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Structure of radial null geodesics in higher dimensional dust collapse

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We investigate here the occurrence and nature of a naked singularity for the inhomogeneous gravitational collapse of higher dimensional Tolman-Bondi dust clouds. The final state of collapse, a black hole or naked singularity, turns out to depend on the order of the first nonvanishing derivative of the density at the center. The study of the collapse along radial null geodesics seems to suggest that higher dimensions ($D\geqslant 6$) respect the CCH, if one chooses a smooth analytic density profile as initial data.

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I. INTRODUCTION

The cosmic censorship hypothesis (CCH) of Penrose [1,2] says that, in generic situations, all spacetime singularities arising from regular initial data are always hidden behind event horizons and hence invisible to outside observers (no naked singularities). This hypothesis plays a fundamental role in the theory of black holes and has been recognized as one of the most important open problems in classical general relativity. There exist many exact solutions of Einstein's equations which admit naked singularities. The models studied so far include the collapse of dust [3], radiation [4], perfect fluid [5], and imperfect fluids [6].

The possible existence of dimensions greater than 4 has been seriously considered in recent times. This has come about from approaches in particle physics to the unification of all forces including gravitation such as Kaluza-Klein theories and superstring theory. Quite recently there has been a great deal of interest in models where the size of the extra dimensions is much larger than the Planck length [7]. It is now important to consider the evolution of the extra dimensions since the observed strength of the gravitational force is directly dependent on the size of the extra dimensions (cf. Ref. [8]).

From the viewpoint of the CCH, one would like to know the effect of extra dimensions on the existence of a naked singularity. In this context, one question which could naturally arise is, what happens in higher dimensions (HD) which are currently being considered in view of their relevance for string theory and other field theories? Would the examples of a naked singularity in four dimensions (4D) go over to HD or not? Does the CCH hold in higher dimensional spacetime? Also it is well known that as the dimensions increase two simultaneous but opposite effects set in: one increase in inhomogeneity and the other a strengthening of gravitational field. The former would facilitate the formation of a naked singularity while the latter favors the formation of a black hole. It is well known that [9] in the 4D case the final state of collapse, a black hole or naked singularity, depends on the order of the first nonvanishing derivative of density at the center. We want to see whether there is any relation between the order of the leading nonvanishing derivative of density and dimension. Does the order of the first nonvanishing derivative of density get affected by increase in dimension of the space? We shall try to find out the answers to these questions in this work.

In the next section we discuss the nature of singularities in higher dimensional Tolman-Bondi spacetime. In Sec. IV we try to check whether a family of geodesics can terminate at the singularity with a given root as a tangent. We conclude with some concluding remarks.

II. NAKED SINGULARITIES IN THE HIGHER DIMENSIONAL TOLMAN-BONDI SPACETIMES

To facilitate the discussion we give a brief summary of the higher dimensional Tolman-Bondi solution. For a detailed study, the reader may refer to Refs. [3,9,10].

Let us consider the metric for $(N+2)$-dimensional spacetime with spherical symmetry [10]:

$$ds^2=-dt^2+\frac{R'^2}{1+f(r)}dr^2+R^2\ d\Omega^2, \qquad (1)$$

where

$$d\Omega^2=d\theta_1^2+\sin^2\theta_1\ d\theta_2^2+\sin^2\theta_1\sin^2\theta_2\ d\theta_3^2+\cdots +\sin^2\theta_1\sin^2\theta_2\sin^2\theta_3\cdots\sin^2\theta_{N-1}\ d\theta_N^2, \qquad (2)$$

is the metric on the N sphere and $N=D-2$ (where D is the total number of dimensions), together with the stress-energy tensor for dust

$$T_{ab}=\varepsilon(t,r)\,\delta_a^t\delta_b^t, \qquad (3)$$

where $u_a=\delta_a^t$ is the $(N+2)$-dimensional velocity and R is the area radius at time t of the shell having the comoving coordinate r.

