Tachyon dynamics and the effective action approximation

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Recently effective actions have been extensively used to describe tachyon condensation in string theory. While the various effective actions which have appeared in the literature have very similar properties for static configurations, they differ for time-dependent tachyons. In this paper we discuss general properties of nonlinear effective Lagrangians which are first order in derivatives. In particular we show that some observed properties, such as asymptotically vanishing pressure, are rather generic features, although the quantative features differ. On the other hand we argue that certain features of marginal tachyon profiles are beyond the reach of any first order Lagrangian description. We also point out that an effective action, proposed earlier, captures the dynamics of tachyons well.

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I. INTRODUCTION

The decay of unstable D-branes is an important and challenging problem in string theory. In order to understand such processes it is important to obtain a reliable description of the dynamics of the tachyons which arise from open strings on unstable D-branes. In the last few years considerable progress has been made starting with Sen's proposal to identify the closed string vacuum with the tachyon vacuum on unstable D-branes in superstring theory (see [1] for a review and references). Furthermore, a class of time independent kink solutions corresponding to lower dimensional D-branes was identified with marginal boundary deformations of the open string sigma model [1]. More recently Sen has obtained a family of time dependent tachyon solutions as marginal deformations of the open string sigma model [2] (see also [3]). An analysis of the stress tensor obtained from the boundary state for a decaying D-brane [4] shows that the decay of an unstable D-brane results in a gas with finite energy density but vanishing pressure.

An interesting question is then to what extent these features can be obtained from a first derivative effective action for the tachyon [5–7]. Typically a tachyon effective action contains an infinite number of higher derivative terms which, unlike the case for massless string modes, cannot be simply neglected. Therefore no truncation to first derivatives can be expected to capture all of the dynamics of the tachyon. Furthermore, even if the full effective action were known, without some kind of simplifying structure it would be near impossible to obtain concrete results.[1] In addition it is not clear whether an initial value problem can be formulated. However, first derivative truncations of the tachyon effective action have been rather sucessful for describing D-branes as static tachyonic solitons [9–15]. One may therefore hope that a truncated action could also be useful to describe the dynamics of tachyon condensation.

In [7] Sen argued that some qualitative features of full tree-level string theory, such as the asymptotic vanishing of the pressure, are indeed reproduced by a Born-Infeld (BI) type action [16]. Unlike the case of massless gauge fields, the BI-type action has not been inferred from on-shell string theory and marginal tachyon profiles do not solve the equations of motion. On the other hand there is a well defined prescription to extract the tachyon effective action from boundary string field theory (BSFT) [17–19], [11–15]. Unfortunately this action is known explictly only for constant and linear tachyon profiles and the marginal tachyon profiles are also not solutions of the BSFT effective action. Nevertheless, BSFT has been quite sucessful in describing some aspects of tachyon condensation to lower dimensional D-branes [11,12,14]. Furthermore, it was shown in [20,21] that some qualitative features of the tachyon dynamics obtained from the BSFT action are consistent with conformal field theory results. However, while the various effective actions that have appeared in the literature are remarkably similar for static profiles, they could hardly differ more for time dependent ones. This is illustrated in Fig. 1 where the kinetic part of the effective action is plotted as a function of $\partial_\mu T \partial^\mu T$ for three different proposals (the details will be explained below).

Given this state of affairs, it is of interest establish to what extent these properties are generic in non-linear, first derivative effective actions for the tachyon. In addition we would like to determine the most appropriate behavior of the effective action for time dependent configurations. The purpose of this paper is establish general properties of suitable first derivative tachyon actions derived under a minimal set of reasonable theoretical assumptions. We find that some observed qualitative phenomena in tachyon condensation are rather generic and are reproduced by a large class of effective ac-

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[1]We note that [8] recently presented a very interesting analysis of tachyon dynamics in p-adic string theory, where the effective action is known to all orders in derivatives and is amenable to numerical analysis.

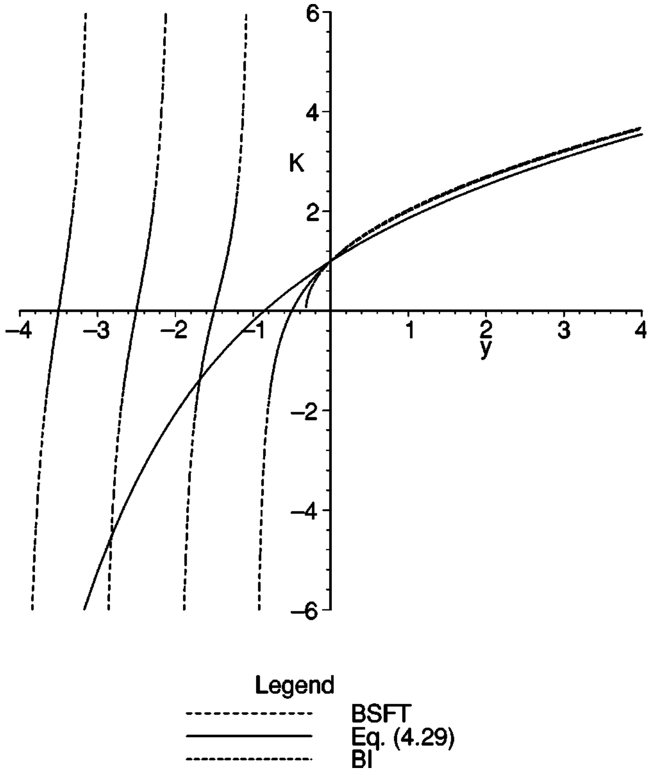

FIG. 1. $K(y)$ from various effective actions.

