k_T factorization of exclusive processes

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We prove the k_T factorization theorem in perturbative QCD (PQCD) for exclusive processes by considering $\pi\gamma^* \to \gamma(\pi)$ and $B \to \gamma(\pi) l \bar{\nu}$. The relevant form factors are expressed as the convolution of hard amplitudes with two-parton meson wave functions in the impact parameter b space, b being conjugate to the parton transverse momenta k_T. The point is that on-shell valence partons carry longitudinal momenta initially, and acquire k_T through collinear gluon exchanges. The b-dependent two-parton wave functions with an appropriate path for the Wilson links are gauge-invariant. The hard amplitudes, defined as the difference between the parton-level diagrams of on-shell external particles and their collinear approximation, are also gauge-invariant. We compare the predictions for two-body nonleptonic B meson decays derived from k_T factorization (the PQCD approach) and from collinear factorization (the QCD factorization approach).

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I. INTRODUCTION

Both collinear and k_T factorizations are the fundamental tools of perturbative QCD (PQCD), where k_T denotes parton transverse momenta. For inclusive processes, consider deeply inelastic scattering (DIS) of a hadron, carrying a momentum p, by a virtual photon, carrying a momentum q. Collinear factorization [1] and k_T factorization [2–4] apply when DIS is measured at a large and small Bjorken variable $x_B \equiv -q^2/(2p\cdot q)$, respectively. The cross section is written as the convolution of a hard subprocess with a hadron distribution function in a parton momentum fraction x in the former, and in both x and k_T in the latter. When x_B is small, $x \geqslant x_B$ can reach a small value, at which k_T is of the same order of magnitude as the longitudinal momentum xp, and not negligible. For exclusive processes, such as hadron form factors, collinear factorization was developed in [5–8]. The range of a parton momentum fraction x, contrary to that in the inclusive case, is not experimentally controllable, and must be integrated over between 0 and 1. Hence, the end-point region with a small x is not avoidable. If there is no end-point singularity developed in a hard amplitude, collinear factorization works. If such a singularity occurs, indicating the breakdown of collinear factorization, k_T factorization should be employed. Since the k_T factorization theorem was proposed [9,10], there have been wide applications to various processes [11]. However, a rigorous proof is not yet available.

Based on the concepts of collinear and k_T factorizations, the PQCD [12–15] and QCD factorization (QCDF) [16] approaches to exclusive B meson decays have been developed, respectively. While applying collinear factorization to the semileptonic decay $B \to \pi \ell \bar{\nu}$ at large recoil, an end-point singularity from $x \to 0$ was observed [17]. Some authors then concluded that PQCD is not applicable to these decays even in the heavy quark limit [16]. According to the above explanation, this conclusion is obviously too strong. We would rather conclude that it is collinear factorization which fails, and that exclusive B meson decays demand k_T factorization. Retaining the dependence on the parton transverse momentum k_T, and resumming the resultant double logarithms $\alpha_s \ln^2 k_T$ into a Sudakov form factor [12], the singularity does not exist. PQCD is then self-consistent and reliable as an expansion in a small coupling constant α_s [18–20].

In this paper we shall prove the factorization theorem with the k_T dependence included into two-parton meson wave functions and into hard amplitudes. In our previous works we have proposed a simple all-order proof of the collinear factorization theorem for the exclusive process $\pi\gamma^* \to \gamma(\pi)$ and $B \to \gamma(\pi) l \bar{\nu}$ up to the two-parton twist-3 level [21]. The proof of the k_T factorization theorem follows similar procedures. We stress that it is more convenient to perform k_T factorization in the impact parameter b space, in which infrared divergences in radiative corrections can be extracted from parton-level diagrams explicitly. We shall explain how to construct a gauge-invariant b-dependent meson wave function defined as a nonlocal matrix element with a special path for the Wilson link. Evaluating this matrix element in perturbation theory, the infrared divergences in the parton-level diagrams are exactly reproduced.

We emphasize that predictions for a physical quantity from the k_T factorization theorem are gauge-invariant, even though three-parton wave functions are not included. The valence partons, carrying only longitudinal momenta, are initially on-shell. They acquire the transverse degrees of freedom through collinear gluon exchanges, before participating in hard scattering. Therefore, the parton-level amplitudes are gauge-invariant. A hard amplitude, derived from the parton-level amplitudes with the gauge-invariant and infrared-divergent meson wave function being subtracted, is then gauge-invariant and infrared-finite. At last, we obtain gauge-invariant and infrared-finite predictions by convoluting the

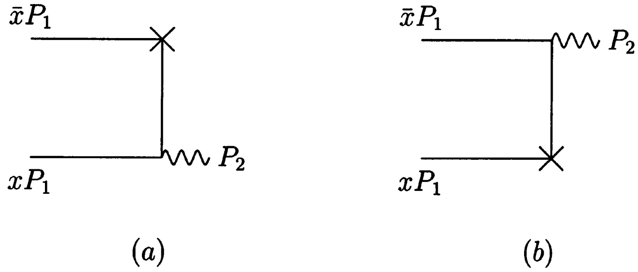

FIG. 1. Lowest-order diagrams for the process $\pi\gamma^* \to \gamma$.

hard amplitude with a model wave function, which is determined from nonperturbative methods (such as lattice QCD and QCD sum rules).

II. FACTORIZATION OF $\pi\gamma^* \to \gamma(\pi)$

We first prove the k_T factorization theorem for the exclusive process $\pi\gamma^* \to \gamma$. This process, though containing no end-point singularity, is simple and appropriate for a demonstration. The momentum P_1 (P_2) of the initial-state pion (final-state photon) is chosen as

$$P_1=(P_1^+,0,\mathbf{0}_T)=\frac{Q}{\sqrt{2}}(1,0,\mathbf{0}_T),$$

$$P_2=(0,P_2^-,\mathbf{0}_T)=\frac{Q}{\sqrt{2}}(0,1,\mathbf{0}_T). \tag{1}$$

We concentrate on the kinematic region with large $Q=\sqrt{-q^2}$, $q=P_2-P_1$ being the momentum transfer from the virtual photon, in which the scattering mechanism is governed by PQCD. The lowest-order diagrams are displayed in Fig. 1. Assume that the on-shell u and $\bar{u}$ quarks carry the fractional momenta $\bar{x}P_1$ and xP_1, respectively, with $\bar{x}\equiv 1-x$. The reason for considering an arbitrary x will become clear later. Figure 1(a) gives the parton-level amplitude

$$\mathcal{G}^{(0)}(x)=-ie^2\bar{u}(xP_1)\not{\epsilon}\frac{\not{P}_2-x\not{P}_1}{(P_2-xP_1)^2}\gamma_\mu u(\bar{x}P_1), \tag{2}$$

where ϵ denotes the polarization vector of the outgoing photon. Figure 1(b) leads to the same result.

