Space-time orbifold: A toy model for a cosmological singularity

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We explore bosonic strings and type II superstrings in the simplest time dependent backgrounds, namely orbifolds of Minkowski space by time reversal and some spatial reflections. We show that there are no negative norm physical excitations. However, the contributions of negative norm virtual states to quantum loops do not cancel, showing that a ghost-free gauge cannot be chosen. The spectrum includes a twisted sector, with strings confined to a ‘‘conical’’ singularity which is localized in time. Since these localized strings are not visible to asymptotic observers, interesting issues arise regarding unitarity of the *S* matrix for scattering of propagating states. The partition function of our model is modular invariant, and for the superstring, the zero momentum dilaton tadpole vanishes. Many of the issues we study will be generic to time-dependent cosmological backgrounds with singularities localized in time, and we derive some general lessons about quantizing strings on such spaces.

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I. INTRODUCTION

Time-dependent space-times are difficult to study, both classically and quantum mechanically. For example, nonstatic solutions are harder to find in general relativity, while the notion of a particle is difficult to define clearly in field theory on time-dependent backgrounds. Quantum mechanical strings propagating on time-dependent spaces can develop many subtle problems including difficulties with unitarity and ghosts in the physical spectrum. Nevertheless, the apparent observation of a cosmological constant from supernovae measurements [1], and an attendant expansion of the universe, requires us to understand clearly how time dependence of cosmological backgrounds is incorporated into string theory. In related theoretical developments, recent work has explored the physics of de Sitter space [2], as well as new pictures of the early universe in which a collision of branes forms the observable cosmic structures [3]. In the latter models, and in the pre-big-bang scenarios [4], a stringy resolution of an initial singularity is proposed to permit an extension of space-time to an era before the big bang. In view of all this it is worthwhile to investigate perturbative string theory in singular cosmological backgrounds.

Perturbative string theory is most easily studied in flat, translationally invariant space. The simplest nonhomogeneous spaces in which it is well defined are orbifolds of flat space in which some Euclidean directions are quotiented by a discrete subgroup of the isometry group [5]. When the action of the discrete group has fixed points, the orbifold has conical singularities, as well as new light states (the so-called twisted sectors) which are confined to these defects. Condensing twisted sector states can resolve the conical singularities in many cases such as the classic example R^4/Z_2 where four Euclidean directions are identified under reflections.

Can we find consistent backgrounds in string theory by identifying points in space-time rather than just in space? One simple example is the Bañados-Teitelboim-Zanelli (BTZ) black hole of three dimensional gravity which is obtained by quotienting AdS_3 by a boost [6].[1] Such orbifolds bear a relation to the kinds of identifications discussed in the context of resolving singularities separating contracting and expanding phases of some cosmological models [3]. Likewise, some coset Wess-Zumino-Witten (WZW) models are consistent time-dependent string backgrounds [9]. Also, string theory on orbifolds with time identified under $t \rightarrow t+1$ (i.e., circular time) has been studied in [10] and the resulting timelike *T* duality has been studied in [11]. Space-time singularities in string theory were studied in [12]. In this paper, we will seek simple models of time-dependent spaces and of cosmological singularities by constructing space-time orbifolds in which we identify space-time under both time reversal and reflections in some directions. Generally speaking, string theories defined on such spaces are threatened by a number of pathologies including potential ghosts in the physical spectrum and problems with unitarity. In fact, all known proofs of the no-ghost theorem explicitly require

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[1]The consistency of string theory on AdS_3 itself is nontrivial, e.g. for the no-ghost theorem and modular invariance see [7]. For work on string theory in BTZ black holes, see e.g. [8].

time-independent backgrounds [13]. Also, supersymmetry is generally broken and so there may be a danger of tadpoles at one loop and instabilities like tachyons could occur.[2] Part of our goal is to explore the many subtleties that beset such constructions in string theory.

We study bosonic and type II superstrings on $R^{1,d}/Z_2$, in which we have identified space-time by time reversal and reflections. When $d=0$, only time is identified and the space has an initial singularity at $t=0$. When $d\geqslant 1$ the background geometry is a space-time cone with a "conical" singularity at $t=x_1=\cdots=x_d=0$. String theory on such spaces is defined by projecting onto the sector of the Hilbert space that is invariant under these discrete transformations, and including possible twisted sectors localized at the orbifold fixed point at $t=x_1=\cdots=x_d=0$, and which therefore do not propagate. After this projection, quantum mechanics is consistent with closed time-like loops in the geometry. We find that the physical states are ghost-free when $d+1\geqslant 9$ for the bosonic string and that there is no restriction on d for the superstrings. In type II superstrings, when $d+1=4$, there is a "massless" twisted sector in which physical states satisfy the on-shell condition $|\vec{p}|^2=0$.[3] It is possible that condensing these states would resolve the conical singularity, and so we focus on the $d+1=4$ case.

We compute the partition function when $d+1=4$ and find that it is zero. Likewise the one loop zero-momentum tadpoles vanish suggesting that we have a consistent string background at this order in string perturbation theory. However, negative norm states (although not present in the on-shell physical spectrum) make a contribution to the partition function—their virtual effects do not cancel between the matter and ghost sectors as they do in the standard R^4/Z_2 orbifold. This shows that it is not possible to choose a ghost-free gauge in which all computations are carried out in terms of positive norm states. We expect that this will be generally true for string theory in time-dependent backgrounds.[4]

We conclude the paper by discussing several novel subtleties introduced by the localization in time of a sector of physical states, and by summarizing lessons learned from our work about time-dependent backgrounds and cosmological singularities in string theory.

II. SPACE-TIME ORBIFOLDS

We study space-time orbifolds constructed by identifying Minkowski space under time reversal and reflections in some spatial directions. As we will see below, the resulting geometry can be interpreted as a space-time cone. After the identifications the covering space has some closed time-like loops. However, because the orbifold prescription projects onto states in Hilbert space that are symmetric under the identifications, quantum mechanical evolution remains consistent.

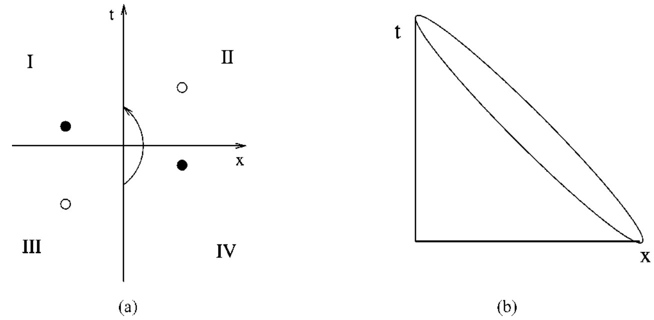

FIG. 1. A space-time cone.

