High-energy neutrino conversion into an electron-W pair in a magnetic field and its contribution to neutrino absorption

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We calculate the conversion rate of high-energy neutrinos propagating in a constant magnetic field into an electron-W pair ($\nu \to W + e$) from the imaginary part of the neutrino self-energy. Using the exact propagators in a constant magnetic field, the neutrino self-energy has been calculated to all orders in the field within the Weinberg-Salam model. We obtain a compact formula in the limit of $B \ll B_{cr} \equiv m^2/e$. We find that above the process threshold $E^{(th)} \approx 2.2 \times 10^{16}$ eV$\times(B_{cr}/B)$ this contribution to the absorption of neutrinos yields an asymptotic absorption length ≈ 1.1 m $\times(B_{cr}/B)^2 \times (10^{16}$ eV$/E)$.

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I. INTRODUCTION

The study of creation, propagation, energy loss, and absorption of neutrinos in a magnetic field is important in several astrophysical contexts and in early cosmology [1]. The neutrino self-energy and dispersion relation is modified in magnetized media [2–4], and processes where neutrinos radiate electron-positron pairs ($\nu \to \nu + e^+ + e^-$) [5–7] or gammas ($\nu \to \nu + \gamma$) [7–9] have been investigated in the range of energies where it is possible to use an effective four-fermion interaction and rates have been obtained in the limits of weak and strong magnetic fields. As an example of the importance of a macroscopic magnetic field as an effective source of energy loss for energetic neutrinos, we recall that the estimate [5] for the rate $\nu \to \nu + e^+ + e^-$ in the strong magnetic field near the surface of a neutron star is about ten times the rate of pair production in the Coulomb field near the nucleus of metallic iron.

Conversion of neutrinos in W-lepton pairs in the presence of magnetic fields ($\nu_l \to W + l$) should be considered when studying the propagation of neutrinos of sufficiently high energies: we shall show that in this limit this process gives an important contribution to neutrino absorption. A similar process $\nu\gamma \to lW^+$ has been studied by Seckel [10], who shows that at energies above the threshold for W production this process is competitive with $\nu\nu$ scattering at the same center of mass energies.

The process we are considering, where an extremely energetic neutrino creates a real W, is a second order process in the weak coupling constant g, while the radiation of a e^+e^- pair through a virtual W or a virtual Z is a fourth order process. Therefore, there is an energy above which the conversion into an electron-W pair becomes the dominant process. In addition, since we use the electroweak Lagrangian and not an effective low-energy theory, our result is valid also at very high energies, much higher than the W mass; actually, the W-electron decay rate contributes significantly to neutrino absorption only in this limit. Notice that eventually the real W decays and that in about 10.5% of cases the final state contains a neutrino of the same flavor (e.g., $\nu_e \to W + e \to \nu_e e^+ e^-$), so that the process can be thought of as the radiation of a lepton pair; in about 21% of cases the final state contains a neutrino of different flavor (e.g., $\nu_e \to W + e \to \nu_\mu \mu e$), and in the remaining 68.5% of cases the final state contains hadrons.

In this paper we use Schwinger's proper time method [11] to calculate the neutrino self-energy in homogeneous magnetic fields and then we obtain the probability of decay into a W-electron pair by extracting the imaginary part of the self-energy. A similar strategy was used by Tsai and Erber [12] to extract the photon pair creation probability from the vacuum polarization in intense homogeneous magnetic fields.

In Sec. II we briefly review the notation and derive the one-loop neutrino self-energy in constant magnetic field [2]; in Sec. III we obtain the imaginary part of the self-energy and the rate of W-electron pair creation in a magnetic field. The ensuing discussion and conclusions are in Sec. IV.