tions, although the exact quantitative predictions differ significantly. On the other hand we find that some global properties of marginal tachyon profiles are beyond the scope of first derivative effective actions.

In this context we also analyze a tachyon action proposed earlier [22] which closely resembles the other effective actions for time independent configurations, but has the advantage that marginal deformations are solutions to the equations of motion. Furthermore, the time-dependent marginal tachyon profiles in [2] are also exact classical solutions for this action. We find that the equation of state obtained from this action is in good quantitative agreement with the exact results of conformal field theory. Finally the potential cosmological relevance of scalar field actions with non-standard kinetic terms has been pointed out in [23]. It is therefore interesting to analyze tachyon effective actions from this point of view. This was done in [24] for the BI-type action and in [20] for the BSFT action. We will see that cosmological implications of our tachyon action are qualitatively similar to these results, although the details are again different.

II. TACHYONS AS MARGINAL DEFORMATIONS

Finding the general tachyon solution in string theory would require detailed knowledge of open superstring field theory. However, using the fact that these solutions correspond to conformally invariant backgrounds for the open string sigma model, a subset of open string tachyon solutions on a non-Bogomol'nyi-Prasad-Sommerfield (BPS) D-brane in type II string theory can be obtained as exactly marginal deformations of the boundary conformal field theory. Specifically, as is well known [25–30],

$$T(x^p)=\chi\sin(x^p/\sqrt{2\alpha'}),\quad T(x^p)=\chi\cos(x^p/\sqrt{2\alpha'})\tag{1}$$

are exactly marginal deformations of the open string Polyakov action on a non-BPS Dp-brane. The physical properties of this open string background are encoded in the corresponding boundary state $|B,+\rangle-|B,-\rangle$ [31,4], where[2]

$$|B,\epsilon\rangle=|B,\epsilon\rangle_{x^p,\psi^p}\otimes|B,\epsilon\rangle_{x^\mu,\psi^\mu}\otimes|B,\epsilon\rangle_{ghosts},\quad \mu\neq p,$$
$$\epsilon=\pm.\tag{2}$$

Here $|B\rangle_{x^\mu,\psi^\mu}$ and $|B\rangle_{ghosts}$ are the same as for a $p-1$ brane, while $|B\rangle_{x^p,\psi^p}$ describes the x^p dependence of the marginal deformation. In particular, the energy momentum tensor for a marginal kink profile can be obtained by evaluating the matrix element of the boundary state $|B\rangle$ with the closed string graviton state $\langle g_{\mu\nu}|$. Concretely we have

$$T_{\mu\nu}(x)=-\frac{\tilde{\tau}_p}{2}[A_{\mu\nu}(x)+B(x)\eta_{\mu\nu}].\tag{3}$$

Here $\tilde{\tau}_p=\sqrt{2}\tau_p$ is the tension of a non-BPS D-brane while $A_{\mu\nu}$ and B parametrize the level $(\frac{1}{2},\frac{1}{2})$ part of the boundary state [4],

$$|B\rangle_{(1/2,1/2)}=\sum_{k_p}[\tilde{A}_{\mu\nu}(k_p)\psi^\mu_{-1/2}\psi^\nu_{-1/2}+\tilde{B}(k_p)(\overline{\beta}_{-1/2}\gamma_{-1/2}$$
$$-\beta_{-1/2}\overline{\gamma}_{-1/2})]|\Omega,k\rangle,\tag{4}$$

with

$$|\Omega,k\rangle=(c_0+\overline{c}_0)c_1\overline{c}_1e^{-\phi(0)}e^{-\overline{\phi}(0)}|k\rangle\tag{5}$$

and $|k\rangle$ is the closed string Fock vacuum with momentum k. Using similar arguments as in [4] we then find, for a $\cos(x^p/\sqrt{2\alpha'})$ kink,

$$A_{\mu\nu}(x)=-f(x^p)\eta_{\mu\nu},\quad \mu,\nu\neq p,$$
$$A_{pp}(x)=-g(x^p),\quad B(x)=-f(x^p),\tag{6}$$

where

$$f(x^p)=1+2\sum_{n=1}^{\infty}[-\sin^2(\chi\pi)]^n\cos\left(n\sqrt{\frac{2}{\alpha'}}x^p\right)$$
$$=\frac{1-\sin^4(\chi\pi)}{1+2\sin^2(\chi\pi)\cos\left(\sqrt{\frac{2}{\alpha'}}x^p\right)+\sin^4(\chi\pi)}\tag{7}$$

and

[2] Our metic convention is $(-,+,\ldots,+)$ and $\mu,\nu=0,\ldots,p$ label the non-BPS Dp-brane world volume.

$$g(x^p)=2-2\sin^2(\pi\chi)-f(x^p). \tag{8}$$

In particular T_{pp} is independent of x^p as it should be. For χ close to 1/2 the energy density is peaked about $x^p=\sqrt{\alpha'}(2n+1)\pi/\sqrt{2}$.

