

## Polarization of $F1$ strings into D2 branes

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We give matrix and supergravity descriptions of type IIA  $F$  strings polarizing into cylindrical D2 branes. When a RR four-form field strength  $F_4$  is turned on in a supersymmetric fashion (with 4 supercharges), a complete analysis of the solutions reveals the existence of a moduli space of  $F1 \rightarrow D2$  polarizations for some fractional strengths of the perturbation, and of no polarization whatsoever for all other strengths of the perturbation. This is a very intriguing phenomenon, whose physical implications we can only speculate about. In the matrix description of the polarization we use the non-Abelian Born-Infeld action in an extreme regime, where the commutators of the fields are much larger than 1. The validity of the results we obtain provides a direct confirmation of this action, although it does not confirm or disprove the symmetrized trace prescription.

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### I. INTRODUCTION

Recently, several papers have appeared describing states where strings and branes are polarized into higher dimensional branes, following the ideas in [1]. Usually the higher brane is supported against collapse by the presence of the field with which it couples. Nevertheless, only rarely is the back reaction of this field on the other supergravity fields taken into account. This is because taking back reaction into account does not usually affect the existence of a polarization ground state in the case of polarization into objects two or more dimensions higher.<sup>1</sup> Nevertheless, when one studies the polarization of a string or brane into a brane one dimension higher [such as  $F1 \rightarrow D2$  or  $D4 \rightarrow$  Neveu-Schwarz 5-brane (NS5)] the effect of this back reaction on the physics is crucial.

The polarization of  $N$  long fundamental strings into a tubular D2 brane in the presence of a four-form background  $F_{0xyz} = m$  was first studied in [2]. Ignoring back reaction, the potential was found to be of the form

$$\frac{V(R)}{L} = \frac{1}{2\pi g_s} \left( \sqrt{R^2 + g_s^2 N^2} - \frac{m}{2} R^2 \right), \quad (1)$$

where  $R$  is the radius of the tube. This naive potential seems to imply that for any  $F_{0xyz}$  the potential is unbounded below at large  $R$ , and the strings have a finite life time as they can tunnel into a D2 brane. Furthermore, if one expands the square root for  $g_s N > R$ ,

$$\frac{V(R)}{L} \approx \frac{1}{2\pi g_s} \left( g_s N + \frac{R^2}{2g_s N} - \frac{m}{2} R^2 \right) \quad (2)$$

one can also see that for  $F_{0xyz} > 1/g_s N$  this potential implies that the strings are not stable classically. Thus, a large number of coincident  $F1$  strings could spontaneously disappear

from the spectrum when a tiny  $F_4$  is present. We got this puzzling result because the back reaction of  $F_4$  on the dilaton and graviton was not taken into account. This back reaction gives a term in Eq. (2) of the form

$$\frac{V_{back\ reaction}(R)}{L} \sim Nm^2 R^2, \quad (3)$$

which usually prevents the decay of the strings. Finding the exact coefficient of this term is crucial if one wants to understand the physics of this system. A very large coefficient may prohibit polarization entirely, while a very small one leaves the puzzle about the strings disappearing at infinity unresolved.

There are several ways to find this coefficient. The hard and straightforward way is to compute the back reaction explicitly using the supergravity equations of motion. This has been done for other polarization cases in [4,5]. A slicker approach is to use supersymmetry [6–8] and to find this coefficient by completing the square in the less naive version of Eq. (2). One can ascertain how much supersymmetry is preserved by this  $F_4$  background either by doing a supergravity computation [9,10] or by relating this background to a boundary field theory via an AdS-CFT (conformal field theory) type correspondence [11,12]. Since the latter method only involves examining the supersymmetry of a two dimensional field theory, we will be using it here. Section. III will be devoted to this subject.

Another aspect which makes the study of  $F1 \rightarrow D2$  polarizations transparent is the existence of a matrix description for the resulting state. This is the subject of Sec. II. As shown in [1] and expanded in [6],  $Dp \rightarrow D(p+2)$  polarizations can be understood in terms of the matrices describing the position of the  $Dp$  branes becoming noncommutative. This description lacks however in other polarization cases.

At first glance, to give a matrix description to  $F1 \rightarrow D2$  polarizations involves using matrix string theory [13].<sup>2</sup> Nev-

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<sup>1</sup>Although it affects the decay into this state as shown in [6,3].

<sup>2</sup>Matrix string theory descriptions of the  $F1$  string polarization into D2 branes were also considered in [15,14].

ertheless, a much simpler way to describe these polarizations is to add a small number of D0 branes to the configuration, and to use the D0 brane noncommutative coordinates to describe it. Our system can be recovered by taking the D0 brane density to zero.<sup>3</sup>

This configuration of strings blowing up into a tubular D2 brane in the presence of D0 brane charge was considered recently by Mateos and Townsend [19]. In a following paper [20], a matrix model description for the supertubes was found. Nevertheless, this matrix model description does not work in the limit we are interested in (when the D0 contribution to the energy vanishes). To describe this limit one must use the full non-Abelian Born-Infeld action of the D0 branes [21,1,22,23]. We should note that in this limit the field strength  $F$  is much larger than one.

