

# Black hole formation in the grazing collision of high-energy particles

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(Received 30 August 2002; published 15 January 2003)

We numerically investigate the formation of  $D$ -dimensional black holes in high-energy particle collisions with the impact parameter and evaluate the total cross section of black hole production. We find that the formation of an apparent horizon occurs when the distance between the colliding particles is less than 1.5 times the effective gravitational radius of each particle. Our numerical result indicates that although both the one-dimensional hoop and the  $(D-3)$ -dimensional volume corresponding to the typical scale of the system give a fairly good condition for horizon formation in the higher-dimensional gravity, the  $(D-3)$ -dimensional volume provides a better condition to judge the existence of the horizon.

DOI: 10.1103/PhysRevD.67.024009

PACS number(s): 04.50.+h, 04.20.Cv, 04.70.Bw, 11.10.Kk

## I. INTRODUCTION

The brane world scenario has been paid much attention in the context of the unified theory of elementary particles. This scenario regards our space as a 3-brane with large extra dimensions on which gauge particles and interactions are confined. In this scenario, the Planck energy can be at the  $O$  (TeV) scale [1]. One of the consequences of lowering the Planck scale is that the properties of a small black hole whose radius is smaller than the size of the extra dimensions are substantially altered; the black hole is well described as a  $D$ -dimensional black hole centered on the brane, but extending out into the  $(D-4)$ -dimensional extra space and the radius of such a black hole is  $\sim 10^{32}$  times larger than that of the usual black hole with the same mass. Hence it becomes much easier to produce black holes using a future planned accelerator such as the CERN Large Hadron Collider, and this possibility has been discussed by many authors [2].

The possibility of producing black holes by the collision of particles can be estimated by using the hoop conjecture [3]. It states that in four-dimensional Einstein gravity, an apparent horizon forms when and only when the mass  $M$  of the system gets compacted into a region whose circumference  $C$  satisfies

$$H_D \equiv \frac{C}{2\pi r_h(M)} \lesssim 1, \quad (1)$$

where  $r_h(M)$  is the Schwarzschild horizon radius for the mass  $M$ . If we assume that the inequality (1) gives the condition for black hole formation in the higher-dimensional gravity where  $r_h(M)$  is now given as

$$r_h(M) = \left( \frac{16\pi G_D M}{(D-2)\Omega_{D-2}} \right)^{1/(D-3)} \quad (2)$$

with the volume of the unit  $(D-2)$ -sphere  $\Omega_{D-2}$  and the gravitational constant  $G_D$ , we can evaluate the impact parameter  $b$  of the colliding particles which leads to black hole

production. The circumference that surrounds two particles at the instant of collision is  $C \sim 2b$ . By setting  $2b/2\pi r_h(2\mu) \sim 1$ , where  $2\mu$  is the center-of-mass energy of the system, the maximal impact parameter  $b_{\max}$  that leads to the black hole formation becomes  $b_{\max} \sim r_h(2\mu)$ . By introducing a numerical factor  $F(D)$  close to unity, the total cross section for black hole production is written in the following form:

$$\sigma_{\text{b.h. production}} = F(D) \pi r_h^2(2\mu). \quad (3)$$

Obtaining  $F(D)$  is necessary to improve experimental predictions of the black hole production in collider physics and observations of ultrahigh-energy cosmic rays.

In our previous paper [4], we investigated the black hole formation in high-energy head-on collisions of particles in the  $D$ -dimensional gravity and found that  $H_D$  becomes a parameter to judge the existence of the apparent horizon for all  $D$ . Although this implies that the hoop conjecture holds for this system, we do not know the condition for the horizon formation in the higher-dimensional spacetime. Recently, Ida and Nakao [5] investigated momentarily static initial data in the five-dimensional spacetime and found that a spindle distribution of matter can lead to black hole formation even if the hoop  $C$  is arbitrarily larger than  $2\pi r_h(M)$ . This provides a counterexample for the ‘‘only when’’ part of the hoop conjecture. They showed that the isoperimetric inequality (characteristic area)  $\leq 4\pi r_h^2(M)$  is satisfied on the horizon and conjectured that for a characteristic  $(D-3)$ -dimensional volume  $V_{D-3}$  of the system, the inequality

$$\frac{V_{D-3}}{G_D M} \lesssim 1 \quad (4)$$

would become a condition for the horizon formation. This ‘‘ $(D-3)$ -volume conjecture’’ (volume conjecture) is one candidate that provides a better condition for the horizon formation in the higher-dimensional spacetime than the hoop conjecture and it is worth investigating.

