Gauge invariance and form factors for the decay $B \to \gamma l^+ l^-$

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We analyze the form factors for the $B \to \gamma l^+ l^-$ weak transition. We show that making use of the gauge invariance of the $B \to \gamma l^+ l^-$ amplitude, the structure of the form factors in the resonance region, and their relations at large values of the photon energy results in efficient constraints on the behavior of the form factors. Based on these constraints, we propose a simple parametrization of the form factors and apply it to the lepton forward-backward (FB) asymmetry in the $B \to \gamma l^+ l^-$ decay. We find that the behavior of the FB asymmetry as a function of the photon energy, as well as the location of its zero, depend only weakly on the $B \to \gamma$ form factors, and thus constitutes a powerful tool for testing the standard model.

I. INTRODUCTION

Recently, the decay $B \to \gamma l^+ l^-$ has been the subject of a number of investigations [1–5], where it has been pointed out that this process may serve as an important probe of the standard model (SM) and possible extensions. However, knowledge of the long-distance QCD effects, which are inherently nonperturbative, is important to extract quantitative information on the underlying short-distance interactions.

From the analyses of the decay $B \to K^* l^+ l^-$ it is known [6,7] that the uncertainties due to the hadronic form factors are considerably reduced if one considers asymmetries such as the forward-backward (FB) asymmetry of the lepton. This empirical observation later received an explanation within the large energy effective theory (LEET) [8]. According to LEET, all the heavy-to-light meson transition form factors are given at leading order in $1/M_B$ and $1/E$ (E is the energy of the light meson) in terms of a few universal form factors, and thanks to that the form factor effects largely drop out from the asymmetries.

As for the radiative dilepton decay $B \to \gamma l^+ l^-$, one expects the same to be true. However, a surprisingly strong dependence of the FB asymmetry on the specific form factor model can be found in the existing literature [1–3], which needs better understanding.

In this paper, we analyze the form factors for the $B \to \gamma$ transition induced by vector, axial-vector, tensor, and pseudotensor currents.

We show that important relations between form factors of different currents arise as a consequence of the gauge invariance of the $B \to \gamma$ amplitude. We derive an exact relation between the form factors of tensor and pseudotensor currents at $q^2 = 0$, where q^2 is the dilepton invariant mass in the decay $B \to \gamma l^+ l^-$. We note that the form factors from a recent sum-rule calculation of Ref. [4] are inconsistent with this exact relation.

We investigate the behavior of the various form factors at large q^2 and find interesting relations corresponding to the resonance contributions to the form factors. We argue that these contributions signal substantial corrections to the Isgur-Wise relations valid to $1/m_b$ accuracy at large q^2 [9].

Combined with the relations among the form factors from LEET, the results obtained provide strong restrictions on the $B \to \gamma$ form factors. We propose a simple model for the form factors which is valid over the full range of the photon energy, and which satisfies all known constraints. An important remark is in order here: It has been shown recently that a proper account of collinear and soft gluons leads to a different effective theory—the so-called soft-collinear effective theory (SCET) [10] which supersedes the LEET. Important for our discussion is that interactions with collinear gluons preserve the *relations* for the soft part of the heavy-to-light form factors from LEET [11] [the differences appear in the $O(\alpha_s)$ part]. Our analysis is therefore fully compatible with SCET.

As an application of our form factor model, we examine the FB asymmetry in the decay $B_s \to \gamma \mu^+ \mu^-$. We find that this asymmetry, and particularly its zero arising in the SM, can be predicted with small theoretical uncertainties.

II. FORM FACTORS FOR THE $B \to \gamma$ TRANSITION

We are concerned with the amplitudes of the $B \to \gamma$ transition resulting from various quark currents. In our analysis, we adopt the following conventions:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu], \quad \epsilon^{0123} = -1, \quad (1)$$

and accordingly

$$\sigma_{\mu\nu}\gamma^5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}. \quad (2)$$

A. Transition to a virtual photon

We start with the amplitude describing the transition of the $B_q (q = s, d)$ meson with momentum p to a virtual photon

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with momentum k. In this case, the form factors depend on two variables: that is, the photon virtuality k^2 and the square of the momentum transfer $(p-k)^2$. As we shall see, gauge invariance and the absence of singularities in the amplitude lead to several relations among the form factors at $k^2=0$, thereby reducing the number of independent form factors for the transition to a *real* photon.

(i) For the $B_q \to \gamma^*$ transition induced by the *axial-vector current*, the gauge-invariant amplitude (with respect to the photon) contains three form factors and can be written in the form[1]

$$\langle \gamma^*(k)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}_q(p)\rangle$$
$$= ie\varepsilon^{*\alpha}(k)\left\{\left(g_{\mu\alpha}-\frac{k_\alpha k_\mu}{k^2}\right)f+p_\mu\left(p_\alpha-\frac{k\cdot p}{k^2}k_\alpha\right)a_1\right.$$
$$\left. +k_\mu\left(p_\alpha-\frac{k\cdot p}{k^2}k_\alpha\right)a_2\right\}, \tag{3}$$

where we have explicitly written the gauge-invariant Lorentz structures. In the above, ε^α denotes the polarization vector of the photon, $e=\sqrt{4\pi\alpha}$, and the form factors are defined according to Ref. [12]. Since the amplitude is a regular function at $k^2=0$, the requirement of gauge invariance results in the following constraints on the form factors at $k^2=0$:

$$f+(k\cdot p)a_2=0, \quad a_1=0. \tag{4}$$

(ii) For the transition induced by the *vector current*, the amplitude is parametrized in terms of a single form factor g; namely,

$$\langle \gamma^*(k)|\bar{q}\gamma_\mu b|\bar{B}_q(p)\rangle = 2eg\varepsilon^{*\alpha}(k)\epsilon_{\mu\alpha\rho\sigma}p^\rho k^\sigma. \tag{5}$$