It is interesting to ask whether the validity of the result we get (which agrees exactly with the supergravity result and the one in [19]) confirms the validity of the symmetrized trace prescription. The answer is probably negative, because our  $F$ 's are very simple, and their commutators vanish. Thus the computation is probably insensitive to the inability of the symmetrized trace prescription to produce the right terms when commutators of  $F$ 's are involved [24].

Anticipating, we will find that the polarization of  $F1$  strings into a tubular brane is impossible except for values of  $F_4$  proportional to fractions whose denominator is smaller than  $N$ . For these values a number of strings polarizes into one or more tubular D2 branes. The radius of these polarization configuration is a modulus.

The polarization of  $F1$  strings into D2 branes has also been studied recently in Melvin-type backgrounds. In [16] and [17] exact solutions of  $F1$ -D2 spherical configurations were found, and the energy of these configurations was computed. For small radii, the polarization potential has an  $r^2$  behavior, as in our case. For large  $r$  the potential is unbounded below, most likely because of the instability of the Melvin background. If one tries to set the coefficient of the  $r^2$  term to zero (to reproduce the moduli space we obtain in the supersymmetric case), one reaches the ‘‘Melvin limit’’ of the  $F_2$  flux [18]. It would be interesting to explore if this phenomenon is responsible for the moduli space of polarization vacua in our case as well.

## II. MATRIX DESCRIPTION

It is possible to describe  $F$  strings with D0 charge blown up into a tubular D2 brane using the D0 brane matrix degrees of freedom. This description is valid both with and without an  $F_4$  flux turned on [20,25]. In order to give an ensemble of D0 branes a D2 charge, one has to take an ansatz where three of the D0 brane coordinates are not commuting.  $F1$  charge can be given as well, by having the noncommuting coordinates be time dependent. This is expected both from the form of the non-Abelian coupling with  $B_2$ , which contains a time derivative of the coordinates, and also by noticing [19] that a

tubular D2 brane with string and D0 charge has a nonzero angular momentum.

Let us assume to have  $N$  D0 branes uniformly distributed along the  $z$  direction. An ansatz for the  $N \times N$  matrices which gives the right charges is [20,25]

$$Z_{ij} = l \delta_{ij} j, \quad X = R(a + a^\dagger), \quad Y = iR(a^\dagger - a),$$

$$a_{ij} = e^{-i\omega t} \delta_{i-1,j}, \quad a_{ij}^\dagger = e^{i\omega t} \delta_{i+1,j}. \quad (4)$$

In the infinite volume limit, the matrices  $X, Y$  and  $Z$  are infinite, and obey several useful identities:

$$-iF_{zx} \equiv [Z, X] = -i l Y, \quad -iF_{zy} \equiv [Z, Y] = i l X,$$

$$iF_{xy} \equiv [X, Y] = 0$$

$$F_{tx} \equiv \dot{X} = \omega Y, \quad F_{ty} \equiv \dot{Y} = -\omega X. \quad (5)$$

Note that we work in the normalization where the matrices are dimensionless. To obtain the physical quantities corresponding to dimensionless symbols (such as  $R, l$  and  $\omega$ ) one has to multiply by the appropriate powers of  $l_s = \sqrt{2\pi\alpha'}$ . Since  $l$  is the separation between the individual D0 branes,  $lN$  is the size of the  $z$  direction, which we take to be very large. The local D0 charge density is  $Q_0 = 1/l$ . For weak fields the  $F$  string charge density along the  $z$  direction is given by [22]

$$Q_1 = \frac{T_0 l_s}{i l N} \text{STr}(\dot{X}^i [X^i, Z]) = 4 T_0 l_s \omega R^2, \quad (6)$$

and is quantized. As we will see shortly, this quantization proceeds differently for large string charge.

The D2 dipole charge is proportional to

$$\frac{1}{-i l N} \text{Tr}(X[Y, Z] + Y[Z, X] + Z[X, Y]) = 4 R^2. \quad (7)$$

The first two terms in the left-hand side are nonzero, while the third is zero. This means there is local D2 charge in the  $xz$  and  $yz$  planes, but not in the  $xy$  plane, exactly as it should be for a cylindrical D2 brane.

As the matrices  $X, Y$ , and  $Z$  are infinite in the large volume limit, the cyclicity of the trace is not respected. Without any  $F_4$  turned on, the action is given [21] by

<sup>3</sup>We thank W. Taylor for this suggestion.

$$L_{NBI} = -T_0 \text{STr} \left( \sqrt{-\det \begin{pmatrix} -1 & F_{tx} & F_{ty} & 0 \\ -F_{tx} & 1 & 0 & F_{xz} \\ -F_{ty} & 0 & 1 & F_{yz} \\ 0 & -F_{xz} & -F_{yz} & 1 \end{pmatrix}} \right). \quad (8)$$