In this paper, we consider the grazing collisions of particles in the  $D$ -dimensional Einstein gravity and investigate the formation of apparent horizons. Eardley and Giddings [6] developed a method of finding apparent horizons for this

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system. The problem was reduced to a boundary-value problem for Poisson's equation in a  $(D-2)$ -dimensional flat space and they solved it analytically for the  $D=4$  case. We solve this problem numerically for  $D>4$  and obtain the maximal impact parameter  $b_{\max}$  for black hole formation and the factor  $F(D)$  in Eq. (3). Then using these solutions, we discuss the condition of horizon formation for this system from the viewpoint of the hoop conjecture and the volume conjecture.

This paper is organized as follows. In Sec. II, we briefly review the method of finding apparent horizons in the system of the grazing collision of particles. In Sec. III, we present our numerical results and discuss the two conjectures for this system. Section IV is devoted to summary and discussion.

## II. APPARENT HORIZONS IN THE GRAZING COLLISION OF HIGH-ENERGY PARTICLES

To simplify the situation, we consider black holes with horizon radius smaller than the size of extra dimensions. This enables us to ignore the effect of the brane tension and the geometry of the extra dimensions. The metric with a high-energy point particle is obtained by infinitely boosting a Schwarzschild black hole with fixed total energy  $\mu$ . The resulting system becomes a massless point particle accompanied by a plane-fronted gravitational shock wave which is the Lorentz-contracted longitudinal gravitational field of the particle. Combining two shock waves, we can set up the collision of high-energy two particles moving in the  $\pm z$  directions. This system was originally developed by D'Eath and Payne [7] and recently analyzed in [4,6]. The metric of this system outside the future light cone of the colliding shocks is given as

$$ds^2 = -dudv + (H_{ik}^{(+)}H_{jk}^{(+)} + H_{ik}^{(-)}H_{jk}^{(-)} - \delta_{ij})dx^i dx^j,$$

$$H_{ij}^{(+)} = \delta_{ij} + \frac{u}{2}\Theta(u)\nabla_i\nabla_j\Phi(\mathbf{x}-\mathbf{x}_+), \quad (5)$$

$$H_{ij}^{(-)} = \delta_{ij} + \frac{v}{2}\Theta(v)\nabla_i\nabla_j\Phi(\mathbf{x}-\mathbf{x}_-),$$

where  $u=t-z$ ,  $v=t+z$ ,  $\Theta$  is the Heviside step function, and  $\mathbf{x}\equiv(x^i)$  is the point in flat  $(D-2)$ -space  $(x_1, \dots, x_{D-2})$  that is transverse to the direction of particle motion. The function  $\Phi(\mathbf{x})$  depends only on  $r\equiv|\mathbf{x}|=\sqrt{x_i x_i}$  and takes the form

$$\Phi(\mathbf{x}) = -8G_4\mu \log r \quad \text{for } D=4 \quad (6)$$

$$\Phi(\mathbf{x}) = \frac{16\pi\mu G_D}{\Omega_{D-3}(D-4)} \frac{1}{r^{D-4}} \quad \text{for } D>4. \quad (7)$$