(iii) For the $B_q \to \gamma^*$ transition induced by the *pseudotensor current*, the amplitude can be written in terms of three form factors:

$$\langle \gamma^*(k)|\bar{q}\sigma_{\mu\nu}\gamma_5 b|\bar{B}_q(p)\rangle$$
$$= e\varepsilon^{*\alpha}(k)\left\{\left[\left(g_{\alpha\nu}-\frac{k_\alpha k_\nu}{k^2}\right)p_\mu-\left(g_{\alpha\mu}-\frac{k_\alpha k_\mu}{k^2}\right)p_\nu\right]g_1\right.$$
$$+(g_{\alpha\nu}k_\mu-g_{\alpha\mu}k_\nu)g_2+\left(p_\alpha-\frac{k\cdot p}{k^2}k_\alpha\right)$$
$$\left. \times(k_\mu p_\nu-p_\nu k_\mu)g_0\right\}. \tag{6}$$

At $k^2=0$, gauge invariance leads to the condition

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[1]Notice that for the B-meson transition to a virtual photon the amplitude of the axial-vector current contains a contact term, which is proportional to the charge of the B meson. The contact term is thus present for the *charged* B-meson transition, but is absent in the case of a *neutral* B meson. For a detailed discussion of form factors and contact terms in this amplitude, see Ref. [12].

$$g_1-(k\cdot p)g_0=0. \tag{7}$$

(iv) The amplitude for the transition induced by the *tensor current* can be obtained from Eq. (6) by applying the identity in Eq. (2), and is given by

$$\langle \gamma^*(k)|\bar{q}\sigma_{\mu\nu}b|\bar{B}_q(p)\rangle$$
$$= ie\varepsilon^{*\alpha}(k)\left\{\left(\epsilon_{\mu\nu\alpha\rho}p^\rho-\frac{k_\alpha}{k^2}\epsilon_{\mu\nu\sigma\rho}k^\sigma p^\rho\right)g_1\right.$$
$$\left. +\epsilon_{\mu\nu\alpha\sigma}k^\sigma g_2+\left(p_\alpha-\frac{k\cdot p}{k^2}k_\alpha\right)\epsilon_{\mu\nu\rho\sigma}p^\rho k^\sigma g_0\right\}. \tag{8}$$

B. Transition to a real photon

For the transition to a real photon, the matrix element of the vector current is given by Eq. (5) while the amplitude for the axial-vector current, employing the relation in Eq. (4), reads

$$\langle \gamma(k)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}_q(p)\rangle = -ie\varepsilon^{*\alpha}(k)[g_{\mu\alpha}(p\cdot k)$$
$$-p_\alpha k_\mu]a_2(k^2=0). \tag{9}$$

As for tensor and pseudotensor currents, their matrix elements have the form

$$\langle \gamma(k)|\bar{q}\sigma_{\mu\nu}\gamma_5 b|\bar{B}_q(p)\rangle$$
$$= e\varepsilon^{*\alpha}(k)\{(g_{\alpha\nu}k_\mu-g_{\alpha\mu}k_\nu)g_2+([g_{\alpha\nu}(p\cdot k)-p_\alpha k_\nu]p_\mu$$
$$-[g_{\alpha\mu}(p\cdot k)-p_\alpha k_\mu]p_\nu)g_0\},$$

$$\langle \gamma(k)|\bar{q}\sigma_{\mu\nu}b|\bar{B}_q(p)\rangle$$
$$= ie\varepsilon^{*\alpha}(k)\{\epsilon_{\mu\nu\alpha\sigma}k^\sigma g_2+[p_\alpha\epsilon_{\mu\nu\rho\sigma}p^\rho k^\sigma$$
$$-(p\cdot k)\epsilon_{\mu\nu\rho\alpha}p^\rho]g_0\}. \tag{10}$$

It should be noted that in the case of a real photon the gauge-invariant amplitudes of the pseudotensor and tensor currents contain two Lorentz structures and not only one as stated in Ref. [13]. Multiplying Eq. (10) by $(p-k)^\nu$, we arrive at the following set of amplitudes that describe the $B \to \gamma$ transition:

$$\langle \gamma(k)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}_q(p)\rangle$$
$$= ie\varepsilon^{*\alpha}(k)[g_{\mu\alpha}(p\cdot k)-p_\alpha k_\mu]\frac{F_A}{M_{B_q}},$$

$$\langle \gamma(k)|\bar{q}\gamma_\mu b|\bar{B}_q(p)\rangle$$
$$= e\varepsilon^{*\alpha}(k)\epsilon_{\mu\alpha\rho\sigma}p^\rho k^\sigma \frac{F_V}{M_{B_q}},$$