II. NEUTRINO SELF-ENERGY IN A CONSTANT MAGNETIC FIELD

In this section we review the calculation of the one-loop neutrino self-energy in a homogeneous magnetic field, using the exact fermion and gauge boson propagators in a constant magnetic field [2]. We consider a magnetic field with magnitude B pointing along the positive z direction. The only

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nonvanishing components of the electromagnetic field strength tensor $F^{\mu\nu}$ are $F^{12}=-F^{21}=B$. The exact expressions for the electron $S(x',x'')$ [11,13] and W boson $G^{\mu\nu}(x',x'')$ [2] propagators in a constant magnetic field are obtained using Schwinger's proper time method:

$$S(x',x'')=\phi^*(x',x'')\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(x'-x'')}S(k), \tag{1}$$

$$G^{\mu\nu}(x',x'')=\phi(x',x'')\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(x'-x'')}G^{\mu\nu}(k), \tag{2}$$

where the translationally invariant parts of the propagators are

$$S(k)=i\int_0^\infty\frac{ds}{\cos eBs}\exp\left[-is\left(m^2-i\epsilon+k_\parallel^2+k_\perp^2\frac{\tan eBs}{eBs}\right)\right]\times\left[(m-\not{k}_\parallel)e^{-ieBs\sigma_3}-\frac{\not{k}_\perp}{\cos eBs}\right] \tag{3}$$

and

$$G^{\mu\nu}(k)=i\int_0^\infty\frac{ds}{\cos eBs}\exp\left[-is\left(k_\parallel^2+k_\perp^2\frac{\tan eBs}{eBs}\right)\right]\times\left\{e^{-is(M^2-i\epsilon)}[g_\parallel^{\mu\nu}+(e^{2eFs})_\perp^{\mu\nu}]+\left[\left(k^\mu+k_\lambda F^{\mu\lambda}\frac{\tan eBs}{B}\right)\left(k^\nu+k_\rho F^{\rho\nu}\frac{\tan eBs}{B}\right)+i\frac{e}{2}(F^{\mu\nu}-g_\perp^{\mu\nu}B\tan eBs)\right]\left(\frac{e^{-is(M^2-i\epsilon)}-e^{-is(xM^2-i\epsilon)}}{M^2}\right)\right\}. \tag{4}$$

In the rest of the paper we shall drop the infinitesimal imaginary contribution to the masses $-i\epsilon$, which determines the correct boundary conditions; if necessary it can be easily reintroduced: $m^2\to m^2-i\epsilon$ and $M^2\to M^2-i\epsilon$. In our notation, $-e$ and m are the charge and mass of the electron, M the W mass, x the gauge parameter, $\sigma_3=\sigma^{12}=(i/2)[\gamma^1,\gamma^2]$, and the metric is $g^{\mu\nu}=\mathrm{diag}(-1,+1,+1,+1)$. We choose the electromagnetic vector potential to be $A_\mu=-\frac{1}{2}F_{\mu\nu}x^\nu$ and, therefore, the phase factor in Eqs. (1) and (2), which is independent of the integration path, is [13]

$$\phi(x',x'')=\exp\left[ie\int_{x''}^{x'}dx_\mu A^\mu(x)\right]=\exp\left(i\frac{e}{2}x''_\mu F^{\mu\nu}x'_\nu\right). \tag{5}$$

For any four-vector a^μ we use the notation $a_\parallel^\mu=(a^0,0,0,a^3)$ and $a_\perp^\mu=(0,a^1,a^2,0)$; it is easy to show that

$$(e^{2eFs})^{\mu\nu}=(g^{\mu\nu})_\perp\cos(2eBs)+\frac{F^{\mu\nu}}{B}\sin 2(eBs). \tag{6}$$

Note that the W and Goldstone scalar propagators were obtained in Ref. [2] by introducing a new electromagnetic gauge invariant gauge fixing term (EGF) which is manifestly invariant under electromagnetic gauge transformations. The advantage of the EGF gauge is that the electromagnetic potential has no cross couplings with the W and Goldstone fields and, therefore, these two fields do not mix in the presence of a magnetic field.

