the celebrated (partial conservation of axial victor current) relation [21]. Here we do not argue on this problem further, since our view is that this problem does not exist within the framework of the SU(2) CQSM, as discussed in Ref. [19].] Summarizing our understanding about this problem up to this point, the ordering ambiguity of the collective operator, in principle, exists, but a physically reasonable time-order-keeping quantization procedure leads to the desired $O(\Omega^1)$ contribution to $g_A^{(3)}$, while causing no problem at least in the flavor SU(2) version of the CQSM. However, Praszałowicz *et al.* noticed an unpleasant feature of the time-order-keeping quantization procedure in the flavor SU(3) version of the CQSM [22]. That is, it leads to nondiagonal elements in the moment of inertia tensor of the soliton, which may destroy the basic theoretical framework of the soliton model. Since there is no such problem in the SU(2) CQSM, the cause of this trouble seems to be attributed to the incompatibility of the time-order-keeping quantization procedure with the basic dynamical assumption of the SU(3) CQSM, i.e., the so-called trivial embedding of the SU(2) soliton configuration followed by the SU(3) symmetric collective quantization. In the absence of satisfactory resolution to this problem, they advocated to use a phenomenologically favorable procedure, which amounts to dropping some theoretically contradictory terms by hand. In the present study, we shall basically follow this procedure. As we shall discuss below, however, the operator ordering problem is even more complicated in our study of quark distribution functions, since we must handle here quark bilinear operators which are nonlocal also in time coordinates.

In our formulation of the $O(\Omega^1)$ contribution to the distribution function, the ordering problem arises when handling the product of operators

$$\Omega_{\alpha\beta}(z_0')[A^\dagger(0) O_a A(z_0)]_{\gamma\delta}, \tag{44}$$

in Eq. (22). In the previous paper, we adopted the ordering

$$\begin{aligned}
&\Omega_{\alpha\beta}(z_0')[A^\dagger(0) O_a A(z_0)]_{\gamma\delta} \\
&\quad \to [\theta(z_0',0,z_0) + \theta(z_0',z_0,0)]\Omega_{\alpha\beta}\tilde{O}_{\gamma\delta} + [\theta(0,z_0,z_0') \\
&\qquad + \theta(z_0,0,z_0')]\tilde{O}_{\gamma\delta}\Omega_{\alpha\beta} + \theta(0,z_0',z_0) \\
&\qquad \times (O_a)_{\gamma'\delta'} A^\dagger_{\gamma\gamma'} \Omega_{\alpha\beta} A_{\delta'\delta} + \theta(z_0,z_0',0) \\
&\qquad \times (O_a)_{\gamma'\delta'} A_{\delta'\delta} \Omega_{\alpha\beta} A^\dagger_{\gamma\gamma'},
\end{aligned} \tag{45}$$

because it is a procedure faithful to the time order of all the relevant collective operators. In consideration of the existence of operator-ordering ambiguity in quantization, we use here a somewhat simpler ordering procedure specified as

$$\begin{aligned}
&\Omega_{\alpha\beta}(z_0')[A^\dagger(0) O_a A(z_0)]_{\gamma\delta} \\
&\quad \to [\theta(z_0',0,z_0) + \theta(z_0',z_0,0)]\Omega_{\alpha\beta}\tilde{O}_{\gamma\delta} + [\theta(0,z_0,z_0') \\
&\qquad + \theta(z_0,0,z_0')]\tilde{O}_{\gamma\delta}\Omega_{\alpha\beta} + \theta(0,z_0',z_0)\tfrac{1}{2}\{\Omega_{\alpha\beta},\tilde{O}_{\gamma\delta}\} \\
&\qquad + \theta(z_0,z_0',0)\tfrac{1}{2}\{\Omega_{\alpha\beta},\tilde{O}_{\gamma\delta}\}.
\end{aligned} \tag{46}$$

The difference between the new and the old quantization procedures turns out to be that $O_{B'}^{(1)}$ term in Eq. (67) of Ref. [8] is absent in the new procedure. The operator-ordering ambiguity occurs also for the quantity $\frac{1}{2}\{\Omega,\tilde{O}_a\}_{\gamma\delta}$ in Eq. (22), which corresponds to the first-order rotational correction arising from the nonlocality (in time) of the operator $A^+(0) O_a A(z_0)$. To explain it, we first recall the quantization rule of the SU(3) collective rotation given as

$$\Omega = \frac{1}{2}\Omega_a \lambda_a, \tag{47}$$

with

$$J_a \equiv -R_a = \begin{cases} I_1 \Omega_a - \frac{2}{\sqrt{3}} \Delta m_s K_1 D_{8a} & (a=1,2,3) \\ I_2 \Omega_a - \frac{2}{\sqrt{3}} \Delta m_s K_2 D_{8a} & (a=4,5,6,7) \\ \sqrt{3}/2 & (a=8). \end{cases} \quad (48)$$

Here R_a is the right rotation generator also familiar in the SU(3) Skyrme model. Note that only $a=1,2,3$ component of $J_a = -R_a$ can be interpreted as the standard angular momentum operators. In the above equations, I_1, I_2 and K_1, K_2 are the components of the moment-of-inertia tensor of the soliton defined by

$$I_{ab} = \frac{N_c}{2} \sum_{m \geqslant 0, n<0} \frac{\langle n|\lambda_a|m\rangle\langle m|\lambda_b|n\rangle}{E_m - E_n}, \quad (49)$$

$$K_{ab} = \frac{N_c}{2} \sum_{m \geqslant 0, n<0} \frac{\langle n|\lambda_a|m\rangle\langle m|\lambda_b \gamma^0|n\rangle}{E_m - E_n}, \quad (50)$$

which reduce to the form

$$I_{ab} = \mathrm{diag}(I_1, I_1, I_1, I_2, I_2, I_2, I_2, 0), \quad (51)$$

$$K_{ab} = \mathrm{diag}(K_1, K_1, K_1, K_2, K_2, K_2, K_2, 0), \quad (52)$$

because of the hedgehog symmetry. Setting $\Delta m_s = 0$, for the moment, to keep the discussion below simpler, we obtain

$$\{\tilde{O}_a, \Omega\} = \frac{1}{2I_1}\{D_{ab}\lambda_b \bar{O}, J_i \lambda_i\} + \frac{1}{2I_2}\{D_{ab}\lambda_b \bar{O}, J_K \lambda_K\}, \quad (53)$$

where the summation over the repeated indices is understood with i running from 1 to 3, and with K from 4 to 7. To keep compliance with the new operator-ordering procedure (46) explained above, we assume the symmetrization of the operator products as

$$D_{ab} J_c \to \tfrac{1}{2}\{D_{ab}, J_c\}, \quad (54)$$

$$J_c D_{ab} \to \tfrac{1}{2}\{D_{ab}, J_c\}, \quad (55)$$

prior to quantization. This amounts to the replacement

$$\{\tilde{O}_a, \Omega\} \to \{\tilde{O}_a, \Omega\}^S, \quad (56)$$

with

$$\{\tilde{O}_a, \Omega\}^S = \frac{1}{2I_1}\{D_{ab}, J_i\}\{\lambda_b, \lambda_i\} + \frac{1}{2I_2}\{D_{ab}, J_K\}\{\lambda_b, \lambda_K\}. \quad (57)$$

Now collecting all the terms, which are first order in Ω, we arrive at the following expression for the $O(\Omega^1)$ effective operator to be sandwiched between the rotational wave functions as in Eq. (27). It is given by

$$O^{(1)}[\xi_A] = O_A^{(1)} + O_B^{(1)} + O_C^{(1)}, \quad (58)$$

where

$$O_A^{(1)} = M_N \frac{N_c}{2} \sum_{m>0, n \leqslant 0} \frac{1}{E_m - E_n} [\langle n|\tilde{O}_a(\delta_n + \delta_m)|m\rangle\langle m|\Omega|n\rangle + \langle n|\Omega|m\rangle\langle m|\tilde{O}_a(\delta_n + \delta_m)|n\rangle], \quad (59)$$

$$O_B^{(1)} = M_N \frac{N_c}{2} \left(\sum_{m \leqslant 0, n \leqslant 0} - \sum_{n>0, m>0} \right) \frac{1}{E_m - E_n} [\langle n|\tilde{O}_a(\delta_n - \delta_m)|m\rangle\langle m|\Omega|n\rangle + \langle n|\Omega|m\rangle\langle m|\tilde{O}_a(\delta_n - \delta_m)|n\rangle], \quad (60)$$

while

$$O_C^{(1)} = \frac{1}{2I_1} J_i \frac{N_c}{2} \frac{d}{dx} \left(\sum_{n \leqslant 0} - \sum_{n>0} \right) \langle n|\lambda_i \bar{O}\delta_n|n\rangle + \frac{1}{2I_2} J_K \frac{N_c}{2} \frac{d}{dx} \left(\sum_{n \leqslant 0} - \sum_{n>0} \right) \langle n|\lambda_K \bar{O}\delta_n|n\rangle \quad (61)$$

for the flavor-singlet case, and

$$O_C^{(1)} = \frac{1}{4I_1}\{D_{ab}, J_i\} \frac{N_c}{2} \frac{d}{dx} \left(\sum_{n \leqslant 0} - \sum_{n>0} \right) \langle n|\{\lambda_b, \lambda_i\}\bar{O}\delta_n|n\rangle + \frac{1}{4I_2}\{D_{ab}, J_K\} \frac{N_c}{2} \frac{d}{dx} \left(\sum_{n \leqslant 0} - \sum_{n>0} \right) \times \langle n|\{\lambda_b, \lambda_K\}\bar{O}\delta_n|n\rangle \quad (62)$$

for the flavor-nonsinglet case. As was done in Ref. [8], it is convenient to treat $O_A^{(1)}$ and $O_B^{(1)}$ in a combined way, i.e., in such a way that it is given as a sum of two parts, respectively, containing symmetric and antisymmetric pieces with respect to the collective space operators D_{ab} and J_c as

$$O_A^{(1)}+O_B^{(1)}=O_{\{A,B\}}^{(1)}+O_{[A,B]}^{(1)}. \tag{63}$$

For obtaining the explicit forms of $O_{\{A,B\}}^{(1)}$ and $O_{[A\cdot B]}^{(1)}$, we will treat the two cases separately. First is the case in which O_a is a flavor-singlet operator as $O_a=\bar{O}$. In this case, we have

$$O_{\{A,B\}}^{(1)}=-M_N\frac{N_c}{4I_1}J_i\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\times[\langle n|\bar{O}\delta_n|m\rangle\langle m|\lambda_i|n\rangle+\langle n|\lambda_i|m\rangle\langle m|\bar{O}\delta_n|n\rangle], \tag{64}$$

$$O_{[A,B]}^{(1)}=0. \tag{65}$$

On the other hand, if O_a is a flavor-nonsinglet operator such as $O_a=\lambda_a\bar{O}$, we find

$$\begin{aligned}O_{\{A,B\}}^{(1)}=&-M_N\frac{N_c}{4I_1}\frac{1}{2}\{D_{ab},J_i\}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\\&\times\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\langle m|\lambda_i|n\rangle+\langle n|\lambda_i|m\rangle\\&\times\langle m|\lambda_b\bar{O}\delta_n|n\rangle]-M_N\frac{N_c}{4I_2}\frac{1}{2}\{D_{ab},J_K\}\\&\times\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\\&\times\langle m|\lambda_K|n\rangle+\langle n|\lambda_K|m\rangle\langle m|\lambda_b\bar{O}\delta_n|n\rangle]\end{aligned} \tag{66}$$

and

$$\begin{aligned}O_{[A,B]}^{(1)}=&-M_N\frac{N_c}{4I_1}\frac{1}{2}[D_{ab},J_i]\left(\sum_{m>0,n\leqslant 0}+\sum_{m\leqslant 0,n>0}\right)\\&\times\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\langle m|\lambda_i|n\rangle-\langle n|\lambda_i|m\rangle\\&\times\langle m|\lambda_b\bar{O}\delta_n|n\rangle]-M_N\frac{N_c}{4I_2}\frac{1}{2}[D_{ab},J_K]\\&\left(\sum_{m>0,n\leqslant 0}-\sum_{m\leqslant 0,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\\&\times\langle m|\lambda_K|n\rangle-\langle n|\lambda_K|m\rangle\langle m|\lambda_b\bar{O}\delta_n|n\rangle].\end{aligned} \tag{67}$$