$$\langle \gamma(k)|\bar{q}\sigma_{\mu\nu}\gamma_5 b|\bar{B}_q(p)\rangle(p-k)^\nu$$
$$= e\varepsilon^{*\alpha}(k)[g_{\mu\alpha}(p\cdot k)-p_\alpha k_\mu]F_{TA},$$

$$\langle \gamma(k)|\bar{q}\sigma_{\mu\nu}b|\bar{B}_q(p)\rangle (p-k)^{\nu}$$

$$= ie\varepsilon^{*\alpha}(k)\epsilon_{\mu\alpha\rho\sigma}p^{\rho}k^{\sigma}F_{TV}, \tag{11}$$

where the dimensionless form factors F_i are given in terms of the form factors g, a_2, g_0, g_2 at $k^2 = 0$:

$$F_V = 2M_{B_q}g, \quad F_A = -M_{B_q}a_2, \tag{12}$$

$$F_{TV} = -g_2 - [p^2 - (p \cdot k)]g_0, \quad F_{TA} = -g_2 - (p \cdot k)g_0. \tag{13}$$

For a real photon in the final state, the form factors depend on the square of the momentum transfer, $q^2 \equiv (p-k)^2$. Equivalently, one may consider the form factors as functions of the photon energy in the B-meson rest frame,

$$E = \frac{M_B}{2}\left(1 - \frac{q^2}{M_B^2}\right). \tag{14}$$

For massless leptons, the kinematically accessible range is

$$0 \leqslant q^2 \leqslant M_B^2, \quad 0 \leqslant E \leqslant E_{\max} = M_B/2. \tag{15}$$

Then, rewriting Eq. (13) in the form

$$F_{TV} = F_{TA} - M_B(M_B - 2E)g_0, \tag{16}$$

we derive the following *exact* relation:

$$F_{TA}(E_{\max}) = F_{TV}(E_{\max}). \tag{17}$$

We stress that the equality of the form factors F_{TA} and F_{TV} is valid only at $E = E_{\max}$.

C. LEET and form factors at large E

Interesting relations between the form factors emerge in the limit where the initial hadron is heavy and the final photon has a large energy [8]. In this case, the form factors may be expanded in inverse powers of Λ_{QCD}/M_B and Λ_{QCD}/E. As a result, to leading-order accuracy, one finds

$$F_V \simeq F_A \simeq F_{TA} \simeq F_{TV} \simeq \zeta_{\perp}^{\gamma}(E, M_B), \tag{18}$$

where $\zeta_{\perp}^{\gamma}(E, M_B)$ is the universal form factor for the $B_q \rightarrow \gamma$ transition (cf. Ref. [8]).[2] We emphasize that this relation is violated by terms of order $O(\Lambda_{\text{QCD}}/M_B)$ and $O(\Lambda_{\text{QCD}}/E)$, as well as by radiative corrections of $O(\alpha_s)$ [14].

For large values of E, the energy dependence of the universal $B \rightarrow \gamma$ form factor can be obtained from perturbative QCD, the result being [13]

$$\zeta_{\perp}^{\gamma}(E, M_B) \propto f_B M_B/E. \tag{19}$$

D. Heavy quark symmetry and form factors at large q^2

As found by Isgur and Wise [9], in the region of large $q^2 \simeq M_B^2$ the form factors satisfy the relations

$$g_2 \simeq -2M_B g, \quad g_0 \simeq \frac{1}{2M_B}(2g + a_2), \tag{20}$$

which are valid at leading order in Λ_{QCD}/m_b.[3] Combining these expressions with Eqs. (12) and (13), we derive the following result for the tensor-type form factors in the region of small E:

$$F_{TV} \simeq F_V - \frac{M_B - E}{2M_B}(F_V - F_A),$$

$$F_{TA} \simeq F_V - \frac{E}{2M_B}(F_V - F_A). \tag{21}$$

It is obvious that these relations are compatible with those in Eq. (18), which emerge in the region where E is large, and hence are valid to leading order in Λ_{QCD}/m_b in the full range of E. Yet the leading-order Λ_{QCD}/m_b relations in Eq. (21) may not be sufficient to understand the behavior of the form factors $F_{V,A}$ and $F_{TA,TV}$ in the region where $q^2 \simeq M_B^2$.

To explain this point, it is useful to express the form factors F_i in terms of the Wirbel-Stech-Bauer (WSB) form factors [15]. The advantage of the WSB form factors is that each one has definite spin and parity, and hence contains contributions of resonances with the corresponding quantum numbers. Explicitly, we have

$$F_V = 2V^{B\rightarrow\gamma}, \quad F_A = \frac{M_B}{E}A_1^{B\rightarrow\gamma},$$

$$F_{TV} = 2T_1^{B\rightarrow\gamma}, \quad F_{TA} = \frac{M_B}{E}T_2^{B\rightarrow\gamma}. \tag{22}$$

Notice that the relation $F_{TV} = F_{TA}$ at $q^2 = 0$ (or $E = M_B/2$) is just the well-known relation $T_1(0) = T_2(0)$. Then, making use of the constraints in Eqs. (4) and (7), which are due to electromagnetic gauge invariance, we obtain exact relations between the WSB form factors that are relevant for the transition into a real photon; namely,

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[2]For a massive particle in the final state one has two universal form factors $\zeta_{\perp}(E, M_B)$ and $\zeta_{\parallel}(E, M_B)$. The latter does not contribute in the case of a massless final vector particle. We also note the relation $\zeta_{\perp}^{B_u\rightarrow\gamma}(E, M_B) = Q_u/Q_d\zeta_{\perp}^{B_d\rightarrow\gamma}(E, M_B)$, where $Q_u/Q_d = -2$.