The factorization in the fermion flow is achieved by inserting the Fierz identity

$$I_{ij}I_{lk}=\frac{1}{4}I_{ik}I_{lj}+\frac{1}{4}(\gamma^\alpha)_{ik}(\gamma_\alpha)_{lj}+\frac{1}{4}(\gamma^5\gamma^\alpha)_{ik}(\gamma_\alpha\gamma^5)_{lj}$$
$$+\frac{1}{4}(\gamma^5)_{ik}(\gamma^5)_{lj}+\frac{1}{8}(\gamma^5\sigma^{\alpha\beta})_{ik}(\sigma_{\alpha\beta}\gamma^5)_{lj}, \tag{3}$$

with I being the identity matrix and $\sigma_{\alpha\beta}\equiv i[\gamma_\alpha,\gamma_\beta]/2$. For the momenta chosen in Eq. (1), only the structure $\gamma^5\gamma^\alpha$ with $\alpha=+$ contributes to the wave function at leading twist (twist 2). The other structures contribute at higher twists, and the factorization of the corresponding wave functions is similar.

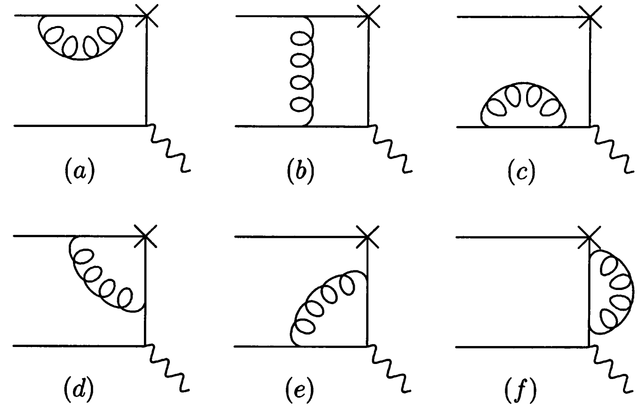

FIG. 2. $O(\alpha_s)$ corrections to Fig. 1(a).

Equation (2) is then factorized into

$$\mathcal{G}^{(0)}(x)=\psi^{(0)}(x)\mathcal{H}^{(0)}(x), \tag{4}$$

where

$$\psi^{(0)}(x)=\frac{1}{4P_1^+}\bar{u}(xP_1)\gamma^5\not{n}_-u(\bar{x}P_1),$$

$$\mathcal{H}^{(0)}(x)=ie^2\frac{\mathrm{tr}(\not{\epsilon}\not{P}_2\gamma_\mu\not{P}_1\gamma^5)}{2xP_1\cdot P_2}, \tag{5}$$

with the dimensionless vector $n_-=(0,1,\mathbf{0}_T)$ on the light cone, defined as the lowest-order distribution amplitude and hard amplitude in perturbation theory, respectively. Note that $\mathcal{G}^{(0)}(x)$, $\psi^{(0)}(x)$, and $\mathcal{H}^{(0)}(x)$ do not depend on a transverse momentum in the $O(\alpha_s^0)$ factorization.

A. $O(\alpha_s)$ factorization

Next we consider the $O(\alpha_s)$ radiative corrections to Fig. 1(a) shown in Figs. 2(a)–2(f), where the gluon carries the loop momentum l. As stated in [21], there are two types of infrared divergences, soft and collinear, which arise from l with the components

$$l^+\sim l^-\sim l_T\sim\bar{\Lambda},$$

$$l^+\sim Q,\ \ l^-\sim\bar{\Lambda}^2/Q,\ \ l_T\sim\bar{\Lambda}, \tag{6}$$

respectively. Here $\bar{\Lambda}$, being of $O(\Lambda_{\mathrm{QCD}})$, represents a small scale. Below we work out the factorization of the collinear enhancement from l parallel to P_1 without integrating out the transverse components l_T. The prescription is basically similar to that for collinear factorization. The wave function and the hard amplitude then become l_T-dependent through collinear gluon exchanges.

We derive the $O(\alpha_s)$ k_T factorization formula, written as the convolution over the momentum fraction ξ and over the impact parameter b:

$$\mathcal{G}^{(1)}(x)=\sum_{i=a}^{f}\mathcal{G}_i^{(1)}(x),$$

$$\mathcal{G}_i^{(1)}(x)=\int d\xi \frac{d^2b}{(2\pi)^2}\phi_i^{(1)}(x,\xi,b)H^{(0)}(\xi,b)+\psi^{(0)}(x)\mathcal{H}_i^{(1)}(x). \quad (7)$$

The above expression, with the $O(\alpha_s)$ wave functions $\phi_i^{(1)}(x,\xi,b)$ and $H^{(0)}(\xi,b)$ specified, defines the $O(\alpha_s)$ hard amplitudes $\mathcal{H}_i^{(1)}(x)$, which do not contain collinear divergences. It is now obvious why we consider an arbitrary x for the parton-level diagrams in Figs. 1 and 2: we can obtain the functional form of $\mathcal{H}_i^{(1)}(x)$ in x. Equation (7) is a consequence of our assertion that partons acquire transverse degrees of freedom through collinear gluon exchanges: $\mathcal{H}^{(1)}$, convoluted with the lowest-order l_T-independent $\psi^{(0)}$, is then identical to that in collinear factorization. As explained later, this consequence is crucial for constructing gauge-invariant hard amplitudes.

Figures 2(a) and 2(c) are self-energy corrections to the external lines. In this case, the loop momentum l does not flow through the hard amplitude. The $O(\alpha_s)$ wave functions extracted from these two diagrams are the same as in the collinear factorization [21]. We simply quote the results

$$\phi_a^{(1)}(x,\xi,b)=\frac{-ig^2C_F}{4P_1^+}\int \frac{d^4l}{(2\pi)^4}\bar{u}(xP_1)\gamma^5\not{n}_-\frac{1}{\bar{x}\not{P}_1}\times\gamma^\nu\frac{\bar{x}\not{P}_1+\not{l}}{(\bar{x}P_1+l)^2}\gamma_\nu u(\bar{x}P_1)\frac{1}{l^2}\delta(\xi-x), \quad (8)$$

$$\phi_c^{(1)}(x,\xi,b)=\frac{-ig^2C_F}{4P_1^+}\int \frac{d^4l}{(2\pi)^4}\bar{u}(xP_1)\gamma^\nu\frac{x\not{P}_1-\not{l}}{(xP_1-l)^2}\times\gamma_\nu\frac{1}{x\not{P}_1}\gamma^5\not{n}_-u(\bar{x}P_1)\frac{1}{l^2}\delta(\xi-x). \quad (9)$$

The loop integrand associated with Fig. 2(b) is given by

$$I_b^{(1)}=e^2g^2C_F\bar{u}(xP_1)\gamma^\nu\frac{x\not{P}_1-\not{l}}{(xP_1-l)^2}\not{\epsilon}\frac{\not{P}_2-x\not{P}_1+\not{l}}{(P_2-xP_1+l)^2}\times\gamma_\mu\frac{\bar{x}\not{P}_1+\not{l}}{(\bar{x}P_1+l)^2}\gamma_\nu u(\bar{x}P_1)\frac{1}{l^2}. \quad (10)$$

Inserting the Fierz identity, we obtain the wave function

$$\phi_b^{(1)}(x,\xi,b)=\frac{ig^2C_F}{4P_1^+}\int \frac{d^4l}{(2\pi)^4}\bar{u}(xP_1)\times\frac{\gamma^\nu(x\not{P}_1-\not{l})\gamma^5\not{n}_-(\bar{x}\not{P}_1+\not{l})\gamma_\nu}{(xP_1-l)^2(\bar{x}P_1+l)^2l^2}\times u(\bar{x}P_1)\delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\mathbf{l}_T\cdot\mathbf{b}}. \quad (11)$$

The Fourier transformation introduces the additional factor $\exp(-i\mathbf{l}_T\cdot\mathbf{b})$ into the wave function $\phi_b^{(1)}$ compared to the result in collinear factorization [21], since the hard amplitude depends on l_T in this case.