We note that the boundary state and, in particular, all components of the stress tensor are periodic in χ with period 1. This periodicity can also bee seen by analyzing the spectrum [32] or by analyzing the cylinder diagram on an orbifold [33]. The origin of this periodicity can be traced to the fact that the vertex operators $\exp(ix^p/\sqrt{2\alpha'})$, $\exp(-ix^p/\sqrt{2\alpha'})$ and $i\partial x^p$ form an $so(3)$ current alegbra. Here $i\partial x^p$ generates translations along the x^p direction. Since the exponential of these operators appears in the path integral, the resulting correlation functions gain a periodic dependence on the coefficient χ due to the compactness of $so(3)$.

Time dependent boundary perturbations can be obtained by analytic continuation of the above profile [4]. Concretely, by performing the double Wick rotation $x^p\to ix^0, x^0\to -ix^p$ we find that $T=\chi\sinh(x^0/\sqrt{2\alpha'})$ and $T=\chi\cosh(x^0/\sqrt{2\alpha'})$ are, at the tree level, exactly marginal, time dependent tachyon profiles. Note that in the $\sinh(x^0/\sqrt{2\alpha'})$ case we must also Wick rotate $\chi\to -i\chi$ to obtain a real profile. Therefore we expect that the peridocity observed for χ in the space-like case is now broken. Indeed for space-like marginal deformations we saw that the perodicity in χ came from an $so(3)$ current alegbra of the x^p free boson conformal field theory (CFT). If we Wick rotate to a time-like free boson, this becomes an $so(2,1)$ current algebra, which has only one compact direction. This must correspond to the deformations generated by $\chi\cosh(x^0/\sqrt{2\alpha'})$ since these arise from Wick rotating $\chi\cos(x^p/\sqrt{2\alpha'})$ which does not require that χ be also Wick rotated. In addition $i\partial x^0$ now generates time translations which are no longer periodic.

The stress tensor for the time-like $\chi\cosh(x^0/\sqrt{2\alpha'})$ profiles is again given by Eq. (3) with [4]

$$A_{00}(x)=2-2\sin^2(\pi\chi)-f(x^0),$$

$$A_{ij}(x)=-f(x^0)\delta_{ij}, \quad B(x)=-f(x^0), \tag{9}$$

and

$$f(x^0)=\frac{1-\sin^4(\chi\pi)}{1+2\sin^2(\chi\pi)\cosh\left(\sqrt{\frac{2}{\alpha'}}x^0\right)+\sin^4(\chi\pi)}. \tag{10}$$

For time dependent marginal deformations the conserved energy $E=-T_{00}$ is

$$E=\sqrt{2}\tau_p[1-\sin^2(\chi\pi)], \tag{11}$$

while the pressure $p=-T_{ii}$ vanishes as the tachyon approaches the vacuum configuration. More specifically, as $x^0\to\infty$,

$$p\simeq -E\frac{1+\sin^2(\chi\pi)}{\sin^2(\chi\pi)}e^{(-\sqrt{2/\alpha'})x^0}. \tag{12}$$

If we consider marginal deformations of the form $\chi\sinh(x^0/\sqrt{2\alpha'})$, then we find similar expressions but with $\sin^2(\chi\pi)$ replaced by $-\sinh^2(\chi\pi)$ and $e^{\pm(\sqrt{2/\alpha'})x^0}$ replaced by $-e^{\pm(\sqrt{2/\alpha'})x^0}$ [4].

III. TACHYON EFFECTIVE ACTIONS

In the previous section we saw that in principle the physical observables of a marginal tachyon profile are encoded in the boundary state. An interesting question is then to what extent can these properties be reproduced by an effective field theory for the tachyon. A reliable effective action formulation would facilitate more complicated calculations, such as the dynamics in the presence of evolving closed string modes. For example in [7] Sen showed that for time dependent tachyon solutions in a BI-type action the pressure does indeed vanish asymptotically. However, as we remarked in the Introduction, this particular form for the effective action is not derived from string theory and one might therefore question the predictions obtained from it. In view of this we consider in this section the predictions of a rather general class of effective actions.