The Einstein equations for the collapsing cloud are

$$\varepsilon(t,r)=\frac{NF'}{2R^NR'} \qquad (4)$$

and

$$\dot{R}^2=\frac{F(r)}{R^{N-1}}+f(r) \qquad (5)$$

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(we have set $8\pi G/C^4=1$).

Here the overdot and prime denote partial derivatives with respect to t and r, respectively. The quantity $F(r)$ arises as a free function from the integration of the Einstein equations and can be interpreted physically as the total mass of the collapsing cloud within a coordinate radius r. $f(r)$ is also another free function of r and is called the energy function. Since in the present discussion we are concerned with gravitational collapse, we take $\dot{R}(t,r)<0$.

For physical reasons, one assumes that energy density $\varepsilon(t,r)$ is non-negative everywhere. The epoch $R=0$ denotes a physical singularity where the spherical shell of matter collapses to zero radius and where the density $\varepsilon(t,r)$ blows up to infinity. The time $t=t_s(r)$ corresponds to the value $R=0$ where the area of the shell of matter at a constant value of coordinate r vanishes. The singularity curve $t=t_s(r)$ corresponds to the time when the matter shells meet the physical singularity.

This specifies the ranges of the coordinates for the metric (1):

$$0\leqslant r<\infty, \quad -\infty<t<t_s(r). \tag{6}$$

For simplicity, we consider the marginally bound case $f(r)=0$.

Hence, Eq. (5) yields

$$\dot{R}^2=\frac{F(r)}{R^{N-1}}. \tag{7}$$

Since we are considering the collapsing case, we take

$$\dot{R}=\frac{-\sqrt{F}}{R^{(N-1)/2}}. \tag{8}$$

Integration of Eq. (8) yields

$$R^{(N+1)/2}=\left[r^{(N+1)/2}-\frac{(N+1)}{2}\sqrt{F}t\right], \tag{9}$$

where we have used the freedom in the scaling of the comoving coordinate r to set $R(0,r)=r$ at the starting epoch of the collapse so that the physical area radius R increases monotonically in r, and with $R'=1$ there are no shell crossings on the initial surface.

We will be interested here only in the central shell-focussing singularity at $R=0$, $r=0$ which is a gravitationally strong singularity, as opposed to the shell crossing ones which are weak and through which the spacetime may sometimes be extended [11].

It follows from Eq. (4) that the function $F(r)$ becomes fixed once the initial density distribution $\varepsilon(0,r)=\rho(r)$ is given, i.e.,

$$F(r)=\frac{2}{N}\int \rho(r)r^N\,dr. \tag{10}$$

We assume that initial density profile $\rho(r)$ has the series expansion [9]

$$\rho(r)=\rho_0+\rho_1 r+\frac{\rho_2 r^2}{2!}+\frac{\rho_3 r^3}{3!}+\cdots+\frac{\rho_n r^n}{n!}+\cdots \tag{11}$$

near the center $r=0$, which can be substituted in Eq. (10) to yield

$$F=F_0 r^{N+1}+F_1 r^{N+2}+F_2 r^{N+3}+\cdots, \tag{12}$$

where

$$F_n=\frac{2}{N}\,\frac{\rho_n}{n!(N+1+n)}, \tag{13}$$

and ρ_n is the nth derivative of density and n takes integral values $0,1,2,\cdots$. We note that the first nonvanishing derivative in the series expansion (11) should be negative, as we will consider only those density functions which decreases as one moves away from the center.

As $t_s(r)$ gives the time at which area radius R becomes zero it follows from Eq. (9) that

$$t_s(r)=\left(\frac{2}{N+1}\right)\frac{r^{(N+1)/2}}{\sqrt{F}}. \tag{14}$$

The Kretschmann scalar $K=R_{abcd}R^{abcd}$ for the metric (1) is given by

$$K=\frac{AF'^2}{R^{2N}R'^2}-\frac{BFF'}{R^{2N+1}R'}+\frac{CF^2}{R^{2N+2}}, \tag{15}$$

where A,B,C are some constants. It is seen from Eqs. (4) and (15) that the energy density and Kretschmann scalar both diverge at the shell labeled r indicating the presence of a scalar polynomial curvature singularity at r.