The integrand associated with the two-particle irreducible diagram in Fig. 2(d) is given by

$$I_d^{(1)}=-e^2g^2C_F\bar{u}(xP_1)\not{\epsilon}\frac{\not{P}_2-x\not{P}_1}{(P_2-xP_1)^2}\gamma^\nu\times\frac{\not{P}_2-x\not{P}_1+\not{l}}{(P_2-xP_1+l)^2}\gamma_\mu\frac{\bar{x}\not{P}_1+\not{l}}{(\bar{x}P+l)^2}\gamma_\nu u(\bar{x}P_1)\frac{1}{l^2}. \quad (12)$$

To collect the leading contribution, γ^ν and γ_ν must be γ^- and $\gamma_-=\gamma^+$, respectively. In the collinear region the following approximation holds:

$$(\not{P}_2-x\not{P}_1)\gamma^\nu(\not{P}_2-x\not{P}_1+\not{l})\approx 2P_2^\nu\not{P}_2, \quad (13)$$

where the l^- and l_T terms, being power-suppressed compared to P_2^-, have been dropped.

The factorization of the collinear enhancement from Fig. 2(d) requires further approximation for the product of the two internal quark propagators [21],

$$\frac{2P_2^\nu}{(P_2-xP_1)^2(P_2-xP_1+l)^2}\approx\frac{n_-^\nu}{n_-\cdot l}\left[\frac{1}{(P_2-xP_1)^2}-\frac{1}{(P_2-xP_1+l)^2}\right], \quad (14)$$

where the numerator $2P_2^\nu$ comes from Eq. (13), and the factor $n_-^\nu/n_-\cdot l$ is exactly the Feynman rule associated with a Wilson line in collinear factorization. Similarly, we have neglected the power-suppressed terms, such as l^2 and $xP_1\cdot l$. The first (second) term on the right-hand side of Eq. (14) corresponds to the case without (with) the loop momentum l flowing through the hard amplitude.

The above eikonal approximation also applies to Fig. 2(e). Hence, the extracted $O(\alpha_s)$ wave functions are written as

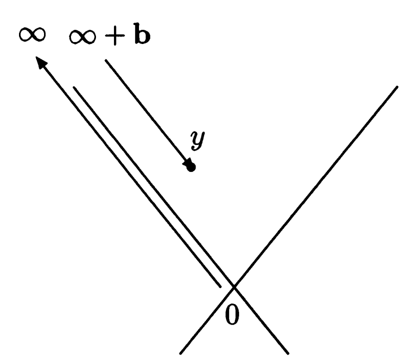

FIG. 3. The path for the Wilson link in a b-dependent two-parton meson wave function.

$$\phi_d^{(1)}(x,\xi,b)=\frac{-ig^2C_F}{4P_1^+}\int\frac{d^4l}{(2\pi)^4}\bar{u}(xP_1)\gamma^5\not{n}_-\frac{\bar{x}\not{P}_1+\not{l}}{(\bar{x}P_1+l)^2}\gamma_\nu u(\bar{x}P_1)\frac{1}{l^2}\frac{n_-^\nu}{n_-\cdot l}\left[\delta(\xi-x)-\delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\mathbf{l}_T\cdot\mathbf{b}}\right],\quad(15)$$

$$\phi_e^{(1)}(x,\xi,b)=\frac{ig^2C_F}{4P_1^+}\int\frac{d^4l}{(2\pi)^4}\bar{u}(xP_1)\gamma_\nu\frac{x\not{P}_1-\not{l}}{(xP_1-l)^2}\gamma^5\not{n}_-u(\bar{x}P_1)\frac{1}{l^2}\frac{n_-^\nu}{n_-\cdot l}\left[\delta(\xi-x)-\delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\mathbf{l}_T\cdot\mathbf{b}}\right],\quad(16)$$

where the first (second) term in the brackets is associated with the first (second) term on the right-hand side of Eq. (14). Because of the Fourier transformation, the second terms acquire the additional factor $\exp(-i\mathbf{l}_T\cdot\mathbf{b})$ compared to the results in collinear factorization.

Figure 2(f) does not exhibit a collinear enhancement, since the radiative gluon gives a self-energy correction to the off-shell internal line. Hence, we have $\phi_f^{(1)}(x,\xi,b)=0$. It is easy to observe that the soft divergences cancel among the $O(\alpha_s)$ radiative corrections. In the soft region of l we have $\exp(-i\mathbf{l}_T\cdot\mathbf{b})\approx 1$ and $l^+\approx 0$, and the two terms in Eqs. (15) and (16) cancel. Similarly, the soft divergences cancel among Figs. 2(a)–2(c). This is the reason we discuss only the factorization of the collinear enhancements.

The above $O(\alpha_s)$ wave functions can be reproduced by the $O(\alpha_s)$ terms of the following nonlocal matrix element in the b space:

$$\phi(x,\xi,b)=i\int\frac{dy^-}{2\pi}e^{-i\xi P_1^+y^-}\langle 0|\bar{u}(y)\gamma_5\not{n}_-P\times\exp\left[-ig\int_0^y ds\cdot A(s)\right]u(0)|\bar{u}(xP_1)u(\bar{x}P_1)\rangle,\quad(17)$$

with the coordinate $y=(0,y^-,\mathbf{b})$. The path for the Wilson link is composed of three pieces: from 0 to ∞ along the direction of n_-, from ∞ to $\infty+\mathbf{b}$, and from $\infty+\mathbf{b}$ back to y along the direction of $-n_-$ as displayed in Fig. 3. We show that the first piece corresponds to the eikonal line associated with the first terms in Eqs. (15) and (16). Fourier transforming the gauge field $A(s)$ into $\tilde{A}(l)$, we have

$$-ig\int_0^\infty dz\exp[iz(n_-\cdot l+i\epsilon)]n_-\cdot\tilde{A}(l)=g\frac{n_-^\alpha}{n_-\cdot l}\tilde{A}_\alpha(l).\quad(18)$$

The field $\tilde{A}(l)$, contracted with other gauge fields, gives the propagator of the gluon attaching the eikonal line. The second piece does not contribute because of the appropriate choice of the small imaginary constant $+i\epsilon$ in the above expression. The third piece corresponds to the eikonal line associated with the second terms in Eqs. (15) and (16). The additional Fourier factor $\exp(-i\mathbf{l}_T\cdot\mathbf{b})$ is a consequence of the shift by $\mathbf{b}$ from the first piece:

$$-ig\int_\infty^{y^-}dz\exp[iz(n_-\cdot l+i\epsilon)-i\mathbf{l}_T\cdot\mathbf{b}]n_-\cdot\tilde{A}(l)=-g\frac{n_-^\alpha}{n_-\cdot l}e^{-i\mathbf{l}_T\cdot\mathbf{b}}e^{il^+y^-}\tilde{A}_\alpha(l),\quad(19)$$

where the Fourier factor $\exp(il^+y^-)$ leads to the function $\delta(\xi-x+l^+/P_1^+)$.