Typically, the full tachyon effective action will be nonlocal and thus difficult to handle. In what follows will consider effective actions which involve only first derivatives of the tachyon field. This class of actions includes, in particular, all suggestions which have appeared in the literature so far. For a non-BPS brane it is known that the action and in particular the potential V must be an even function of the tachyon field T. Therefore the most general form for the effective action of a real T in $p+1$ dimensions that depends on at most first order derivatives and is even in T is given by

$$S=\int d^{p+1}x\mathcal{L}\equiv\int d^{p+1}x\sum_{\alpha,\beta=0}^{\infty}c_{\alpha\beta}T^{2\alpha}(\partial_\mu T\partial^\mu T)^\beta. \tag{13}$$

From Noether's theorem it follows that for any static, codimension-1 solution of the equations of motion of an action of the type (13) will have the integral of motion

$$T_{pp}=\frac{\delta\mathcal{L}}{\delta T'}T'-\mathcal{L}=V_0, \tag{14}$$

where V_0 is a constant and T depends only on x^p. In addition the energy density $T_{00}=\mathcal{L}$ is trivially conserved. For a time dependent but spatially homogeneous solution one again finds two conserved quantities but now the conservation of energy leads to a non-trivial condition

$$T_{00}=-\frac{\delta\mathcal{L}}{\delta\dot{T}}\dot{T}+\mathcal{L}=-E_0, \tag{15}$$

where E_0 is a constant.

The simplest condition one may want to impose is that the effective action reproduce the correct perturbative tachyon mass near $T=0$, that is $\mathcal{L}\sim-(\partial T)^2+(1/2\alpha')T^2+\cdots$. However this also illustrates one problem of trunctating to first derivative actions since integration by parts allows us to write the kinetic term as $T\partial^2 T$ and it could therefore be modified by higher derivative terms. Indeed several proposals for the effective action do not reproduce the correct tachyon mass.

The next issue that we want to discuss is whether the marginal deformations in the last section can be solutions of a first derivative effective action. As shown in [22], the requirement that $T=\chi\sin(x^p/\sqrt{2\alpha'})$ solves the field equations completely determines the general action (13) in terms of an arbitrary potential $V(T)=f(T^2/2\alpha')$, i.e.

$$\mathcal{L}=\sum_{\gamma=0}^{\infty}\frac{1}{\gamma!}\frac{1}{2\gamma-1}\frac{d^\gamma f(t)}{dt^\gamma}(\partial_\mu T\partial^\mu T)^\gamma, \tag{16}$$

where $t\equiv T^2/2\alpha'$. Evaluating the resulting kink equation (14) one then finds

$$V_0=\frac{\delta\mathcal{L}}{\delta T'}T'-\mathcal{L}=\sum_{\gamma=0}^{\infty}\frac{1}{\gamma!}\frac{d^\gamma f(t)}{dt^\gamma}[(T')^2]^\gamma$$
$$=f\left(\frac{T^2}{2\alpha'}+(T')^2\right). \tag{17}$$

Thus, assuming that V is nowhere constant, we see that the only regular static solutions are

$$T=\chi\sin\left(\frac{x^p-x_0}{\sqrt{2\alpha'}}\right), \tag{18}$$

for arbitrary x_0 and χ. In addition it is easy to see that, by taking the limit $\chi\to 0$, this condition also ensures that the correct perturbative mass for the tacyhon is reproduced.

Let us now look at the periodicity properties of the physical observables on marginal profiles. As discussed in Sec. II the stress tensor for marginal deformations is periodic in χ. For a space-like kink $T=\chi\sin(x^p/\sqrt{2\alpha'})$, periodicity of the conserved energy T_{00} in χ implies that $\mathcal{L}$ is periodic in χ for all values in x^p. This implies that $\mathcal{L}(T,T')=F(T^2/2\alpha'+T'^2)$, where F is a periodic function. On the other hand periodicity of the conserved momentum T_{pp} implies that

$$\frac{\delta\mathcal{L}}{\delta T'}T'-\mathcal{L}=2\frac{dF(z)}{dz}T'^2-F(z) \tag{19}$$

is periodic in χ. These two conditions are, however, incompatible. Similar comments apply to the case of time-dependent solutions. Thus, we conclude that while marginal deformations can be solutions of first derivative effective actions, the periodicity in χ of the all observables cannot be reproduced by any first derivative effective action. This problem, which was also observed in [33], originates in the fact that the string ground state at $\chi=0$ is not the same as the string ground state in the new vacuum at $\chi=1/2$ due to spectral flow. Therefore to see this periodicity the infinite tower of massive strings modes has to be taken into account. Generally we then expect that while effective actions capture the physics near a given string theory vacuum, global properties are typically beyond such approximations. In particular we then expect that the effective action (16) will only be reliable for $\chi\ll 1$.