A. Conical space-time geometries

Consider identifying time (X^0) and d spatial directions under reflections:

$$X^a \rightarrow -X^a \quad (a=0,\ldots,d), \tag{1}$$

leaving all other directions unaffected. Figure 1 shows the resulting space-time cone when $d=1$. Points in opposite quadrants of the $X-T$ plane are identified as in Fig. 1. Therefore the quadrants II and IV (or I and II) may be taken as "fundamental" regions with independent physics. Identifying these regions along the T (or X) axis produces the cone in Fig. 1(b) with a singular point at $T=X=0$.

The proper distance on the covering space between a point (T,X) and its image $(-T,-X)$ is $ds^2=4(X^2-T^2)$. This is time-like in the region inside the light cone emanating from the point $T=X=0$ on the covering space. As a result there are closed time-like curves in this geometry, such as the one in Fig. 1(a). In the orbifold construction which we will describe below, it is not immediately obvious that such loops pose a fundamental problem since we are instructed to project to states in the Hilbert space that are invariant under the space-time identifications, i.e., we project onto quantum mechanical wave functions that satisfy $\psi(x,t)=\psi(-x,-t)$. As a result, the classic paradoxes of "killing one's own grandmother" are avoided. However, other subtleties could arise. For example, the precise definition of observables and S matrices on such spaces remains to be understood. Likewise in the presence of closed null curves there is a potential danger of a divergent stress tensor in a second quantized field theory. However, this is usually mitigated by supersymmetry which we will have in the bulk of spacetime. This paper is intended as a preliminary exploration of space-time cones, and so we will not address the second quantization of theories in such spaces. Rather we will study the quantum mechanics of free strings which is well defined for the reasons described above. We hope to return to a general study of closed timelike curves in string theory in a later publication.

A picture of time evolution on the cone is provided in Fig. 2(a) where we have folded regions II and IV along the X axis and identified the negative and positive directions along the

[2]Of course, string theorists have learned over the past few years that the dynamics of tachyons in some cases may be under control and may even be of cosmological interest.

[3]Actually, this implies that $\vec{p}=0$ since the twisted sector states are localized in time and so only carry momenta in the unorbifolded Euclidean directions.

[4]This is reminiscent of mixing between the matter and ghost sector conformal field theories (CFTs) discussed in [14].

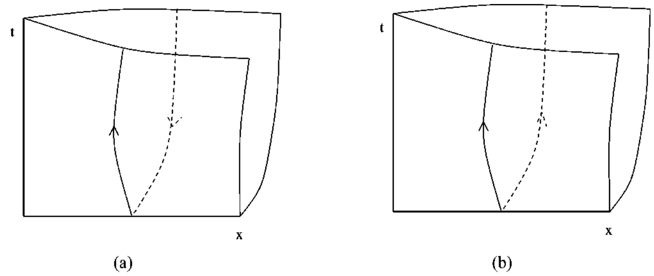

FIG. 2. Time evolution on the cone.

time axis, to make a cone. It is natural then to describe the evolution of states on the cone with respect to the time direction inherited from the positive time direction in quadrants II and IV of the parent manifold. The line $x=0$ appears to have time "running both ways," but this is simply because we have projected onto states that are time reversal invariant on the $X=0$ axis.

Constructing the cone by gluing the X axis of quadrants I and II yields a similar picture with two "sheets" glued together on the T and X axis. At first sight the time inherited from the covering space gives evolution moving "up" on both sheets in Fig. 2(b), with the boundary condition that the wave-functions on both sheets approach the same value on a big-bang-like surface at $T=0$. However, on the X axis of the covering space the orbifold identifications also imply that $\partial\psi(x,t)/\partial t|_{t=0} = -\partial\psi(-x,t)/\partial t|_{t=0}$. Therefore, on the cone, with time evolving "up" on both sheets, although wave functions on both sheets agree on the initial surface, their time derivatives are opposites of each other. Therefore it seems more natural once again to describe the evolution of states with respect to a continuous time as in Fig. 2(a).

B. Euclidean world sheets and Lorentzian backgrounds

As we have discussed, we will construct string theory on our space-time orbifold by projecting onto states of strings in Minkowski space that are invariant under the discrete identifications. In Lorentzian space-times the signature of the string worldsheet must be $(-1,1)$ in order for classical string propagation to exist. (The 2D equations of motion are solved by equating the world sheet metric with the metric induced from space-time.) Nevertheless, the standard techniques of string theory involve analytically continuing the world sheet to Euclidean signature in order to exploit the techniques of two-dimensional conformal field theory and complex geometry. In static backgrounds we might imagine continuing the space-time to Euclidean signature at the same time, but this is not possible in time-dependent backgrounds such as ours. Our analysis in this paper is done with a Lorentzian signature world sheet except our discussion of modular invariance where we formally continue the world sheet to Euclidean signature. The resulting path integral (with a Euclidean world sheet and Lorentzian target space) is not strictly speaking well-defined because the action is not bounded from below. Nevertheless, it appears to be finite in our case, and we use it formally to discuss modular invariance. Subtleties in defining the Polyakov path integral in Lorentzian signature have been discussed by Mathur in [15].

III. BOSONIC STRING THEORY ON THE LORENTZIAN ORBIFOLD

Before studying superstrings on space-time orbifolds we examine the 26-dimensional bosonic string propagating on $R^{1,d}/Z_2$. This already contains the distinctive features of the Lorentzian orbifold. In particular, we show that it is possible to obtain a ghost-free physical spectrum and a modular invariant partition function, but that virtual negative norm states make un-cancelled contributions to quantum loops. This is a reflection of the time dependence of the string theory background.

Consider flat 26-dimensional Minkowski space with points identified under the Z_2 action,

$$X^a \to -X^a \quad (a=0\cdots d);$$

$$X^i \to X^i \quad (i=d+1\cdots 25). \tag{2}$$

This action has a fixed $(25-d)$-dimensional hyper-plane, given by $X^a=0$. To get consistent string propagation on this space-time, we project the conventional bosonic string Hilbert space onto its Z_2 invariant subspace. This gives the untwisted sector of the orbifold theory. In addition, there is a twisted sector corresponding to strings that are closed only under the identifications made by the orbifold group. Again, we project out twisted sector states that are not invariant under the orbifold action. The twisted strings are trapped around the tip of the cone in Fig. 1(b), which is a $(25-d)$-dimensional hyper-plane localized at an instant in time. The untwisted strings can propagate in the bulk.