At last, for the evaluation of the lowest-order hard amplitude, we neglect only the minus component l^- in the denominator [see the second term on the right-hand side of Eq. (14)],

$$(P_2-xP_1+l)^2\approx-(2\xi P_1\cdot P_2+l_T^2).\quad(20)$$

Note that in collinear factorization both l^- and l_T are dropped. The b-dependent hard amplitude is then given by

$$H^{(0)}(\xi,b)=\int d^2l_T\mathcal{H}^{(0)}(\xi,l_T)\exp(i\mathbf{l}_T\cdot\mathbf{b}),$$

$$\mathcal{H}^{(0)}(\xi,l_T)=ie^2\frac{tr(\not{\epsilon}\not{P}_2\gamma_\mu\not{P}_1\gamma^5)}{2\xi P_1\cdot P_2+l_T^2}.\quad(21)$$

Equivalently, the above $\mathcal{H}^{(0)}(\xi,l_T)$ is derived by considering an off-shell $\bar{u}$ quark, which carries the momentum $\xi P_1-\mathbf{l}_T$, and the leading structure $\not{P}_1\gamma_5$ associated with the pion, which is the same as in collinear factorization.

B. All-order factorization

In this subsection we present the all-order proof of the k_T factorization theorem for the process $\pi\gamma^*\to\gamma$, and construct the parton-level wave function in Eq. (17). The proof is similar to that for collinear factorization, if it is performed in the impact parameter b space. It will be observed that collinear factorization is the $b\to 0$ limit of k_T factorization. Therefore, we just highlight the differences, and for the rest of the details we refer the reader to [21]. The idea of the proof is based on induction. The factorization of the $O(\alpha_s)$ collinear enhancements has been derived in the previous subsection. Consider $G^{(0)}(x,b)$ and $G^{(1)}(x,b)$ defined via

$$\mathcal{G}^{(0),(1)}(x)\equiv\mathcal{G}^{(0),(1)}(x,k_T=0)=\int\frac{d^2b}{(2\pi)^2}G^{(0),(1)}(x,b),\quad(22)$$

which indicates that the integration over the variable b corresponds to an amplitude with $k_T=0$ for external particles. The $O(\alpha_s)$ hard amplitude $H^{(1)}(\xi,b)$ is defined similarly via $\mathcal{H}^{(1)}(\xi)$. We obtain the factorization formula up to $O(\alpha_s)$,

$$G^{(0)}(x,b)+G^{(1)}(x,b)=\int d\xi[\phi^{(0)}(x,\xi,b)+\phi^{(1)}(x,\xi,b)]\times[H^{(0)}(\xi,b)+H^{(1)}(\xi,b)], \quad (23)$$

with $\phi^{(0)}(x,\xi,b)=\psi^{(0)}(x)\delta(\xi-x)$. The summation over all the diagrams is understood.

Assume that the factorization theorem holds up to $O(\alpha_s^N)$,

$$G^{(j)}(x,b)=\sum_{i=0}^{j}\int d\xi\phi^{(i)}(x,\xi,b)H^{(j-i)}(\xi,b), \quad j=1,\ldots,N, \quad (24)$$

where $\phi^{(i)}(x,\xi,b)$ is given by the $O(\alpha_s^i)$ terms in the perturbative expansion of Eq. (17). $H^{(j-i)}(\xi,b)$ stands for the $O(\alpha_s^{j-i})$ infrared-finite hard amplitude. Equations (23) and (24) approach the expressions in collinear factorization as $b\to 0$ as stated above. We shall show that the $O(\alpha_s^{N+1})$ diagrams $\mathcal{G}^{(N+1)}$ in the momentum space are written as the convolution of the $O(\alpha_s^N)$ diagrams $\mathcal{G}^{(N)}$ with the $O(\alpha_s)$ wave function by employing the Ward identity,

$$l_\mu G^\mu(l,k_1,k_2,\ldots,k_n)=0, \quad (25)$$

where G^μ represents a physical amplitude with an external gluon carrying the momentum l and with n external quarks carrying the momenta $k_1,k_2,\ldots,k_n$. All these external particles are on the mass shell. It is known that factorization of a QCD process in momentum, spin, and color spaces requires summation of many diagrams. With the Ward identity, the diagram summation can be handled in an elegant way.

Look for the gluon in a complete set of $O(\alpha_s^{N+1})$ diagrams $\mathcal{G}^{(N+1)}$, one of whose ends attaches the outermost vertex on the upper u quark line in the pion. Let α denote the outermost vertex, and β denote the attachments of the other end of the identified gluon inside the rest of the diagrams. There are two types of collinear configurations associated with this gluon, depending on whether the vertex β is located on an internal line with a momentum along P_1. The quark spinor adjacent to the vertex α is $u(\bar{x}P_1)$. If β is not located on a collinear line along P_1, the component γ^+ in γ^α and the minus component of the vertex β give the leading contribution. If β is located on a collinear line along P_1, β cannot be minus, and both α and β label the transverse components. This configuration is the same as of the self-energy correction to an on-shell particle.

According to the above classification, we decompose the tensor $g_{\alpha\beta}$ appearing in the propagator of the identified gluon into

$$g_{\alpha\beta}=\frac{n_{-\alpha}l_\beta}{n_-\cdot l}-\delta_{\alpha\perp}\delta_{\beta\perp}+\left(g_{\alpha\beta}-\frac{n_{-\alpha}l_\beta}{n_-\cdot l}+\delta_{\alpha\perp}\delta_{\beta\perp}\right). \quad (26)$$

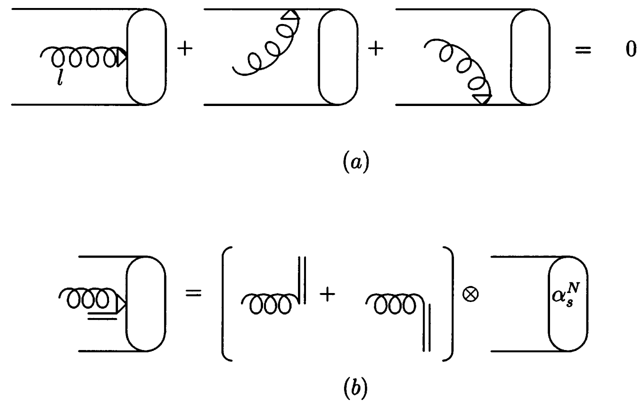

FIG. 4. (a) Ward identity. (b) Factorization of $\mathcal{G}^{(N+1)}$.