We point out that these expressions also are not completely free from operator-ordering ambiguities. If we symmetrize the order of two operators $\Omega_{\alpha\beta}$ and $\tilde{O}_{\gamma\delta}$ in the first and the second term of Eq. (67), the antisymmetric term $O_{[A,B]}^{(1)}$ does not appear from the first. A favorable aspect of this symmetrization procedure is that it does not cause an internal inconsistency of the SU(3) CQSM, which was first pointed out by Praszałowicz *et al.* [22] Unfortunately, however, it also eliminates the phenomenologically welcome first-order rotational correction to $g_A^{(3)}$, the sprout of which is contained in the first term of Eq. (67). As repeatedly emphasized, the presence of this novel $1/N_c$ correction itself is nothing incompatible with any symmetry of strong interaction. However it is not a completely satisfactory procedure, we therefore retain only the first term of Eq. (67) and abandon the second one, which precisely corresponds to the symmetry-preserving approach advocated by Praszałowicz *et al.*

Now we consider the concrete case again. For the flavor-singlet unpolarized distribution, we find there exists no $O(\Omega^1)$ contribution, i.e.,

$$q^{(0)}(x,\Omega^1)=0. \tag{68}$$

In the flavor-nonsinglet case, the $O(\Omega^1)$ contribution consists of two terms as

$$O^{(1)}[\xi_A]=O_{\{A,B\}}^{(1)}+O_C^{(1)}. \tag{69}$$

Here

$$\begin{aligned}O_{\{A,B\}}^{(1)}=&\frac{1}{2}\{D_{ab},R_i\}M_N\frac{N_c}{4I_1}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|\lambda_i|n\rangle+\langle n|\lambda_i|m\rangle\\&\times\langle m|\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle]+\frac{1}{2}\{D_{ab},R_K\}M_N\frac{N_c}{4I_2}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle\\&\times\langle m|\lambda_K|n\rangle+\langle n|\lambda_K|m\rangle\langle m|\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle]\\=&\sum_{i=1}^{3}\{D_{ai},R_i\}\frac{M_N}{I_1}\frac{1}{3}\sum_{j=1}^{3}\frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_j|m\rangle\langle m|\lambda_j(1+\gamma^0\gamma^3)\delta_n|n\rangle\\&+\sum_{K=4}^{7}\{D_{aK},R_K\}\frac{M_N}{I_2}\frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_4|m\rangle\langle m|\lambda_4(1+\gamma^0\gamma^3)\delta_n|n\rangle.\end{aligned} \tag{70}$$

In deriving the last equality, we have made use of the generalized hedgehog symmetry of the static soliton configuration. The explicit summation symbol for the repeated indices has been restored here for clarity. For the second contribution to $O^{(1)}[\xi_A]$, we have

$$O_C^{(1)} = -\sum_{i=1}^{3}\{D_{8i},R_i\}\frac{1}{2I_1}\frac{1}{3}\sum_{j=1}^{3}\frac{N_c}{2}\frac{d}{dx}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_j|m\rangle\langle m|\lambda_j(1+\gamma^0\gamma^3)\delta_n|n\rangle$$
$$-\sum_{K=4}^{7}\{D_{8K},R_K\}\frac{1}{2I_2}\frac{N_c}{2}\frac{d}{dx}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_4|m\rangle$$
$$\times\langle m|\lambda_4(1+\gamma^0\gamma^3)\delta_n|n\rangle. \tag{71}$$

Here, we have used the identities

$$\frac{1}{3}\sum_{j=1}^{3}\sum_{m=all,M(n)}\langle n|\lambda_j|m\rangle\langle m|\lambda_j(1+\gamma^0\gamma^3)\delta_n|n\rangle = \sum_{M(n)}\langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle \tag{72}$$

and

$$\sum_{m=all,M(n)}\langle n|\lambda_4|m\rangle\langle m|\lambda_4(1+\gamma^0\gamma^3)\delta_n|n\rangle = \frac{1}{2}\sum_{M(n)}\langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle. \tag{73}$$

Here and hereafter, $\Sigma_{M(n)}$ stands for the summation over the third component of the grand spin of the eigenstate n. The second identity can be proved as follows:

$$\sum_{m=all,M(n)}\langle n|\lambda_4|m\rangle\langle m|\lambda_4(1+\gamma^0\gamma^3)\delta_n|n\rangle$$
$$=\sum_{M(n)}\langle n|\lambda_4^2(1+\gamma^0\gamma^3)\delta_n|n\rangle$$
$$=\sum_{M(n)}\left\langle n\left|\left(\frac{2}{3}-\frac{1}{2\sqrt{3}}\lambda_8+\frac{1}{2}\lambda_3\right)(1+\gamma^0\gamma^3)\delta_n\right|n\right\rangle$$
$$=\sum_{M(n)}\left\langle n\left|\left(\frac{2}{3}-\frac{1}{2\sqrt{3}}\frac{1}{\sqrt{3}}\right)(1+\gamma^0\gamma^3)\delta_n\right|n\right\rangle$$
$$=\frac{1}{2}\sum_{M(n)}\langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle, \tag{74}$$

where the generalized hedgehog symmetry is used again. Now combining $O_{\{A,B\}}^{(1)}$ and $O_C^{(1)}$ terms, the $O(\Omega^1)$ contribution to the flavor-nonsinglet ($a=3$ or 8) unpolarized distribution function can be expressed as

$$q^{(a)}(x;\Omega^1) = \left\langle\sum_{i=1}^{3}\{D_{ai},R_i\}\right\rangle_p k_1(x)$$
$$+\left\langle\sum_{K=4}^{7}\{D_{aK},R_K\}\right\rangle_p k_2(x), \tag{75}$$

with

$$k_1(x) = M_N\frac{1}{2I_1}\frac{N_c}{2}\frac{1}{3}\sum_{j=1}^{3}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)$$
$$\times\langle n|\lambda_j|m\rangle\left\langle m\left|\lambda_j(1+\gamma^0\gamma^3)\left(\frac{\delta_n}{E_m-E_n}-\frac{1}{2}\delta_n'\right)\right|n\right\rangle \tag{76}$$

and

$$k_2(x) = M_N\frac{1}{2I_2}\frac{N_c}{2}\left(\sum_{m+all,n\leqslant 0} - \sum_{m=all,n>0}\right)\langle n|\lambda_4|m\rangle$$
$$\times\left\langle m\left|\lambda_4(1+\gamma^0\gamma^3)\left(\frac{\delta_n}{E_m-E_n}-\frac{1}{2}\delta_n'\right)\right|n\right\rangle. \tag{77}$$

Here we have used the notation $\delta_n \equiv \delta(xM_N - E_n - p_3)$ and $\delta_n' \equiv \delta'(xM_N - E_n - p_3)$. Turning to the longitudinally polarized distributions, the $O(\Omega^1)$ contribution to the flavor-singlet distribution consists of two terms as

$$O^{(1)}[\xi_A] = O_{\{A,B\}}^{(1)} + O_C^{(1)}, \tag{78}$$

where

$$O_{\{A,B\}}^{(1)} = -2J_3M_N\frac{N_c}{4I_1}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)$$
$$\times\frac{1}{E_m-E_n}\langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle \tag{79}$$

and

$$O_C^{(1)} = 2J_3 \frac{d}{dx} \frac{N_c}{8I_1} \left(\sum_{m=all,n\leq 0} - \sum_{m=all,n>0} \right) \langle n|\lambda_3|m\rangle \times \langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle. \quad (80)$$

Combining the two terms, we have

$$\Delta q^{(0)}(x:\Omega^1) = \langle 2J_3\rangle_{p\uparrow} e(x), \quad (81)$$

with

$$e(x) = M_N \frac{N_c}{4I_1} \left(\sum_{m=all,n\leq 0} - \sum_{m=all,n>0} \right) \langle n|\lambda_3|m\rangle \times \left\langle m \left| (\gamma_5+\Sigma_3) \left(\frac{\delta_n}{E_m-E_n} - \frac{1}{2}\delta_n' \right) \right| n \right\rangle. \quad (82)$$

The $O(\Omega^1)$ contribution to the flavor-nonsinglet polarized distribution is a little more complicated. It generally consists of three terms, i.e., $O_{\{A,B\}}^{(1)}, O_{[A,B]}^{(1)}$, and $O_C^{(1)}$. Using the two identities,

$$\sum_{m=all,M(n)} \frac{1}{E_m-E_n} [\langle n|\lambda_b(\gamma_5+\Sigma_3)\delta_n|m\rangle\langle m|\lambda_i|n\rangle + \langle n|\lambda_i|m\rangle\langle m|\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle]$$
$$= \delta_{b8}\delta_{i3} \sum_{m=all,M(n)} \frac{1}{E_m-E_n} 2\langle n|\lambda_3|m\rangle\langle m|\lambda_8(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$= \frac{2}{\sqrt{3}} \delta_{b8}\delta_{i3} \sum_{m=all,M(n)} \frac{1}{E_m-E_n} \langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle \quad (83)$$

and

$$\sum_{m=all,M(n)} \frac{1}{E_m-E_n} [\langle n|\lambda_b(\gamma_5+\Sigma_3)\delta_n|m\rangle\langle m|\lambda_K|n\rangle + \langle n|\lambda_K|m\rangle\langle m|\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle]$$
$$= 4d_{3Kb} \sum_{m=all,M(n)} \frac{1}{E_m-E_n} \langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle, \quad (84)$$

we obtain

$$O_{\{A,B\}}^{(1)} = -\frac{2}{\sqrt{3}} \frac{1}{2}\{D_{a8},J_3\} M_N \frac{N_c}{4I_1} \left(\sum_{m=all,n\leq 0} - \sum_{m=all,n>0} \right) \frac{1}{E_m-E_n} \langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$-4d_{3KK} \frac{1}{2}\{D_{aK},J_K\} M_N \frac{N_c}{4I_2} \left(\sum_{m=all,n\leq 0} - \sum_{m=all,n>0} \right) \frac{1}{E_m-E_n} \langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle. \quad (85)$$