In superstring theory all of the tachyon effective actions proposed to date take the specific form[3]

$$\mathcal{L}=-V(T)K(\partial_\mu T\partial^\mu T). \tag{20}$$

Certainly without some kind of simplifying structure even an action which has been truncated to be only first order in derivatives becomes intractable. Therefore in what follows we restrict our attention to these forms for $\mathcal{L}$. We note here that an effective action of this form is compatible with T-duality if the transverse scalars and world volume vector fields are included as $\mathcal{L}=-V(T)\sqrt{-\det(G_{\mu\nu}+\mathcal{F}_{\mu\nu})}K((G+\mathcal{F})^{\mu\nu}\partial_\mu T\partial_\nu T)$ or in the BI form $\mathcal{L}=-V(T)\sqrt{-\det(G_{\mu\nu}+\mathcal{F}_{\mu\nu}+\kappa_{BI}\partial_\mu T\partial_\nu T)}$ [34,35].

Sen's conjectures state that the potential is nowhere negative, vanishes in the true open string vacuum and has a maximum value of $\sqrt{2}\tau_p$ at $T=0$. Furthermore, there should be no perturbative open string excitations about the true vacuum. Although one might think that action of the form (20) would ensure this condition, this is not true in general. Assuming that for small y, $K(y)\sim 1+\kappa_1 y+\cdots$ we check the perturbative excitations around $V=0$ by introducing a new tachyon variable

$$d\varphi=\sqrt{V(T)}dT. \tag{21}$$

The effective action now looks like

$$\mathcal{L}=-V(\varphi)K\left(\frac{\partial_\mu\varphi\partial^\mu\varphi}{V}\right). \tag{22}$$

Note that this change of variable relates the BI action [16] to the proposal of [36]. Expanding Eq. (22) for $(\partial\varphi)^2\ll V$ [i.e. $(\partial T)^2\ll 1$] leads to

$$\mathcal{L}\simeq-\kappa_1\partial_\mu\varphi\partial^\mu\varphi-V(\varphi). \tag{23}$$

The mass squared of elementary excitations around the tachyon vacuum is then given by

$$M^2=\frac{2}{\kappa_1}\frac{d^2V(\varphi)}{d\varphi^2}\bigg|_{V=0}=\frac{2}{\kappa_1}\frac{1}{\sqrt{V}}d^2\sqrt{\frac{V}{dT^2}}\bigg|_{V=0}. \tag{24}$$

The absence of perturbative excitations implies that this mass is infinite. It is not hard to convince oneself that M will be infinite provided that V vanishes faster than $V\sim e^{-aT}$ if the minimum is at infinite T, or faster than $V\sim(T-T_0)^2$ if the minimum is at a finite value of T. Note that $V\sim e^{-aT}$

[3]Although we note that for the bosonic string the action does not factorize [11,12,22].

appears in the effective action of a D-brane in *bosonic* string theory. However, the effective actions of bosonic BSFT [11,12,22] do not have the form (20) and hence Eq. (24) does not apply. Indeed one can check that the resulting mass in these cases is infinite. Nevertheless, even if the form (20) is assumed, plane-wave excitations for φ are absent due to non-linearities in the kinetic term [7]. It has also been pointed out that the Hamiltonian formalism is in fact better suited for analyzing the dynamics in the true vacuum [37]. However, this cannot be done here without choosing a specific form for K.

Another requirement that the effective action should satisfy is that it should reproduce the correct tension for kink solutions which are identified with a BPS D$(p-1)$-brane. More generally if $T=uf(x^p)$ is an off-shell profile, then $u\to\infty$ in the infra-red limit. The world sheet theory is expected to run to that of N BPS D$(p-1)$-branes and anti-D$(p-1)$-branes. Here N is the number of times T interpolates between the vacua as $u\to\infty$. Since this should be true for all profiles, this suggest that at large u, i.e. at large $\partial_p T$, the action becomes topological. This will be the case if $K\simeq\kappa_\infty\sqrt{(\partial T)^2}$ as $(\partial T)^2\to+\infty$. Recall that we normalize $K(0)=1$ and $V(0)=\sqrt{2}\tau_p$ so that κ_∞ is not arbitrary. In this case the energy for the profile $T=uf(x^p)$ is, in the limit $u\to\infty$,

$$E=\kappa_\infty\int_{-\infty}^{\infty}dx^p V(uf(x^p))u|f'(x^p)|=N\kappa_\infty\int dT\,V(T). \tag{25}$$

It is not hard to see that this property is indeed shared by all non-linear, first derivative superstring tachyon actions proposed thus far [13–16,22]. It was also pointed out in [6] that a square-root form for time independent configurations is needed to ensure that the fluctuations about a kink background have finite masses. A square-root form also ensures that the effective action for the relative center of mass coordinates of the D$(p-1)$-branes has a BI form [22]. In addition, to agree with the interpretation as N seperate BPS D$(p-1)$-branes and anti-D$(p-1)$-branes this energy should be $E=2\pi\sqrt{\alpha'}N$. Hence we find a constraint on the area of the potential between two minima and the large $(\partial T)^2$ behavior of K. This condition is indeed satisfied by the proposals of [13–15,22].