One can plug in the  $F_{ij}$  defined in Eq. (5), and evaluate the determinant above. By using the fact that the determinant of a matrix with noncommuting entries<sup>4</sup> takes all orderings into account, or alternatively the fact that  $X$  and  $Y$  commute, we notice that the terms proportional to  $X^2Y^2$  and their permutations cancel. The only combination of matrices which appears under the square root is  $X^2 + Y^2$ . Since this combination is proportional to  $\mathbf{1}$ , the Lagrangian simply becomes

$$L = -T_0 N \sqrt{1 + 4l^2 R^2 - 4\omega^2 R^2}. \quad (9)$$

In order to obtain a Hamiltonian from this Lagrangian, one has to find the conjugate momentum corresponding to the string charge:

$$Q_1 = \frac{l_s}{N} \frac{\delta L}{\delta \omega} = \frac{4T_0 l_s \omega R^2}{\sqrt{1 + 4l^2 R^2 - 4\omega^2 R^2}}, \quad (10)$$

where the normalization of the string charge with respect to  $\delta L / \delta \omega$  was found by comparing the small string charge limit of Eq. (10) with Eq. (6). The Hamiltonian is therefore

$$H = \omega \frac{\delta L}{\delta \omega} - L = T_0 N \sqrt{1 + \frac{Q_1^2 g_s^2}{8\pi R^2} \sqrt{1 + 4l^2 R^2}}, \quad (11)$$

the same as the one found in [19]. This formula is valid for all values of the D0,  $F_1$  and D2 charges. We can take the limit when the D0 charge vanishes ( $N \rightarrow 0$  keeping the length of the tube  $lN$  fixed) and the string charge dominates to obtain

$$\frac{H}{N l l_s} = \frac{Q_1}{l_s^2} + \frac{4\pi R^2}{l_s Q_1 g_s^2}. \quad (12)$$

The first term is the energy of  $Q_1$  coincident static strings, and the second one reproduces exactly the Born-Infeld contribution to the action (27) when one identifies the parameters in the matrix and supergravity descriptions:

$$r^2 = x^2 + y^2 = \frac{\text{STr}(X^2 + Y^2)}{N} l_s^2 = 4R^2 l_s^2. \quad (13)$$

This exact agreement provides a confirmation of the non-Abelian Born-Infeld action, as well as of the non-Abelian coupling with the  $B$  field found in [22]. Nevertheless, it does

not imply anything about the validity of the symmetrized trace prescription because the only matrix whose trace we had to take was  $\mathbf{1}$ .

The presence of  $F_4$  modifies the action by the addition of

$$L_{F_4} = T_0 \text{STr}(C_{0zx} F_{zx} + C_{0zy} F_{zy}) = F_{0xyz} 2l N T_0 R^2. \quad (14)$$

The terms in Eqs. (12) and (14) correspond to the terms found in the naive action (2). Unfortunately, the back reaction term cannot be found in the matrix description because the  $(F_4)^2$  couplings in the non-Abelian D brane action are not known.

### III. THE SUPERGRAVITY ANALYSIS

Let us for the beginning explore the supergravity background created by a large number of coincident  $F$  strings. By a generalization of the AdS-CFT correspondence in the spirit of [12], the two-dimensional field theory living on the world volume of these coincident  $F$  strings is dual to string theory living in the near-horizon background of these strings. As explained in [26], turning on an operator in the boundary theory Hamiltonian corresponds to turning on a non-normalizable mode in the bulk. One can study the effect of the operator on the boundary theory by examining the supergravity dual which the corresponding non-normalizable mode creates. Nevertheless the study of this theory, although straightforward once all the computations are done, is not our primary interest. Rather, we are looking for a supergravity background with a nonzero  $F_4$  which preserves 4 of the original 16 supercharges, and we are using this correspondence only as a tool to keep control on the supersymmetry.

As the fermions of the type IIA  $F$  string in static gauge transform in the  $\mathbf{8s}$  of the  $\text{SO}(8)$   $R$  symmetry group, a fermion bilinear transforms in the  $\mathbf{35}_+$  or the  $\mathbf{28}$  of this group. The bilinear in the  $\mathbf{35}_+$  corresponds to a self dual Ramond-Ramond (RR) 4-form field strength on the space transverse to the strings. This 4-form polarizes the strings into D4 branes, creating the dimensional reduction of the  $\text{M2} \rightarrow \text{M5}$  setup in [7]. The fermion bilinear in the  $\mathbf{28}$  corresponds to a bulk RR 2 form and 6 form field strengths on the transverse space. The 6 form on the transverse space is equivalent by Hodge duality to a 4 form with 2 legs along the strings, which is what we need for  $F_1 \rightarrow \text{D2}$  polarization.