Next, the $O_C^{(1)}$ term is given by

$$O_C^{(1)} = \frac{1}{2}\{D_{ac},J_i\} \frac{N_c}{8I_1} \frac{d}{dx} \left(\sum_{n\leq 0} - \sum_{n>0} \right) \langle n|\{\lambda_c,\lambda_i\}(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$+ \frac{1}{2}\{D_{ac},J_K\} \frac{N_c}{8I_2} \frac{d}{dx} \left(\sum_{n\leq 0} - \sum_{n>0} \right) \langle n|\{\lambda_c,\lambda_K\}(\gamma_5+\Sigma_3)\delta_n|n\rangle, \quad (86)$$

where i runs from 1 to 3, while K runs from 4 to 7. To rewrite this term, we use two identities,

$$\sum_{M(n)} \langle n|\{\lambda_c,\lambda_i\}(\gamma_5+\Sigma_3)\delta_n|n\rangle = \frac{2}{\sqrt{3}} \delta_{c8}\delta_{i3} \sum_{m=all,M(n)} \langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle \quad (87)$$

and

$$\sum_{M(n)} \langle n|\{\lambda_c,\lambda_K\}(\gamma_5+\Sigma_3)\delta_n|n\rangle = 4d_{3cK} \sum_{m=all,M(n)} \langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle, \quad (88)$$

which will be proved in Appendix A. We are then led to

$$O_C^{(1)}=\frac{2}{\sqrt{3}}\frac{1}{2}\{D_{a8},J_3\}\frac{N_c}{8I_1}\frac{d}{dx}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$+4d_{3KK}\frac{1}{2}\{D_{aK}^{(8)},J_K\}\frac{N_c}{8I_2}\frac{d}{dx}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle. \tag{89}$$

Combining the $O^{(1)}_{\{A,B\}}$ and $O^{(1)}_C$ terms, we obtain

$$O^{(1)}_{\{A,B\}}+O^{(1)}_C=\frac{2}{\sqrt{3}}\frac{1}{2}\{D_{a8},J_3\}e(x)+4d_{3KK}\frac{1}{2}\{D_{aK},J_K\}s(x), \tag{90}$$

where $e(x)$ is defined in Eq. (82), while $s(x)$ is defined by

$$s(x)=-M_N\frac{N_c}{4I_2}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\langle n|\lambda_4|m\rangle\left\langle m\left|\lambda_4(\gamma_5+\Sigma_3)\left(\frac{\delta_n}{E_m-E_n}-\frac{1}{2}\delta_n'\right)\right|n\right\rangle. \tag{91}$$

The remaining antisymmetric term, which is already familiar in the SU(2) CQSM, is given by

$$O^{(1)}_{[A,B]}=-D_{a3}h(x), \tag{92}$$

with

$$h(x)=-i\varepsilon_{3ij}M_N\frac{N_c}{8I_1}\left(\sum_{m>0,n\leqslant 0}+\sum_{m\leqslant 0,n>0}\right)$$
$$\times\frac{1}{E_m-E_n}[\langle n|\lambda_j(\gamma_5+\Sigma_3)\delta_n|m\rangle\langle m|\lambda_i|n\rangle$$
$$-\langle n|\lambda_i|m\rangle\langle m|\lambda_j(\gamma_5+\Sigma_3)\delta_n|n\rangle]. \tag{93}$$

The $O(\Omega^1)$ contribution to the flavor-nonsinglet polarized distribution then becomes

$$\Delta q^{(a)}(x:\Omega^1)=\langle -D_{a3}\rangle_{p\uparrow}h(x)$$
$$+\left\langle 4\sum_{K=4}^{7}d_{3KK}\frac{1}{2}\{D_{aK},J_K\}\right\rangle_{p\uparrow}s(x)$$
$$+\frac{2}{\sqrt{3}}\left\langle\frac{1}{2}\{D_{a8},J_3\}\right\rangle_{p\uparrow}e(x). \tag{94}$$

At this stage, it would be convenient to summarize the complete forms of the unpolarized and longitudinally polarized distribution functions up to the first order in Ω. First, for the unpolarized distribution, the flavor-singlet distribution is given by

$$q^{(0)}(x)=\langle 1\rangle_p f(x), \tag{95}$$

whereas the flavor-nonsinglet distributions ($a=3$ or 8) are given as

$$q^{(a)}(x)=\left\langle\frac{D_{a8}}{\sqrt{3}}\right\rangle_p f(x)+\left\langle\sum_{i=1}^{3}\{D_{ai},R_i\}\right\rangle_p k_1(x)$$
$$+\left\langle\sum_{K=4}^{7}\{D_{aK},R_K\}\right\rangle_p k_2(x). \tag{96}$$

Using the proton matrix elements of the relevant collective operators:

$$\langle D_{38}/\sqrt{3}\rangle_p=\frac{1}{30},\quad \langle D_{88}/\sqrt{3}\rangle_p=\frac{\sqrt{3}}{10}, \tag{97}$$

$$\left\langle\sum_{i=1}^{3}\{D_{3i},R_i\}\right\rangle_p=\frac{7}{10},\quad \left\langle\sum_{i=1}^{3}\{D_{8i},R_i\}\right\rangle_p=\frac{\sqrt{3}}{10}, \tag{98}$$

$$\left\langle\sum_{K=4}^{7}\{D_{3K},R_K\}\right\rangle_p=\frac{1}{5},\quad \left\langle\sum_{K=4}^{7}\{D_{8K},R_K\}\right\rangle_p=\frac{3\sqrt{3}}{5}, \tag{99}$$

we finally arrive at

$$q^{(0)}(x)=f(x), \tag{100}$$

$$q^{(3)}(x)=\tfrac{1}{30}f(x)+\tfrac{7}{10}k_1(x)+\tfrac{1}{5}k_2(x), \tag{101}$$

$$\frac{1}{\sqrt{3}}q^{(8)}(x)=\frac{1}{10}f(x)+\frac{1}{10}k_1(x)+\frac{3}{5}k_2(x). \tag{102}$$

These three distribution functions are enough to give the flavor decomposition of the unpolarized distribution functions:

$$u(x)=\frac{1}{3}q^{(0)}(x)+\frac{1}{2}q^{(3)}(x)+\frac{1}{2\sqrt{3}}q^{(8)}(x), \tag{103}$$

$$d(x) = \frac{1}{3} q^{(0)}(x) - \frac{1}{2} q^{(3)}(x) + \frac{1}{2\sqrt{3}} q^{(8)}(x), \tag{104}$$

$$s(x) = \frac{1}{3} q^{(0)}(x) - \frac{1}{\sqrt{3}} q^{(8)}(x). \tag{105}$$

The first moment sum rules for the unpolarized distribution functions are connected with the quark-number conservation laws. The verification of them is therefore an important check of the internal consistency of a theoretical formalism. We first point out that the three basic distribution functions of the model, i.e., $f(x), k_1(x), k_2(x)$, satisfy the sum rules

$$\int_{-1}^{1} f(x) dx = 3, \tag{106}$$

$$\int_{-1}^{1} k_1(x) dx = 1, \tag{107}$$

$$\int_{-1}^{1} k_2(x) dx = 1. \tag{108}$$

Using Eqs. (100)–(105) together with these sum rules, it is an easy task to show that

$$\int_{-1}^{1} q^{(0)}(x) dx = 3, \tag{109}$$

$$\int_{-1}^{1} q^{(3)}(x) dx = 1, \tag{110}$$

$$\int_{-1}^{1} q^{(8)}(x) dx = 1, \tag{111}$$

and

$$\int_{-1}^{1} u(x) dx = \int_{0}^{1} [u(x) - \overline{u}(x)] dx = 2, \tag{112}$$

$$\int_{-1}^{1} d(x) dx = \int_{0}^{1} [d(x) - \overline{d}(x)] dx = 1, \tag{113}$$

$$\int_{-1}^{1} s(x) dx = \int_{0}^{1} [s(x) - \overline{s}(x)] dx = 0, \tag{114}$$

which are just the desired quark-number conservation laws.

Incidentally, the unpolarized distribution functions in the SU(2) CQSM are given in the following form:

$$u(x) = \tfrac{1}{2} q^{(0)}(x) + \tfrac{1}{2} q^{(3)}(x), \tag{115}$$

$$d(x) = \tfrac{1}{2} q^{(0)}(x) - \tfrac{1}{2} q^{(3)}(x), \tag{116}$$

$$s(x) = 0, \tag{117}$$

where

$$q^{(0)}(x) = f(x), \tag{118}$$

$$q^{(3)}(x) = k_1(x), \tag{119}$$

with $f(x)$ and $k_1(x)$ being the same functions as appear in the SU(3) CQSM.

Next, the $O(\Omega^0 + \Omega^1)$ contributions to the longitudinally polarized distribution functions can be summarized as

$$\Delta q^{(0)}(x) = \langle 2 J_3 \rangle_{p\uparrow} e(x), \tag{120}$$

for the flavor-singlet distributions, and

$$\begin{aligned} \Delta q^{(a)}(x) = &\langle -D_{a3} \rangle_{p\uparrow} [g(x) + h(x)] \\ &+ \left\langle 4 \sum_{K=4}^{7} d_{3KK} \frac{1}{2} \{ D_{aK}, J_K \} \right\rangle_{p\uparrow} s(x) \\ &+ \frac{2}{\sqrt{3}} \left\langle \frac{1}{2} \{ D_{a8}, J_3 \} \right\rangle e(x), \end{aligned} \tag{121}$$

for the nonsinglet distributions. Using the matrix elements of the relevant collective space operators between the spin-up proton state,

$$\begin{gathered} \langle -D_{33} \rangle_{p\uparrow} = \frac{7}{30}, \quad \langle -D_{83} \rangle_{p\uparrow} = \frac{\sqrt{3}}{30}, \\ \left\langle 4 \sum_{K=4}^{7} d_{3KK} D_{3K} J_K \right\rangle_{p\uparrow} = \frac{7}{15}, \quad \left\langle 4 \sum_{K=4}^{7} d_{3KK} D_{8K} J_K \right\rangle_{p\uparrow} = \frac{\sqrt{3}}{15}, \\ \langle D_{38} J_3 \rangle_{p\uparrow} = \frac{\sqrt{3}}{60}, \quad \langle D_{88} J_3 \rangle_{p\uparrow} = \frac{\sqrt{3}}{20}, \end{gathered} \tag{122}$$

we obtain

$$\Delta q^{(0)}(x) = e(x), \tag{123}$$

$$\Delta q^{(3)}(x) = \tfrac{1}{30} e(x) + \tfrac{7}{30} [g(x) + h(x)] + \tfrac{7}{15} s(x), \tag{124}$$

$$\frac{1}{\sqrt{3}} \Delta q^{(8)}(x) = \frac{1}{10} e(x) + \frac{1}{30} [g(x) + h(x)] + \frac{1}{15} s(x). \tag{125}$$

In terms of these three functions, the longitudinally polarized distribution functions with each flavor are given by

$$\Delta u(x) = \frac{1}{3} \Delta q^{(0)}(x) + \frac{1}{2} \Delta q^{(3)}(x) + \frac{1}{2\sqrt{3}} \Delta q^{(8)}(x), \tag{126}$$

$$\Delta d(x) = \frac{1}{3}\Delta q^{(0)}(x) - \frac{1}{2}\Delta q^{(3)}(x) + \frac{1}{2\sqrt{3}}\Delta q^{(8)}(x), \tag{127}$$

$$\Delta s(x) = \frac{1}{3}\Delta q^{(0)}(x) - \frac{1}{\sqrt{3}}\Delta q^{(8)}(x). \tag{128}$$

For comparison, we also show the corresponding theoretical formulas obtained within the framework of the SU(2) CQSM:

$$\Delta u(x) = \tfrac{1}{2}\Delta q^{(0)}(x) + \tfrac{1}{2}\Delta q^{(3)}(x), \tag{129}$$

$$\Delta d(x) = \tfrac{1}{2}\Delta q^{(0)}(x) + \tfrac{1}{2}\Delta q^{(3)}(x), \tag{130}$$

$$\Delta s(x) = 0, \tag{131}$$

where

$$\Delta q^{(0)}(x) = e(x), \tag{132}$$

$$\Delta q^{(3)}(x) = \tfrac{1}{3}[g(x) + h(x)]. \tag{133}$$

We recall here that, as a consequence of the new operator-ordering procedure adopted in the present paper, one noteworthy difference with the previous treatment arises, concerning the $O(\Omega^1)$ contribution to the isovector distribution $\Delta q^{(3)}(x)$. Namely, the $[\Delta u(x) - \Delta d(x)]^{(1)}_{B'+C}$ term in Eq. (114) of Ref. [8] is totally absent in the new formulation here. We shall numerically check that the effect of this change on the final predictions for the longitudinally polarized distributions is very small.