The outgoing radial null geodesics of Eq. (1) are given by

$$\frac{dt}{dr}=R'. \tag{16}$$

Let $u=r^\alpha$ $(\alpha>1)$. Then

$$\frac{dR}{du}=\frac{1}{\alpha r^{\alpha-1}}\left(\dot{R}\frac{dt}{dr}+R'\right). \tag{17}$$

By virtue of Eqs. (8) and (16) the above equation leads to

$$\frac{dR}{du}=\frac{R'}{\alpha r^{\alpha-1}}\left[1-\frac{\sqrt{F}}{R^{(N-1)/2}}\right]=\frac{R'}{\alpha r^{\alpha-1}}\left[1-\sqrt{\frac{\Lambda}{X^{N-1}}}\right]$$
$$=U(X,u), \tag{18}$$

where

$$\Lambda=\frac{F}{u^{N-1}}, \quad X=\frac{R}{u}. \tag{19}$$

It is clear that $R=0$, $u=0$ is a singular point of Eq. (18). If there are outgoing radial null geodesics terminating in the past at the singularity with a definite tangent, then at the singularity we have $dR/du>0$. Hence apparent horizon for

$(N+2)$ dimensional spacetime is given by $R=F^{1/(N-1)}$. In order to check whether the singularity is naked, we examine the null geodesic equations for the tangent vectors $K^a = dx^a/dk$, where k is an affine parameter along the geodesics.

The radial null geodesics of the spacetime (1) are given by

$$K^t = \frac{dt}{dk} = \frac{P}{R}, \tag{20}$$

$$K^r = \frac{dr}{dk} = \frac{K^t}{R'} = \frac{P}{RR'}, \tag{21}$$

where the function $P(t,r)$ satisfies the differential equation

$$\frac{dP}{dk} + P^2\left(\frac{\dot{R}'}{R'R} - \frac{\dot{R}}{R^2} - \frac{1}{R^2}\right) = 0. \tag{22}$$

Differentiation of Eq. (9) yields

$$R' = \frac{X\eta r^{\alpha-1}}{N+1} + \left[\frac{N+1-\eta}{N+1}\right]\frac{1}{X^{(N-1)/2} r^{(\alpha-1)(N-1)/2}}, \tag{23}$$

where $\eta = rF'/F$.

Since we are interested in the behavior of η near the center, we can simplify η further to get

$$\eta(r) = (N+1) + \eta_1 r + \eta_2 r^2 + \eta_3 r^3 + \cdots, \tag{24}$$

where

$$\eta_1 = \frac{F_1}{F_0}, \quad \eta_2 = \frac{2F_2}{F_0} - \frac{F_1^2}{F_0^2},$$

$$\eta_3 = \frac{3F_3}{F_0} - \frac{3F_1F_2}{F_0^2} + \frac{F_1^3}{F_0^3}, \quad \text{etc.} \tag{25}$$

If all the derivatives ρ_n of the density vanish for $n \leqslant (q-1)$, and the qth derivative is the first nonvanishing derivative, then T_η^q, the qth term in the expansion for η, is

$$T_\eta^q = \frac{qF_q r^q}{F_0}. \tag{26}$$

Here q takes the values 1,2,3, etc. In this case, we can write $\eta(r)$ as

$$\eta(r) = (N+1) + \frac{qF_q}{F_0} r^q + O(r^{q+1}). \tag{27}$$

We use expression for $[(N+1)-\eta]$ from Eq. (27) keeping only terms up to the order q and substitute in Eq. (23) to get

$$R' = r^{(\alpha-1)}\left[\frac{\eta X}{N+1} - \frac{qF_q}{(N+1)F_0 X^{(N-1)/2} r^{(\alpha-1)(N+1)/2}} r^q\right]. \tag{28}$$