#### A. The background

The string frame supergravity background of  $N$  fundamental strings is

<sup>4</sup>The determinant is defined as

$$\det M = \epsilon^{a_1 a_2 \dots a_n} \epsilon^{a'_1 a'_2 \dots a'_n} M_{a_1 a'_1} M_{a_2 a'_2} \dots M_{a_n a'_n}.$$

$$\begin{aligned}
ds^2 &= Z^{-1}(-dx_0^2 + dx_1^2) + dx_\perp^2 \\
e^\Phi &= g_s Z^{-1/2} \\
B &= Z^{-1} dx^0 \wedge dx^1,
\end{aligned} \tag{15}$$

where  $Z$  is a harmonic function on the transverse space which becomes

$$Z = 1 + \frac{R^6}{r^6}, \quad R^6 = 32\pi^2 N g_s^2 \alpha'^3, \tag{16}$$

when the strings are coincident. If  $g_s > 1$ , the supergravity solution is everywhere valid and it is not dual to anything. However, for the beginning we are interested in a regime where we could use the AdS-CFT type of duality discussed above, and thus we will focus for now on<sup>5</sup>  $g_s > 1$ . For  $r^6 > g_s^2 N$  the dilaton becomes large, and physics starts being described by the supergravity solution of  $N$  coincident M2 branes. For  $1 < r^6 < g_s^2 N$  supergravity in the background (15) gives a weakly coupled description of the physics, and the 1 in the harmonic function can be ignored. We will work in this regime. For even smaller radii,  $r < 1$ , the theory has a weakly coupled description in terms of an orbifold CFT, analyzed in [13].

### B. The perturbation

We are interested in perturbing the above background with the RR fields which could cause polarization. As explained above, these fields are  $F_2$  and  $F_6$  on the space transverse to the strings. If one defines<sup>6</sup>  $F_6 \equiv *(F_4 - C_1 \wedge H_3)$ , the type IIA equations of motion become

$$\begin{aligned}
d*F_2 - H_3 \wedge F_6 &= 0 \\
dF_6 - H_3 \wedge F_4 &= 0 \\
-d*F_6 = d(F_4 - C_1 \wedge H_3) &= -F_2 \wedge H_3 \\
dF_2 &= 0.
\end{aligned} \tag{17}$$

Combining Eq. (15) with Eq. (17) and expressing the ten dimensional Hodge dual  $*$  in terms of the 8 dimensional transverse space Hodge dual  $*_8$ , the equations satisfied by the first order perturbations become

$$\begin{aligned}
d[Z^{-1}(*_8 F_2 - F_6)] &= 0, \quad dF_6 = 0, \\
d[Z^{-1}(*_8 F_6 - F_2)] &= 0, \quad dF_2 = 0.
\end{aligned} \tag{18}$$

We should note that acting on even forms  $*_8^2 = 1$ , while  $*_8^2 = -1$ . To find the perturbations with the same  $R$ -symmetry

<sup>5</sup>Once we establish the form of the supersymmetric supergravity perturbation, we can also use it for  $g_s < 1$  (as long as the 1 in the harmonic function can be ignored) since the supersymmetry of the background does not depend on  $g_s$ .

<sup>6</sup>This is not the canonical definition of  $F_6$  but it is the most convenient to work with.

transformation properties as the fermion bilinear it is convenient to group the 8 transverse coordinates and the fermions into 4 complex pairs:

$$\begin{aligned}
z^1 &= x^2 + ix^3, \quad z^2 = x^4 + ix^5, \\
z^3 &= x^6 + ix^7, \quad z^4 = x^8 + ix^9, \\
\Lambda^1 &= \lambda^1 + i\lambda^2, \quad \Lambda^2 = \lambda^3 + i\lambda^4, \\
\Lambda^3 &= \lambda^5 + i\lambda^6, \quad \Lambda^4 = \lambda^7 + i\lambda^8.
\end{aligned} \tag{19}$$

Under the rotations  $z^i \rightarrow e^{i\phi_i} z^i$  the fermions transform as

$$\begin{aligned}
\Lambda^1 &\rightarrow e^{i(\phi_1 - \phi_2 + \phi_3 + \phi_4)/2} \Lambda^1 \\
\Lambda^2 &\rightarrow e^{i(\phi_1 + \phi_2 + \phi_3 - \phi_4)/2} \Lambda^2 \\
\Lambda^3 &\rightarrow e^{i(\phi_1 - \phi_2 - \phi_3 - \phi_4)/2} \Lambda^3 \\
\Lambda^4 &\rightarrow e^{i(\phi_1 + \phi_2 - \phi_3 + \phi_4)/2} \Lambda^4.
\end{aligned} \tag{20}$$

Thus a fermion bilinear of the form  $\text{Re}(\bar{\Lambda}^1 \Lambda^2)$  has the same  $\text{SO}(8)$  transformation properties as  $T_2 = \text{Re}(dz^2 d\bar{z}^4)$ ,  $V_2$  (defined in the Appendix) and their Hodge duals<sup>7</sup>  $T_6$  and  $V_6$ .