Similarly as in the case of the unpolarized distributions, we can write down the first-moment sum rules also for the longitudinally polarized distributions. No exact conservation law follows from these sum rules, however. As a matter of course, this does not mean there is no useful sum rule for the spin-dependent distributions. For example, the celebrated Bjorken sum rule [23,24] for the isovector part of the longitudinally polarized distribution functions has an important phenomenological significance, although it is not a sort of relation which gives an exact conservation laws for some quantum numbers.

C. Δm_s correction to PDF

Our strategy for estimating the SU(3) symmetry breaking effects is to use the first-order perturbation theory in Δm_s, i.e., the mass difference between the s and u,d quarks. There are several such corrections that are all first order in Δm_s. The first comes from Eq. (23) containing the SU(3) symmetry breaking part of the effective Dirac Hamiltonian $\Delta H_{\alpha\beta}$. Following Ref. [25], this SU(3) symmetry breaking correction is hereafter referred to as the "dynamical Δm_s correction." The second correction originates from the term (22), which is first order in Ω, if it is combined with the quantization rule (48) of the SU(3) collective rotation. In fact, one can easily convince that the replacement

$$\Omega_{\alpha\beta} = \frac{1}{2}\Omega_a(\lambda_a)_{\alpha\beta} \to \frac{1}{2}\left(\frac{J_i}{I_1} + \frac{2}{\sqrt{3}}\Delta m_s \frac{K_1}{I_1} D_{8i}\right)(\lambda_i)_{\alpha\beta} + \frac{1}{2}\left(\frac{J_K}{I_2} + \frac{2}{\sqrt{3}}\Delta m_s \frac{K_2}{I_2} D_{8K}\right)(\lambda_K)_{\alpha\beta} \tag{134}$$

brings about terms proportional to the mass difference Δm_s. This SU(3) symmetry breaking correction, which comes from the Δm_s correction to the SU(3) quantization rule, will be called the "kinematical Δm_s correction." The third correction is brought about by the mixing of the SU(3) irreducible representations, describing the baryon states as collective rotational states. Since this mixing occurs also at the first order in Δm_s, we must take it into account. This last SU(3) symmetry breaking correction will be called the "representation-mixing Δm_s correction." In the following, we shall treat these three corrections in order. The answer will be given in the form

$$q(x;\Delta m_s) = \int \Psi^{(n)*}_{YTT_3;JJ_3}[\xi_A] O^{(\Delta m_s)}[\xi_A] \Psi_{YTT_3;JJ_3}[\xi_A] d\xi_A, \tag{135}$$

where the effective collective space operator consists of three parts:

$$O^{(\Delta m_s)}[\xi_A] = O^{(\Delta m_s)}_{dyn} + O^{(\Delta m_s)}_{kin} + O^{(\Delta m_s)}_{rep}. \tag{136}$$

First to evaluate $O^{(\Delta m_s)}_{dyn}$ by using Eq. (23), the ordering

$$\begin{aligned}\Delta H_{\alpha\beta}(z_0')[A^\dagger(0) O_a A(z_0)]_{\gamma\delta} &\to [\theta(z_0',0,z_0) + \theta(z_0',z_0,0)]\Delta H_{\alpha\beta}\tilde{O}_{\gamma\delta} \\ &\quad + [\theta(0,z_0,z_0') + \theta(z_0,0,z_0')]\tilde{O}_{\gamma\delta}\Delta H_{\alpha\beta} \\ &\quad + \theta(0,z_0',z_0)\tfrac{1}{2}\{\Delta H_{\alpha\beta},\tilde{O}_{\gamma\delta}\} \\ &\quad + \theta(z_0,z_0',0)\tfrac{1}{2}\{\Delta H_{\alpha\beta},\tilde{O}_{\gamma\delta}\},\end{aligned} \tag{137}$$

is used in conformity with the rule (46). After carrying out the integration over the variables $\boldsymbol{R},\boldsymbol{z}',z_0'$ and over z_0, we are led to the following answer for the dynamical Δm_s correction:

$$O^{(\Delta m_s)} = -M_N \frac{N_c}{2}\left(\sum_{m=all,n\leq 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m - E_n} \times [\langle n|\Delta H|m\rangle\langle m|\tilde{O}\delta_n|n\rangle + \langle n|\tilde{O}\delta_n|m\rangle\langle m|\Delta H|n\rangle], \tag{138}$$

where $\tilde{O} = \overline{A^\dagger\lambda_a A O}$ and

$$\Delta H = \Delta m_s \gamma^0 A^\dagger\left(\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8\right)A = \Delta m_s\gamma^0\left(\frac{1}{3} - \frac{1}{\sqrt{3}}D_{8c}\lambda_c\right). \tag{139}$$

In deriving the above equation, we have used the fact that the collective operators contained in ΔH and $\widetilde{O}$ commute with each other.

In the case of flavor-singlet unpolarized distribution, the above general formula gives

$$O_{dyn}^{(\Delta m_s)} = -M_N \frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\frac{1}{3}\Delta m_s[\langle n|(1-\sqrt{3}D_{8c}\lambda_c)\gamma^0|m\rangle$$
$$\times\langle m|(1+\gamma^0\gamma^3)\delta_n|n\rangle + \langle n|(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|(1-\sqrt{3}D_{8c}\lambda_c)\gamma^0|n\rangle]$$
$$= -\frac{1}{3}\Delta m_s M_N \frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|(1-\sqrt{3}D_{88}\lambda_8)\gamma^0|\rangle$$
$$\times\langle m|(1+\gamma^0\gamma^3)\delta_n|n\rangle + \langle n|(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|(1-\sqrt{3}D_{88}\lambda_8)\gamma^0|n\rangle]. \qquad (140)$$

Using the generalized hedgehog symmetry, we therefore arrive at

$$q^{(0)}(x:\Delta m_s^{dyn}) = -\tfrac{4}{3}\langle 1-D_{88}\rangle \Delta m_s I_1 \widetilde{k_0}(x), \qquad (141)$$

with

$$\widetilde{k_0}(x) = \frac{1}{I_1}\frac{N_c}{4}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\gamma^0|m\rangle\langle m|(1+\gamma^0\gamma^3)\delta_n|n\rangle. \qquad (142)$$

Next we turn to the flavor-nonsinglet unpolarized distributions ($a=3$ or 8). The general formula (140) gives

$$O_{dyn}^{(\Delta m_s)} = -M_N \frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\frac{1}{3}\Delta m_s[\langle n|(1-\sqrt{3}D_{8c}\lambda_c)\gamma^0|m\rangle\langle m|D_{ab}\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle$$
$$+\langle n|D_{ab}\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|(1-\sqrt{3}D_{8c}\lambda_c)\gamma^0|n\rangle] \qquad (143)$$
$$= -\frac{1}{3}\Delta m_s M_N \frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\{D_{ab}[\langle n|\gamma^0|m\rangle\langle m|\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle + \langle n|\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle$$
$$\times\langle m|\gamma^0|n\rangle] - \sqrt{3}D_{8c}D_{ab}\langle n|\lambda_c\gamma^0|m\rangle\langle m|\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle - \sqrt{3}D_{ab}D_{8c}\langle n|\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|\lambda_c\gamma^0|n\rangle\}. \qquad (144)$$

It is easy to show that the contribution of the first term of Eq. (144), (i.e., the term proportional to D_{ab}), to $q^{(a)}(x,\Delta m_s^{dyn})$ is given by

$$-\frac{4\Delta m_s I_1}{3}\left\langle \frac{D_{a8}}{\sqrt{3}}\right\rangle_{p\uparrow}\widetilde{k_0}(x) \qquad (145)$$

with $\widetilde{k_0}(x)$ given by Eq. (142). The manipulation of the remaining two terms is a little more complicated. First, we notice that, since D_{8c} and D_{ab} commute, we can write

$$D_{8c}D_{ab}\langle n|\lambda_c\gamma^0|m\rangle\langle m|\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle + D_{ab}D_{8c}\langle n|\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|\lambda_c\gamma^0|n\rangle$$
$$= \tfrac{1}{2}\{D_{ab},D_{8c}\}[\langle n|\lambda_b(1+\gamma^0\gamma^3)\delta_n|m\rangle\langle m|\lambda_c\gamma^0|n\rangle + \langle n|\lambda_c\gamma^0|m\rangle\langle m|\lambda_b(1+\gamma^0\gamma^3)\delta_n|n\rangle]. \qquad (146)$$

We now consider the two parts separately. For the parts where the indices b and c run from 1 to 3, the diagonal matrix element of $D_{ab}D_{8c}$ between the spin-up proton state can be expressed as

$$\langle D_{ab}D_{8c}\rangle_{p\uparrow}^{b,c=1,2,3\ \ part} = \langle D_{a3}D_{8c}\rangle_{p\uparrow}\delta_{b,3}\delta_{c,3} + \frac{1}{4}\langle D_{a,1+i2}D_{8,1-i2}\rangle_{p\uparrow}\delta_{b,1-i2}\delta_{c,1+i2} + \frac{1}{4}\langle D_{a,1-i2}D_{8,1+i2}\rangle_{p\uparrow}\delta_{b,1+i2}\delta_{c,1-i2}. \qquad (147)$$

Noting the equalities

$$\langle D_{a,1+i2} D_{8,1-i2} \rangle_{p\uparrow} = \langle D_{a,1-i2} D_{8,1+i2} \rangle_{p\uparrow} = 2 \langle D_{a3} D_{83} \rangle_{p\uparrow}, \tag{148}$$

we can prove that

$$\langle D_{ab} D_{8c} \rangle_{p\uparrow}^{b,c=1,2,3 \;\; part} = \langle D_{a3} D_{83} \rangle_{p\uparrow} (\delta_{b,1}\delta_{c,1} + \delta_{b,2}\delta_{c,2} + \delta_{b,3}\delta_{c,3}). \tag{149}$$

Using a similar relation for the product of operators $D_{8c} D_{ab}$, we then get

$$\langle \{ D_{ab} D_{8c} \} \rangle_{p\uparrow}^{b,c=1,2,3 part} = \frac{1}{3} \left\langle \sum_{i=1}^{3} \{ D_{ai} D_{8i} \} \right\rangle_{p\uparrow} (\delta_{b,1}\delta_{c,1} + \delta_{b,2}\delta_{c,2} + \delta_{b,3}\delta_{c,3}). \tag{150}$$