The orbifold above has the novel feature that it includes a reflection in the time direction, destroying the global time-like isometry of flat space-time. This means that we cannot perform quantization by going to light-cone gauge. The alternative is to use the covariant BRST formalism. However, in the absence of a light-cone gauge choice, the absence of negative-norm states in the physical spectrum is no longer evident, especially in view of the nonapplicability of the known proofs of no-ghost theorem [13]. In the following, we will mostly be concerned with this issue. In the covariant formalism, we work with world-sheet fields X^μ ($\mu=0,\ldots,25$) and the reparametrization ghosts b and c. In the untwisted sector $X^\mu(\sigma+2\pi,\tau)=X^\mu(\sigma,\tau)$, and the mode expansion is[5]

$$X^\mu = x^\mu + p^\mu \tau + i\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-in(\tau-\sigma)} + i\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}e^{-in(\tau+\sigma)}. \tag{3}$$

The (tachyonic) ground state $|p^a,p^i\rangle$ carries momentum in both orbifolded and unorbifolded directions and the Hilbert space of states is constructed by acting with creation operators on the ground state. Half of the states with nonzero p^a are projected out of the spectrum. For example, of the states $\alpha_{-1}^a\tilde{\alpha}_{-1}^i|p^a,p^i\rangle$ and $\alpha_{-1}^a\tilde{\alpha}_{-1}^i|-p^a,p^i\rangle$, only the linear com-

[5]We will work in $\alpha'=2$ units in this paper.

bination $\alpha^a_{-1}\tilde{\alpha}^i_{-1}(|p^a,p^i\rangle - |-p^a,p^i\rangle)$ is retained. When $p^a=0$, only the Z_2 invariant combinations of the oscillators acting on the vacuum are kept. Hence, $\alpha^a_{-1}\tilde{\alpha}^i_{-1}|0,p^i\rangle$ is projected out but $\alpha^a_{-1}\tilde{\alpha}^b_{-1}|0,p^i\rangle$ is retained.

In the twisted sector, the fields X^a satisfy the antiperiodic boundary condition $X^a(\sigma+2\pi)=-X^a(\sigma)$, with the mode expansion given by

$$X^a = i\sum_r \frac{\alpha^a_r}{r} e^{-ir(\tau-\sigma)} + i\sum_r \frac{\tilde{\alpha}^a_r}{r} e^{-ir(\tau+\sigma)}, \tag{4}$$

where r is half odd-integral. The twisted sector is localized at the orbifold fixed plane at $X^a=0$. In particular, for any value of the parameter τ, the twisted string world sheet does not propagate too far out in time X^0. This is similar to an instanton. The mode expansion for X^i in directions transverse to the orbifold is the same as in the untwisted sector. Consequently, the ground state in the twisted sector carries a momentum p^i only in the transverse directions. The Hilbert space is built by acting with creation operators on the ground state and projecting onto the Z_2 invariant subspace.

The ghosts b and c are not affected by the orbifold and have the same mode expansions in both sectors:

$$b(\sigma,\tau) = \sum_n b_n e^{-in(\tau-\sigma)}, \quad c(\sigma,\tau) = \sum_n c_n e^{-in(\tau-\sigma)}; \tag{5}$$

and similarly for right-movers $\tilde{b}$ and $\tilde{c}$.

A. Physical states

The BRST operator Q_B is given by

$$Q_B = \sum_n (c_n L^m_{-n} + \tilde{c}_n \tilde{L}^m_{-n}) + \sum_{m,n} \frac{(m-n)}{2} :(c_m c_n b_{-m-n} + \tilde{c}_m \tilde{c}_n \tilde{b}_{-m-n}): + a(c_0+\tilde{c}_0), \tag{6}$$

where L^m are the Virasoro generators in the matter sector and a is the zero point energy. Physical states are elements of the Becchi-Roaet-Stora-Tyutin (BRST) cohomology, i.e., they obey $Q_B|\psi\rangle=0$ subject to the equivalence relation $|\psi\rangle \sim |\psi\rangle + Q_B|\phi\rangle$, where $|\phi\rangle$ is an arbitrary state.

It is now easy to see that the physical spectrum does not contain negative norm states. In the untwisted sector, after the orbifold projection, the states form a subspace of the Fock space of the parent theory. Furthermore, the orbifold action (2) commutes with the BRST operator (6). This means that the space of physical states of the orbifold theory is a subspace of the space of physical states of the parent theory, and hence is free of negative norm states. More explicitly, for $p^a \neq 0$, one can easily establish a correspondence between states in the parent theory and those of the orbifold theory by appropriately choosing symmetrized or antisymmetrized momentum wave functions.

To see that the twisted sector physical states do not have negative norms, recall that the BRST condition $Q_B|\psi\rangle=0$, along with $b_0|\psi\rangle=0$, implies (see, for example, [16])

$$(L^m_0 + L^{gh}_0 - a)|\psi\rangle = 0, \tag{7}$$

where L^{gh}_0 is the ghost Virasoro generator. In terms of the twisted sector number operators, we have

$$L^m_0 + L^{gh}_0 = \frac{1}{2}p^i p_i + \sum_{n=1}^{\infty} n\left(N_{bn}+N_{cn}+\sum_{i=d+1}^{25} N_{in}\right) + \sum_{r=1/2}^{\infty}\sum_{a=0}^{d} rN_{ar}, \tag{8}$$

and the twisted sector zero-point energy is $a=[26-(d+1)-2]/24-(d+1)/48=(15-d)/16$. Then for a physical state $|\psi,p^i\rangle$, with momentum p^i in the unorbifolded directions, this implies

$$\frac{1}{2}p^i p_i + \sum_{n=1}^{\infty} n\left(N_{bn}+N_{cn}+\sum_{i=d+1}^{25} N_{in}\right) + \sum_{r=1/2}^{\infty}\sum_{a=0}^{d} rN_{ar} = (15-d)/16. \tag{9}$$

Since the left-hand side is always positive, d is restricted to $d \leqslant 15$ in order to allow for any physical states in the twisted sector. Furthermore, since $(15-d)/16<1$, a twisted sector physical state will not contain c, b and X^i excitations. For $1 \leqslant d \leqslant 7$, the physical spectrum will always contain a negative norm state corresponding to $\alpha^0_{-1/2}$. However, for $d \geqslant 8$ there are no negative norm states in the twisted sector physical spectrum which, for $15 \geqslant d \geqslant 8$, contains only the ground state $|0,p^i\rangle$. In particular, $p^i=0$ for $d=15$.[6]

B. Partition function and virtual ghosts

Although there are no negative norm physical states (for the right range of d), the orbifold theory may still contain negative norm virtual states running in loops. This can be studied by looking at the one-loop partition function. Before considering the orbifold case, we recall the partition function of the closed bosonic string in 26-dimensional Minkowski space,[7]

[6]For $d=15$, the state in the twisted sector is physical only when $p^i=0$. If this state at $p^i \neq 0$ were BRST exact, it would be orthogonal to all other physical states. Amplitudes involving such a state would then have to be proportional to $\delta^{(9-d)}(p^i)$. As argued in [16], since amplitudes in field theory and string theory never have this kind of a behavior, such a state with $p^i=0$ should not be part of the physical spectrum. However, this is *not* true for the twisted sector state on the Lorentzian orbifold. This is because the state with nonzero p^i is not BRST exact since it is not even BRST closed $(Q_B|p^i\neq 0\rangle \neq 0)$, as it does not satisfy the Virasoro constraint. So the above argument does not apply and the zero-momentum physical twisted state should be retained.