We substitute the above expression for R' in Eq. (18) to yield

$$\frac{dR}{du} = \frac{1}{\alpha}\left[1 - \sqrt{\frac{\Lambda}{X^{(N-1)}}}\right]\left[\frac{\eta X}{N+1} - \frac{\Theta}{X^{(N-1)/2}}\right] = U(X,u), \tag{29}$$

where

$$\Theta = \frac{qF_q}{(N+1)F_0 r^{(\alpha-1)(N+1)/2}} r^q. \tag{30}$$

Let us consider the limit X_0 of the tangent X along the null geodesic terminating at the singularity at $R=0$, $u=0$.

Thus,

$$X_0 = \lim_{\substack{R\to 0\\ u\to 0}} \frac{R}{u} = \lim_{\substack{R\to 0\\ u\to 0}} \frac{dR}{du} = \lim_{\substack{R\to 0\\ u\to 0}} U(X,u). \tag{31}$$

If a real and positive value of X_0 satisfies the above equation then the singularity could be naked. If the singularity is naked, some α exists such that at least one finite positive value of X_0 exists which solves the algebric equation

$$V(X_0) = 0, \tag{32}$$

where

$$V(X) = U(X,0) - X$$
$$= \frac{1}{\alpha}\left[1 - \sqrt{\frac{\Lambda_0}{X^{N-1}}}\right]\left[\frac{\eta_0 X}{N+1} - \frac{\Theta_0}{X^{(N-1)/2}}\right] - X, \tag{33}$$

where

$$\Lambda_0 = \lim_{r\to 0} \Lambda, \quad \eta_0 = \lim_{r\to 0} \eta.$$

Note that this root equation method picks up only the geodesics behaving as $X = R/r^\alpha = \text{const}$.

There may be the possibility of the existence of geodesics which have different behaviors than are assumed. To find such geodesics, we must solve the null geodesic equation [12].

The constant α can be determined by the requirement that Θ_0, the limiting value of Θ as $r \to 0$, should not be equal to zero or infinity. This gives

$$q = (\alpha-1)(N+1)/2, \quad \text{i.e.,} \quad \alpha = \frac{2q}{N+1} + 1, \tag{34}$$

which implies

$$\Theta_0 = \frac{qF_q}{(N+1)F_0}. \tag{35}$$

Using Eqs. (12) and (34), the limiting value of the function Λ is found to be

$$\Lambda_0 = 0, \quad q < (N+1)/(N-1)$$
$$= F_0, \quad q = (N+1)/(N-1)$$
$$= \infty, \quad q > (N+1)/(N-1). \tag{36}$$

We note that q is the order of the first nonvanishing derivative of density. Since Λ_0 takes different values for different choices of q, the nature of the roots depends on the first nonvanishing derivative of density at the center.

So we analyze the various cases in $(N+2)$-dimensional spacetimes one by one.

(A) First consider $N=3$ (i.e., 5D). We shall consider various cases of density profile in this spacetime.

Case (i): $\rho_1 \neq 0$. In this case, $q=1$, $\alpha=3/2$, $\Lambda_0=0$, $\Theta_0 = F_1/4F_0$, $\eta_0=4$. Hence Eq. (33) yields

$$X_0^2 = \frac{-F_1}{2F_0} = \frac{-2\rho_1}{5\rho_0}. \tag{37}$$

Because of the assumption that the density decreases away from the center, $\rho_1 < 0$ and so X_0 will be positive and thus the singularity is naked.