[3]It should be noted that at large q^2 there are in general three independent relations between the $B \rightarrow V$ form factors. The third relation, however, is automatically satisfied in the case of a photon in the final state, due to the gauge-invariance constraints in Eqs. (4) and (7).

$$T_2^{B\to\gamma}=\frac{2E}{M_B}T_3^{B\to\gamma}, \quad A_1^{B\to\gamma}=\frac{2E}{M_B}A_2^{B\to\gamma}. \qquad (23)$$

By virtue of these relations, we may rewrite Eq. (22) in the form

$$F_A=2A_2^{B\to\gamma}, \quad F_{TA}=2T_3^{B\to\gamma}, \qquad (24)$$

which exhibits the absence of a singular behavior of F_A and F_{TA}.

We now examine the analytic structure of the form factors near $q^2=M_B^2$, starting with V and T_1, which have a pole at $q^2=M_{B*}^2$. This pole is located very close to the upper boundary of the physical region, $q^2=M_B^2$, since $M_{B*}-M_B=45$ MeV$\sim O(\Lambda_{QCD}^2/m_b)$. Moreover, as shown in Refs. [12,16], the residues of the form factors V and T_1 in the pole at $q^2=M_{B*}^2$ are equal in the heavy quark limit $m_b\to\infty$. As a consequence, F_V and F_{TV} should be approximately equal and rise steeply as $q^2\to M_B^2$.

As for the form factors F_A and F_{TA}, they are expected to have qualitatively different behavior near $q^2=M_B^2$. Indeed, the masses of the resonances that correspond to A_2 and T_3 [Eq. (24)], denoted by B^{**}, are expected to be several hundred MeV higher than M_B, since $M_{B**}-M_B\sim O(\Lambda_{QCD})$. Thus, singularities of the form factors F_A and F_{TA} are much farther from the physical region, compared to the form factors F_V and F_{TV}. Consequently, F_A and F_{TA} are relatively flat as $q^2\to M_B^2$.

It is now clear why the leading-order Λ_{QCD}/m_b relations (21) are not useful for understanding the behavior of the form factors near $q^2=M_B^2$. As a matter of fact, at leading order in Λ_{QCD}/m_b all the $b\bar{q}$ resonances with different spins have the same masses, so that all the form factors F_i have poles at $q^2=M_B^2$ (i.e., at $E=0$). This picture is fully consistent with Eq. (21), but it is far from reality, since $O(\Lambda_{QCD}/m_b)$ corrections to the form factor relations in Eq. (21) become crucial in the region near $q^2=M_B^2$. We expect, then, the following relation between the form factors near $q^2=M_B^2$:

$$F_A\simeq F_{TA}\ll F_V\simeq F_{TV}, \qquad (25)$$

in agreement with the resonance location.

E. The general picture of the $B\to\gamma$ form factors

Combining the above information on the form factors, the following picture of the $B\to\gamma$ form factors emerges.

At $E=E_{max}$, the form factors F_{TA} and F_{TV} are equal, $F_{TA}(E_{max})=F_{TV}(E_{max})$.

In the region where $E\gg\Lambda_{QCD}$, the form factors obey the LEET relation, which is valid to $O(\Lambda_{QCD}/m_b)$, $O(\Lambda_{QCD}/E)$, and $O(\alpha_s)$ accuracy:

$$F_V\simeq F_A\simeq F_{TA}\simeq F_{TV}\propto f_B M_B/E. \qquad (26)$$

At large $q^2\simeq M_B^2$ (i.e., at small E), the following relation for the form factors should hold:

TABLE I. Parameters of the $B_d\to\gamma$ form factors, as defined in Eq. (28).

Parameter	F_V	F_{TV}	F_A	F_{TA}
β (GeV^{-1})	0.28	0.30	0.26	0.33
Δ (GeV)	0.04	0.04	0.30	0.30

$$F_A\simeq F_{TA}\ll F_V\simeq F_{TV}. \qquad (27)$$

We expect these relations to work with 10–15 % accuracy.

Given these features, we now turn to the analysis of existing predictions for the $B\to\gamma$ form factors.

1. Form factors F_A and F_V

The form factors F_A and F_V for the $B_d\to\gamma$ transition have been calculated in Ref. [12] within the dispersion approach of Ref. [17].[4] For large and intermediate values of E, these form factors can be well parametrized by a particularly simple formula:

$$F(E)=\beta\frac{f_B M_B}{\Delta+E}, \qquad (28)$$

with $f_B=0.2$ GeV, $M_B=5.28$ GeV, and $\beta_V-\beta_A=O(1/m_b)$ in accord with LEET. In order to use this formula for the form factors in the full range of E, we should take into account that the form factors F_V and F_A have, respectively, poles at $q^2=M_{B*}^2$ and $q^2=M_{B**}^2$, so that $\Delta_V=M_{B*}-M_B$ and $\Delta_A=M_{B**}-M_B$. Although the masses of the positive-parity states with higher spins B^{**} are not known, we can use data on D mesons to estimate the mass difference $\Delta_A\simeq 0.3$–0.4 GeV; here we take $\Delta_A=0.3$ GeV. The numerical parameters are listed in Table I.