The most straightforward way to examine the supersymmetry preserved by a specific fermion bilinear is to examine the free 2-dimensional boundary theory which has 8 bosons  $X_\mu$  in  $\mathfrak{8}_v$  of  $\text{SO}(8)$  and both the left and right moving fermions  $\lambda_a$  and  $\tilde{\lambda}_a$  in  $\mathfrak{8}_s$ . The original supersymmetry transformations are

$$\begin{aligned}
\delta X^\mu &= -\epsilon_a \Gamma_{aa}^\mu \lambda_a - \tilde{\epsilon}_a \Gamma_{aa}^\mu \tilde{\lambda}_a \\
\delta \lambda_a &= \partial X_\mu \Gamma_{aa}^\mu \epsilon_a + F_\mu \Gamma_{aa}^\mu \tilde{\epsilon}_a \\
\delta \tilde{\lambda}_a &= \bar{\partial} X_\mu \Gamma_{aa}^\mu \tilde{\epsilon}_a - F_\mu \Gamma_{aa}^\mu \epsilon_a \\
\delta F^\mu &= -\tilde{\epsilon}_a \Gamma_{aa}^\mu \bar{\partial} \lambda_a + \epsilon_a \Gamma_{aa}^\mu \partial \tilde{\lambda}_a,
\end{aligned} \tag{21}$$

where  $\Gamma_{aa}^\mu$  are  $8 \times 8$  matrices given in [29]. It is a straightforward exercise to show that adding a fermion bilinear,

<sup>7</sup>The standard properties of these tensors are reviewed in the Appendix.



$\text{Re}(\bar{\Lambda}^1 \Lambda^2)$ , together with its bosonic partner preserves 4 of the original 16 supercharges<sup>8</sup>  $\epsilon_{\tilde{a}}$  and  $\tilde{\epsilon}_{\tilde{a}}$ .

Thus, a 2 form and a dual 6 form corresponding to the fermion bilinear can be written as combinations of  $T_2$  and  $V_2$ , and respectively  $T_6$  and  $V_6$  multiplied by powers of  $r$ . Plugging this ansatz in Eq. (18), we find 4 solutions. Two of them correspond to a mode which is not a chiral primary, and the other two, one non-normalizable and the other non-normalizable correspond to the fermion bilinear we are interested in. Adding the fermion bilinear to the boundary theory Hamiltonian corresponds [26] to adding to the background (15) the non-normalizable solution:

$$\begin{aligned} F_2 &= \frac{m}{\sqrt{\alpha'}} Z(2T_2 - 6V_2), \\ F_6 &= -\frac{m}{\sqrt{\alpha'}} Z(6T_6 - 6V_6), \end{aligned} \quad (22)$$

where  $m$  is proportional to the coefficient of the fermion bilinear added to the boundary Lagrangian.

A tubular D2 brane in this geometry couples with  $C_3 + B_2 \wedge C_1$ . In our background, this combination can be found using

$$\begin{aligned} d(C_3 + C_1 \wedge B_2) &= F_4 - C_1 \wedge H_3 + B_2 \wedge F_2 \\ &= -Z^{-1}(*_8 F_6 - F_2) \wedge dx^0 \wedge dx^1 \\ &= 2mT_2 \wedge dx^0 \wedge dx^1. \end{aligned} \quad (23)$$

Since  $Z^{-1}(*_8 F_6 - F_2)$  is a harmonic form, it only depends on its value at infinity, which is given by the boundary theory. Therefore the expression above remains true when the  $F1$  strings are coincident or when they are distributed. This implies that the WZ term in the action of a cylindrical D2 brane is independent of where the  $F1$  strings are.

### C. Probing the perturbed solution

In order to find the self-interaction potential of a cylindrical D2 brane with large  $F1$  charge  $N$ , one has to find first the potential felt by a D2 brane probe with smaller  $F1$  charge  $1 \ll n \ll N$ , and then to build the D2 brane by bringing test D2

branes from  $\infty$ . Anticipating, we will find that similarly to other Polchinski-Strassler type problems [6–8,27,28], the probe potential will be independent of the distribution of the strings; thus the self-interaction potential of a tubular D2 brane containing all the  $N$  strings will be the same as the probe brane potential with  $n$  replaced by  $N$ .

A cylindrical D2 brane can be given  $F1$  charge by turning on a nontrivial  $F_{01}$  on its world volume. The quantized  $F1$  string charge in a nontrivial  $G_{\mu\nu}$ ,  $\Phi$  and  $B_{\mu\nu}$  is

$$\begin{aligned} \Pi &= \frac{\delta L}{\delta F_{01}} \\ &= \frac{2\pi r}{\sqrt{2\pi}(2\pi\alpha')^{3/2}g_s} \\ &\quad \times Z^{1/2} \frac{2\pi\alpha'(B_{01} + 2\pi\alpha'F_{01})}{\sqrt{|G_{00}|G_{11} - (B_{01} + 2\pi\alpha'F_{01})^2}} \\ &= n. \end{aligned} \quad (24)$$

If we call

$$\begin{aligned} A &\equiv \frac{B_{01} + 2\pi\alpha'F_{01}}{\sqrt{|G_{00}|G_{11} - (B_{01} + 2\pi\alpha'F_{01})^2}} \\ &= \frac{\sqrt{2\pi}(2\pi\alpha')^{1/2}ng_s}{2\pi rZ^{1/2}}, \end{aligned} \quad (25)$$

the Hamiltonian density per unit string length will be

$$H_{BI} = \frac{2\pi rZ^{1/2}Z^{-1}}{\sqrt{2\pi}(2\pi\alpha')^{3/2}g_s} (\sqrt{A^2 + 1} - A). \quad (26)$$