[7]More precisely, the definition of $Z(\tau,\bar{\tau})$ is

$$Z(\tau,\bar{\tau}) = \mathrm{Tr}(-1)^F c_0 b_0 \tilde{c}_0 \tilde{b}_0 q^{H_L} \bar{q}^{H_R} \tag{10}$$

where $(-1)^F$ anticommutes with all the ghost fields. In the following, we implicitly assume that the trace is taken with $(-1)^F c_0 b_0 \tilde{c}_0 \tilde{b}_0$ inserted.

$$Z(\tau,\overline{\tau})=\mathrm{Tr}\, q^{H_L}\overline{q}^{H_R}\sim\frac{V_{26}}{\tau_2^{12}}\left(\frac{1}{|q^{1/24}\prod_m (1-q^m)|^2}\right)^{26}\times\left|q^{1/24}\prod_m (1-q^m)\right|^4=\frac{V_{26}}{\tau_2^{12}}\left(\frac{1}{|q^{1/24}\prod_m (1-q^m)|^2}\right)^{24},$$

where $H_L=L_0-a$, $H_R=\widetilde{L}_0-a$, a is the zero point energy, and $q=e^{2\pi i\tau}$. V_{26} is a space-time volume factor related to the continuum normalization of the momentum integral and Π_m is a short hand for $\Pi_{m=1}^{\infty}$. Here, the contributions from the negative and positive norm ghost states cancel the contributions from the time-like and one space-like oscillators, respectively, giving the same result as we would get in the light-cone gauge. To verify that this really is how the cancellations work, one can compute the following closely related quantity:

$$S(\tau,\overline{\tau})=\mathrm{Tr}(-1)^s q^{H_L}\overline{q}^{H_R}\sim\frac{V_{26}}{\tau_2^{12}}\frac{\left|q^{1/24}\prod_m (1-q^m)\right|^2\left|q^{1/24}\prod_m (1+q^m)\right|^2}{\left(\left|q^{1/24}\prod_m (1-q^m)\right|^2\right)^{25}\left(\left|q^{1/24}\prod_m (1+q^m)\right|^2\right)}=\frac{V_{26}}{\tau_2^{12}}\left(\frac{1}{|q^{1/24}\prod_m (1-q^m)|^2}\right)^{24}.$$

The insertion $(-1)^s$ ensures that negative norm states contribute with a negative sign in the trace. The equality $Z(\tau,\overline{\tau})=S(\tau,\overline{\tau})$ then reflects the fact that the negative norm ghost contribution really did cancel that of the time-like oscillators.[8]

We now proceed to the partition function of the orbifold $R^{1,d}/Z_2$. In general, in the absence of a timelike isometry care is necessary in defining string amplitudes by analytic continuation. Indeed, very little of a general nature is known about perturbative string theory in time-dependent backgrounds. Here we take the approach of naively continuing the torus amplitude to Euclidean signature on the string world sheet. The resulting formal object is analyzed below and seems to be well-defined. A partition function, being a vacuum amplitude, is a space-time scalar. Therefore, the trace in it extends to the space-time index of the states, i.e., the conjugate to $\alpha^{\mu}_{-n}|p\rangle$ appears as $\langle p|\alpha_{\mu,n}$. The commutators then involve δ^{μ}_{ν} rather than $\eta_{\mu\nu}$ and time-like oscillators contribute in the same way as space-like ones. The partition function then has the same form as that of the Euclidean orbifold R^{1+d}/Z_2, and is given by

$$Z(\tau,\overline{\tau})=\mathrm{Tr}_U\frac{1+\hat{g}}{2}q^{H_L}\overline{q}^{H_R}+\mathrm{Tr}_T\frac{1+\hat{g}}{2}q^{H_L}\overline{q}^{H_R} \tag{11}$$

$$=\frac{V_{25-d}}{2}\left(\frac{1}{\sqrt{\tau_2}\left|q^{1/24}\prod_m (1-q^m)\right|^2}\right)^{24-(d+1)}\left[\frac{V_{d+1}}{\left(\sqrt{\tau_2}\left|q^{1/24}\prod_m (1-q^m)\right|^2\right)^{d+1}}+\frac{1}{2^{d+1}\left|q^{1/24}\prod_m (1+q^m)\right|^{2(d+1)}}\right.$$
$$\left.+\frac{1}{\left|q^{-1/48}\prod_m (1-q^{m-1/2})\right|^{2(d+1)}}+\frac{1}{\left|q^{-1/48}\prod_m (1+q^{m-\frac{1}{2}})\right|^{2(d+1)}}\right], \tag{12}$$

where $\hat{g}$ is the Z_2 action on the Hilbert space and Tr_U and Tr_T denote traces over untwisted and twisted sector states respectively. The first term contains a V_{d+1} from the momenta of the untwisted sector states. Only the zero modes contribute to the second term leading to a factor of $1/2^{d+1}$. Since the last two terms arise from the twisted sectors, there is no contribution from momentum in the $d+1$ orbifolded directions. The first terms is modular invariant by itself, and the last three transform into each other under modular transformations as is standard in noncompact orbifolds.

Although the expression for the partition function of the Lorentzian orbifold $R^{1,d}/Z_2$ is the same as that of the Euclidean orbifold R^{d+1}/Z_2, they embody very different physics. In the Euclidean orbifold, the negative norm ghost contribution always cancels against time-like oscillators, indicating the possibility of choosing a gauge (the light-cone gauge), in which there are no negative norm states. However, in the Lorentzian orbifold $R^{1,d}/Z_2$, virtual negative norm states make uncancelled contributions to the partition function. As a check of this, one can again look at

$$S(\tau,\overline{\tau})=\mathrm{Tr}_U(-1)^s\frac{1+\hat{g}}{2}q^{H_L}\overline{q}^{H_R}+\mathrm{Tr}_T(-1)^s\frac{1+\hat{g}}{2}q^{H_L}\overline{q}^{H_R}. \tag{13}$$

The difference, if any, between $S(\tau,\overline{\tau})$ and $Z(\tau,\overline{\tau})$ can only arise because of differing contributions from the negative norm states and therefore, we concentrate on these parts of

[8]We thank C. Vafa for this argument.