The maximal difference between the form factors at $E=0$ is at the level of 50%. The difference between F_A and F_V is around 10% for $E\gtrsim 0.7$ GeV, indicating Λ_{QCD}/m_b corrections to the LEET relation between the form factors at the level of 5–10 %.

2. Form factors F_{TA} and F_{TV}

There are several calculations of the $B\to\gamma$ form factors F_{TA} and F_{TV} available in the literature [4,5]. Let us check whether the results for the form factors satisfy the constraints derived above.

(i) The light-cone sum rule calculation of Ref. [4] predicts the form factors $F_{TV}\gg F_{TA}$ for large values of E, including E_{max}, which points to a very strong violation of the LEET

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[4]The form factors $F_{A,V}$ for the $B_u\to\gamma$ transition have been calculated in Refs. [18,19] using light-cone sum rules. These results agree with the results from the dispersion approach [12]. It should be noted that the form factors $F_{A,V}$ for the $B_u\to\gamma$ transition have the opposite sign and are approximately twice as big as the corresponding form factors $F_{A,V}$ for the $B_d\to\gamma$ transition; see the discussion in Ref. [12].

relations. More importantly, the *exact* relation in Eq. (17) between the form factors at E_{max} is also drastically violated. Thus, we conclude that the calculation of form factors performed in Ref. [4] cannot be trusted.

(ii) The quark model calculation of Ref. [5] satisfies the exact constraint in Eq. (17), with values of the form factors $F_{TA} = F_{TV} = 0.115$ at $q^2 = 0$ (or, equivalently, $E_{max} = M_B/2$) [20]. Taking into account the LEET relation (18), this value is in agreement with our results for the form factors $F_A = 0.09$ and $F_V = 0.105$ at $q^2 = 0$. On the other hand, there are several features of the predicted form factors that do not seem realistic. First, as can be seen from Fig. 1 of Ref. [5], the form factors F_{TA} and F_{TV} differ considerably, with $F_{TA} \gg F_{TV}$ for the values of $E \simeq 0.5$–1 GeV. This signals a very strong violation of the LEET condition, with corrections of the order of several hundred percent. Let us recall that the form factors may indeed be very different in the region $q^2 \simeq M_B^2$, since F_{TV} contains a pole at $q^2 = M_{B*}^2$ while F_{TA} does not. But then one would expect the relation $F_{TV} \gg F_{TA}$, opposite to the one obtained in Ref. [5]. Second, the form factors F_{TA} and F_{TV} of Ref. [5] vanish at $q^2 = M_B^2$. Taking into account that the form factor F_{TV} contains a pole at $q^2 = M_{B*}^2$, it seems very unlikely that the form factor F_{TV} vanishes at $q^2 = M_B^2$. Therefore, the predictions of Ref. [5] for values of $q^2 \gtrsim 10$ GeV^2 cannot be considered very reliable.

To sum up: There are no fully convincing results for the form factors F_{TA} and F_{TV}; thus, for the analysis of the FB asymmetry, we prefer to rely upon a simple model for the $B \rightarrow \gamma$ transition form factors which satisfies explicitly all the constraints discussed above.

F. Model for the $B \rightarrow \gamma$ form factors

We assume the ansatz in Eq. (28) to be valid for all $B_{d,s} \rightarrow \gamma$ form factors with their own constants. Together with the condition in Eq. (16), this leads to the following relation between the parameters of F_{TA} and F_{TV}:

$$\frac{\beta_{TV}}{\Delta_{TV} + M_B/2} = \frac{\beta_{TA}}{\Delta_{TA} + M_B/2}, \tag{29}$$

such that $\beta_{TV} - \beta_{TA} = O(1/m_b)$. Furthermore, according to our arguments mentioned above, we set

$$\Delta_{TV} = \Delta_V, \quad \Delta_{TA} = \Delta_A. \tag{30}$$

The remaining parameter to be fixed is the constant β_{TV}, for which we write

$$\beta_{TV} = (1 + \delta)\beta_V, \tag{31}$$

and choose $\delta = 0.1$ according to the result of Ref. [5]. This completes our simple model for the form factors which are consistent with the exact relations at E_{max}, LEET at large E, and heavy quark symmetry at small E. Table I contains the various numerical parameters for the $B_d \rightarrow \gamma$ form factors,
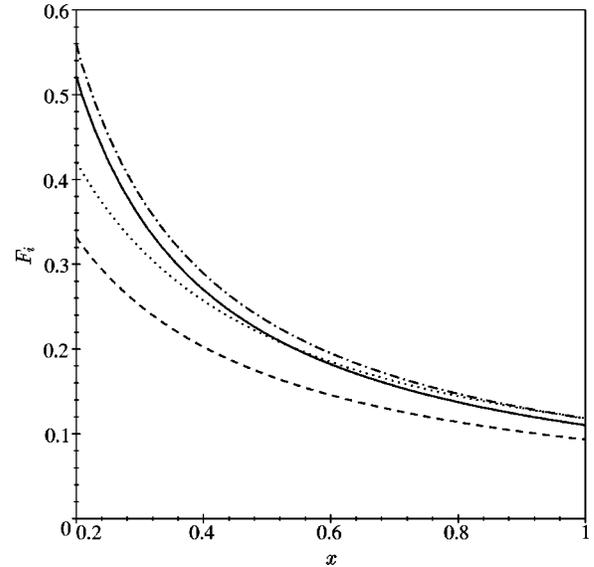