This relation is then used to derive the equality,

$$\begin{aligned} &\sum_{b,c=1}^{3} \frac{1}{2} \langle \{ D_{ab} D_{8c} \} \rangle_{p\uparrow} \frac{N_c}{2} \left(\sum_{m=all, n\le 0} - \sum_{m=all, n>0} \right) \frac{1}{E_m - E_n} [\langle n | \lambda_b (1+\gamma^0\gamma^3) \delta_n | m \rangle \langle m | \lambda_c \gamma^0 | n \rangle \\ &\quad + \langle n | \lambda_c \gamma^0 | m \rangle \langle m | \lambda_b (1+\gamma^0\gamma^3) \delta_n | n \rangle] \\ &= \left\langle \sum_{i=1}^{3} \{ D_{ai}, D_{8i} \} \right\rangle_{p\uparrow} \frac{1}{3} \sum_{j=1}^{3} \frac{N_c}{2} \left\langle \left(\sum_{m=all, n\le 0} - \sum_{m=all, n>0} \right) \right\rangle \frac{1}{E_m - E_n} \langle n | \lambda_j \gamma^0 | m \rangle \langle m | \lambda_j (1+\gamma^0\gamma^3) \delta_n | n \rangle. \end{aligned} \tag{151}$$

Next, for the parts where b and c run from 4 to 7, we use the identities

$$\begin{aligned} \sum_{m=all, M(n)} \langle n | \lambda_4 \gamma^0 | m \rangle \langle m | \lambda_4 (1+\gamma^0\gamma^3) \delta_n | n \rangle &= \sum_{m=all, M(n)} \langle n | \lambda_5 \gamma^0 | m \rangle \langle m | \lambda_5 (1+\gamma^0\gamma^3) \delta_n | n \rangle \\ &= \sum_{m=all, M(n)} \langle n | \lambda_6 \gamma^0 | m \rangle \langle m | \lambda_6 (1+\gamma^0\gamma^3) \delta_n | n \rangle \\ &= \sum_{m=all, M(n)} \langle n | \lambda_7 \gamma^0 | m \rangle \langle m | \lambda_7 (1+\gamma^0\gamma^3) \delta_n | n \rangle. \end{aligned} \tag{152}$$

Using these relations, we find that

$$\begin{aligned} &\sum_{b,c=4}^{7} \frac{1}{2} \langle \{ D_{ab}, D_{8c} \} \rangle_{p\uparrow} \frac{N_c}{2} \left(\sum_{m=all, n\le 0} - \sum_{m=all, n>0} \right) \frac{1}{E_m - E_n} \\ &\times [\langle n | \lambda_b (1+\gamma^0\gamma^3) \delta_n | m \rangle \langle m | \lambda_c \gamma^0 | n \rangle + \langle n | \lambda_c \gamma^0 | m \rangle \langle m | \lambda_b (1+\gamma^0\gamma^3) \delta_n | n \rangle] \\ &= \left\langle \sum_{K=4}^{7} \{ D_{aK}, D_{8K} \} \right\rangle_{p\uparrow} \frac{N_c}{2} \left(\sum_{m=all, n\le 0} - \sum_{m=all, n>0} \right) \frac{1}{E_m - E_n} \langle n | \lambda_4 \gamma^0 | m \rangle \langle m | \lambda_4 (1+\gamma^0\gamma^3) \delta_n | n \rangle. \end{aligned} \tag{153}$$

Now combining the above three contributions, the dynamical Δm_s corrections to the flavor-nonsinglet unpolarized distributions can be written in the form

$$q^{(a)}(x;\Delta m_s^{dyn}) = -\frac{4 \Delta m_s I_1}{3} \left\langle \frac{D_{a8}}{\sqrt{3}} \right\rangle_{p\uparrow} \tilde{k}_0(x) + \frac{2 \Delta m_s I_1}{\sqrt{3}} \left\langle \sum_{i=1}^{3} \{ D_{ai}, D_{8i} \} \right\rangle_{p\uparrow} \tilde{k}_1(x) + \frac{2 \Delta m_s I_2}{\sqrt{3}} \left\langle \sum_{K=4}^{7} \{ D_{aK}, D_{8K} \} \right\rangle_{p\uparrow} \tilde{k}_2(x), \tag{154}$$

where $\tilde{k}_0(x)$ is already defined in Eq. (142), while

$$\tilde{k}_1(x) = M_N \frac{N_c}{4 I_1} \frac{1}{3} \sum_{j=1}^{3} \left(\sum_{m=all, n\le 0} - \sum_{m=all, n>0} \right) \times \frac{1}{E_m - E_n} \langle n | \lambda_j \gamma^0 | m \rangle \langle m | \lambda_j (1+\gamma^0\gamma^3) \delta_n | n \rangle, \tag{155}$$

$$\tilde{k}_2(x) = M_N \frac{N_c}{4 I_2} \left(\sum_{m=all, n\le 0} - \sum_{m=all, n>0} \right) \times \frac{1}{E_m - E_n} \langle n | \lambda_4 \gamma^0 | m \rangle \langle m | \lambda_4 (1+\gamma^0\gamma^3) \delta_n | n \rangle. \tag{156}$$

Next, we consider the longitudinally polarized distributions. The flavor-singlet part is easily obtained in the form

$$\Delta q^{(0)}(x;\Delta m_s^{dyn}) = -\frac{4\Delta m_s I_1}{\sqrt{3}}\langle D_{83}\rangle_{p\uparrow}\tilde{e}(x), \tag{157}$$

with

$$\tilde{e}(x) = -M_N\frac{N_c}{4I_1}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_3\gamma^0|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle. \tag{158}$$

The flavor-nonsinglet part is again slightly more complicated. From the general formula (144), we get

$$\begin{aligned} O_{dyn}^{(\Delta m_s)} &= -M_N\frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\frac{1}{3}\Delta m_s \\ &\quad\times[\langle n|(1-\sqrt{3}D_{8c}\lambda_c)\gamma^0|m\rangle\langle m|D_{ab}\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle + \langle n|D_{ab}\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle\langle m|(1-\sqrt{3}D_{8c}\lambda_c)\gamma^0|n\rangle] \\ &= -\frac{1}{3}\Delta m_s M_N\frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\times\{D_{ab}[\langle n|\gamma^0|m\rangle\langle m|\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle \\ &\quad+\langle n|\lambda_b(\gamma_5+\Sigma_3)\delta_n|m\rangle\langle m|\gamma^0|n\rangle] - \sqrt{3}D_{8c}D_{ab}\langle n|\lambda_c\gamma^0|m\rangle\langle m|\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle \\ &\quad-\sqrt{3}D_{ab}D_{8c}\langle n|\lambda_b(\gamma_5+\Sigma_3)\delta_n|m\rangle\langle m|\lambda_c\gamma^0|n\rangle\}. \end{aligned} \tag{159}$$

Similarly as before, the contribution of the first term (proportional to D_{ab}) to $\Delta q^{(0)}(x;\Delta m_s^{dyn})$ is found to be

$$\frac{4\Delta m_s I_1}{3}\langle D_{a3}\rangle_{p\uparrow}\tilde{e}(x), \tag{160}$$

with $\tilde{e}(x)$ given by Eq. (158). On the other hand, the remaining two terms can be rewritten in the form

$$\begin{aligned} &\frac{1}{\sqrt{3}}\Delta m_s M_N\frac{N_c}{2}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n} \\ &\quad\times\frac{1}{2}\{D_{ab},D_{8c}\}[\langle n|\lambda_b(\gamma_5+\Sigma_3)\delta_n|m\rangle\langle m|\lambda_c\gamma^0|n\rangle + \langle n|\lambda_c\gamma^0|m\rangle\langle m|\lambda_b(\gamma_5+\Sigma_3)\delta_n|n\rangle]. \end{aligned} \tag{161}$$

First by confining to the terms in which either or both of b and c run from 1 to 3, there are only two possibilities to survive, i.e., $b=8,c=3$ or $b=3,c=8$. The contributions of these terms to $q^{(a)}(x;\Delta m_s^{dyn})$ are found to be

$$\begin{aligned} &\frac{4\Delta m_s I_1}{3}\langle D_{a3}D_{88}\rangle_{p\uparrow}M_N\frac{N_c}{4I_1}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\gamma^0|m\rangle\langle m|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle \\ &\quad+\frac{4\Delta m_s I_1}{3}\langle D_{a8}D_{83}\rangle_{p\uparrow}M_N\frac{N_c}{4I_1}\left(\sum_{m=all,n\leqslant 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_3\gamma^0|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle. \end{aligned} \tag{162}$$

In order to evaluate the remaining contributions in which b and c run from 4 to 7, we use the identity

$$\sum_{m=all,M(n)} \langle n|\lambda_4\gamma^0|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle = \sum_{m=all,M(n)} \langle n|\lambda_5\gamma^0|m\rangle\langle m|\lambda_5(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$= -\sum_{m=all,M(n)} \langle n|\lambda_6\gamma^0|m\rangle\langle m|\lambda_6(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$= -\sum_{m=all,M(n)} \langle n|\lambda_7\gamma^0|m\rangle\langle m|\lambda_7(\gamma_5+\Sigma_3)\delta_n|n\rangle, \quad (163)$$

together with the familiar relation

$$d_{344}=d_{355}=-d_{366}=-d_{377}=\tfrac{1}{2}. \quad (164)$$

This enables us to express the corresponding contribution to $q^{(a)}(x;\Delta m_s^{dyn})$ in the following form:

$$\frac{2\Delta m_s I_2}{\sqrt{3}}\left\langle 4\sum_{K=4}^{7} d_{3KK}D_{aK}D_{8K}\right\rangle_{p\uparrow} M_N\frac{N_c}{4I_2}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_4\gamma^0|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle. \quad (165)$$

Now, by collecting the various terms explained above, the dynamical Δm_s correction to the flavor-nonsinglet longitudinally polarized distribution functions can be expressed as

$$\Delta q^{(a)}(x;\Delta m_s^{dyn}) = \frac{4\Delta m_s I_1}{3}\langle D_{a3}(1-D_{88})\rangle_{p\uparrow}\tilde{f}(x) - \frac{4\Delta m_s I_1}{3}\langle D_{a8}D_{83}\rangle_{p\uparrow}\tilde{e}(x) - \frac{2\Delta m_s I_2}{\sqrt{3}}\left\langle 4\sum_{K=4}^{7} d_{3KK}D_{aK}D_{8K}\right\rangle_{p\uparrow}\tilde{s}(x), \quad (166)$$

where $\tilde{e}(x)$ is defined in Eq. (158), while $\tilde{f}(x)$ and $\tilde{s}(x)$ are given by

$$\tilde{f}(x) = -M_N\frac{N_c}{4I_1}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\gamma^0|m\rangle\langle m|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle,$$

$$\tilde{s}(x) = -M_N\frac{N_c}{4I_2}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_4\gamma^0|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle. \quad (167)$$

Next we turn to the kinematical Δm_s correction, which originates from the first-order correction with respect to Δm_s in the collective quantization rule (48). Putting this rule into the operator Ω contained in Eq. (22), we are led to a simple rule for obtaining the kinematical Δm_s correction to $O_{kin}^{(\Delta m_s)}$, i.e.,

$$\frac{J_i}{2I_1} \to \frac{1}{2}\frac{2}{\sqrt{3}}\Delta m_s\frac{K_1}{I_1}D_{8i}, \quad (168)$$

$$\frac{J_K}{2I_2} \to \frac{1}{2}\frac{2}{\sqrt{3}}\Delta m_s\frac{K_2}{I_2}D_{8K}. \quad (169)$$