In the remainder of this paper we focus our attention on electron-type neutrinos; the generalization to μ and τ neutrinos is straightforward. For the purpose of this work, it would seem convenient to work in the unitary gauge ($x\to\infty$), where the unphysical scalars disappear. However, the W propagator is quite cumbersome in this gauge. We prefer to work in the Feynman gauge ($x=1$), where the expression of the propagator is much simpler. In principle the choice of the Feynman gauge carries the price of calculating an additional bubble diagram: the one with the Goldstone scalar. But this scalar bubble diagram does not contribute to leading order, since it is suppressed by a factor of $m^2/M^2\approx 4.04\times10^{-11}$, and can be neglected. Therefore, we only need to calculate the bubble diagram with a W boson:

$$\Sigma_W(x',x'')=\frac{ig^2}{2}\gamma_R\gamma_\mu S(x',x'')\gamma_\nu\gamma_L G^{\mu\nu}(x',x'') \tag{7}$$

where

$$\gamma_R=\frac{1+\gamma_5}{2},\qquad \gamma_L=\frac{1-\gamma_5}{2}. \tag{8}$$

This expression is translationally invariant, since the W and the electron carry opposite charge and therefore the phase factor ϕ in the W propagator cancels the phase factor ϕ^* in the electron propagator. We can write $\Sigma_W(p)$ in momentum space using

$$\Sigma_W(x',x'')=\int\frac{d^4p}{(2\pi)^4}e^{ip\cdot(x'-x'')}\Sigma_W(p) \tag{9}$$

as

$$\Sigma_W(p)=-\frac{ig^2}{2}\int\frac{d^4k}{(2\pi)^4}\int_0^\infty\frac{ds_1}{\cos z_1}\int_0^\infty\frac{ds_2}{\cos z_2}e^{-is_1[m^2+q_\parallel^2+q_\perp^2(\tan z_1)/z_1]}e^{-is_2[M^2+k_\parallel^2+k_\perp^2(\tan z_2)/z_2]}\gamma_R\gamma_\mu\times\left[(m-\not{q}_\parallel)e^{-iz_1\sigma_3}-\frac{\not{q}_\perp}{\cos z_1}\right][g_\parallel^{\mu\nu}+(e^{2eFs_2})_\perp^{\mu\nu}]\gamma_\nu\gamma_L \tag{10}$$

where

$$q=p-k, \quad z_1=eBs_1, \quad z_2=eBs_2. \tag{11}$$

We now do the straightforward γ algebra, change variables from s_i to z_i, translate the k variables of integration as follows:

$$(k_\parallel,k_\perp)\rightarrow\left(k_\parallel+\frac{z_1}{z_1+z_2}p_\parallel,k_\perp+\frac{\tan z_1}{\tan z_1+\tan z_2}p_\perp\right), \tag{12}$$

and, finally, perform the four Gaussian integrations over the shifted variables k. The result is

$$\Sigma_W(p)=\frac{g^2}{(4\pi)^2}\int_0^\infty\int_0^\infty\frac{dz_1dz_2}{(z_1+z_2)\sin(z_1+z_2)}\times e^{-i(m^2/eB)[z_1+z_2M^2/m^2+(z_1+z_2)\varphi_0]}\times\left[\frac{z_2}{z_1+z_2}\not{p}_\parallel e^{i\sigma_3(z_1+2z_2)}+\frac{\sin z_2}{\sin(z_1+z_2)}\not{p}_\perp\right]\gamma_L+(\text{c.t.}) \tag{13}$$

where

$$\varphi_0=\frac{z_1z_2}{(z_1+z_2)^2}\frac{p_\parallel^2}{m^2}+\frac{\sin z_1\sin z_2}{(z_1+z_2)\sin(z_1+z_2)}\frac{p_\perp^2}{m^2} \tag{14}$$

and the appropriate counterterms (c.t.) are defined such that

$$(\text{c.t.})=-\Sigma_W(p)|_{B=0,\not{p}=0}-\not{p}\left[\frac{\partial\Sigma_W(p)}{\partial\not{p}}\right]_{B=0,\not{p}=0}. \tag{15}$$

The B-independent counterterms are unimportant for the purpose of this work, since they do not contribute to the imaginary part of the self-energy.