The first term on the right-hand side extracts the first type of collinear enhancements, since the lightlike vector $n_{-\alpha}$ selects the plus component of γ^α, and the dominant component $l_{\beta=-}$ in the collinear region selects the minus component of the vertex β. The components $l_{\beta=+,\perp}$ do not change the collinear structure, since they are negligible in the numerators compared to the leading terms proportional to P_1^+ and P_2^-. This can be confirmed by contracting l_β to Figs. 2(d) and 2(e), from which Eq. (14) is obtained. The second term extracts the second type of collinear enhancements. The last term does not contribute a collinear enhancement due to the equation of motion for the u quark. We shall concentrate on the factorization of $\mathcal{G}_\parallel^{(N+1)}$ corresponding to the first term on the right-hand side of Eq. (26), and the factorization associated with the second term can be included simply by following the procedure in [21].

Those diagrams with Figs. 2(a) and 2(b) as the $O(\alpha_s)$ subdiagrams are excluded from the set of $\mathcal{G}_\parallel^{(N+1)}$ as discussing the first type of collinear configurations, since the identified gluon does not attach a line parallel to P_1. Consider the physical amplitude, in which the two on-shell quarks and one on-shell gluon carry the momenta $\bar{\xi}P_1$, xP_1, and l, respectively. Figure 4(a), describing the Ward identity, contains a complete set of contractions of l_β, since the second and third diagrams have been added back. The second and third diagrams in Fig. 4(a) lead to

$$l_\beta\frac{1}{\bar{\xi}\not{P}_1-\not{l}}\gamma^\beta u(\bar{\xi}P_1)=\frac{1}{\bar{\xi}\not{P}_1-\not{l}}(\not{l}-\bar{\xi}\not{P}_1+\bar{\xi}\not{P}_1)u(\bar{\xi}P_1)=-u(\bar{\xi}P_1), \quad (27)$$

$$l_\beta\bar{u}(xP_1)\gamma^\beta\frac{1}{x\not{P}_1-\not{l}}=-\bar{u}(xP_1), \quad (28)$$

respectively. The terms $u(\bar{\xi}P_1)$ and $\bar{u}(xP_1)$ at the ends of the above expressions correspond to the $O(\alpha_s^N)$ diagrams.

Figure 4(b) shows that the diagrams $\mathcal{G}_\parallel^{(N+1)}$ associated with the first term in Eq. (26) are factorized into the convolution of the parton-level $O(\alpha_s^N)$ diagrams $\mathcal{G}^{(N)}$ with the $O(\alpha_s)$ collinear piece extracted from Fig. 2(d). The double line represents the Wilson line. The first diagram means that the gluon momentum does not flow into $\mathcal{G}^{(N)}$, while in the second diagram the gluon momentum does. Similar reasoning applies to the identified gluon, one of whose ends attaches the outermost vertex of the lower $\bar{u}$ quark line. Substituting Eq. (24) into $G^{(N)}(\xi,b)$ in the b space on the right-hand side of Fig. 4(b), and following the procedure in [21], we arrive at

$$G^{(N+1)}(x,b)=\sum_{i=0}^{N+1}\int d\xi\phi^{(i)}(x,\xi,b)H^{(N+1-i)}(\xi,b), \tag{29}$$

with the infrared-finite $O(\alpha_s^{N+1})$ hard amplitude $H^{(N+1)}$. Equation (29) implies that all the collinear enhancements in the process $\pi\gamma^*\to\gamma$ can be factorized into the wave function in Eq. (17) order by order.

C. Gauge invariance

We now demonstrate the gauge invariance of the k_T factorization theorem. Equation (17) is explicitly gauge-invariant because of the presence of the Wilson link from 0 to y [3,22]. Below we argue that hard amplitudes in k_T factorization are also gauge-invariant. Equation (7) approaches the collinear factorization under the approximation

$$\phi^{(1)}(x,\xi,b)\approx\phi^{(1)}(x,\xi,0)\equiv\psi^{(1)}(x,\xi), \tag{30}$$

with $\psi^{(1)}(x,\xi)$ being the distribution amplitude in collinear factorization. The integration of the hard amplitude $H^{(0)}(\xi,b)$ over b gives $\mathcal{H}^{(0)}(\xi,l_T=0)$. Hence, we have the collinear factorization formula,

$$\mathcal{G}^{(1)}(x)=\int d\xi\psi^{(1)}(x,\xi)\mathcal{H}^{(0)}(\xi)+\psi^{(0)}(x)\mathcal{H}^{(1)}(x), \tag{31}$$

where the summation over the diagrams has been suppressed. Since $\mathcal{G}^{(1)}(x)$, $\psi^{(1)}(x,\xi)$, and $\mathcal{H}^{(0)}(\xi)$ are gauge-invariant in collinear factorization, $\mathcal{H}^{(1)}(x)$ is gauge-invariant. From Eq. (7), the gauge invariance of $\phi^{(1)}(x,\xi,b)$ stated above, together with the gauge invariance of $\mathcal{G}^{(1)}(x)$ and $\mathcal{H}^{(1)}(x)$, then imply the gauge invariance of $H^{(0)}(\xi,b)$. Similarly, the k_T factorization formula of $O(\alpha_s^2)$,

$$\mathcal{G}^{(2)}(x)=\int d\xi\frac{d^2b}{(2\pi)^2}[\phi^{(2)}(x,\xi,b)H^{(0)}(\xi,b)+\phi^{(1)}(x,\xi,b)H^{(1)}(\xi,b)]+\psi^{(0)}(x)\mathcal{H}^{(2)}(x), \tag{32}$$

leads to the gauge invariance of $H^{(1)}(\xi,b)$: Both $\mathcal{G}^{(2)}(x)$ and $\psi^{(0)}(x)\mathcal{H}^{(2)}(x)$ are gauge-invariant in collinear factorization, and all $\phi^{(i)}(x,\xi,b)$ are gauge-invariant as explained previously. The gauge invariance of $H^{(0)}(\xi,b)$ stated above then implies the gauge invariance of $H^{(1)}(\xi,b)$. Therefore, the hard amplitudes in k_T factorization are gauge-invariant at all orders.

Equation (17) plays the role of an infrared regulator for parton-level diagrams. A hard amplitude then corresponds to the regularized parton-level diagrams. After determining the gauge-invariant infrared-finite hard amplitude $H(x,b)$, we convolute it with the physical two-parton pion wave function, whose all-order gauge-invariant definition is given by

$$\phi(x,b)=i\int\frac{dy^-}{2\pi}e^{-ixP_1^+y^-}\langle 0|\bar{u}(y)\gamma_5 \not{n}_- P\exp\left[-ig\int_0^y ds\cdot A(s)\right]u(0)|\pi(P_1)\rangle. \tag{33}$$

The valence-quark state $|\bar{u}(xP_1)u(\bar{x}P_1)\rangle$ has been replaced by the pion state $|\pi(P_1)\rangle$, and the pion decay constant f_π has been omitted. The relevant form factor F for the process $\pi\gamma^*\to\gamma$ is then expressed as

$$F=\int dx\frac{d^2b}{(2\pi)^2}\phi(x,b)H(x,b). \tag{34}$$

We conclude that predictions derived from the k_T factorization theorem are gauge-invariant and infrared-finite.