Taking care of the fact that the collective operator contained in $\tilde{O}$ commutes with D_{8i} as well as D_{8K}, we obtain

$$O_{kin}^{(\Delta m_s)} = -D_{8i}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_1}{I_1} M_N \frac{N_c}{4}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\bar{O}\delta_n|m\rangle\langle m|\lambda_i|n\rangle + \langle n|\lambda_i|m\rangle\langle m|\bar{O}\delta_n|n\rangle],$$
$$+D_{8i}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_1}{I_1}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\langle n|\lambda_i \bar{O}\delta_n|n\rangle, \tag{170}$$

for the flavor-singlet distributions in which $\tilde{O}_a = A^\dagger \lambda_0 A \bar{O} = \bar{O}$. On the other hand, the flavor-nonsinglet part becomes

$$O_{kin}^{(\Delta m_s)} = -D_{ab}D_{8i}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_1}{I_1} M_N \frac{N_c}{4}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\langle m|\lambda_i|n\rangle + \langle n|\lambda_i|m\rangle\langle m|\lambda_b\bar{O}\delta_n|n\rangle]$$
$$-D_{ab}D_{8K}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_2}{I_2} M_N \frac{N_c}{4}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\langle m|\lambda_K|n\rangle + \langle n|\lambda_K|m\rangle\langle m|\lambda_b\bar{O}\delta_n|n\rangle]$$
$$+D_{ab}D_{8i}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_1}{I_1}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\left\langle n\left|\frac{1}{2}\{\lambda_b\bar{O},\lambda_i\}\right|n\right\rangle$$
$$+D_{ab}D_{8K}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_2}{I_2}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\left\langle n\left|\frac{1}{2}\{\lambda_b\bar{O},\lambda_K\}\right|n\right\rangle. \tag{171}$$

Here, the last two terms of the above equation are rewritten by using the relations,

$$\sum_{M(n)}\langle n|\{\lambda_b,\lambda_i\}\bar{O}\delta_n|n\rangle = 2\delta_{bi}\sum_{M(n)}\langle n|\bar{O}\delta_n|n\rangle + \frac{2}{\sqrt{3}}\delta_{b8}\delta_{i3}\sum_{M(n)}\langle n|\lambda_3\bar{O}\delta_n|n\rangle \tag{172}$$

and

$$\sum_{M(n)}\langle n|\{\lambda_b,\lambda_K\}\bar{O}\delta_n|n\rangle = \delta_{bK}\sum_{M(n)}\langle n|\bar{O}\delta_n|n\rangle + 2\delta_{bK}d_{3KK}\sum_{M(n)}\langle n|\lambda_3\bar{O}\delta_n|n\rangle, \tag{173}$$

which will be proved in Appendix B. We thus get for the flavor-nonsinglet case,

$$O_{kin}^{(\Delta m_s)} = -D_{ab}D_{8i}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_1}{I_1} M_N \frac{N_c}{4}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\langle m|\lambda_i|n\rangle + \langle n|\lambda_i|m\rangle\langle m|\lambda_b\bar{O}\delta_n|n\rangle]$$
$$-D_{ab}D_{8K}\frac{2}{\sqrt{3}}\Delta m_s \frac{K_2}{I_2} M_N \frac{N_c}{4}\left(\sum_{m=all,n\le 0} - \sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\times[\langle n|\lambda_b\bar{O}\delta_n|m\rangle\langle m|\lambda_K|n\rangle + \langle n|\lambda_K|m\rangle$$
$$\times\langle m|\lambda_b\bar{O}\delta_n|n\rangle] + \frac{2\Delta m_s}{\sqrt{3}}\frac{K_1}{I_1}D_{ai}D_{8i}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\langle n|\bar{O}\delta_n|n\rangle$$
$$+\frac{\Delta m_s}{\sqrt{3}}\frac{K_2}{I_2}D_{aK}D_{8K}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\langle n|\bar{O}\delta_n|n\rangle + \frac{2\Delta m_s}{3}\frac{K_1}{I_1}D_{a8}D_{83}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\langle n|\lambda_3\bar{O}\delta_n|n\rangle$$
$$+\frac{2\Delta m_s}{\sqrt{3}}\frac{K_2}{I_2}\sum_{K=4}^{7} d_{3KK}D_{aK}D_{8K}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\le 0} - \sum_{n>0}\right)\langle n|\lambda_3\bar{O}\delta_n|n\rangle. \tag{174}$$

Let us first consider the unpolarized case. From the general formula (170), it is easy to see that the kinematical Δm_s correction to the flavor-singlet unpolarized distribution identically vanishes, i.e.,

$$q^{(0)}(x;\Delta m_s^{kin})=0. \tag{175}$$

On the other hand, by using the identities

$$\sum_{M(n)} \langle n|(1+\gamma^0\gamma^3)\delta_n|n\rangle = \frac{1}{3}\sum_{j=1}^{3}\sum_{m=all,M(n)} \langle n|\lambda_j|m\rangle\langle m|\lambda_j(1+\gamma^0\gamma^3)\delta_n|n\rangle$$

$$=2\sum_{m=all,M(n)} \langle n|\lambda_4|m\rangle\langle m|\lambda_4(1+\gamma^0\gamma^3)\delta_n|n\rangle, \tag{176}$$

the kinematical Δm_s correction to the flavor-nonsinglet unpolarized distribution can be expressed in the form

$$q^{(a)}(x;\Delta m_s^{kin})=-\frac{2\Delta m_s I_1}{\sqrt{3}}\frac{K_1}{I_1}\left\langle \sum_{i=1}^{3}\{D_{ai},D_{8i}\}\right\rangle_p k_1(x)-\frac{2\Delta m_s I_2}{\sqrt{3}}\frac{K_2}{I_2}\left\langle \sum_{i=4}^{7}\{D_{aK},D_{8K}\}\right\rangle_p k_2(x). \tag{177}$$

Here $k_1(x)$ and $k_2(x)$ are the same functions as appeared in Eq. (75).

The kinematical Δm_s correction to the flavor-singlet longitudinally polarized distribution can similarly be evaluated as

$$\Delta q^{(0)}(x;\Delta m_s^{kin})=-\langle D_{83}\rangle_{p\uparrow}\frac{4\Delta m_s}{\sqrt{3}}\frac{K_1}{I_1}M_N\frac{N_c}{4}\left(\sum_{m=all,\leqslant 0}-\sum_{m=all,n>0}\right)$$
$$\times\frac{1}{E_m-E_n}\langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle+\langle D_{83}\rangle_{p\uparrow}\frac{4\Delta m_s}{\sqrt{3}}\frac{K_1}{I_1}\frac{N_c}{8}\frac{d}{dx}\left(\sum_{m=all,\leqslant 0}-\sum_{m=all,n>0}\right)\langle n|\lambda_3|m\rangle$$
$$\times\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle=\frac{4\Delta m_s I_1}{\sqrt{3}}\frac{K_1}{I_1}\langle D_{83}\rangle_{p\uparrow}e(x), \tag{178}$$

with $e(x)$ defined before in Eq. (82). For the flavor nonsinglet piece, we obtain

$$q^{(a)}(x;\Delta_s^{kin})=-\frac{4\Delta m_s}{3}\frac{K_1}{I_1}\langle D_{a8}D_{83}\rangle_{p\uparrow}M_N\frac{N_c}{4}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)$$
$$\times\frac{1}{E_m-E_n}\langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$-\frac{2\Delta m_s}{\sqrt{3}}\frac{K_2}{I_2}\left\langle 4\sum_{K=4}^{7}d_{3KK}D_{aK}D_{8K}\right\rangle_{p\uparrow}M_N\frac{N_c}{4}\left(\sum_{m=all,n\leqslant 0}-\sum_{m=all,n>0}\right)\frac{1}{E_m-E_n}\langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$+\frac{2\Delta m_s}{3}\frac{K_1}{I_1}\langle D_{a8}D_{83}\rangle_{p\uparrow}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\leqslant 0}-\sum_{n>0}\right)\langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$+\frac{2\Delta m_s}{\sqrt{3}}\frac{K_2}{I_2}\left\langle \sum_{K=4}^{7}d_{3KK}D_{aK}D_{8K}\right\rangle_{p\uparrow}\frac{N_c}{4}\frac{d}{dx}\left(\sum_{n\leqslant 0}-\sum_{n>0}\right)\langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle. \tag{179}$$

To rewrite the last two terms, we use the identities

$$\sum_{M(n)}\langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle=\sum_{m=all,M(n)}\langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle$$
$$=2\sum_{m=all,M(n)}\langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle. \tag{180}$$

This enables us to express $q^{(a)}(x;\Delta m_s^{kin})$ in the form

$$q^{(a)}(x;\Delta m_s^{kin}) = \frac{4\Delta m_s I_1}{3}\frac{K_1}{I_1}\langle D_{a8}D_{83}\rangle_{p\uparrow} e(x) + \frac{2\Delta m_s I_2}{\sqrt{3}}\frac{K_2}{I_2}\left\langle 4\sum_{K=4}^{7} d_{3KK}D_{aK}D_{8K}\right\rangle_{p\uparrow} s(x), \tag{181}$$

with $e(x)$ and $s(x)$ being the functions, respectively, defined in Eqs. (82) and (91).

It is now convenient to express the dynamical and kinematical Δm_s corrections in a combined form. For the unpolarized distributions, this gives

$$q^{(0)}(x;\Delta m_s^{dyn+kin}) = -\frac{4\Delta m_s I_1}{3}\langle 1-D_{88}\rangle_p \tilde{k}_0(x), \tag{182}$$

$$\begin{aligned} q^{(a)}(x;\Delta m_s^{dyn+kin}) &= -\frac{4\Delta m_s I_1}{3}\left\langle \frac{D_{a8}}{\sqrt{3}}\right\rangle_p \tilde{k}_0(x) + \frac{2\Delta m_s I_1}{\sqrt{3}} \\ &\times \left\langle \sum_{i=1}^{3}\{D_{ai},D_{8i}\}\right\rangle_p \left[\tilde{k}_1(x) - \frac{K_1}{I_1}k_1(x)\right] + \frac{2\Delta m_s I_2}{\sqrt{3}} \\ &\times \left\langle \sum_{i=4}^{7}\{D_{aK},D_{8K}\}\right\rangle_p \left[\tilde{k}_2(x) - \frac{K_2}{I_2}k_2(x)\right], \end{aligned} \tag{183}$$

while, for the longitudinally polarized distributions, we have

$$\Delta q^{(0)}(x;\Delta m_s^{dyn+kin}) = -\frac{4\Delta m_s I_1}{\sqrt{3}}\langle D_{83}\rangle_{p\uparrow}\left[\tilde{e}(x) - \frac{K_1}{I_1}e(x)\right], \tag{184}$$

$$\begin{aligned} \Delta q^{(a)}(x;\Delta m_s^{dyn+kin}) &= \frac{4\Delta m_s I_1}{3}\langle D_{83}(1-D_{88})\rangle_{p\uparrow}\tilde{f}(x) \\ &- \frac{4\Delta m_s I_1}{3}\langle D_{a8}D_{83}\rangle_{p\uparrow}\left[\tilde{e}(x) - \frac{K_1}{I_1}e(x)\right] \\ &- \frac{2\Delta m_s I_2}{\sqrt{3}}\left\langle 4\sum_{K=4}^{7} d_{3KK}D_{aK}D_{8K}\right\rangle_{p\uparrow} \\ &\times \left[\tilde{s}(x) - \frac{K_2}{I_2}s(x)\right]. \end{aligned} \tag{185}$$