$Z(\tau,\overline{\tau})$ and $S(\tau,\overline{\tau})$. The contribution from negative norm states in the left moving sector to various terms in Eqs. (11),(13) is given by

$$\mathrm{Tr}_U(\pm 1)^s q^{L_0} \sim \prod_n \frac{(1\mp q^n)}{(1\mp q^n)},$$

$$\mathrm{Tr}_U(\pm 1)^s \hat{g} q^{L_0} \sim \prod_n \frac{(1\mp q^n)}{(1\pm q^n)},$$

$$\mathrm{Tr}_T(\pm 1)^s q^{L_0} \sim \prod_n \frac{(1\mp q^n)}{(1\mp q^{n+1/2})},$$

$$\mathrm{Tr}_T(\pm 1)^s \hat{g} q^{L_0} \sim \prod_n \frac{(1\mp q^n)}{(1\pm q^{n+1/2})}. \tag{14}$$

The upper signs in the four expressions correspond to contributions of negative norms states to $Z(\tau,\overline{\tau})$ and the lower signs correspond to their contributions to $S(\tau,\overline{\tau})$. The factor $(1\mp q^n)$ in the numerator is the contribution from the negative norm ghost states and the factor in the denominator is the contribution from the time-like oscillators.[9] From these expressions, it is clear that the contributions of the negative norm states to $Z(\tau,\overline{\tau})$ and $S(\tau,\overline{\tau})$ are not the same. Hence $S(\tau,\overline{\tau})\neq Z(\tau,\overline{\tau})$, which explicitly shows that the contributions of virtual negative norm states in the partition function do not cancel on the Lorentzian orbifold. This implies that a ghost-free gauge for string theory in such a background does not exist. This is perhaps not surprising: we cannot choose the light cone gauge because our orbifold involves a reflection in the time direction. One might have thought that there is some other gauge in which all calculations can be done in terms of positive norm states, but the analysis above shows that such a gauge does not exist. Nevertheless, as we have shown, there are no negative norm physical states on the orbifold. We expect that these features are generic for theories in time dependent backgrounds.

Since b and c are reparametrization ghosts, their periodicities on the world-sheet torus are fixed by the theory. However, suppose that we regard the X^μ as describing simply a free field theory on the orbifold. Then, in principle, we can introduce a (b,c) system, not as reparametrization ghosts, but with the sole purpose of removing the negative norm states. Then, by assigning appropriate periodicities to the ghosts, depending on the X^μ boundary conditions, it is possible to fully cancel the contributions of negative norm states in all sectors of the theory. However, this will not be a string theory.

Summary. We have found that there are no negative norm physical states in the bosonic string theory on the Lorentzian orbifold $R^{1,d}/Z_2$ when $d+1\geqslant 9$, and the partition function is modular invariant. However, negative norm virtual states make uncancelled contributions to quantum loops. This implies that it is not possible to choose a gauge in which all computations are done in terms of positive norm states. For $9\leqslant d+1\leqslant 16$, the ground state in the twisted sector $|p^i\rangle$ carrying momentum in the unorbifolded directions is physical with $|\vec{p}|^2=(15-d)/8$. For $d+1>16$, there are no physical states in the twisted sector.

IV. TYPE II SUPERSTRINGS ON THE LORENTZIAN ORBIFOLD

We will next move on to type II superstrings. Because the orbifold involves time, we will work in the covariant Ramond-Neveu-Schwarz(RNS) formulation. Now the orbifold action is

$$X^a \to -X^a,\ X^i \to X^i,\quad \psi^a \to -\psi^a,\ \psi^i \to \psi^i, \tag{15}$$

where, $a=0,\ldots,d$ and $i=d+1,\ldots,9$. For technical reasons, we will always consider d odd.

We first look at the untwisted sector. Here, the fermions have the standard mode expansions: $\psi^\mu(\sigma_-)=\Sigma_r \psi_r^\mu e^{-ir(\tau-\sigma)}$, with similar expressions for left-movers $\tilde{\psi}^\mu(\sigma_+)$. The sum is over $r\in Z+\frac{1}{2}$ in the NS sector and $r\in Z$ in the R sector. The bosons have the mode expansions (3). The zero point energy $a=a_B+a_F$ is $a=\frac{1}{2}$ in the NS sector and $a=0$ in the R sector. The NS sector ground state is a tachyonic scalar $|p^a,p^i\rangle_{NS}$, whereas the R ground state is a massless spinor $|p^a,p^i\rangle_R$. The orbifold operation acts on the R vacuum as

$$|p^a,p^i\rangle_R \to (\Gamma_{11})^{d+1}\Gamma^0\Gamma^1\cdots\Gamma^d|-p^a,p^i\rangle_R. \tag{16}$$

After the orbifold projection, the invariant states have momentum wave functions of definite symmetry, $|p^a,p^i\rangle \pm|-p^a,p^i\rangle$, depending on the $(d+1)$-dimensional chirality of the R ground state and the oscillator numbers.

As in the bosonic orbifold of the previous section, the physical untwisted orbifold states form a subspace in the space of physical states of the parent type II theory. Consequently, the untwisted sector is free of physical negative norm states.

The supersymmetry of the physical untwisted spectrum (for odd d) can be illustrated as follows. Let $\mathcal{S}^{(1,9)}$ denote an $SO(1,9)$ spin-field of definite chirality that relates the NS and R ground states in type II theory, $|p\rangle_R \sim \mathcal{S}^{(1,9)}|p\rangle_{NS}$. Then the space-time supersymmetry current in the unorbifolded theory has the form $J\sim e^{-\phi/2}\mathcal{S}^{(1,9)}$, where $e^{-\phi/2}$ is the spin-field for the $\beta,\ \gamma$ ghost system. Suppose that $\mathcal{S}^{(1,9)}$ has positive chirality and we denote $SO(n)$ spinors of ± 1 chirality by $\mathcal{S}_\pm^{(n)}$. As the orbifold breaks $SO(1,9)$ to $SO(1,d)\times SO(9-d)$, the positive chirality spin-field decomposes as

$$\mathcal{S}_+^{(1,9)}=\mathcal{S}_+^{(1,d)}\otimes\mathcal{S}_+^{(9-d)}+\mathcal{S}_-^{(1,d)}\otimes\mathcal{S}_-^{(9-d)}.$$

Then, under the orbifold projection (16), the piece with positive $SO(1,d)$ chirality survives and the orbifold inherits a

[9] Note that for the Euclidean orbifold, the contributions from the time-like oscillator is always $1/(1\mp q^n)$ which cancels the contribution from the negative norm ghost oscillators resulting in the equality $Z(\tau,\overline{\tau})=S(\tau,\overline{\tau})$ for the Euclidean orbifold.

supercurrent $J_{orb} \sim e^{-\phi/2} \mathcal{S}_{+}^{(1,d)} \otimes \mathcal{S}_{+}^{(9-d)}$ from the parent theory. This proves the supersymmetry of the untwisted sector, while showing that the amount of supersymmetry has been reduced.