The k_T factorization theorem for the pion form factor involved in the process $\pi\gamma^*\to\pi$ can be proved in the same way. The $O(\alpha_s)$ factorization is similar to the collinear factorization performed in [21]. The only difference is the extra Fourier factor $\exp(-i\mathbf{l}_T\cdot\mathbf{b})$ associated with the diagrams, in which the loop momentum flows through the hard amplitude. Following the steps in Sec. II A, the eikonal line can be constructed from the diagrams with collinear gluons attaching the hard amplitude and the outgoing pion. The decomposition in Eq. (26) and the whole procedure presented above then apply. That is, the all-order proof is also similar to that of collinear factorization [21]. Compared to the process $\pi\gamma^*\to\gamma$, the structures γ_5 and $\gamma_5\sigma^{\alpha\beta}$ from the Fierz identity contribute, and the corresponding twist-3 pion wave functions appear.

III. FACTORIZATION OF $B\to\gamma(\pi)l\bar{\nu}$

In this section we prove the k_T factorization theorem for the radiative decay $B\to\gamma l\bar{\nu}$, retaining the transverse degrees of freedom of internal particles, and construct the B meson wave function in the impact parameter b space. We shall discuss only the $O(\alpha_s)$ factorization, and demonstrate that the all-order factorization can be proved in a way similar to collinear factorization [21]. The momentum P_1 of the B meson and the momentum P_2 of the outgoing on-shell photon are chosen as

$$P_1=\frac{M_B}{\sqrt{2}}(1,1,\mathbf{0}_T),\quad P_2=\frac{M_B}{\sqrt{2}}(0,\eta,\mathbf{0}_T), \tag{35}$$

where the photon energy fraction η is large enough to justify the applicability of PQCD. Assume that the light spectator quark in the B meson carries the momentum k. In collinear factorization, only the plus component k^+ is relevant through the inner product $k\cdot P_2$ [23]. The lowest-order diagrams for the $B\to\gamma\ell\bar{\nu}$ decay are displayed in Fig. 1, but with the upper quark (virtual photon) replaced by a b quark (W boson).

Below we shall concentrate on Fig. 1(a), because Fig. 1(b) is power-suppressed. Figure 1(a) gives the parton-level amplitude,

$$\mathcal{G}^{(0)}(x)=e\bar{u}(k)\not{\epsilon}\frac{\not{P}_2-\not{k}}{(P_2-k)^2}\gamma_\mu(1-\gamma_5)b(P_1-k), \quad (36)$$

which does not depend on a transverse momentum. Inserting the Fierz identity in Eq. (3) into Eq. (36), we obtain Eq. (4) with

$$\psi^{(0)}(x)=\frac{1}{4P_1^+}\bar{u}(k)\gamma_5\not{n}_-b(P_1-k),$$

$$\mathcal{H}^{(0)}(x)=-e\frac{\mathrm{tr}[\not{\epsilon}\not{P}_2\gamma_\mu(1-\gamma_5)\not{n}_+\gamma^5]P_1^+}{2xP_1\cdot P_2},$$

$$=-e\frac{\mathrm{tr}[\not{\epsilon}\not{P}_2\gamma_\mu(1-\gamma_5)(\not{P}_1+M_B)(\not{n}_+/\sqrt{2})\gamma^5]}{2xP_1\cdot P_2}, \quad (37)$$

with the dimensionless vector $n_+=(1,0,\mathbf{0}_T)$ on the light cone. We have dropped the higher-power term $\not{k}$ in the numerator, and the momentum fraction x is defined by $x=k^+/P_1^+$. For the B meson wave functions, there are two leading-twist components associated with the structures $\gamma_5\gamma^\pm$. For the $B\to\gamma l\bar{\nu}$ decay, we choose the structure $\gamma_5\gamma^+=\gamma_5\not{n}_-$, since $\not{\epsilon}$ in Eq. (37) involves $\gamma_\perp$, and only the structure $\gamma^-\gamma_5=\not{n}_+\gamma_5$ contributes to the hard amplitude.

Next we consider the $O(\alpha_s)$ radiative corrections to Fig. 1(a) shown in Figs. 2(a)–2(f). We discuss the factorization of the soft divergence from the loop momentum $l^\mu\sim(\bar{\Lambda},\bar{\Lambda},\bar{\Lambda})$, where $\bar{\Lambda}$ can be regarded as the B meson and b quark mass difference, $\bar{\Lambda}=M_B-m_b$. The dependence of the B meson wave function on the transverse momentum is generated by soft gluon exchanges. The analysis is similar to that in Sec. II, and we obtain Eq. (7). The factorization of the two-particle reducible diagrams in Figs. 2(a)–2(c) is straightforward. Take Fig. 2(b) as an example, which gives the integrand

$$I_b^{(1)}=ieg^2C_F\bar{u}(k)\gamma^\nu\frac{\not{k}-\not{l}}{(k-l)^2}\not{\epsilon}\frac{\not{P}_2-\not{k}+\not{l}}{(P_2-k+l)^2}\times\gamma_\mu(1-\gamma_5)\frac{\not{P}_1-\not{k}+\not{l}+m_b}{(P_1-k+l)^2-m_b^2}\gamma_\nu b(P_1-k)\frac{1}{l^2}. \quad (38)$$

Employing the eikonal approximation in the heavy-quark limit, we have

$$\frac{\not{P}_1-\not{k}+\not{l}+m_b}{(P_1-k+l)^2-m_b^2}\gamma_\nu b(P_1-k)\approx\frac{v_\nu}{v\cdot l}b(P_1-k), \quad (39)$$

with the velocity $v=P_1/M_B$. The $O(\alpha_s)$ wave function extracted from Eq. (38) is then written as

$$\phi_b^{(1)}(x,\xi,b)=\frac{ig^2C_F}{4P_1^+}\int\frac{d^4l}{(2\pi)^4}\bar{u}(k)\frac{\gamma^\nu(\not{k}-\not{l})}{(k-l)^2l^2}\times\gamma_5\not{n}_-b(P_1-k)\frac{v_\nu}{v\cdot l}\delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\mathbf{l}_T\cdot\mathbf{b}}. \quad (40)$$