Last, we consider the asymptotic form of the pressure for homogenous, time-dependent tachyon approaching the minimum of the potential. We then have, for the conserved energy,

$$E=-T_{00}=2V(T)K'\dot{T}^2+V(T)K((\partial T)^2), \tag{26}$$

while the pressure is given by

$$p=-T_{ii}=-V(T)K((\partial T)^2). \tag{27}$$

Because $V(T)$ vanishes at its minimum, energy conservation implies that $\dot{T}^2$ necessarily approaches a singular point of K' or K as T condenses to $V=0$. Now, if this singularity is at some finite value of $\dot{T}$, then, since K' will diverge faster than K, the first term in Eq. (26) will dominate and the second term will vanish. Therefore p must also approach zero at this point. If the singular point of K or K' is at infinity, then p vanishes unless $yK'(y)$ and $K(y)$ have the same asymptotic behavior—that is, if $K(y)\sim y^n$ as $y=-\dot{T}^2\to-\infty$. In this latter case, as $V\to 0$, $p\to -E/(2n+1)$ is non-vanishing. Thus, unless $K(y)$ has a power law behavior for rapidly varying time dependent tachyons, the pressure vanishes as the tachyon condenses, although whether or not this happens exponentially quickly or not will depend on the choice of K.

IV. TACHYON DYNAMICS

In the previous section we outlined various properties that the effective action for the tachyonic mode of a non-BPS D-brane is expected to have. In particular, if we require that the marginal deformations (1) be solutions to the equations of motion, then the effective action is uniquely determined by a choice of potential $V(T)$. As proposed in [22], to fix this ambiguity we take the exact potential found in boundary string field theory [13–15],

$$V(T)=\sqrt{2}\tau_p e^{-T^2/2\alpha'}, \tag{28}$$

where τ_p is the tension of a BPS Dp-brane. The resulting Lagrangian that we construct then takes the form [22]

$$\mathcal{L}=-\sqrt{2}\tau_p e^{-T^2/2\alpha'}[e^{-\partial_\mu T\partial^\mu T}+\sqrt{\pi\partial_\mu T\partial^\mu T}\,\mathrm{erf}(\sqrt{\partial_\mu T\partial^\mu T})]. \tag{29}$$

One can then check that this action satisfies all the properties discussed in the previous section (with the exception of periodicity which we argued could not be captured by any first derivative effective action).

For static configurations this action is in remarkable agreement with the BSFT action [13–15]

$$\mathcal{L}=\frac{1}{\sqrt{2}}\tau_p e^{-T^2/2\alpha'}4^{(\partial T)^2}(\partial T)^2\frac{\Gamma((\partial T)^2)^2}{\Gamma(2(\partial T)^2)}. \tag{30}$$

This is somewhat surprising since the BSFT action is derived by simply assuming a linear tachyon profile. This good agreement can be viewed as a test of the BSFT action for non-trivial space-like kinks, since these are solutions to the equations of motion of Eq. (29). However, for time-like solutions the two actions differ considerably. In particular, while Eq. (29) is smooth for all $(\partial T)^2$ the BSFT action has poles at negative integer values of $(\partial T)^2$. However, there is no reason to believe that these poles are physically important since the BSFT action is derived for linear profiles but these are not solutions to the equations of motion. Furthermore, the fact that the action (29) and the BSFT action differ substantially for time dependent solutions suggests that, in contrast to space-like marginal profiles, the BSFT effective action cannot be trusted. The action (29) also agrees well with the BI form [16]

$$\mathcal{L}=\sqrt{2}\tau_p e^{-T^2/2\alpha'}\sqrt{1+\kappa_{BI}(\partial T)^2}, \tag{31}$$

for $(\partial T)^2 > 0$ if we take $\kappa_{BI} = \pi$. However, again they differ substantially for time dependent profiles where $(\partial T)^2 < 0$ and therefore similar comments apply. In particular the BI form imposes a maximum value of $|\dot{T}|$. These three forms for the function K are plotted in Fig. 1.

From the construction of the effective action in the last section it is clear that

$$T(x^0) = A\sinh\left(\frac{x^0}{\sqrt{2\alpha'}}\right) + B\cosh\left(\frac{x^0}{\sqrt{2\alpha'}}\right) \tag{32}$$

is an exact solution of the equation of motion. In fact we can say more by analyzing the energy momentum tensor for the action (29):

$$T_{\mu\nu} = -\sqrt{2}\tau_p e^{-T^2/2\alpha'}\left[\frac{\sqrt{\pi}}{\sqrt{(\partial T)^2}}\partial_\mu T \partial_\nu T \mathrm{erf}[\sqrt{(\partial T)^2}] - \eta_{\mu\nu}\{e^{-(\partial T)^2} + \sqrt{\pi(\partial T)^2}\mathrm{erf}[\sqrt{(\partial T)^2}]\}\right]. \tag{33}$$

Let us now consider a homogenous, but otherwise arbitrary, time dependent tachyon configuration. Then the energy takes the simple form

$$E = -T_{00} = \sqrt{2}\tau_p e^{-T^2/2\alpha' + \dot{T}^2}. \tag{34}$$

Conservation of energy then implies that Eq. (32) is the only regular solution of the equation of motion. Of course, the same result can be obtained by analytic continuation from Eq. (17). In particular, as the tachyon rolls to the minimum $\dot{T}$ diverges in agreement with the conformal field theory approach. This is in contrast with the BI-type and BSFT actions where $\dot{T}$ approaches a constant [4,20,21].