Case (ii): $\rho_1=0$, $\rho_2 \neq 0$. In this case $q=2$, $\alpha=2$, $\Lambda_0 = F_0$, $\Theta_0 = F_2/2F_0$. Equation (33) then leads to

$$\frac{2X_0^3}{F_0^{3/2}} + \frac{2X_0^2}{F_0} + \frac{F_2 X_0}{F_0^{5/2}} - \frac{F_2}{F_0^2} = 0. \tag{38}$$

We define $y = X_0/\sqrt{F_0}$, $\xi = F_2/F_0^2$. Equation (38) then becomes

$$2y^3 + 2y^2 + y\xi - \xi = 0. \tag{39}$$

Numerical calculations show that the above equation has positive real roots (in fact 2) if

$$\xi \leqslant \frac{1-\sqrt{5}}{9-4\sqrt{5}}, \quad \text{i.e.,} \quad \xi \leqslant -22.18033. \tag{40}$$

Thus whenever $\xi \leqslant -22.18033$, the central singularity is naked, and it is covered if ξ is greater than this number.

Case (iii): $\rho_1=0$, $\rho_2=0$, $\rho_3 \neq 0$ (i.e., $q \geqslant 3$). In this case $\alpha \geqslant 5/2$, $\Lambda_0 = \infty$, and Eq. (33) does not have positive real roots and hence collapse ends into a black hole.

Case (iv): $\rho_1=\rho_2=\rho_3=0$, $\rho_4 \neq 0$. In this situation, $q=4$, $\alpha=3$, $\Lambda_0=\infty$. So positive values of X_0 cannot satisfy Eq. (33) for the roots, hence the singularity is covered.

(B) Next consider $N \geqslant 4$ (i.e., spacetimes where the dimensions are greater than or equal to 6).

Case (i): $\rho_1 \neq 0$. In this case $q=1$, $\alpha=(N+3)/(N+1)$, $\Lambda_0=0$, $\Theta_0 = F_1/(N+1)F_0$. Hence Eq. (33) gives

$$X_0 = \left(\frac{-F_1}{2F_0}\right)^{2/(N+1)}. \tag{41}$$

Since F_1 is negative, X_0 will be positive and hence the singularity is naked.

Case (ii): When $\rho_1=0$, $\rho_2 \neq 0$. In this case $q=2$, $\Lambda_0 = \infty$, and hence Eq. (33) does not have real positive roots. Thus in all the spacetimes where the dimensions are greater than or equal to 6, the singularity is naked only for the models where $\rho_1 < 0$.

Case (iii): $\rho_1=\rho_2=0$, $\rho_3 \neq 0$. In this case $q=3$, and it can be seen from Eq. (36) that $\Lambda_0=\infty$. Hence Eq. (33) cannot be satisfied for any positive value of X_0. Therefore the collapse ends with a black hole.

Case (iv): $\rho_1=\rho_2=\rho_3=0$, $\rho_4 \neq 0$. Here $q=4$ and hence by same reasoning explained in case (iii), the singularity is covered.

III. THE FAMILY OF SINGULAR GEODESICS

We follow the method described by Christodolou [13] to check whether a family of outgoing null geodesics terminates at the singularity in the past with given root X_0 as a tangent. Suppose that a real positive root $X=X_0$ satisfies $V(X)=0$.

From $X=R/u$, we can write

$$\frac{dX}{du} = \frac{U(X,u)-X}{u}. \tag{42}$$

We could then write

$$U(X) = (X-X_0)(h_0-1) + h(X), \tag{43}$$

where h_0 is a constant defined by

$$h_0 = \left(\frac{dU}{dX}\right)_{X=x_0}, \tag{44}$$

and the function $h(X)$ contains hither-order terms in $(X - X_0)$ such that

$$h(X_0) = \left(\frac{dh}{dX}\right)_{X=x_0} = 0. \tag{45}$$

Equation (42) can be written as

$$\frac{dX}{du} - (X-X_0)\frac{(h_0-1)}{u} = \frac{B}{u}, \tag{46}$$

where

$$B = B(X,u) = U(X,u) - U(X,0) + h(X) \tag{47}$$

is such that $B(X_0,0)=0$.