The loop integrands associated with Figs. 2(d) and 2(e) are given by

$$I_d^{(1)}=-ieg^2C_F\bar{u}(k)\not{\epsilon}\frac{\not{P}_2-\not{k}}{(P_2-k)^2}\gamma^\nu\frac{\not{P}_2-\not{k}+\not{l}}{(P_2-k+l)^2}\gamma_\mu(1-\gamma_5)\frac{\not{P}_1-\not{k}+\not{l}+m_b}{(P_1-k+l)^2-m_b^2}\gamma_\nu b(P_1-k)\frac{1}{l^2}, \quad (41)$$

$$I_e^{(1)}=ieg^2C_F\bar{u}(k)\gamma_\nu\frac{\not{k}-\not{l}}{(k-l)^2}\not{\epsilon}\frac{\not{P}_2-\not{k}+\not{l}}{(P_2-k+l)^2}\gamma^\nu\frac{\not{P}_2-\not{k}}{(P_2-k)^2}\gamma_\mu(1-\gamma_5)b(P_1-k)\frac{1}{l^2}, \quad (42)$$

respectively. Neglecting the subleading terms proportional to $\not{k}$ and $\not{l}$ in the numerators in comparison with $\not{P}_2$, we have the eikonal approximation

$$\frac{\not{P}_2-\not{k}}{(P_2-k)^2}\gamma^\nu\frac{\not{P}_2-\not{k}+\not{l}}{(P_2-k+l)^2}\approx\frac{n_-^\nu}{n_-\cdot l}\left[\frac{1}{(P_2-k)^2}-\frac{1}{(P_2-k+l)^2}\right]\not{P}_2, \quad (43)$$

similar to Eq. (14). Inserting the Fierz identity, we extract the $O(\alpha_s)$ wave functions,

$$\phi_d^{(1)}(x,\xi,b)=\frac{-ig^2C_F}{4P_1^+}\int\frac{d^4l}{(2\pi)^4}\bar{u}(xP_1)\gamma_5\not{n}_-b(P_1-k)\frac{1}{l^2}\frac{n_-\cdot v}{n_-\cdot l\,v\cdot l}\left[\delta(\xi-x)-\delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\mathbf{l}_T\cdot\mathbf{b}}\right], \quad (44)$$

$$\phi_e^{(1)}(x,\xi,b)=\frac{ig^2C_F}{4P_1^+}\int\frac{d^4l}{(2\pi)^4}\overline{u}(xP_1)\gamma_\nu\frac{\not k-\not l}{(k-l)^2}\gamma_5\not n_- b(P_1-k)\frac{1}{l^2}\frac{n_-^\nu}{n_-\cdot l}\left[\delta(\xi-x)-\delta\left(\xi-x+\frac{l^+}{P_1^+}\right)e^{-i\mathbf{l}_T\cdot\mathbf{b}}\right]. \tag{45}$$

The eikonal approximation in Eq. (39) has been applied. Figure 2(f) does not have the soft divergence due to the off-shell internal quark.

It is obvious that the above $O(\alpha_s)$ parton-level wave functions are similar to those derived in Sec. II: the eikonal line in n_- is the same as in collinear factorization, and the integrands contain the additional Fourier factor $\exp(-i\mathbf{l}_T\cdot\mathbf{b})$, when the loop momentum flows through the hard amplitude. The decomposition in Eq. (26) and the procedure for the all-order proof presented in Sec. II apply to the $B\to\gamma l\overline{\nu}$ decay. We construct a gauge-invariant light-cone B meson wave function,

$$\phi_+(x,b)=i\int\frac{dy^-}{2\pi}e^{-ixP_1^+y^-}\langle 0|\overline{u}(y)\gamma_5\gamma^+ P \times\exp\left[-ig\int_0^y ds\cdot A(s)\right]b_v(0)|B(P_1)\rangle, \tag{46}$$

where b_v is the rescaled b quark field characterized by the velocity v. The lowest-order hard amplitude in the b space is given by Eq. (21) with

$$\mathcal{H}^{(0)}(\xi,l_T) =-e\frac{\mathrm{tr}[\not\epsilon\not P_2\gamma_\mu(1-\gamma_5)(\not P_1+M_B)(\not n_+/\sqrt{2})\gamma^5]}{2\xi P_1\cdot P_2+l_T^2}, \tag{47}$$

where the momentum fraction ξ is defined by $\xi=(k^+-l^+)/P_1^+$. The above expression can be derived by considering an off-shell $\overline{u}$ quark of the momentum $(\xi P_1^+,0,-\mathbf{l}_T)$, and the leading structure $(\not P_1+M_B)(\not n_+/\sqrt{2})\gamma^5$ associated with the B meson, which is the same as in collinear factorization.

As emphasized in the Introduction, the semileptonic decay $B\to\pi l\overline{\nu}$, because of the end-point singularities (the failure of collinear factorization), demands k_T factorization. Its all-order proof is also performed in the same way. Note that for this mode, both the leading-twist B meson wave functions $\phi_\pm$, associated with the structures $\gamma_5\gamma^\pm$, contribute [21].

IV. DISCUSSION

We have explained that the range of a parton momentum fraction x in exclusive processes, contrary to that in inclusive processes, is not experimentally controllable. Hence, the end-point region with a small x is not avoidable. If a hard amplitude develops an end-point singularity in collinear factorization, implying the importance of the end-point region, k_T factorization must be employed. Exclusive B meson decays belong to this category, for which k_T factorization is a more appropriate tool. We have proved the k_T factorization theorem for the processes $\pi\gamma^*\to\gamma(\pi)$ and $B\to\gamma(\pi)l\overline{\nu}$ in this paper. The proof performed in the impact parameter b space indicates that collinear factorization is the $b\to 0$ limit of k_T factorization.

The prescriptions for determining wave functions and hard amplitudes in the k_T factorization theorem are summarized as follows.

(i) A two-parton b-dependent wave function is factorized from parton-level diagrams in the same way as in collinear factorization (for example, under the same eikonal approximation), but the loop integrand is associated with an additional Fourier factor $\exp(-i\mathbf{l}_T\cdot\mathbf{b})$, when the loop momentum l flows through a hard amplitude.

(ii) A k_T-dependent hard amplitude is obtained in the same way as in collinear factorization, but considering off-shell external partons, which carry the fractional momenta $k=xP+\mathbf{k}_T(k^2=-k_T^2)$, P being the external meson momenta. Then Fourier transform this hard amplitude into the b space.

(iii) The insertion of the Fierz identity to separate the fermion flow between a wave function and a hard amplitude is the same as in collinear factorization. Take the process $\pi\gamma^*\to\pi$ discussed in Sec. II as an example. Up to the twist-3 accuracy for the initial pion, adopt the structures $\gamma_5\gamma^+$, γ_5, and $\gamma_5\sigma^{\alpha\beta}$ with $\alpha,\beta=\pm$, i.e., without the $\perp$ components.

Under the above prescriptions, the Wilson link for the b-dependent wave function is the same as in collinear factorization, but with a shift $\mathbf{b}$ between the two pieces of paths along the light cone. Both the b-dependent two-parton meson wave functions and hard amplitudes are gauge-invariant in k_T factorization, without introducing three-parton wave functions. Therefore, predictions for a physical quantity obtained from the k_T factorization theorem are gauge-invariant. For inclusive processes in small x_B physics, the gauge invariance of the unintegrated gluon distribution function and of the hard subprocess of Reggeized gluons, being also off-shell by $-k_T^2$, is ensured in a similar way. The distinction is that the structures of γ matrices from the Fierz identity are replaced by eikonal vertices, which contain only the longitudinal components [3].