Let us now consider T_{ij}. From Eq. (33) we have

$$T_{ij} = \sqrt{2}\delta_{ij}\tau_p e^{-T^2/2\alpha'}[e^{\dot{T}^2} + i\sqrt{\pi\dot{T}^2}\mathrm{erf}(i\sqrt{\dot{T}^2})]. \tag{35}$$

Now, for large y,

$$\sqrt{\pi}\mathrm{erf}(iy) \simeq i\, e^{y^2}\left[\frac{1}{y} + \frac{1}{2y^3} + O\left(\frac{1}{y^5}\right)\right], \tag{36}$$

so that

$$T_{ij} \simeq \frac{T_{00}}{2\dot{T}^2}\delta_{ij} \quad \text{for} \quad \dot{T} \to \infty. \tag{37}$$

Thus, the action (29) predicts that at large times the tachyon condensation produces a gas with non-vanishing energy and vanishing pressure. In particular for large x^0, where $T \simeq \chi e^{x^0/\sqrt{2\alpha'}}/2$,

$$p \simeq \frac{2E}{\chi^2} e^{-(\sqrt{2/\alpha'})x^0}. \tag{38}$$

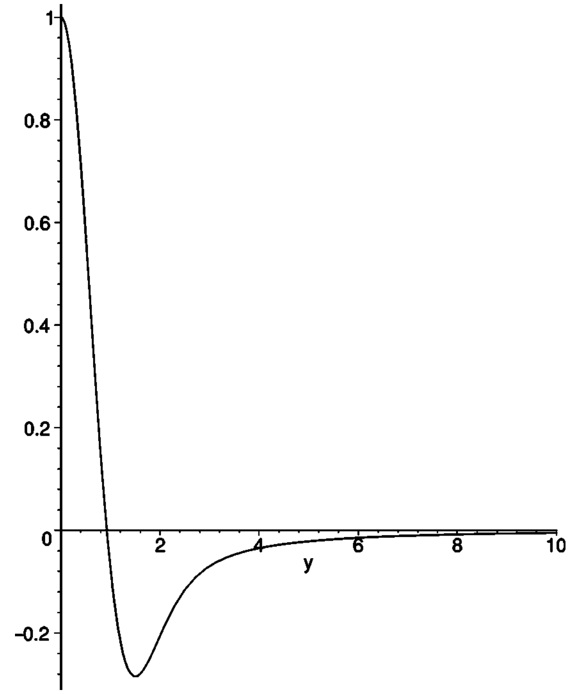

FIG. 2. $\exp(-y^2)[\exp(y^2) + i\sqrt{\pi}y\,\mathrm{erf}(iy)]$.

This exponential fall off agrees exactly with the string theory result from the boundary state [4]. Note that this prediction is different from that obtained using the BSFT effective action [20], where the square of x^0 appears in the exponential. One can also see that the BI form $K = \sqrt{1 + \kappa_{BI}(\partial T)^2}$ with the potential (28) predicts that $(x^0)^2$ appears in the exponential. In addition for $\chi \ll 1$ we find the same dependence of p and E on χ as predicted from the boundary state (12). On the other hand, in the boundary state approach, the pressure is always negative, whereas here we find that the pressure approaches zero from above (see Fig. 2). The same phenomenon was also observed in [20,21] for the BSFT effective action. Indeed it is clear from Eq. (12) that the sign of the pressure as $V \to 0$ is the opposite of the sign of $K(-\dot{T}^2)$, which is negative in BSFT and Eq. (29), but positive for a BI-type action. There one of the speculations was that this difference could be due to the fact that the solution of the BSFT differ from the exact marginal deformations. This possibility can be excluded here as the solutions to our action are precisely the marginal deformations, although higher derivative terms could lead to corrections in our action as well.

Finally we make some comments on the interpretation of these solutions. For the static solutions $T = \chi\cos(x^p/\sqrt{2\alpha'})$ the energy density is not spatially homogeneous but is peaked about $x^p = \sqrt{\alpha'/2}\pi(2n+1)$ for integer n, becoming more sharply peaked as $\chi \to \infty$. However, by construction the action (16) is independent of χ and therefore the total energy, evaluated over a single period $2\pi\sqrt{2\alpha'}$, is $4\pi\sqrt{\alpha'}\tau_p$ for all χ. For $\chi = 1/2$ this represents a configuration of BPS D$(p-1)$-branes at each odd n and anti-D$(p-1)$ branes at each even n. The interpretation of other values of χ is less clear; however, these are no longer marginal deformations once string loop corrections are considered [33]. Of course, the solutions involving the marginal deformation $T = \chi \sin(x^p/\sqrt{2\alpha'})$ can be obtained from the previous ones by translation along x^p.