Equation (46) can be integrated to get

$$X - X_0 = Au^{h_0-1} + u^{h_0-1}\int Bu^{-h_0}\,du, \tag{48}$$

where A is a constant of integration that labels different geodesics. If $h_0<1$ then there is only one radial null geodesic terminating at the singularity described by $A=0$. On the other hand, if $h_0>1$, then there are infinitely many families of radial null geodesics terminating at the singularity with each curve being labeled by different values of constant A.

The form of Eq. (46) is similar to the form of the null geodesic equation of TBL spacetime given in Refs. [11–13]. Here one may apply the contraction mapping principle to Eq. (46) to show the existence and uniqueness of a solution to Eq. (46). For complete analysis the reader may refer to Refs. [11–13].

For $(N+2)$-dimensional spacetime, h_0 can be calculated from Eq. (33) as

$$h_0=\left(\frac{dU}{dX}\right)_{X=X_0}$$
$$=\frac{1}{\alpha}\left[\left(1-\frac{\sqrt{\Lambda_0}}{X_0^{(N-1)/2}}\right)\left(1+\frac{(N-1)}{2}\frac{\Theta_0}{X_0^{(N+1)/2}}\right)+\left(X-\frac{\Theta_0}{X_0^{(N-1)/2}}\right)\left(\frac{(N-1)}{2}\frac{\sqrt{\Lambda_0}}{X_0^{(N+1)/2}}\right)\right]. \quad (49)$$

Since we have taken under consideration the collapse in HD spacetimes $(D>4)$ where only the first two leading derivatives of density at the center plays the role of deciding the nature of the singularity, we consider these cases one by one.

(i) Let $\rho_1\neq 0$. Then $\Lambda_0=0$, $q=1$, $\alpha=(N+3)/(N+1)$, $\Theta_0=F_1/(N+1)F_0$, $X_0=(-F_1/2F_0)^{2/(N+1)}$. Substituting these quantities in Eq. (49) we get

$$h_0=\frac{2}{N+3}<1.$$

Hence there will be only one outgoing null geodesic coming out from the singularity having X_0 as a tangent.

(ii) Next consider $\rho_2\neq 0$. For higher dimensional dust collapse this case is applicable only to the 5D case, where the collapse leads to a naked singularity when $\xi=F_2/F_0^2<-22.18033$. In this case, $N=3$, $q=2$, $\alpha=2$, $\Lambda_0=F_0$, $\Theta_0=F_2/2F_0$. We substitute these quantities in Eq. (49) to get

$$h_0=\frac{1}{2}\left[1+\frac{F_2}{2F_0X_0^2}-\frac{F_2}{\sqrt{F_0}X_0^3}\right]. \quad (50)$$

Using Eq. (39) this can be written as

$$h_0=-\frac{(\xi+2y^2)}{4y^3}. \quad (51)$$

In particular for $\xi=-40$ [satisfying condition (40)] there are two positive roots to Eq. (39), namely, $y_1=1.1386363$ and $y_2=3.25599$.

From Eq. (51) it can be seen that

$$[h_0]_{y=y_1}=6.634$$

and

$$[h_0]_{y=y_2}=0.13$$

Thus along smaller root we have $h_0>1$ and we have an infinite family of radial null geodesic coming out along this direction, while along larger root we have $h_0<1$ and hence we can have only one radial null geodesic coming out along this direction (with $A=0$).