We now turn to the twisted sector. The twisted bosons have the mode expansion (4). Fermions in the twisted sector satisfy boundary conditions:

$$\text{NS:}\quad \psi^a(\sigma+2\pi)=\psi^a(\sigma),\quad \psi^i(\sigma+2\pi)=-\psi^i(\sigma); \tag{17}$$

$$\text{R:}\quad \psi^a(\sigma+2\pi)=-\psi^a(\sigma),\quad \psi^i(\sigma+2\pi)=\psi^i(\sigma). \tag{18}$$

These lead to the mode expansions:

$$\text{NS:}\, \psi^a(\sigma_-)=\sum_{n\in Z} \psi_n^a\, e^{-in(\tau-\sigma)},$$

$$\psi^i(\sigma_-)=\sum_{r\in Z+1/2} \psi_r^i\, e^{-ir(\tau-\sigma)}; \tag{19}$$

$$\text{R:}\quad \psi^a(\sigma_-)=\sum_{r\in Z+1/2} \psi_r^a\, e^{-ir(\tau-\sigma)},$$

$$\psi^i(\sigma_-)=\sum_{n\in Z} \psi_n^i\, e^{-in(\tau-\sigma)}. \tag{20}$$

The periodicities and mode expansions are reversed along the orbifolded directions compared to the unorbifolded ones. The twisted NS sector has fermion zero modes along the orbifold and the corresponding ground state $|p^i\rangle_{NS}^T$ is a $SO(1,d)$ spinor and a $SO(9-d)$ scalar. The twisted R sector ground state $|p^i\rangle_R^T$ is a spinor under $SO(9-d)$ and a scalar under $SO(1,d)$. Some more details can be found in Appendix B.

Using the mode expansions, the Virasoro generators L_m and the world sheet supercurrents G_r and F_n can be worked out. These are summarized in Appendix A. To identify the physical spectrum, one also needs the zero point energies, $a=a_B+a_F$. In the NS sector, the world sheet bosonic and fermionic sectors contribute as

$$a_B=-\frac{d+1}{48}+\frac{9-d}{24}-\frac{2}{24},$$

$$a_F=-\frac{d+1}{24}+\frac{9-d}{48}-\frac{2}{48}. \tag{21}$$

Here, $-2/24$ is the contribution from the b,c ghosts and $-2/48$ is the contribution from the NS sector β,γ ghosts. In the twisted Ramond sector, a_B is as above and the fermions give,

$$a_F=-\frac{(9-d)}{24}+\frac{d+1}{48}+\frac{2}{24}, \tag{22}$$

where 2/24 is from the Ramond sector β,γ ghosts. In total then,

$$a=a_B+a_F=\frac{3-d}{8}\quad (\text{twisted, NS}), \tag{23}$$

$$a=a_B+a_F=0\qquad (\text{twisted, R}). \tag{24}$$

The zero point energy vanishes for any value of d in the twisted Ramond sector.

A. Twisted sector physical states

The content of the twisted sector physical spectrum is determined by the super-Virasoro constraints,

$$(L_m-a\,\delta_m)|\text{phys}\rangle=0\quad (m\geqslant 0),$$

$$G_r|\text{phys}\rangle=0\quad (r\geqslant \tfrac{1}{2},\text{NS}),$$

$$F_n|\text{phys}\rangle=0\quad (n\geqslant 0,\text{R}), \tag{25}$$

with the generators given in Appendix A. As in the bosonic case, the L_0 constraint gives

$$p^i p_i+\sum_l lN_l=\frac{3-d}{8}\quad (\text{NS sector}), \tag{26}$$

$$=0\quad (\text{R sector}). \tag{27}$$

Here p^i is the momentum carried by the twisted sector state in the unorbifolded direction, and $\Sigma_l lN_l$ schematically represents the combined sum over the bosonic, fermionic and ghost number operators in the twisted sector. Note that the minimum nonzero value of this sum is $\frac{1}{2}$, while the right-hand side is always less than $\frac{1}{2}$. Therefore, physical twisted states cannot have any oscillator excitations. In particular, they will be free of negative norms. In the twisted NS sector, there are no physical states for $d>3$. For $d\leqslant 3$, the twisted NS ground state $|p^i\rangle_{NS}^T$ is physical with $p^i p_i=(3-d)/8$. In particular, for the case of $d=3$, this ground state has $p^i=0$.

This state also trivially satisfies all the other physical state constraints in Eq. (25). In the R sector, the only physical state, for any d, is the Ramond ground state at zero momentum, $|p^i=0\rangle_R^T$. This also satisfies the remaining constraints in Eq. (25). In particular, the F_0 constraint gives $p_i\Gamma^i|p^i\rangle_R^T=0$, which is normally the Dirac equation reducing the number of spinor components by half. In our case, since $p^i=0$, it does not impose a constraint. Thus, e.g. in $d=3$ the twisted R sector vacuum has twice as many components as the twisted sector NS vacuum (see Appendix B).

The Gliozzi-Scherk-Olive(GSO) projection results in the NS sector ground state, $|p^i\rangle_{NS}^T$, having the same $SO(1,d)$ chirality in the left and right moving sectors.[10] In the twisted R sector, the ground state $|p^i\rangle_R^T$, has the same (opposite) $SO(9-d)$ chirality in the left and right moving sector for type IIB (type IIA) string theory.

[10]Recall we consider odd d so chirality is well defined.

In general, the bosonic and fermionic degrees of freedom in the twisted sector will not match. For the special case of $d=3$, the twisted sector NS ground state is a chiral spinor of $SO(1,3)$ and the R sector ground state is a chiral spinor of $SO(6)$. These spinors have different dimensionalities and as a result Bose-Fermi degeneracy of the space-time spectrum is broken in the twisted sector.[11]

B. Partition function and tadpoles

The one-loop partition function, as in the bosonic case, does not distinguish between space-like and time-like oscillators.[12] Therefore, the result for superstrings on the Lorentzian orbifold $R^{1,d}/Z_2$ will be the same as that for the Euclidean orbifold R^{d+1}/Z_2. This is in spite of the fact that the spectra in the two cases are very different, especially in the twisted sector. For definiteness, we look at the case of $d=3$.

The torus partition function for the orbifold is given by

$$Z(\tau,\bar{\tau})=\mathrm{Tr}_U\frac{(1+\hat{g})}{2}q^{H_L}\bar{q}^{H_R}+\mathrm{Tr}_T\frac{(1+\hat{g})}{2}q^{H_L}\bar{q}^{H_R},$$

where $\hat{g}$ is the representation of the Z_2 orbifold action on the Fock space, and Tr_U and Tr_T represent traces taken over the untwisted and the twisted sectors. We also need to sum over the four different spin structures of the torus in both sectors. The contributions from the b,c and β,γ ghosts will cancel the contributions from two unorbifolded Euclidean directions. Then, for the $d=3$ case, the result after the relative sign factors for the contributions from different spin structures have been chosen, is

$$Z=\frac{V_6}{2\tau_2^2\eta^4\bar{\eta}^4}\sum_{h,g=0}^{1}\frac{(V_4)^{(1-h)(1-g)}Z_b^{(h,g)}}{(16)^{(1-h)g}}\times\sum_{a,b=0}^{1}(-1)^{(a+b+ab)}\frac{\theta^2\begin{bmatrix}a\\b\end{bmatrix}\theta\begin{bmatrix}a+h\\b+g\end{bmatrix}\theta\begin{bmatrix}a-h\\b-g\end{bmatrix}}{2\eta^4}$$
$$\times\sum_{\bar{a},\bar{b}=0}^{1}(-1)^{(\bar{a}+\bar{b}+\lambda\bar{a}\bar{b})}\frac{\bar{\theta}^2\begin{bmatrix}\bar{a}\\\bar{b}\end{bmatrix}\bar{\theta}\begin{bmatrix}\bar{a}+h\\\bar{b}+g\end{bmatrix}\bar{\theta}\begin{bmatrix}\bar{a}-h\\\bar{b}-g\end{bmatrix}}{2\bar{\eta}^4},\tag{28}$$