For time dependent solutions the energy density, ϵ, is spatially homogeneous. In particular for $T = \chi \cosh(x^0/\sqrt{2\alpha'})$ we have $\epsilon = \sqrt{2}\tau_p e^{-\chi^2/\sqrt{2\alpha'}}$, which is less than the false vacuum energy density, while for $T = \chi \sinh(x^0/\sqrt{2\alpha'})$ the energy density $\epsilon = \sqrt{2}\tau_p e^{\chi^2/\sqrt{2\alpha'}}$ is greater than the false vacuum energy density. These solutions are no longer related by temporal or spatial translation. Indeed while both solutions start and end in the vacuum as $t \to \pm\infty$ the $\cosh(x^p/\sqrt{2\alpha'})$ solutions never pass over the energy barrier at $T=0$ whereas the $\sinh(x^p/\sqrt{2\alpha'})$ solutions travel from one vacuum to the other. As we discussed in Sec. III, in the full string theory there is a periodic dependence on χ for the $\chi\cosh(x^0/\sqrt{2\alpha'})$ solutions but not for the $\chi\sinh(x^0/\sqrt{2\alpha'})$ solutions.

V. COUPLING TO GRAVITY

The relevance of scalar field action with higher than quadratic derivative terms for cosmology has been recognized a while ago [23], where it was argued that scalar field actions of the form $S = V(T)K((\partial T)^2)$ can produce inflationary scenarios (k-inflation) as well as late stage cosmological acceleration (k-essence). It is therefore interesting to analyze our tachyon action from this point of view. Related analyses were carried out in [20,24,38–41] for the BI-type and BSFT effective actions. We consider a general first derivative effective action of the form (20) minimally coupled to gravity:

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2\kappa^2}R - V(T)K(\partial_\mu T \partial^\mu T)\right). \tag{39}$$

In $d = p+1$ dimensions and with T assumed to be spatially homogeneous and time dependent the metric then takes the usual FRW form

$$ds^2 = -dt^2 + a(t)^2 ds^2_{d-1}, \tag{40}$$

where ds^2_{d-1} is a spatial manifold with constant curvature k. For simplicity we consider the spatially flat case $k=0$. A convenient set of evolution equations is then simply (recall that one of Einstein's equations is not independent)

$$H^2 = \frac{\kappa^2}{(d-1)(d-2)}\epsilon, \quad \dot{\epsilon} = -(d-1)H(\epsilon + p), \tag{41}$$

where $H \equiv \dot{a}/a$ is the Hubble constant and ϵ and p are the energy density and pressure, respectively. If we now substitute our tachyon action, these general formulas become

$$H^2 = \frac{2\kappa^2}{(d-1)(d-2)}V(\sqrt{T^2 - 2\alpha'\dot{T}^2}),$$

$$\frac{d}{dt}[V(\sqrt{T^2 - 2\alpha'\dot{T}^2})] = -(d-1)H[V(T)K(-\dot{T}^2) + V(\sqrt{T^2 - 2\alpha'\dot{T}^2})]. \tag{42}$$

In particular at late times $\dot{T} \gg 1$ and we can approximate $VK \simeq -V(\sqrt{T^2 - 2\alpha'\dot{T}^2})/2\dot{T}^2$ so that $VK \ll -V(\sqrt{T^2 - 2\alpha'\dot{T}^2})$. Hence we see that $V(\sqrt{T^2 - 2\alpha'\dot{T}^2}) = C^2 a^{1-d}$ for a constant C and therefore

$$a(t) \sim t^{2/(d-1)}. \tag{43}$$

Thus the outcome of this analysis is identical with that obtained in [24] for the BI action and describes matter dominated expansion.

For intermediate times it is convenient to consider the master equation

$$\dot{\epsilon} = -\kappa\sqrt{\frac{d-1}{d-2}}\sqrt{\epsilon}(\epsilon + p). \tag{44}$$

From Fig. 2 we then see that the evolution starts off with an inflationary phase $p = -\epsilon$ and then transforming smoothly into a matter dominated expansion (43) for late times. This qualitative behavior is the same as found in [20] for the BSFT action.

VI. CONCLUSION

In this paper we have discussed the general properties of first derivative tachyon effective actions. For example we showed that the asymptotic vanishing of the pressure for time-dependent tachyon profiles is relatively generic, although the details vary. On the other hand we argued that the periodicity of the stress tensor under marginal deformations cannot be reproduced by any first derivative effective action. We also studied in detail the first derivative effective action that we proposed in [22] and showed that it reproduces many of the expected features of tachyon dynamics, including several correct quantitative features. However, it seems appropriate to mention the more pessimistic note that one could interpret the large discrepancies among the various proposed effective actions for time dependent tachyons, compared with their striking similarity for static profiles, as an indication that the effective action approach will not be as successful in the time dependent case. Indeed it has recently been observed from the boundary state that time dependent tachyons also couple to massive closed string fields with an exponentially increasing strength [42], so that the truncation to low level string modes is potentially artificial.

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