IV. STRENGTH OF SINGULARITY

Following Clarke and Krolak [14] a sufficient condition for a singularity to be strong in the sense of Tiple [15] is that, at least along one null geodesic (with affine parameter k), we should have in the limit of approach to the singularity

$$\lim_{k\to 0} k^2R_{ab}K^aK^b>0, \quad (52)$$

where K^a is the tangent to null geodesics and R_{ab} is the Ricci tensor. For our $(N+2)$-dimensional dust model, along radial null geodesics, we find that

$$\lim_{k\to 0} k^2R_{ab}K^aK^b=\frac{N\eta_0\Lambda_0}{2\alpha X_0^{N+3}}\lim_{k\to 0}\left(\frac{kP}{r^{2\alpha}}\right)^2. \quad (53)$$

Using Eqs. (20)–(22) and the L-Hospital rule we find that

$$\lim_{k\to 0}\frac{kP}{r^{2\alpha}}=\frac{\alpha X_0^3}{\alpha X_0+2\sqrt{\Lambda_0}(\alpha-1)} \quad \text{if} \quad \lim_{k\to 0} P=P_0=0,\infty,$$
$$=\frac{X_0^2}{2}, \qquad \text{otherwise}. \quad (54)$$

Hence using Eqs. (36), (53), and (54) we get

$$\lim_{k\to 0} k^2R_{ab}K^aK^b=0, \quad q<\frac{N+1}{N-1}$$
$$>0, \quad q\geqslant\frac{N+1}{N-1}. \quad (55)$$

Equation (55) is a general statement for $(N+2)$-dimensional dust collapse, regarding the curvature strength along radial null geodesics. Thus with the help of Eq. (55) and earlier calculations from this section, we conclude that strong curvature naked singularities occur only in 4D and 5D cases for the density profiles where $\rho_3\neq 0$ and $\rho_2\neq 0$, respectively, and there are infinite families of RNGs terminating at the singularity in the past (along smaller root).

In the spacetime, where dimension is greater than or equal to 6 (i.e., $N\geqslant 4$) naked singularities occur only for the density profile where $\rho_1<0$ and these singularities are of weak curvature type. Further there is only one RNG terminating at the singularity as the value of h_0 is less than 1 in these cases.

V. CONCLUDING REMARKS

We have generalized the earlier work given in Refs. [9,11] to higher dimensional Tolman-Bondi spacetimes. It is interesting to note that in the 4D case, the leading three derivatives of density at the center play the role of deciding the nature of the singularity, whereas in the 5D case the leading two derivatives decide the nature of the singularity and for spacetimes where the dimensions are greater than or equal to 6 only the first derivative of density plays this role. Thus as the dimension of the spacetime increases, the calculation of less numbers of derivatives of the density at the center are required to decide the nature of the singularity. This might be the effect of the increase in strength of gravity (as the gravitational force is directly proportional to the size of the extra dimensions)

If one considers the analytic initial data, in the case of the collapse of a dust cloud, this amounts to demanding analyticity of the density function. The initial density $\rho(r)$ then must contain only even powers in r, and we have

$$\rho(r)=\rho_0+\rho_2 r^2+\rho_4 r^4+\cdots.$$

Since the first derivative of density at the center ρ_1 is absent in the above analytic density profile, there could not be a naked singularity in spacetimes where the dimensions are greater than or equal to 6. Thus one may argue that in these spacetimes (for $D\geqslant 6$) the cosmic censorship hypothesis holds, if the analytic density function is chosen as an initial data.

There are previous works on this same subject [16,17]. In particular, in Ref. [17] it has been claimed that the higher dimensional Tolman-Bondi collapse admits naked singularity of strong curvature in any higher dimensions, whereas in the present work we have shown that in all the higher dimensional spacetimes where $D\geqslant 6$, the naked singularity is weak along radial null geodesics. Different conclusions on the same subject arise due to the fact that the models considered in both papers are different. The difference between the two classes of models is similar to the difference between Newman's work [11] (if the assumption of evenness of density function is dropped) and the work given in Ref. [18]. The main difference between these two classes of models is that with the scaling of coordinates given by $R(0,r)=r$, the density and other functions such as Kretschmann's scalar are smooth on the $t=0$ hypersurface in the case of our models whereas in the models considered in Ref. [17], there is a singularity at $r=0$ on this surface. It thus follows that the class being treated here is different from the models treated by Ghosh and Beesham [17]. This is discussed in detail in Ref. [19] (while making the comparison with Newman's work) in the case of 4D spacetime.