where $\lambda=0,1$ for type IIA, IIB superstring. This is the same as the Euclidean case (see, for example, [17]). The θ functions are defined as

$$\theta\begin{bmatrix}a\\b\end{bmatrix}\equiv\theta\begin{bmatrix}a\\b\end{bmatrix}(0|\tau)=\sum_{n\in Z}q^{(1/2)(n-a/2)^2}e^{-\pi ib(n-a/2)},$$

and Z_b is the contribution from the bosonic sector,

$$Z_b^{(0,0)}=\frac{1}{\tau_2^2\eta^4\bar{\eta}^4},\quad Z_b^{(h,g)}=\frac{\eta^2\bar{\eta}^2}{\theta^2\begin{bmatrix}1-h\\1-g\end{bmatrix}\bar{\theta}^2\begin{bmatrix}1-h\\1-g\end{bmatrix}},\quad (h,g)\neq(0,0).$$

V_6 and V_4 are volume factors entering the continuum normalization of the momentum integrals parallel and transverse to the orbifold. $h=0$ for the twisted sector and $g=0$ for terms without the operator $\hat{g}$. The contributions from the world sheet fermions vanish in each one of the four (h,g) sectors separately, due to the Jacobi identity,

$$\frac{1}{2}\sum_{a,b=0}^{1}(-1)^{a+b+ab}\prod_{i=1}^{4}\theta\begin{bmatrix}a+h_i\\b+g_i\end{bmatrix}=-\prod_{i=1}^{4}\theta\begin{bmatrix}1-h_i\\1-g_i\end{bmatrix},$$

coupled with $\theta[{}^1_1]=0$. Hence $Z(\tau,\bar{\tau})=0$, without having to fix the relative factor V_4. The vanishing of the partition function in particular implies its modular invariance. All this looks rather surprising considering the difference between the Euclidean and Lorentzian orbifolds. As in the bosonic case, the difference can be made manifest by inserting, in the

[11]In the case of the Euclidean orbifold R^4/Z_2, the Dirac equation in the Ramond sector reduced the fermionic components by half resulting in a Bose-Fermi degenerate spectrum.

[12]As in the bosonic case, care is necessary in general to define string amplitudes in time-dependent backgrounds via analytic continuation. Here we will not analyze this in detail. We naively continue world sheet time to Euclidean signature while evaluating the torus diagram. The resulting formal object appears to be well-defined.

partition function, an operator $(-1)^s$ that changes the sign of all negative norm states. Once again one finds that although the physical spectrum is free of negative norm states, non-physical negative norm states do not decouple in the loops.

The vanishing of the partition function implies that there is no dilaton tadpole, at zero energy momentum [18]. In the absence of the orbifold, a non-zero momentum tadpole vanishes simply by momentum conservation. However, in the space-time orbifold, because of energy-momentum nonconservation at the ‘‘conical’’ singularity, kinematics can allow inserting, on the torus, a dilaton vertex operator carrying nonzero energy and momentum. The vanishing of such tadpoles is not obvious and requires further investigation.

Summary. For type II superstring, we have found that there are no negative norm *physical* states on the Lorentzian orbifold $R^{1,d}/Z_2$. The ground state in the twisted NS sector transforms as a spinor of $SO(1,d)$ and a scalar of $SO(9-d)$. It is only physical when $d\leqslant 3$ and the momentum it carries in the unorbifolded directions has to satisfy $|\vec{p}|^2=(3-d)/8$. In the twisted Ramond sector, the ground state is a $SO(1,d)$ scalar and $SO(9-d)$ spinor. It is physical for any value of d and its momentum in the unorbifolded directions has to vanish: $p^i=0$. The partition function is modular invariant and the zero momentum dilaton tadpole vanishes at one loop.

V. DISCUSSION

In this article we studied two basic issues in string theory about which very little is known—time-dependent backgrounds and cosmological singularities. We chose the simplest possible spaces exhibiting these phenomena, space-time orbifolds of Minkowski space, and showed how simple quotients by time reversal and spatial reflections evade some of the obvious potential pitfalls (tachyons and ghosts in the physical spectrum, zero-momentum tadpoles at one loop, lack of modular invariance etc.).[13] Although there are closed time-like loops in the construction, quantum mechanical evolution is consistent because the orbifold prescription projects onto states that are invariant under the discrete identification.

How is an S matrix defined when a class of physical states is localized in time? An asymptotic observer in models such as ours only observes transition amplitudes between the propagating untwisted sector states. Any such amplitude could involve the emission of arbitrarily many twisted sector states which cannot be observed at late or early times. Therefore it appears that the rules for computing transition amplitudes in space-time orbifolds will require tracing over emissions of states that are localized in time. If so, pure states scattering off a space-time orbifold singularity could emerge as mixed states due to entanglement with an unobservable twisted sector. Perhaps such a mechanism is responsible for creating the observed large entropy of the universe in a cosmological context.

One might wonder whether a loss of unitarity is also implied by the uncanceled contributions of negative norm virtual states in the partition function of our space-time orbifolds. Certainly, this result implies that it is not possible to choose a ghost-free gauge in which all computations are carried out in terms of positive-norm states. We expect that this will be true in many time-dependent backgrounds of string theory—it is at least clear that the ghost-free light-cone gauge cannot be chosen in time-dependent backgrounds. This is in sharp contrast to usual string theories and field theories in static backgrounds. Nevertheless, it is not clear that a loss of unitarity in transition amplitudes is implied. In particular, since our models do not have any *physical* negative norm states, cutting the one loop diagram will not give a transition amplitude to a ghostlike state. In the absence of a general argument connecting negative norm virtual contributions to the partition function and S-matrix unitarity, we require detailed study of amplitudes for propagating untwisted sector states scattering from the orbifold singularity.

There are very interesting subtleties in the computation of correlation functions and transition amplitudes on space-time orbifolds such as ours in which the twisted sectors are localized in time. Because of the localization, we do not expect energy (or momentum in any of the orbifolded directions) to be conserved in interactions between the untwisted and twisted sectors. One important consequence is that (unlike usual spatial orbifolds) kinematics does not forbid a finite momentum tadpole appearing at one loop. We can expect that this issue of finite-momentum tadpoles will persist for time-dependent string backgrounds in general.