In order to more easily isolate the range of variables that give most of the contribution to the integrals and the terms in the integrand that are negligible, we also find it convenient to change integration variables from (z_1,z_2) to (z,u):

$$z=\frac{m^2}{eB}(z_1+z_2)\equiv\frac{1}{\beta}(z_1+z_2)\quad\text{and}$$

$$u=\frac{M^2}{m^2}\frac{z_2}{z_1+z_2}\equiv\frac{1}{\eta}\frac{z_2}{z_1+z_2}, \tag{16}$$

where we have introduced the two parameters $\beta=eB/m^2$ and $\eta=m^2/M^2=4.0376\times10^{-11}$; the resulting expression is

$$\Sigma_W(p)=\frac{g^2}{(4\pi)^2}\left(\frac{eB}{M^2}\right)\int_0^\infty\frac{dz}{\sin\beta z}\times\int_0^{M^2/m^2}du\,e^{-iz[1-\eta u+u+\varphi_0]}\times\left[\eta u\not{p}_\parallel e^{i\sigma_3\beta z(1+\eta u)}+\sin\frac{[\beta z\eta u]}{\sin}\beta z\not{p}_\perp\right]\gamma_L+(\text{c.t.}) \tag{17}$$

with

$$\varphi_0=u(1-\eta u)\frac{p_\parallel^2}{M^2}+\frac{\sin[\beta z(1-\eta u)]\sin[\beta z\eta u]}{\eta\beta z\sin\beta z}\frac{p_\perp^2}{M^2}. \tag{18}$$

III. RATE OF *W*-ELECTRON PAIR CREATION

Because of the oscillating phase $\exp(-iz)$, the main contribution to the integral over z comes from the region where $z\lesssim1$. If we are only interested in neutrinos that travel through a "moderate" magnetic field $eB\ll m^2=eB_{cr}$, which means $\beta\ll1$, we can expand the terms in the integrand in power series of $\beta z\ll1$. Furthermore, since Σ_W is quite small, of order g^2eB/M^2, as can be inferred from the expression (17) or from the explicit calculation in Ref. [2], we can set $\not{p}=0$ in Σ_W.

After this is all done, we obtain

$$\Sigma_W(p)\simeq\frac{g^2}{(4\pi)^2}\left(\frac{eB}{M^2}\right)^2\not{p}_\perp\gamma_L\int_0^\infty dz\,z\int_0^{M^2/m^2}du\,u\left[\frac{2}{3}+\eta u+\frac{1}{3}(\eta u)^2\right]\times e^{-iz[1+u-\eta u+(1/3)(p_\perp^2/m^2)(eB/M^2)^2z^2u^2(1-\eta u)^2]}. \tag{19}$$

Notice that the term in the exponential proportional to z^3 cannot be dropped in general, in spite of the fact that $\beta z\ll1$: the coefficient of $(\beta z)^2$, i.e., $(mp_\perp/M^2)^2$, could be very large when the neutrino is very energetic. In fact, there are two physically interesting regimes that are discriminated by the dimensionless field dynamical parameter

$$\xi\equiv\frac{eBp_\perp}{mM^2}, \tag{20}$$

which can be read from the ratio of the z and z^3 terms in the exponential of Eq. (19) or inferred from kinematical considerations and the momentum change of a virtual e-W pair in a magnetic field.

At low energies, $\xi\ll1$, the z^3 term in the exponential can be dropped, the self-energy is real, and we obtain the result of Ref. [2]; at high energies, $\xi\gg1$, the self-energy acquires an imaginary part that we shall calculate in the following. In this last case we write Σ_W as

$$\Sigma_W(p) = \frac{2}{3}\frac{g^2}{(4\pi)^2}\left(\frac{eB}{M^2}\right)^2 \not{p}_\perp \gamma_L \int_0^\infty dz z \times \int_0^\infty du u e^{-iz[1+u+(1/3)\xi^2 z^2 u^2]} \tag{21}$$

where we have dropped all nonleading terms in ηu and extended the integration in du to ∞, due to the facts that the main contribution to the integral in du comes from the region $u \lesssim 1$ because of the oscillating phase $\exp(-izu)$ and that $\eta = m^2/M^2$ is extremely small.