We summarize below the necessary matrix elements of collective operators. For the unpolarized case, we need

$$\left\langle \frac{D_{38}}{\sqrt{3}}\right\rangle_p = \frac{1}{30}, \quad \left\langle \frac{D_{88}}{\sqrt{3}}\right\rangle_p = \frac{\sqrt{3}}{10}, \tag{186}$$

$$\left\langle \sum_{i=1}^{3}\{D_{3i},D_{8i}\}\right\rangle_p = \frac{2\sqrt{3}}{45}, \quad \left\langle \sum_{i=1}^{3}\{D_{8i},D_{8i}\}\right\rangle_p = \frac{2}{5}, \tag{187}$$

$$\left\langle \sum_{i=4}^{7}\{D_{3K},D_{8K}\}\right\rangle_p = -\frac{2\sqrt{3}}{45}, \quad \left\langle \sum_{i=4}^{7}\{D_{8K},D_{8K}\}\right\rangle_p = \frac{6}{5}, \tag{188}$$

while, for the longitudinally polarized case,

$$\langle D_{33}\rangle_{p\uparrow} = -\frac{7}{30}, \quad \langle D_{83}\rangle_{p\uparrow} = -\frac{\sqrt{3}}{30}, \tag{189}$$

$$\langle D_{33}(1-D_{88})\rangle_{p\uparrow} = -\frac{13}{90}, \quad \langle D_{83}(1-D_{88})\rangle_{p\uparrow} = -\frac{\sqrt{3}}{30}, \tag{190}$$

$$\langle D_{38}D_{83}\rangle_{p\uparrow} = -\frac{1}{45}, \quad \langle D_{88}D_{83}\rangle_{p\uparrow} = 0, \tag{191}$$

$$\left\langle 4\sum_{K=4}^{7} d_{3KK}D_{3K}D_{8K}\right\rangle = -\frac{22\sqrt{3}}{135},$$

$$\left\langle 4\sum_{K=4}^{7} d_{3KK}D_{8K}D_{8K}\right\rangle = -\frac{2}{15}. \tag{192}$$

Because the firstmoment sum rules for the unpolarized distributions are connected with the quark-number conservation laws and since they are shown to be satisfied at the leading $O(\Omega^0+\Omega^1)$ contributions to the distribution functions, one must check whether the above SU(3) symmetry breaking corrections do not destroy these fundamental conservation laws. To verify them, we first notice the relations,

$$\int_{-1}^{1}\tilde{k}_0(x)dx = 0, \tag{193}$$

$$\int_{-1}^{1}\tilde{k}_1(x)dx = \frac{K_1}{I_1}, \tag{194}$$

$$\int_{-1}^{1}\tilde{k}_2(x)dx = \frac{K_2}{I_2} \tag{195}$$

with I_1, I_2 and K_1, K_2 being the basic moments of inertia of the soliton defined in Eqs. (49)–(52). Combining the above relations with the similar sum rules for $k_1(x)$ and $k_2(x)$, we then find that

$$\int_{-1}^{1}\left[\tilde{k}_1(x)-\frac{K_1}{I_1}k_1(x)\right]dx=\frac{K_1}{I_1}-\frac{K_1}{I_1}=0, \tag{196}$$

$$\int_{-1}^{1}\left[\tilde{k}_2(x)-\frac{K_2}{I_2}k_2(x)\right]dx=\frac{K_2}{I_2}-\frac{K_2}{I_2}=0. \tag{197}$$

It is now evident from these relations that

$$\int_{-1}^{1}q^{(0)}(x:\Delta m_s^{dyn+kin})dx=0, \tag{198}$$

$$\int_{-1}^{1}q^{(3)}(x:\Delta m_s^{dyn+kin})dx=0, \tag{199}$$

$$\int_{-1}^{1}q^{(8)}(x:\Delta m_s^{dyn+kin})dx=0, \tag{200}$$

which ensures that there is no contribution from the dynamical plus kinematical Δm_s corrections to the quark-number sum rules.

Since the mass difference between the s and u,d quarks breaks SU(3) symmetry, a baryon state is no longer a member of the pure SU(3) representation but it is generally a mixture of several SU(3) representations. Up to the first order in Δm_s, it can be shown that the proton state is a linear combination of three SU(3) representation as

$$|p\uparrow\rangle=|8,p\uparrow\rangle+c_{\overline{10}}^{N}|\overline{10},p\uparrow\rangle+c_{27}^{N}|27,p\uparrow\rangle. \tag{201}$$

Here, the mixing coefficients are given by

$$c_{\overline{10}}^{N}=-\frac{\sqrt{5}}{15}\left(\alpha+\frac{1}{2}\gamma\right)I_2, \tag{202}$$

$$c_{27}^{N}=-\frac{\sqrt{6}}{25}\left(\alpha-\frac{1}{6}\gamma\right)I_2, \tag{203}$$

where

$$\alpha=\left(-\frac{\overline{\sigma}}{N_c}+\frac{K_2}{I_2}\right)\Delta m_s, \tag{204}$$

$$\gamma=2\left(\frac{K_1}{I_1}-\frac{K_2}{I_2}\right)\Delta m_s, \tag{205}$$

with $\overline{\sigma}$ being the scalar charge of the nucleon given by

$$\overline{\sigma}=N_c\sum_{n\le 0}\langle n|\gamma^0|n\rangle. \tag{206}$$

The representation mixing correction to any nucleon observables can therefore be evaluated based on the formula

$$\langle p\uparrow|\hat{O}|p\uparrow\rangle=\langle 8,p\uparrow|\hat{O}|8,p\uparrow\rangle+2c_{\overline{10}}^{N}\langle\overline{10},p\uparrow|\hat{O}|8,p\uparrow\rangle+2c_{27}^{N}\langle 27,p\uparrow|\hat{O}|8,p\uparrow\rangle+O[(\Delta m_s)^2].$$

Here, as for the effective operator $\hat{O}$, we take the basic $O(\Omega^0+\Omega^1)$ operators, which can be read from Eqs. (95) and (96) for the unpolarized distributions, while from Eqs. (120) and (121) for the longitudinally polarized ones. From Eq. (95), it is easy to verify that there is no representation mixing correction to flavor-singlet unpolarized distribution

$$q^{(0)}(x:\Delta m_s^{rep})=0. \tag{207}$$

On the other hand, the representation mixing correction to the flavor-nonsinglet distribution is given by

$$\begin{aligned}q^{(a)}(x:\Delta m_s^{rep})=2c_{\overline{10}}^{N}\Bigg\{&\left\langle\overline{10},p\uparrow\left|\frac{D_{a8}}{\sqrt{3}}\right|8,p\uparrow\right\rangle f(x)+\left\langle\overline{10},p\uparrow\left|\sum_{i=1}^{3}\{D_{8i},R_i\}\right|8,p\uparrow\right\rangle k_1(x)\\&+\left\langle\overline{10},p\uparrow\left|\sum_{K=4}^{7}\{D_{aK},R_K\}\right|8,p\uparrow\right\rangle k_2(x)\Bigg\}+2c_{27}^{N}\left\langle 27,p\uparrow\left|\frac{D_{a8}}{\sqrt{3}}\right|8,p\uparrow\right\rangle f(x)\\&+\left\langle 27,p\uparrow\left|\sum_{i=1}^{3}\{D_{ai},R_i\}\right|8,p\uparrow\right\rangle k_1(x)\left\langle 27,p\uparrow\left|\sum_{K=4}^{7}\{D_{aK},R_K\}\right|8,p\uparrow\right\rangle k_2(x)\}.\end{aligned} \tag{208}$$

Given below are the matrix elements of the relevant collective operators,

$$\left\langle\overline{10},p\left|\frac{D_{38}}{\sqrt{3}}\right|8,p\right\rangle=-\frac{1}{6\sqrt{5}},\quad\left\langle\overline{10},p\left|\frac{D_{88}}{\sqrt{3}}\right|8,p\right\rangle=\frac{1}{2\sqrt{15}}, \tag{209}$$

$$\left\langle\overline{10},p\left|\sum_{i=1}^{3}\{D_{3i},R_i\}\right|8,p\right\rangle=\frac{1}{2\sqrt{5}},\qquad\left\langle\overline{10},p\left|\sum_{i=1}^{3}\{D_{8i},R_i\}\right|8,p\right\rangle=-\frac{3}{2\sqrt{15}}, \tag{210}$$

$$\left\langle \overline{10},p \left| \sum_{K=4}^{7} \{D_{3K},R_K\} \right| 8,p \right\rangle = 0,$$

$$\left\langle \overline{10},p \left| \sum_{K=4}^{7} \{D_{8K},R_K\} \right| 8,p \right\rangle = 0, \tag{211}$$

and

$$\left\langle 27,p \left| \frac{D_{38}}{\sqrt{3}} \right| 8,p \right\rangle = \frac{1}{15\sqrt{6}}, \quad \left\langle 27,p \left| \frac{D_{88}}{\sqrt{3}} \right| 8,p \right\rangle = \frac{1}{5\sqrt{2}}, \tag{212}$$

$$\left\langle 27,p \left| \sum_{i=1}^{3} \{D_{3i},R_i\} \right| 8,p \right\rangle = \frac{1}{15\sqrt{6}},$$

$$\left\langle 27,p \left| \sum_{i=1}^{3} \{D_{8i},R_i\} \right| 8,p \right\rangle = \frac{1}{5\sqrt{2}}, \tag{213}$$

$$\left\langle 27,p \left| \sum_{K=4}^{7} \{D_{3K},R_K\} \right| 8,p \right\rangle = -\frac{4}{15\sqrt{6}},$$

$$\left\langle 27,p \left| \sum_{K=4}^{7} \{D_{8K},R_K\} \right| 8,p \right\rangle = -\frac{4}{5\sqrt{2}}. \tag{214}$$

Using these, we finally arrive at

$$q^{(0)}(x:\Delta m_s^{rep}) = 0, \tag{215}$$

and

$$q^{(3)}(x:\Delta m_s^{rep}) = -\frac{1}{3\sqrt{5}} c_{\overline{10}}^N [f(x) - 3k_1(x)] + \frac{2}{15\sqrt{15}} c_{27}^N [f(x) + k_1(x) - 4k_2(x)],$$

$$q^{(8)}(x:\Delta m_s^{rep}) = +\frac{1}{\sqrt{15}} c_{\overline{10}}^N [f(x) - 3k_1(x)] + \frac{2}{5\sqrt{2}} c_{27}^N [f(x) + k_1(x) - 4k_2(x)]. \tag{216}$$

Remembering the sum rules for $f(x), k_1(x)$, and $k_2(x)$ given in Eqs. (106), (107), and (108), we can show that

$$\int_{-1}^{1} q^{(0)}(x:\Delta m_s^{rep}) dx = 0, \tag{217}$$

$$\int_{-1}^{1} q^{(3)}(x:\Delta m_s^{rep}) dx = 0, \tag{218}$$

$$\int_{-1}^{1} q^{(8)}(x:\Delta m_s^{rep}) dx = 0, \tag{219}$$

which ensures that the quark-number sum rules are intact by the introduction of the representation mixing Δm_s corrections.