One reason for our focus on the $R^{1,3}/Z_2$ orbifold of the superstring is that this orbifold had ‘‘massless’’ twisted sector states with Euclidean momenta satisfying $\vec{p}^2=0$. In the classic R^4/Z_2 orbifold the massless twisted sector states (for which Lorentzian $\vec{p}^2=0$) correspond to geometric blowup modes which can resolve the singularity. Some condensates of the twisted sector states correspond to parameters of the Eguchi-Hanson Ricci-flat metric on the smooth manifold obtained by replacing the tip of the R^4/Z_2 cone by a sphere. We might hope that some conical space-time singularities can be resolved by similar condensates of ‘‘massless’’ twisted sector states. When the twisted sector states are tachyonic, we might similarly hope that tachyon condensation would resolve the orbifold singularity. Unfortunately, much of the geometric technology of deforming singular manifolds into smooth spaces relies on complex geometry and cannot accommodate a manifold with signature $(1,d)$. For example, the Eguchi-Hanson metric [19] has signature (4,0) and while one can easily obtain a (2,2) signature Ricci-flat metric by analytic continuation from it, a (1,3) signature Ricci-flat metric cannot be obtained in this way.[14] In order to understand

[13]Of course, even if tachyons appear in the physical spectrum, recent experience has taught us that the condensation of the tachyon may be under control and could even perhaps be cosmologically interesting.

[14]A simple generalization of the Eguchi-Hanson metric cannot work because the curvature two form for an Eguchi-Hanson space is self-dual, but in (1,3) signature, the self-duality condition has an extra factor of ‘‘i.’’

cosmological singularities in string theory it is urgent that we develop the mathematics of resolution of singularities of Lorentzian manifolds.[15]

In string theory, the quantum mechanics of a relativistic string is used to compute transition amplitudes and an S matrix for the scattering of conventional multigraviton states. In view of this we have studied the quantum mechanics of strings on space-time orbifolds. Field theories on such spaces raise several new issues. For example, new singularities can potentially arise in correlation functions of operators at space-like separations if the space-time interval between some operators and the orbifold images of others is time-like or null. The rules for defining field theories in such backgrounds remain to be worked out.[16]

We conclude here by summarizing some perspectives from this work about time-dependent backgrounds and cosmological singularities in string theory:

String theories defined on time dependent backgrounds run the risk of having ghosts and tachyons in the physical spectrum.

Even when there are no ghosts in the physical spectrum, negative norm states can make uncancelled contributions to the partition function. In such cases it is not possible to choose a ghost-free gauge like light-cone gauge. This might lead to loss of unitarity, but a more detailed analysis is needed.

The quantum mechanics of strings on space-time orbifolds can be consistently defined even if there are closed time-like loops by projecting onto states invariant under the orbifold group. It would be interesting to consider space-time orbifolds without closed time-like curves, but we expect the issues raised here to persist (see [20]).

The resulting orbifolds can be tachyon and ghost-free and typically contain a twisted sector at a fixed plane localized in time.

Scattering from such an asymptotically unobservable twisted sector could cause transitions from a pure state to a mixed state, generating entropy.

Since energy need not be conserved in a time-dependent background, kinematics does not forbid the production of tadpoles with finite momentum. Hence, the vanishing of these amplitudes must be checked to confirm the existence of a valid solution to string theory.

We expect to return to many of the issues laid out above in a future publication.

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APPENDIX A: TWISTED SECTOR SUPERCONFORMAL GENERATORS

Here we list the superconformal generators in the twisted sector of the $R^{1,d}/Z_2$ orbifold theory. The Virasoro generators are given by

$$L_m^{NS}=L_m^B+L_m^{F,NS}, \qquad L_m^R=L_m^B+L_m^{F,R} \tag{A1}$$

where, in the twisted sector,

$$L_m^B=\frac{1}{2}\sum_{n\in Z}:(\alpha^a_{-n-1/2}\alpha^b_{m+n+1/2}\eta_{ab}+\alpha^i_{-n}\alpha^i_{m+n}): \tag{A2}$$

and

$$L_m^{F,NS}=\frac{1}{2}\sum_{n\in Z}\left(n+\frac{m}{2}\right):\psi^a_{-n}\psi^b_{m+n}\eta_{ab}: + \frac{1}{2}\sum_{r\in Z+1/2}\left(r+\frac{m}{2}\right):\psi^i_{-r}\psi^i_{m+r}: \tag{A3}$$

$$L_m^{F,R}=\frac{1}{2}\sum_{r\in Z+1/2}\left(r+\frac{m}{2}\right):\psi^a_{-r}\psi^b_{m+r}\eta_{ab}: + \frac{1}{2}\sum_{n\in Z}\left(n+\frac{m}{2}\right):\psi^i_{-n}\psi^i_{m+n}:. \tag{A4}$$

The supercurrent components in the twisted sector are

$$G_r=\sum_m :(\psi^a_{-m}\alpha^b_{r+m}\eta_{ab}+\psi^i_{r+m}\alpha^i_{-m}): \tag{A5}$$

$$F_n=\sum_m :(\psi^a_{-m+1/2}\alpha^b_{n+m-1/2}\eta_{ab}+\psi^i_{n+m}\alpha^i_{-m}):. \tag{A6}$$

There are similar expressions for the left movers.

APPENDIX B: TWISTED SECTOR VACUA AS SPINORS

Consider $2n$ world sheet fermion zero modes ψ_0^a satisfying $\{\psi_0^a,\psi_0^b\}=\delta^{ab}$ and commuting with the mass operator. The theory then has an $SO(2n)$ spinor as its degenerate vacuum, which can be constructed as follows. Define $\Gamma^a=\sqrt{2}\psi_0^a$. These then satisfy the Dirac algebra $\{\Gamma^a,\Gamma^b\}=2\delta^{ab}$. The ground state is a representation of this algebra and can be constructed using the standard procedure: For $k=1,\ldots n$, define

$$e_k=\frac{1}{2}(\Gamma_k+i\Gamma_{n+k}), \quad e_k^\dagger=\frac{1}{2}(\Gamma_k-i\Gamma_{n+k}).$$

These satisfy the fermionic algebra, $\{e_k^\dagger,e_l\}=\delta_{kl}$, with other anticommutators vanishing. Start from a state $|0\rangle$ annihilated by the lowering operators. Other components of the ground

[15]Perhaps the geometric difficulty in resolving these singularities is related to the fact that the "massless" twisted sector states are not exactly moduli fields in the low energy theory. They are localized in time and are on shell only at zero momentum.

[16]We are grateful to Nati Seiberg for a discussion of these issues. Also see [20].

state spinor are obtained by using the raising operators on this lowest state; $|0\rangle, e_k^\dagger|0\rangle, e_k^\dagger e_l^\dagger|0\rangle, \cdots, e_1^\dagger e_2^\dagger \cdots {}_n^\dagger|0\rangle$. The degeneracy of a state with p raising operators is the combinatoric factor, ${}^nC_p$ and the total number of states is 2^n; that of a spinor. The chirality operator is $\Gamma_{2n+1}=\Gamma_1\cdots\Gamma_{2n}$. States with even (odd) number of oscillators form a positive (negative) chirality spinor. In the twisted NS sector, $2n=d+1$ and in the twisted R sector $2n=9-d$.

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