The integration in dz of the imaginary part of the self-energy can be performed with the substitution $z = y\sqrt{1+u}/(\xi u)$ in terms of the modified Bessel function

$$K_{2/3}(w) = \sqrt{3}\int_0^\infty y \sin\left[\frac{3}{2}w\left(y+\frac{1}{3}y^3\right)\right]dy \tag{22}$$

obtaining

$$\Im\Sigma_W(p) = -\frac{2}{3}\frac{g^2}{(4\pi)^2}\left(\frac{eB}{M^2}\right)^2 \not{p}_\perp \gamma_L \frac{1}{\sqrt{3}\xi^2} \times \int_0^\infty du \frac{1+u}{u} K_{2/3}\left[\frac{\sqrt{3}}{\xi}\frac{2}{u}\left(\frac{1+u}{3}\right)^{3/2}\right]. \tag{23}$$

The final integration in du yields

$$\Im\Sigma_W(p) = -\frac{g^2}{24\pi}\left(\frac{eB}{M^2}\right)^2\left(1+\sqrt{3}\frac{mM^2}{eBp_\perp}\right)e^{-\sqrt{3}mM^2/(eBp_\perp)}, \tag{24}$$

and, therefore, the absorption coefficient $\alpha = -2p_\perp \Im\Sigma_W(p)$ is

$$\alpha = \frac{g^2}{12\pi\hbar c}\frac{p_\perp c}{\hbar c}\left(\frac{m}{M}\right)^4\left(\frac{B}{B_{cr}}\right)^2\left(1+\sqrt{3}\frac{M}{m}\frac{Mc}{p_\perp}\frac{B_{cr}}{B}\right) \times e^{-\sqrt{3}(M/m)(Mc/p_\perp)(B_{cr}/B)}. \tag{25}$$

From the exponential in Eq. (25) we can read the threshold for the process:[1] if we define $\xi^{(th)} = \sqrt{3}$ then

$$cp_\perp^{(th)} = \sqrt{3}Mc^2\frac{M}{m}\frac{B_{cr}}{B} \approx (2.2\times 10^{16}\ \text{eV})\left(\frac{B_{cr}}{B}\right). \tag{26}$$

For energies well above the threshold, the absorption coefficient has the asymptotic behavior

$$\alpha = \frac{g^2}{12\pi\hbar c}\frac{p_\perp c}{\hbar c}\left(\frac{eB}{M^2}\right)^2 = \frac{g^2}{12\pi\hbar c}\frac{p_\perp c}{\hbar c}\left(\frac{m}{M}\right)^4\left(\frac{B}{B_{cr}}\right)^2 = 0.935\left(\frac{p_\perp c}{10^{16}\ \text{eV}}\right)\left(\frac{B}{B_{cr}}\right)^2\ \text{m}^{-1}, \tag{27}$$

where we have used the numerical value $G_F/(\hbar c)^3 = g^2/[\sqrt{2}(2Mc^2)^2\hbar c] = 1.16639\times 10^{-5}\ \text{GeV}^{-2}$.

If we consider ν_μ (ν_τ) instead of ν_e the corresponding threshold energies are higher by a factor $m_\mu/m_e \approx 206.8$ ($m_\tau/m_e \approx 3478$), while the asymptotic behavior does not change, since it depends only on M and not on m.[2]