Next, we consider the representation mixing correction to the longitudinally polarized distributions. The representation mixing correction to the flavor-singlet distribution is again zero, i.e.,

$$\Delta q^{(0)}(x:\Delta m_s^{rep}) = 0, \tag{220}$$

while, for the flavor-nonsinglet distribution, we have

$$\begin{aligned} \Delta q^{(a)}(x:\Delta m_s^{rep}) &= 2c_{\overline{10}}^N \Bigg\{ \langle \overline{10},p\uparrow | D_{a3} | 8,p\uparrow \rangle [-g(x) - h(x)] \\ &+ \left\langle \overline{10},p\uparrow \left| 4\sum_{i=K}^{4} d_{3KK} \frac{1}{2}\{D_{aK},J_K\} \right| 8,p\uparrow \right\rangle s(x) \\ &+ \left\langle \overline{10},p\uparrow \left| \frac{1}{2}\{D_{a8},J_3\} \right| 8,p\uparrow \right\rangle \frac{2}{\sqrt{3}} e(x) \Bigg\} \\ &+ 2c_{27}^N \Bigg\{ \langle 27,p\uparrow | D_{a3} | 8,p\uparrow \rangle [-g(x) - h(x)] \\ &+ \left\langle 27,p\uparrow \left| 4\sum_{K=4}^{7} d_{3KK} \frac{1}{2}\{D_{aK},J_K\} \right| 8,p\uparrow \right\rangle s(x) \\ &+ \left\langle 27,p\uparrow \left| \frac{1}{2}\{D_{a8},J_3\} \right| 8,p\uparrow \right\rangle \frac{2}{\sqrt{3}} e(x) \Bigg\}. \end{aligned} \tag{221}$$

Here we need the following matrix elements:

$$\langle \overline{10},p\uparrow | D_{33} | 8,p\uparrow \rangle = -\frac{\sqrt{5}}{30}, \tag{222}$$

$$\left\langle \overline{10},p\uparrow \left| 4\sum_{K=4}^{7} d_{3KK} D_{3K} J_K \right| 8,p\uparrow \right\rangle = -\frac{2\sqrt{5}}{15}, \tag{223}$$

$$\langle \overline{10},p\uparrow | D_{38} J_3 | 8,p\uparrow \rangle = -\frac{\sqrt{15}}{60}, \tag{224}$$

$$\langle \overline{10},p\uparrow | D_{83} | 8,p\uparrow \rangle = \frac{\sqrt{15}}{30}, \tag{225}$$

$$\left\langle \overline{10},p\uparrow \left| 4\sum_{K=4}^{7} d_{3KK} D_{8K} J_K \right| 8,p\uparrow \right\rangle = \frac{2\sqrt{15}}{15}, \tag{226}$$

$$\langle \overline{10},p\uparrow | D_{88} J_3 | 8,p\uparrow \rangle = \frac{\sqrt{5}}{20}, \tag{227}$$

and

$$\langle 27,p\uparrow|D_{33}|8,p\uparrow\rangle = -\frac{\sqrt{6}}{270}, \tag{228}$$

$$\left\langle 27,p\uparrow\left|4\sum_{K=4}^{7} d_{3KK}D_{3K}J_K\right|8,p\uparrow\right\rangle = -\frac{4\sqrt{6}}{135}, \tag{229}$$

$$\langle 27,p\uparrow|D_{38}J_3|8,p\uparrow\rangle = \frac{\sqrt{2}}{60}, \tag{230}$$

$$\langle 27,p\uparrow|D_{83}|8,p\uparrow\rangle = -\frac{\sqrt{2}}{30}, \tag{231}$$

$$\left\langle 27,p\uparrow\left|4\sum_{K=4}^{7} d_{3KK}D_{8K}J_K\right|8,p\uparrow\right\rangle = -\frac{4\sqrt{2}}{15}, \tag{232}$$

$$\langle 27,p\uparrow|D_{88}J_3|8,p\uparrow\rangle = \frac{\sqrt{6}}{20}. \tag{233}$$

Using these relations, we finally obtain

$$\Delta q^{(0)}(x:\Delta m_s^{rep}) = 0, \tag{234}$$

$$\Delta q^{(3)}(x:\Delta m_s^{rep}) = +\frac{\sqrt{5}}{15}c_{10}^{N}[g(x)+h(x)-4s(x)-e(x)] + \frac{\sqrt{6}}{135}c_{27}^{N}[g(x)+h(x)-8s(x)+3e(x)], \tag{235}$$

$$\Delta q^{(8)}(x:\Delta m_s^{rep}) = -\frac{\sqrt{15}}{15}c_{10}^{N}[g(x)+h(x)-4s(x)-e(x)] + \frac{\sqrt{2}}{15}c_{27}^{N}[g(x)+h(x)-8s(x)+3e(x)]. \tag{236}$$

III. CONCLUDING REMARKS

We have developed a path integral formulation of the flavor SU(3) CQSM for evaluating quark and antiquark distribution functions in the nucleon. It has been done so as to take over the advantage of the SU(2) model such that the polarization of Dirac-sea quarks in the hedgehog mean field is properly taken into account. This is essential for making reasonable predictions for the hidden strange-quark distributions in the nucleon, which has a totally nonvalence character, as well as the light-flavor sea-quark distribution in the nucleon. The theory as a whole is based on a double expansion in two small parameters. One is the expansion in the collective angular velocity operator Ω of the rotating soliton, which can also be regarded as a $1/N_c$ expansion. The another is the perturbation in the strange- and nonstrange-quark mass difference, which is also thought to be small as compared with the typical energy scale of baryon physics.

As for the SU(3) symmetry breaking corrections, we have taken into account three possible corrections, named the dynamical correction, kinematical correction, and the representation mixing correction, which are all linear order in the mass parameter Δm_s. It was emphasized that the simultaneous account of the dynamical and the kinematical corrections is essential for maintaining the quark-number sum rules. Unfortunately, we encounter a subtle problem in the evaluation of the parton distribution functions at the subleading order of $1/N_c$ expansion, or more concretely, the $O(\Omega^1)$ contribution to the PDF. It arises from an ordering ambiguity of two collective space operators in quantization. In the case of SU(2) CQSM, this ambiguity can be avoided if one adopts a physically plausible time-order-keeping quantization prescription. However, it appears that this particular quantization procedure is not compatible with the fundamental dynamical assumption of the SU(3) CQSM, i.e., the embedding of the SU(3) hedgehog followed by the quantization of soliton rotation in the full SU(3) collective coordinate space. On the other hand, one can avoid this incompatibility, if one adopts the symmetrized ordering of two collective operators before quantization. The price to pay for it is, however, that one loses phenomenologically desirable first-order rotational correction to some flavor-nonsinglet observables, which we know is essential for resolving the long-standing g_A problem in the flavor SU(2) version of the CQSM. Undoubtedly, our understanding of the theoretical aspects of the model is still incomplete and some more work should be done for clarifying these questions.

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APPENDIX A: PROOF OF EQUALITIES (87) AND (88)

Here, let us prove two identities (87) and (88), which we have used in Sec. II. Using the standard SU(3) algebra

$$\{\lambda_c,\lambda_i\} = \frac{4}{3}\delta_{ci} + 2d_{cie}\lambda_e, \tag{A1}$$

we proceed as

$$\begin{aligned}
&\sum_{M(n)} \langle n|\{\lambda_c,\lambda_i\}(\gamma_5+\Sigma_3)\delta_n|n\rangle \\
&= \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{ci}+2d_{cie}\lambda_e\right)(\gamma_5+\Sigma_3)\delta_n\right|n\right\rangle \\
&= 2d_{ci3}\sum_{M(n)} \langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle \\
&= 2d_{338}\delta_{c8}\delta_{i3}\sum_{M(n)} \langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle \\
&= \frac{2}{\sqrt{3}}\delta_{c8}\delta_{i3}\sum_{m=all,M(n)} \langle n|\lambda_3|m\rangle\langle m|(\gamma_5+\Sigma_3)\delta_n|n\rangle,
\end{aligned} \tag{A2}$$

which proves the first identity. To prove the second identity, we first notice that

$$\sum_{M(n)} \langle n|\{\lambda_c,\lambda_K\}(\gamma_5+\Sigma_3)\delta_n|n\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{ci}+2d_{cKe}\lambda_e\right)(\gamma_5+\Sigma_3)\delta_n\right|n\right\rangle = 2d_{3cK}\sum_{M(n)} \langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle. \quad \text{(A3)}$$

Second, we can show that

$$\sum_{m=all,M(n)} \langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle = \sum_{M(n)} \langle n|\lambda_4^2(\gamma_5+\Sigma_3)\delta_n|n\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{2}{3}-\frac{1}{2\sqrt{3}}\lambda_8+\frac{1}{2}\lambda_3\right)(\gamma_5+\Sigma_3)\delta_n\right|n\right\rangle = \frac{1}{2}\sum_{M(n)} \langle n|\lambda_3(\gamma_5+\Sigma_3)\delta_n|n\rangle. \quad \text{(A4)}$$

Combining the above two equations, we therefore obtain

$$\sum_{M(n)} \langle n|\{\lambda_c,\lambda_K\}(\gamma_5+\Sigma_3)\delta_n|n\rangle = 4d_{3cK}\sum_{m=all,M(n)} \langle n|\lambda_4|m\rangle\langle m|\lambda_4(\gamma_5+\Sigma_3)\delta_n|n\rangle, \quad \text{(A5)}$$

which proves the second identity.

APPENDIX B: PROOF OF EQUALITIES (172) AND (173)

Here, we will prove the identities (172) and (173) used in Sec. II. Utilizing the generalized hedgehog symmetry together with the standard SU(3) algebra, we can proceed as follows:

$$\sum_{M(n)} \langle n|\{\lambda_b,\lambda_i\}\bar{O}\delta_n|n\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{bi}+2d_{bie}\lambda_e\right)\bar{O}\delta_n\right|n\right\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{bi}+2d_{bi8}\lambda_8+2d_{bi3}\lambda_3\right)\bar{O}\delta_n\right|n\right\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{bi}+2\delta_{bi}d_{118}\frac{1}{\sqrt{3}} + 2d_{833}\lambda_3\delta_{b8}\delta_{i3}\right)\bar{O}\delta_n\right|n\right\rangle = 2\delta_{bi}\sum_{M(n)} \langle n|\bar{O}\delta_n|n\rangle + \frac{2}{\sqrt{3}}\delta_{b8}\,\delta_{i3}\sum_{M(n)} \langle n|\,\lambda_3\bar{O}\delta_n|n\rangle, \quad \text{(B1)}$$

where the index i runs from 1 to 3. This proves the first identity (172). Similarly, for the second case in which K runs from 4 to 7, we can show that

$$\sum_{M(n)} \langle n|\{\lambda_b,\lambda_K\}\bar{O}\delta_n|n\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{bK}+2d_{bKe}\lambda_e\right)\bar{O}\delta_n\right|n\right\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{bK}+2d_{bK8}\lambda_8+2d_{bK3}\lambda_3\right)\bar{O}\delta_n\right|n\right\rangle = \sum_{M(n)} \left\langle n\left|\left(\frac{4}{3}\delta_{bK}+2\delta_{bK}d_{448}\frac{1}{\sqrt{3}} + 2d_{3KK}\lambda_3\delta_{bK}\right)\bar{O}\delta_n\right|n\right\rangle = 2\delta_{bK}\sum_{M(n)} \langle n|\bar{O}\delta_n|n\rangle + 2\delta_{bK}d_{3KK}\sum_{M(n)} \langle n|\lambda_3\bar{O}\delta_n|n\rangle, \quad \text{(B2)}$$

which proves the second identity (173).

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