We are interested in probes with dominant string charge,  $A \gg 1$ , which is easily realized for  $n > \sqrt{N}$  in the regime where supergravity is valid. In this regime the square root can be expanded, and the main contributions to the Hamiltonian cancel, as they represent the interaction energy between parallel  $F1$  strings. After this cancellation, the subleading term of the Born-Infeld action is

$$V_{BI} = \frac{\pi r^2}{(2\pi\alpha')^2 n g_s^2}. \quad (27)$$

As we mentioned in the previous section, this supergravity result reproduces exactly the one obtained using the non-Abelian Born-Infeld action in the limit of vanishing D0 charge. For a D2 brane of  $R^2 \times S^1$  geometry, this contribution to the action depends neither on the orientation of the  $S^1$ , nor on  $Z$ . It is straightforward to generalize this action for the circle replaced by an ellipse. This can be easily done in a similar way to [6–8,27,28]. If one turns on the bilinear  $\text{Re}(\bar{\Lambda}^1 \Lambda^2)$  which corresponds to  $T_2 = \text{Re}(dz^2 d\bar{z}^4)$ , it is natural to expect the probe to have a minimum if the two semi-axes of the ellipse lie in the two planes spanned by components of  $z^2 = x^4 + ix^5$  and  $z^4 = x^8 + ix^9$ . If the lengths and

<sup>8</sup>On can also examine the supersymmetry preserved by a certain fermion bilinear by doing a 9-11 flip. A bilinear of the form  $\bar{\Lambda}^1 \Lambda^2$  becomes a vector bilinear on the M2 brane when lifting to  $M$  theory. Reducing along a different direction, this becomes a vector bilinear on the D2 brane, of the form  $\bar{\lambda}^1 \gamma^\mu \lambda^2$  which upon  $T$  duality along  $\mu$  becomes again a scalar fermion bilinear on the D1 brane:  $\bar{\lambda}^1 \gamma^5 \lambda^2$ . Reexpressing the complex fermions in real ones, this bilinear is  $\lambda^1 \lambda^3 - \lambda^2 \lambda^4$ , which can be expressed as coming from a superpotential of the form  $W = \phi^1 \phi^3 - \phi^2 \phi^4$ . Thus after changing the sign of a field this superpotential becomes the dimensional reduction of a hyper bilinear in the 2D SYM on the D2 branes. Since that superpotential preserves 4 supercharges ( $\mathcal{N}=2$  in 2+1 dimensions), this is the supersymmetry preserved by our operator.

orientations of the semiaxes are given by the modulus and phase of  $Z_4$  and  $Z_2$ , the result is obtained by just replacing  $r^2$  by  $|Z_4|^2/2 + |Z_2|^2/2$  in Eq. (27).

One can also find the interaction potential coming from the Wess-Zumino action. Using Eq. (23) and integrating over the ellipsoid one finds after a few straightforward steps

$$V_{WZ} = -\frac{2\pi}{(2\pi\alpha')^2} m \operatorname{Re}(Z_4 \bar{Z}_2), \quad (28)$$

which is also independent of the harmonic function  $Z$ . Thus, the first two terms of the action of the probe are

$$V_{BI} + V_{WZ} = \frac{\pi}{2(2\pi\alpha')^2} \left[ \frac{|Z_4|^2 + |Z_2|^2}{ng_s^2} - 4m \operatorname{Re}(Z_4 \bar{Z}_2) \right]. \quad (29)$$

As we said in the Introduction, in addition to these two terms there is another one coming from the back reaction of the  $F_2$  and  $F_6$  perturbations on the metric, dilaton and 2-form. One can see from the equations of motion that this term should be proportional<sup>9</sup> to  $m^2 g_s^2 n r^2$ . Supersymmetry allows us to find this term exactly by simply completing the square in Eq. (29). Thus

$$\begin{aligned} V_{total} &= \frac{\pi}{2(2\pi\alpha')^2} \left[ \frac{|Z_4|^2 + |Z_2|^2}{ng_s^2} - 4m \operatorname{Re}(Z_4 \bar{Z}_2) \right. \\ &\quad \left. + m^2 n g_s^2 (|Z_4|^2 + |Z_2|^2) \right] \\ &= \frac{\pi}{2(2\pi\alpha')^2 n g_s^2} \left[ |Z_4 - Z_2 m n g_s^2|^2 \right. \\ &\quad \left. + |Z_2 - Z_4 m n g_s^2|^2 \right]. \quad (30) \end{aligned}$$

#### D. The full potential

The last term of Eq. (30) is also independent of  $Z$ . Therefore, the potential of the probe tube does not depend on the position of the  $F1$  strings sourcing the geometry. One can now find the self-interaction potential of a D2 brane tube containing *all*  $F1$  strings, in its own geometry, by bringing from infinity small probes and using them to construct the final configuration. Since the potential of each of these small probes is independent of where the rest of the probes are, the final potential will be the sum of the probe potentials, which gives the same result as Eq. (30) with  $n$  replaced by  $N$ . The most general configuration will consist of several D2 tubes with  $F1$  charges  $n_i$ . The potential will be the sum of terms, each of them equal to Eq. (30) with  $n$  replaced by  $n_i$ :