FIG. 1. The predicted x dependence of the $B_d \rightarrow \gamma$ form factors F_V (solid curve), F_A (dashed curve), F_{TA} (dotted curve), and F_{TV} (dash-dotted curve) according to our model, as described in the text ($x \equiv 2E/M_B$).

and Fig. 1 shows the form factors in our model as a function of the scaled photon energy, $x \equiv 2E/M_B$.

For the $B_s \rightarrow \gamma$ transition, we do not know the precise values of the form factors, but we shall assume that the LEET-violating effects in the $B_s \rightarrow \gamma$ form factors have the same structure as those in the $B_d \rightarrow \gamma$ transition. With this assumption, the form factors as given in Table I are sufficient for the analysis of the FB asymmetry in the $B_s \rightarrow \gamma l^+ l^-$ decay presented in the next section.

III. FORWARD-BACKWARD ASYMMETRY IN $B \rightarrow \gamma l^+ l^-$

We now assess the implications of our form factor model for the FB asymmetry of μ^- in the decay $\bar{B}_s \rightarrow \gamma \mu^+ \mu^-$. Referring to Refs. [1,4,5,19], the radiative dilepton decay receives various contributions. The main contribution in the case of light leptons comes from the so-called structure-dependent (SD) part, where the photon is emitted from the external quark line. Contributions coming from photons attached to charged internal lines are suppressed by m_b^2/M_W^2 [19]. The bremsstrahlung contribution due to emission of the photon from the external leptons is suppressed by the mass of the light leptons $l = e, \mu$ and affects the photon energy spectrum only in the low E region [1,5].

Neglecting the bremsstrahlung contributions, the decay is then governed by the effective Hamiltonian describing the $b \rightarrow sl^+ l^-$ decay, together with the form factors parametrizing the $B \rightarrow \gamma$ transition, as discussed in the preceding section. Using the effective Hamiltonian for $b \rightarrow sl^+ l^-$ in the SM [21], the matrix element is ($m_s = 0$)

$$\mathcal{M}_{SD} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[(c_9^{\text{eff}} \bar{l}\gamma^\mu l + c_{10} \bar{l}\gamma^\mu\gamma_5 l) \right.$$

$$\times \langle \gamma(k) | \bar{s}\gamma_\mu P_L b | \bar{B}_s(p) \rangle$$

$$\left. - \frac{2c_7^{\text{eff}} m_b}{q^2} \langle \gamma(k) | \bar{s} i\sigma_{\mu\nu} q^\nu P_R b | \bar{B}_s(p) \rangle \bar{l}\gamma^\mu l \right],$$

$$(32)$$

where $q = p - k$ and $P_{L,R} = (1 \mp \gamma_5)/2$. Within the SM, the Wilson coefficients, including next-to-leading-order corrections [21], have numerical values ($m_t = 166$ GeV)

$$c_7^{\text{eff}} = -0.330, \quad c_9^{\text{eff}} = c_9 + Y(q^2),$$
$$(33)$$
$$c_9 = 4.182, \quad c_{10} = -4.234,$$

where the function Y denotes contributions from the one-loop matrix elements of four-quark operators (see Ref. [21] for details), and has absorptive parts for $q^2 > 4m_c^2$. In addition to the short-distance contributions, there are also $c\bar{c}$ resonant intermediate states such as $J/\psi, \psi'$, etc., which we will take into account by utilizing e^+e^- annihilation data, as described in Ref. [22].

Defining the angle θ between the three-momentum vectors of μ^- and the photon in the dilepton center-of-mass system, and recalling the scaled energy variable $x \equiv 2E/M_{B_s}$ in the B_s rest frame, we obtain the differential decay rate ($\hat{m}_i \equiv m_i/M_{B_s}$)

$$\frac{d\Gamma(\bar{B}_s \to \gamma\mu^+\mu^-)}{dx\, d\cos\theta} = \frac{G_F^2 \alpha^3 M_{B_s}^5}{2^{11}\pi^4} |V_{tb}V_{ts}^*|^2 x^3$$

$$\times \left(1 - \frac{4\hat{m}_\mu^2}{1-x}\right)^{1/2} [B_0(x) + B_1(x)\cos\theta$$

$$+ B_2(x)\cos^2\theta].$$
$$(34)$$

Here, we have summed over the spins of the particles in the final state, and have introduced the auxiliary functions

$$B_0 = (1 - x + 4\hat{m}_\mu^2)(F_1 + F_2) - 8\hat{m}_\mu^2 |c_{10}|^2 (F_V^2 + F_A^2),$$

$$B_1 = 8\left(1 - \frac{4\hat{m}_\mu^2}{1-x}\right)^{1/2} \text{Re}\{c_{10}[c_9^{\text{eff}*}(1-x)F_V F_A$$