There are more differences between the k_T factorizations of inclusive and exclusive processes. Inclusive processes involve a single scale, and only single logarithms. Exclusive processes involve two scales (when a valence parton is soft, another is fast) and double logarithms. That is, no rapidity ordering is assumed [24]. Hence, the required resummation techniques are different. The definition of meson wave functions constructed in this work serves as the starting point of k_T resummation [25–27]. The resultant Sudakov factor smears the end-point singularity in the semileptonic decay $B\to\pi l\bar{\nu}$ by increasing the magnitude of k_T though infinitely many gluon exchanges. The perturbative expansion of decay amplitudes then makes sense. Certainly, this conclusion needs to be justified by evaluating next-to-leading-order corrections in α_s in the k_T factorization theorem. If higher-order contributions converge quickly enough, the PQCD approach to exclusive B meson decays will be theoretically solid.

In our next work we shall construct k_T factorization of two-body nonleptonic B meson decays. Below we briefly compare the phenomenological consequences for these decays derived from collinear and k_T factorizations, mentioning only the CP asymmetry in the $B_d^0\to\pi^+\pi^-$ mode. According to the power counting rules of QCDF [16] based on collinear factorization, the factorizable emission diagram in Fig. 5(a) gives the leading contribution of $O(\alpha_s^0)$, since the $B\to\pi$ form factor $F^{B\pi}$ is not calculable. Because Fig. 5(a) is real, the strong phase arises from the factorizable annihilation diagram in Fig. 5(b), being of $O(\alpha_s m_0/M_B)$, and from the vertex correction in Fig. 5(c), being of $O(\alpha_s)$. For m_0/M_B slightly smaller than unity, Fig. 5(c) is the leading source of strong phases in collinear factorization (QCDF). In k_T factorization, the power counting rules change. The factorizable emission diagram is calculable and of $O(\alpha_s)$ as indicated in Fig. 5(d). The factorizable annihilation diagram has the same power counting as for Fig. 5(b). The vertex correction becomes of $O(\alpha_s^2)$ as shown in Fig. 5(e). Therefore, Fig. 5(b) contributes the leading strong phase in k_T factorization (PQCD). The strong phases from Figs. 5(b) and 5(c) are opposite in sign, and the former has a large magnitude. This is the reason QCDF prefers a small and positive CP asymmetry $C_{\pi\pi}$ [28], while PQCD prefers a large and negative $C_{\pi\pi}\sim-30\%$ [15,29,30]. It is expected that in the near future the two different approaches to exclusive B meson decays, based on collinear and k_T factorizations, could be distinguished by experiments [31,32].

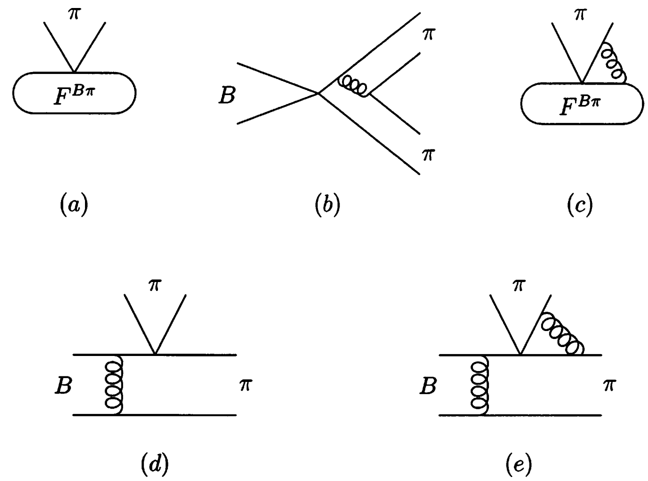

FIG. 5. Diagrams contributing to the $B_d^0\to\pi^+\pi^-$ decay.

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[1] G. Sterman, *An Introduction to Quantum Field Theory* (Cambridge University Press, Cambridge, England, 1993).
[2] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B **242**, 97 (1990); Nucl. Phys. **B366**, 135 (1991).
[3] J.C. Collins and R.K. Ellis, Nucl. Phys. **B360**, 3 (1991).
[4] E.M. Levin, M.G. Ryskin, Yu.M. Shabelskii, and A.G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991).
[5] G.P. Lepage and S.J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. D **22**, 2157 (1980).
[6] A.V. Efremov and A.V. Radyushkin, Phys. Lett. **94B**, 245 (1980).
[7] V.L. Chernyak, A.R. Zhitnitsky, and V.G. Serbo, JETP Lett. **26**, 594 (1977).
[8] V.L. Chernyak and A.R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 544 (1980); Phys. Rep. **112**, 173 (1984).
[9] J. Botts and G. Sterman, Nucl. Phys. **B225**, 62 (1989).
[10] H-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
[11] P. Jain *et al.*, Nucl. Phys. **A666**, 75 (2000); H-n. Li, *ibid.* **A684**, 304 (2001), and references therein.
[12] H-n. Li and H.L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); Phys. Rev. D **53**, 2480 (1996).
[13] C.H. Chang and H-n. Li, Phys. Rev. D **55**, 5577 (1997).
[14] T.W. Yeh and H-n. Li, Phys. Rev. D **56**, 1615 (1997).
[15] Y.Y. Keum, H-n. Li, and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001); Y.Y. Keum and H-n. Li, *ibid.* **63**, 074006 (2001).
[16] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).
[17] A. Szczepaniak, E.M. Henley, and S. Brodsky, Phys. Lett. B **243**, 287 (1990).
[18] T. Kurimoto, H-n. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007 (2002).
[19] H-n. Li, Phys. Rev. D **66**, 094010 (2002).
[20] Z.T. Wei and M.Z. Yang, Nucl. Phys. **B642**, 263 (2002).
[21] H-n. Li, Phys. Rev. D **64**, 014019 (2001); M. Nagashima and H-n. Li, hep-ph/0202127.
[22] J.C. Collins and D.E. Soper, Nucl. Phys. **B194**, 445 (1982).
[23] S. Descotes-Genon and C.T. Sachrajda, hep-ph/0209216; E. Lunghi, D. Pirjol, and D. Wyler, hep-ph/0210091; but see also G.P. Korchemsky, D. Pirjol, and T.M. Yan, Phys. Rev. D **61**, 114510 (2000).
[24] H-n. Li, Phys. Lett. B **405**, 347 (1997); hep-ph/9703328; H-n. Li and J.L. Lim, Eur. Phys. J. C **10**, 319 (1999).

[25] J.C. Collins and D.E. Soper, Nucl. Phys. **B193**, 381 (1981).
[26] H-n. Li, Phys. Rev. D **55**, 105 (1997).
[27] I.V. Musatov and A.V. Radyushkin, Phys. Rev. D **56**, 2713 (1997).
[28] M. Beneke, hep-ph/0207228.
[29] C.D. Lü, K. Ukai, and M.Z. Yang, Phys. Rev. D **63**, 074009 (2001).
[30] Y.Y. Keum, H-n. Li, and A.I. Sanda, hep-ph/0201103; Y.Y. Keum, hep-ph/0209002; hep-ph/0209208; Y.Y. Keum and A.I. Sanda, Phys. Rev. D (to be published), hep-ph/0209014.
[31] Y. Nir, hep-ph/0208080.
[32] J. Rosner, hep-ph/0208243.