<sup>9</sup>The factor  $m^2 g_s^2$  comes from the square of the first order perturbation,  $n$  comes from the dominant  $F$ -string charge of the probe, and the proportionality with  $r^2$  is not hard to derive.

$$V_{many\ tubes} = \sum_i \frac{\pi}{2(2\pi\alpha')^2 n_i g_s^2} \left[ |Z_4^i - Z_2^i m n_i g_s^2|^2 + |Z_2^i - Z_4^i m n_i g_s^2|^2 \right], \quad (31)$$

where  $Z^i$  parametrize the semiaxes of the  $i$ th D2 tube, which contains  $n_i$   $F1$  strings. This potential is very interesting. It implies that for a generic strength of the perturbation, there is only one minimum at  $Z_4 = Z_2 = 0$ . Contrary to what one might expect, increasing the strength of the four-form does not make the polarization to a D2 brane more likely. This is because a large  $F_4$  causes a large backreaction on the graviton, dilaton and 2 form, which do not favor the polarization.

Even more remarkable, for the particular values  $m = \pm 1/n_i g_s^2$ , the zero energy solution is given by  $Z_4^i = \pm Z_2^i$  for the  $i$ th tube, and  $Z_4 = Z_2 = 0$  for the others. Thus, for these values of the perturbation a moduli space of possible polarization vacua opens up;  $n_i$  of the original  $N$  strings can be polarized. The lowest absolute value of  $m$  which allows the polarization to occur is  $m = 1/N g_s^2$ . In this case all  $N$  strings polarize. For  $m = 1/p g_s^2$ ,  $N/2 < p < N$ ,  $p$  of the  $N$  strings polarize into one D2 brane. For  $p = N/2$  it suddenly becomes possible for half the strings to polarize into one D2 brane, and the other half to polarize into another D2 brane. Both D2 tubes can be at any radius. In general, for  $N/(k-1) < p < N/k$ , the ground state will consist of  $k$  concentric D2 branes at different radii, with  $F1$  charge  $p$  each.

#### E. Consistency checks

Before we continue, we should address two small but potentially dangerous issues. First, we have to make sure that the supergravity perturbation fields are weaker than the background fields. The energy of the background and perturbation fields are respectively

$$\begin{aligned} e^{-2\phi} H_{01r}^{(3)} H_{01r}^{(3)} G^{00} G^{11} G^{rr} &\sim \frac{N \alpha'^3}{r^8}, \\ F_{ij}^{(2)} F_{ij}^{(2)} G^{ii} G^{jj} &\sim m^2 Z^2 \sim \frac{m^2 N^2 g_s^4 \alpha'^5}{r^{12}}. \quad (32) \end{aligned}$$

The condition for validity of the perturbation expansion is thus  $r^4 \gg m^2 N g_s^4 \alpha'^2$ . For the smallest value of  $m$  which allows for a moduli space this is equivalent to  $r^4 \gg \alpha'^2/N$  which is trivially satisfied. However, for larger values of  $m$  which allow  $n$  strings to polarize into a D2 brane, this condition is satisfied only for  $r^4 \gg \alpha'^2 N/n^2$ .

The second issue has to do with the attraction between the opposite sides of a tube, or between concentric tubes. Because the large  $F1$ -string tension and the D2 brane tension add in quadratures, the interaction coming from the gravitational D2-D2 attraction has a negligible effect on the energy. What could spoil the interaction is the electric attraction between the opposite sides of the tube.

The issue of this possible interaction is usually not addressed in most of the polarization papers. In principle this can give an energy contribution which may affect the polar-

ization ground state. In our context, since the balance of the energies is so delicate, such a contribution could in principle easily lift the moduli space. One could probably argue that supersymmetry prevents this contribution from affecting the potential, but nevertheless it is instructive to see how it happens.

As we can see from Eq. (23), our D2 brane tube couples with a field combination which is proportional to the harmonic form  $Z^{-1}(*_8F_6 - F_2)$ . The presence of extra D2 brane charge affects the Bianchi identity for  $F_6$  only. Nevertheless,  $Z^{-1}(*_8F_6 - F_2)$  remains harmonic. Therefore the presence of D2 charge (from the opposite side of the tube) does not affect the Wess-Zumino term of the potential. Thus, as we expected, the attraction between the opposite sides of the tube or between different tubes does not have any effect on the energy of the system. The lack of interaction between tubes placed at different radii is characteristic to Polchinski-Strassler type setups, and is also present for the supertubes with D0 charge [30].

Usually the appearance of a moduli space of vacua signals some symmetry enhancement. It is interesting to ask what this symmetry might be. One certain thing is that supersymmetry is not enhanced, since both the field strengths which allow polarization and those which do not allow it preserve 4 supercharges. Thus, this is probably a symmetry which appears in the UV of the mysterious orbifold CFT living on the strings.