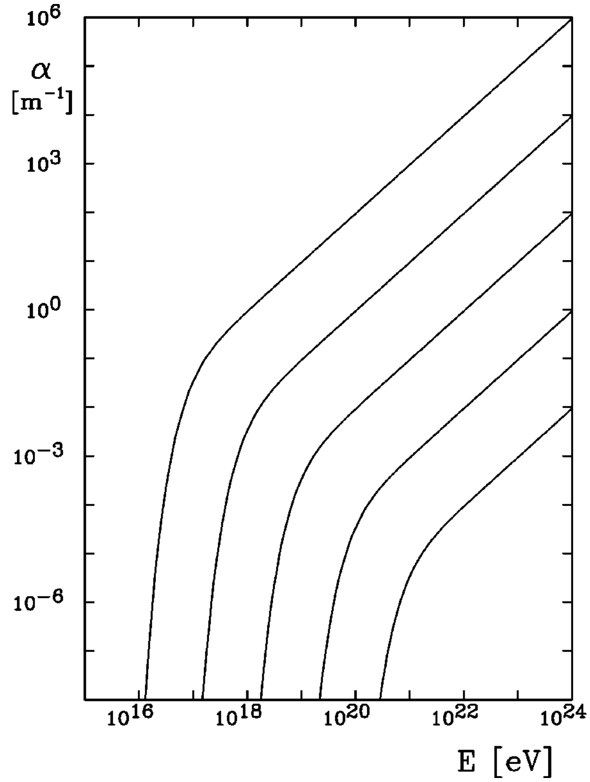

FIG. 1. Neutrino absorption coefficient for the process $\nu_e \to W + e$ in inverse meters as function of the neutrino transverse energy ($E \equiv p_\perp c$) in eV for five values of the magnetic field: $B = 10^{-1}B_{cr}$, $B = 10^{-2}B_{cr}$, $B = 10^{-3}B_{cr}$, $B = 10^{-4}B_{cr}$, $B = 10^{-5}B_{cr}$ going from top to bottom, with $B_{cr} = 4.4\times 10^{10}$ G. The same curves apply for ν_μ (ν_τ), if both the horizonal and the vertical scales are multiplied by $(m_\mu/m_e) = 206.768266$ [$(m_\tau/m_e) = 3477.6$].

IV. DISCUSSION AND CONCLUSIONS

In Fig. 1 we show the absorption coefficient for $\nu_e \to W + e$ as a function of the neutrino transverse energy $E \equiv p_\perp c$

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[1]The kinematical threshold is obviously given by the sum of the masses $M+m$, below which the rate is rigorously zero: this threshold is lost in our expansion, but it is unimportant as long as the effective threshold, below which the process is exponentially suppressed, is very much larger than $M+m$.

[2]In principle the full result would have additional dependences on the lepton mass coming from terms that contain $\eta = (m/M)^2$ and that we have disregarded, since they are extremely small for electrons; these corrections are larger for muons and especially taus, but still small.

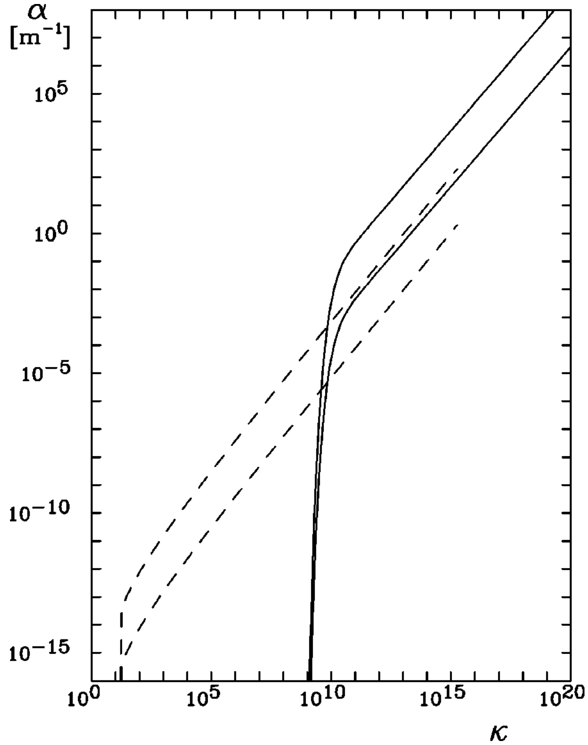

FIG. 2. Neutrino absorption coefficients for the processes $\nu_e \to W+e$ (solid curves) and $\nu \to \nu + e^+ + e^-$ (dashed curves) in inverse meters as function of the dimensionless characteristic parameter $\kappa \equiv eBp_\perp/m^3$ for two values of the magnetic field: $B = 10^{-1}B_{cr}$ and $B = 10^{-3}B_{cr}$ going from top to bottom.