$$+ c_7^{\text{eff}} \hat{m}_b (F_V F_{TA} + F_A F_{TV})]\},$$

$$B_2 = (1 - x - 4\hat{m}_\mu^2)(F_1 + F_2),$$
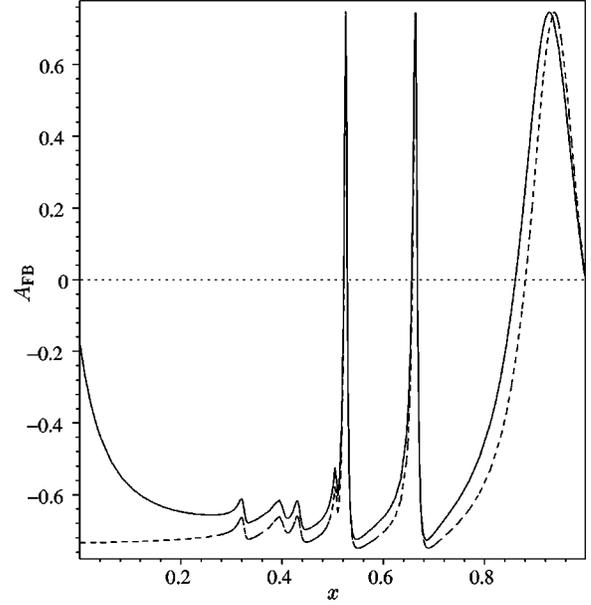$$(35)$$

with the form factors defined in Eq. (11), and

FIG. 2. SM prediction for the FB asymmetry of μ^- in the decay $\bar{B}_s \to \gamma\mu^+\mu^-$ as a function of $x \equiv 2E/M_{B_s}$, using the form factors given in Eq. (28) (solid curve) and including the effects of $c\bar{c}$ resonances. For comparison, we also show the distribution obtained by utilizing the leading-order LEET form factor relation in Eq. (18) (dashed curve).

$$F_1 = (|c_9^{\text{eff}}|^2 + |c_{10}|^2)F_V^2 + \frac{4|c_7^{\text{eff}}|^2 \hat{m}_b^2}{(1-x)^2} F_{TV}^2$$

$$+ \frac{4 \text{Re}(c_7^{\text{eff}} c_9^{\text{eff}*}) \hat{m}_b}{1-x} F_V F_{TV},$$

$$F_2 = (|c_9^{\text{eff}}|^2 + |c_{10}|^2)F_A^2 + \frac{4|c_7^{\text{eff}}|^2 \hat{m}_b^2}{(1-x)^2} F_{TA}^2$$

$$+ \frac{4 \text{Re}(c_7^{\text{eff}} c_9^{\text{eff}*}) \hat{m}_b}{1-x} F_A F_{TA}.$$
$$(36)$$

Recall that the Wilson coefficient c_9^{eff} [Eq. (33)] depends on x via $q^2 = M_{B_s}^2(1-x)$.

The term odd in $\cos\theta$ in Eq. (34) produces a FB asymmetry, defined as[5]

$$A_{FB}(x) = \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{dx\, d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dx\, d\cos\theta}}{\int_0^1 d\cos\theta \frac{d\Gamma}{dx\, d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dx\, d\cos\theta}},$$

$$(37)$$

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[5]Note that the FB asymmetry is equivalent to the asymmetry in the l^+ and l^- energy spectra discussed in Ref. [1]. Our results in Eqs. (34) and (38) agree with those given in that paper.

which is given by

$$A_{\mathrm{FB}}(x) = 3 \left(1 - \frac{4\hat{m}_\mu^2}{1-x} \right)^{1/2}$$

$$\times \frac{\mathrm{Re} \{ c_{10} [c_9^{\mathrm{eff}*}(1-x)F_V F_A + c_7^{\mathrm{eff}} \hat{m}_b (F_V F_{TA} + F_A F_{TV})] \}}{[(F_1+F_2)(1-x+2\hat{m}_\mu^2) - 6\hat{m}_\mu^2 |c_{10}|^2 (F_V^2 + F_A^2)]}. \tag{38}$$

Note that there are also nonfactorizable radiative corrections to the asymmetry which are not contained in the transition form factors.[6]

We plot in Fig. 2 the FB asymmetry as a function of the scaled photon energy x, by using our model for the form factors, Eq. (28), and the universal form factors, Eq. (18). In the latter case, omitting the nonfactorizable corrections the dependence on the form factors drops out completely, and so the asymmetry is fully determined by the Wilson coefficients.

From Fig. 2 one infers an interesting feature of $A_{\mathrm{FB}}(x)$ in the SM: namely, for a given photon energy $x = x_0$, and far from the $c\bar{c}$ resonances, the FB asymmetry vanishes. As can be seen from Fig. 2, the $1/M_B$ and $1/E$ corrections to the form factors, which are taken into account by our form factor model, Eq. (28), shift the location of the zero by only a few percent, and do not change the qualitative picture of the asymmetry for $x \gtrsim 0.4$. Using the numerical values of the standard model Wilson coefficients given in Eq. (33), together with $m_b = 4.4$ GeV, we obtain $x_0 \simeq 0.85$–0.88 depending on the form factors used. Notice again that the location of zero is further affected by nonfactorizable radiative corrections [14].