#### IV. CONCLUSIONS AND FUTURE DIRECTIONS

We have given matrix and supergravity descriptions to the polarization of type IIA fundamental strings in tubular D2 branes. For the matrix description we have used the matrix degrees of freedom of D0 branes and their non-Abelian Born-Infeld action to describe an  $F1 \rightarrow D2$  tubular configuration in a nontrivial  $F_4$  background.

In order to give a supergravity description while maintaining control over the supersymmetry we have turned on a fermion bilinear in the theory living on  $N$  coincident fundamental strings. We have then related this bilinear to a supergravity nonnormalizable mode via an AdS-CFT like correspondence, and showed that the nonnormalizable mode induces  $F1 \rightarrow D2$  polarization. We have found that polarization occurs only when the value of the bulk fields is proportional to fractions whose denominator is smaller than  $N$ ; moreover, in these cases the polarization radius is a modulus. For any other values of the bulk fields there is no polarization.

For the supergravity description we have used the setup and technology developed by Polchinski and Strassler [6]. Although this technology is sometimes heavy, it has certain advantages (such as the control on the supersymmetry and on the back reaction of the fields, or the lack of attraction between the opposite sides of the polarized object) which in our opinion make it the only correct setup for the study of polarization into objects one dimension higher.

One interesting question which one might ask is what happens if one turns on a nonsupersymmetric traceless scalar bilinear in the potential, which might create an instability

along some direction. Understanding the effect of this other piece in the potential can be easily done in our framework (see [8] or [31] for an example). Such a piece makes the strings cease to be a classical ground state, and seems to leave the puzzle in [2] unsolved. The only consolation is that only a very special combination of 4-form and graviton will create this instability. In general however, turning on an  $F^4$  will not affect the stability of the strings.

It is intriguing to see what is the fate of this configurations when one lifts them to M theory. A D2 brane with large  $F$  string charge lifts to an M2 brane helix which winds around the eleventh dimension many times while circulating once around an  $S^1$ . The radius of the large circle is a modulus, so there is no potential for the helix to be stretched in a radially symmetric fashion. Even more puzzling to our intuition is the fact that a state of many such helices, nonconcentric and possibly intertwined preserves 4 supercharges. When one takes the size of the 11'th dimension to infinity these helices should naturally dissolve into the ‘‘Coulomb branch’’ of the theory on parallel M2 branes.

The  $F1 \rightarrow D2$  configuration falls in the general category of objects polarizing into objects one dimension higher. Similar situations can occur in the case of D4 branes polarizing to NS5 branes. We expect a similar potential, which allows a moduli space to open up for a very specific value of the perturbation, and does not allow polarization for any other value. On general grounds one expects the  $D4 \rightarrow NS5$  polarization to happen for  $\mathcal{N}=1$  supersymmetry in  $4+1$  dimensions (8 supercharges), and therefore the moduli space not to be lifted up by quantum corrections. Again, the lift of the configuration in M theory would give a spiral-shaped M5 brane.

The appearance of flat directions is a very intriguing phenomenon, whose origin would be exciting to find. One possible hint to the direction one should search in is the recent investigation by Brecher, Saffin and Emparan [16–18] of the  $F1 \rightarrow D2$  polarization in a Melvin background. The polarization potential they obtain becomes flat only for the critical value of the Melvin field. Since in our case this field is very small, and it appears in combination with other fields, the only way to ascertain if the flat direction has a similar origin is to lift our perturbed solution to M theory.

We have studied a very intriguing system, and found many phenomena which challenge our intuition, such as the presence of a moduli space of polarization vacua or the fact that M2 brane spherical helices in M theory at finite  $R_{11}$  can be supersymmetric. We cannot but hope that in time a more fundamental explanation of these phenomena and other less understood aspects of M theory will be given.

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## APPENDIX

We list several properties of the transverse space antisymmetric 2- and 6-tensors which form a basis for the forms with the  $R$ -symmetry transformation properties of a fermion bilinear:

$$T_6 = *_8 T_2 \quad (\text{A1})$$

$$V_6 = \frac{1}{6!} \left( \frac{x^t x^m}{r^2} T_{tmprqs} + 5 \text{ more} \right) dx^m \wedge dx^n \wedge dx^p \wedge dx^q \wedge dx^r \wedge dx^s, \quad (\text{A2})$$

$$V_2 = \frac{1}{2!} \left( \frac{x^q x^i}{r^2} T_{qj} + 1 \text{ more} \right) dx^i \wedge dx^j, \quad (\text{A3})$$

$$T_2 - V_2 = *_8 V_6, \quad T_6 - V_6 = *_8 V_2, \quad (\text{A4})$$

$$d(\ln r) \wedge V_6 = 0, \quad d(\ln r) \wedge V_2 = 0, \quad (\text{A5})$$

$$d(r^p(6T_6 + pV_6)) = 0, \quad d(r^p(2T_2 + pV_2)) = 0. \quad (\text{A6})$$

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