[1] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969).

[2] R. Penrose, in *Theoretical Principles in Astrophysics and Relativity*, edited by N. R. Lebovitz *et al.* (University of Chicago Press, Chicago, 1978), pp. 217–243.

[3] D. M. Eardley and L. Smarr, Phys. Rev. D **19**, 2239 (1979); D. Christodolou, Commun. Phys. (London) **93**, 171 (1984); P. S. Joshi and T. P. Singh, Phys. Rev. D **51**, 6778 (1995); I. H. Dwivedi and P. S. Joshi, Class. Quantum Grav. **14**, 1223 (1997); R. V. Saraykar and S. H. Ghate, *ibid.* **16**, 281 (1989); S. H. Ghate, R. V. Saraykar, and K. D. Patil, Pramana, J. Phys. **53**, 253 (1999).

[4] W. A. Hiscok, L. G. Williams, and D. M. Eardley, Phys. Rev. D **26**, 751 (1982); Y. Kuroda, Prog. Theor. Phys. **72**, 63 (1984); K. Rajagopal and K. Lake, Phys. Rev. D **35**, 1531 (1987); I. H. Dwivedi and P. S. Joshi, Class. Quantum Grav. **6**, 1599 (1989); P. S. Joshi and I. H. Dwivedi, Gen. Relativ. Gravit. **24**, 129 (1992); J. Lemos, Phys. Rev. Lett. **68**, 1447 (1992); K. D. Patil, R. V. Saraykar, and S. H. Ghate, Pramana, J. Phys. **52**, 553 (1999); S. G. Ghosh and R. V. Saraykar, Phys. Rev. D **62**, 107502 (2000).

[5] A. Ori and T. Piran, Phys. Rev. D **42**, 1068 (1990); P. S. Joshi and I. H. Dwivedi, Commun. Math. Phys. **146**, 333 (1992); T. Harada, Phys. Rev. D **58**, 104015 (1998).

[6] P. Szekeres and V. Iyer, Phys. Rev. D **47**, 4362 (1993); S. Barve, T. P. Singh, and L. Witten, Gen. Relativ. Gravit. **32**, 697 (2000).

[7] V. A. Rubakov, Phys. Usp. **44**, 871 (2001); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998).

[8] James Geddes, Phys. Rev. D **65**, 104015 (2002).

[9] T. P. Singh and P. S. Joshi, Class. Quantum Grav. **13**, 559 (1996); S. Jhingan and P. S. Joshi, Ann. Isr. Phys. Soc. **13**, 357 (1997).

[10] A. Sil and S. Chatterjee, Gen. Relativ. Gravit. **26**, 999 (1994).

[11] R. P. A. C. Newman, Class. Quantum Grav. **3**, 527 (1986).

[12] Hideaki Kudoh, T. Harda, and H. Iguchi, Phys. Rev. D **62**, 104016 (2000).

[13] D. Christodolou, Commun. Phys. **93**, 171 (1984).

[14] C. J. S. Clarke and A. Krolak, J. Geophys. **2**, 127 (1986).

[15] F. J. Tipler, Phys. Lett. **64A**, 8 (1977).

[16] S. G. Ghosh and A. Beesham, Class. Quantum Grav. **17**, 4959 (2000).

[17] S. G. Ghosh and A. Beesham, Phys. Rev. D **64**, 124005 (2001).

[18] I. H. Dwivedi and P. S. Joshi, Class. Quantum Grav. **9**, L69 (1992); P. S. Joshi and T. P. Singh, Gen. Relativ. Gravit. **27**, 921 (1995); P. S. Joshi and T. P. Singh, Phys. Rev. D **51**, 6778 (1995).

[19] P. S. Joshi, *Global Aspects in Gravitation and Cosmology* (Clarendon, Oxford, 1993).