for several values of the magnetic field B: $B=10^{-1}B_{cr}$, $B=10^{-2}B_{cr}$, $B=10^{-3}B_{cr}$, $B=10^{-4}B_{cr}$, and $B=10^{-5}B_{cr}$ with $B_{cr}=m^2/e=4.4\times10^{10}$ G. It is evident that the process has an energy threshold that grows for smaller values of B, as quantitatively described by Eq. (26). If we consider the angular coefficient, which is 1, and the spacing between the curves above the threshold, we find that α grows linearly with E and quadratically with B, in agreement with Eq. (27). The same Fig. 1 is valid for ν_μ (ν_τ), if one multiplies both the horizonal and the vertical scales by $(m_\mu/m_e) = 206.768266$ [$(m_\tau/m_e)=3477.6$].

In Fig. 2 we compare the process $\nu_e \to W+e$ with the process $\nu\to\nu+e^+ + e^-$ which has a much lower threshold; this threshold can be estimated from the dimensionless field dynamical parameter characteristic of this process $\kappa \equiv eBp_\perp/m^3$, which is analogous to ξ of Eq. (20) with the substitution $M\to m$: the threshold is smaller by a factor of about $m^2/M^2 \approx 4\times10^{-11}$. The curves plotted as function of κ show clearly that the thresholds of both processes are functions only of the product $Bp_\perp$ [see Eq. (26)]. For the process $\nu\to\nu+e^+ + e^-$ we use the result shown in Eq. (8) of Ref. [6]:

$$\alpha(\nu\to\nu e^+e^-) = \frac{G_F^2(g_V^2+g_A^2)}{(3\pi)^3} m^4 E\left(\frac{B}{B_{cr}}\right)^2 \times\left(\ln(\kappa) - \frac{\ln(3)}{2} - \gamma_E - \frac{29}{24}\right), \quad (28)$$

which the authors claim is valid for $E \ll M^3/eB$ [$\kappa \ll (M/m)^3 \approx 4\times10^{15}$]: we plot Eq. (8) of Ref. [6] [see Eq. (28)] up to $\kappa=10^{15}$. We show results only for $B=0.1B_{cr}$ (top curves) and $B=0.001B_{cr}$ (bottom curves). We see that the process $\nu\to\nu+e^+ + e^-$ (dashed curves) dominates below the threshold of $\nu\to W+e$ (solid curves), but above this threshold $\nu\to W+e$ is almost two orders of magnitude larger (about a factor of 50). Therefore, above the threshold $E \approx (mM^2)/eB$ [$\kappa\approx(M/m)^2 \approx 2\times10^{10}$], the effective Lagrangian (four-fermion interaction) cannot be used. The rate of $\nu\to\nu+e^+ + e^-$ can instead be estimated using the rate of $\nu\to W+e$, which gives the total absorption times the branching ratio of $W\to\nu_e + e$, which is $(10.66\pm0.20)\%$.

In conclusion, we have calculated the absorption rate of very-high-energy neutrinos in a magnetic field. Our main result is given by the compact formula in Eq. (25) valid for electron neutrinos. The result for muon or tau neutrinos can be obtained by substituting the electron mass m with the muon or tau mass, remembering that the electron mass m appears also in $B_{cr}=m^2/e$.

This process is exponentially suppressed, and, therefore, it can be disregarded, for energies below a threshold energy inversely proportional to the magnetic field; for a field one-tenth of the critical field this energy is of the order of 10^{17} eV [see Eq. (26) and Fig. 1].

Above this threshold the absorption coefficient grows linearly with energy and quadratically with the field as shown in Eq. (27): for a field one-tenth of the critical field and an energy of 10^{18} eV the absorption coefficient is about 1 m^{-1} (see Fig. 1).

Above the threshold this process substitutes the radiation of e^+e^- pairs as the dominant mechanism for ν absorption in a magnetic field (see Fig. 2).

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