We would like to emphasize that the absence of the zero in the SM forward-backward asymmetry in the region $x \gtrsim 0.7$ reported in [3] is due to using the form factors of Ref. [4], inconsistent with the rigorous constraints discussed in the present paper. In view of this, the various distributions and asymmetries calculated in Refs. [2,3] with the form factors of Ref. [4] should be revised.

IV. CONCLUSIONS

We have analyzed the form factors that describe the $B \to \gamma$ transition, and investigated their implications for the FB asymmetry of the muon in the decay $\bar{B}_s \to \gamma \mu^+ \mu^-$, within the SM. Our results are as follows.

1. We have derived an exact relation for the form factors F_{TA} and F_{TV} of the $B \to \gamma$ transition induced by tensor and pseudotensor currents at maximum photon energy:

$$F_{TA}(E_{\max}) = F_{TV}(E_{\max}). \tag{39}$$

2. We have investigated the resonance structure of the form factors at $q^2 \simeq M_B^2$ and found that singularities of F_T

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[6]In the case of the $B \to K^* l^+ l^-$ decay these corrections were analyzed in [14].

and F_{TV} are located much closer to the edge of the physical region (i.e., $q^2 = M_B^2$) than those of the form factors F_{TA} and F_A. Hence we expect F_T and F_{TV} to rise rapidly as $q^2 \to M_B^2$ but F_A and F_{TA} to remain relatively flat, so that at $q^2 \simeq M_B^2$ we have the relation

$$F_A \simeq F_{TA} \ll F_V \simeq F_{TV}. \tag{40}$$

This behavior indicates a strong violation of the Isgur-Wise relations between the form factors at large q^2.

3. We have found a serious discrepancy between the just-mentioned constraints and existing calculations of the form factors F_{TA} and F_{TV} from QCD sum rules [4] and quark models [5]. (i) The form factors of Ref. [4] violate both the exact constraint (39) and the relations expected from the large energy effective theory (18). (ii) The form factors of Ref. [5] signal a very strong violation of the LEET relation [Eq. (18)]. Moreover, the vanishing of the form factors F_{TA} and F_{TV} at $q^2 = M_B^2$ in [5] contradicts the resonance structure of these form factors.

4. We would like to stress that there is an important relation between the universal form factors describing the $B_u \to \gamma$ and $B_d \to \gamma$ transitions:

$$\zeta_\perp^{B_u \to \gamma}(E, M_B) = Q_u / Q_d \, \zeta_\perp^{B_d \to \gamma}(E, M_B), \tag{41}$$

where $Q_{u,d}$ represent the charge of u and d quarks. As a consequence of this relation, the form factors $F_{A,V,TA,TV}$ of the $B_u \to \gamma$ transition have opposite sign, and their moduli are approximately twice as big as the corresponding form factors of the $B_d \to \gamma$ transition.

Furthermore, it is worth emphasizing that this relation has not been properly taken into account in Ref. [4] when using the $B_u \to \gamma$ form factors of Ref. [19] for the description of the $B_{d,s} \to \gamma l^+ l^-$ decay.

5. By using the exact relation between the form factors F_{TA} and F_{TV} at $E_{\max}$, the resonance structure of the form factors in the region $q^2 \simeq M_B^2$, and the LEET relations $F_A \simeq F_V \simeq F_{TA} \simeq F_{TV}$ valid for $E \gg \Lambda_{\mathrm{QCD}}$, we proposed a simple parametrization for the form factors:

$$F_i(E) = \beta_i \frac{M_B f_B}{\Delta_i + E}, \quad i = A, V, TA, TV. \tag{42}$$

The numerical parameters (Table I) have been fixed by utilizing reliable data on the form factors at large and intermediate values of the photon energy.

6. We have applied our form factor model to the FB asymmetry of the muon in $\bar{B}_s \to \gamma \mu^+ \mu^-$ decay. Comparing the distribution of A_{FB} based on these form factors with the one obtained by using the leading-order LEET form factors shows that the behavior of the FB asymmetry remains essentially unchanged in the region $x = 2E/M_{B_s} \gtrsim 0.4$.

Our analysis confirms the result of Ref. [1] that the shape of the FB asymmetry as well as the location of its zero are typical for the SM. We point out that the asymmetries and distributions reported in a number of recent publications

[2,3] should be revised as they are based on the form factors of Ref. [4] which are inconsistent with the rigorous constraints on the form factors.

According to the above results we conclude that the FB asymmetry in the $B \to \gamma l^+ l^-$ decay, particularly its zero arising in the SM, can be predicted with small theoretical uncertainties. This is similar to the decay $B \to K^* l^+ l^-$, where a full next-to-leading-order calculation (second reference in [14]) shows that a measurement of the zero of the FB asymmetry would allow a determination of $c_7^{\text{eff}}/\text{Re}(c_9^{\text{eff}})$ at the 10% level. (This order of magnitude should also hold in the case of the $B \to \gamma l^+ l^-$ decay.)

To sum up, the study of the decay $B_s \to \mu^+ \mu^- \gamma$ at future hadron collider experiments will provide complementary information on the structure of the underlying effective Hamiltonian describing $b \to s l^+ l^-$ transitions.

ACKNOWLEDGMENTS

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