$\mathbf{x}_{\pm}$  denote the points of two particles in  $(D-2)$ -dimensional space and we take

$$\mathbf{x}_{\pm} = (\pm b/2, 0, \dots, 0). \quad (8)$$

The apparent horizon  $\mathcal{S}$  is defined as a closed spacelike  $(D-2)$ -surface on which the outer null geodesic congruence has zero convergence. Eardley and Giddings [6] developed a method of finding the apparent horizon in the union of the two shock waves,  $u=0>v$  and  $v=0>u$ . Their method reduces the problem to finding the  $(D-3)$ -dimensional closed surface  $\mathcal{C}$  and two functions  $\Psi_{\pm}(\mathbf{x})$  on the  $(D-2)$ -dimensional plane satisfying

$$\nabla^2(\Phi_{\pm} - \Psi_{\pm}) = 0 \quad \text{interior to } \mathcal{C}, \quad (9)$$

$$\Psi_{\pm} = 0 \quad \text{on } \mathcal{C}, \quad (10)$$

$$\nabla\Psi_+ \cdot \nabla\Psi_- = 4 \quad \text{on } \mathcal{C}, \quad (11)$$

where  $\Phi_{\pm}\equiv\Phi(\mathbf{x}-\mathbf{x}_{\pm})$ . If we can find a solution of Eqs. (9), (10), and (11), a surface with zero expansion is given as the union of two  $(D-2)$ -surfaces  $v=-\Psi_+(\mathbf{x})$  in  $u=0>v$  and  $u=-\Psi_-(\mathbf{x})$  in  $v=0>u$ .  $\mathcal{C}$  is the intersection of the two  $(D-2)$ -surfaces at  $u=v=0$ . For  $D=4$ , Eardley and Giddings solved the above problem analytically [6]. They used the conformal invariance of the two-dimensional Poisson equation. For  $D>4$ , we cannot use their method because the  $(D-2)$ -dimensional Poisson equation does not have conformal invariance. We must solve the above problem numerically.

In the  $(D-2)$ -dimensional space,  $\mathcal{C}$  and  $\Psi_{\pm}$  have a symmetry about the  $x_1$  axis that connects two source points. Hence solving the apparent horizon is reduced to a two-dimensional problem. We use spherical coordinate  $(r, \theta)$  to represent the point  $\mathbf{x}$  in  $(D-2)$ -dimensional space, where  $r$  is the distance  $|\mathbf{x}|$  from the origin and  $\theta$  is the angle between  $\mathbf{x}$  and the  $x_1$  axis. We parametrize the curve  $\mathcal{C}$  by  $r=g(\theta)$ . The function  $g(\theta)$  and  $\Psi_{\pm}$  have symmetry  $g(\theta)=g(\pi-\theta)$  and  $\Psi_+(r, \theta)=\Psi_-(r, \pi-\theta)$ . We calculate  $\mathcal{C}$  and  $\Psi_{\pm}$  numerically as follows. First, we give a trial curve  $\mathcal{C}$  and solve Eq. (9) under the boundary condition (10). Then, we calculate the value  $\delta=\nabla\Psi_+ \cdot \nabla\Psi_- - 4$  on  $\mathcal{C}$ , and determine a modified boundary  $\mathcal{C}$  using the value of  $\delta$ .  $\mathcal{C}$  and  $\Psi_{\pm}$  converge to the solution of Eqs. (9), (10), and (11) by iterating these two steps. In order to proceed the first step, we introduce a regular function  $h\equiv\Phi_+ - \Psi_+$  that obeys the  $(D-2)$ -dimensional Laplace equation,

$$h_{,rr} + \frac{D-3}{r}h_{,r} + \frac{1}{r^2}h_{,\theta\theta} + \frac{D-4}{r^2}\cot\theta h_{,\theta} = 0. \quad (12)$$

The boundary condition is

$$h|_{r=g(\theta)} = \Phi_+, \quad \left. \frac{\partial h}{\partial \theta} \right|_{\theta=0} = \left. \frac{\partial h}{\partial \theta} \right|_{\theta=\pi} = 0, \quad (13)$$

which comes from Eq. (10) and the symmetry about the  $x_1$  axis. To simplify the boundary condition, we introduce a new coordinate  $(\tilde{r}, \tilde{\theta})$  by

$$r = g(\tilde{\theta})\tilde{r}, \quad \theta = \tilde{\theta}. \quad (14)$$

In this coordinate, Eq. (12) becomes

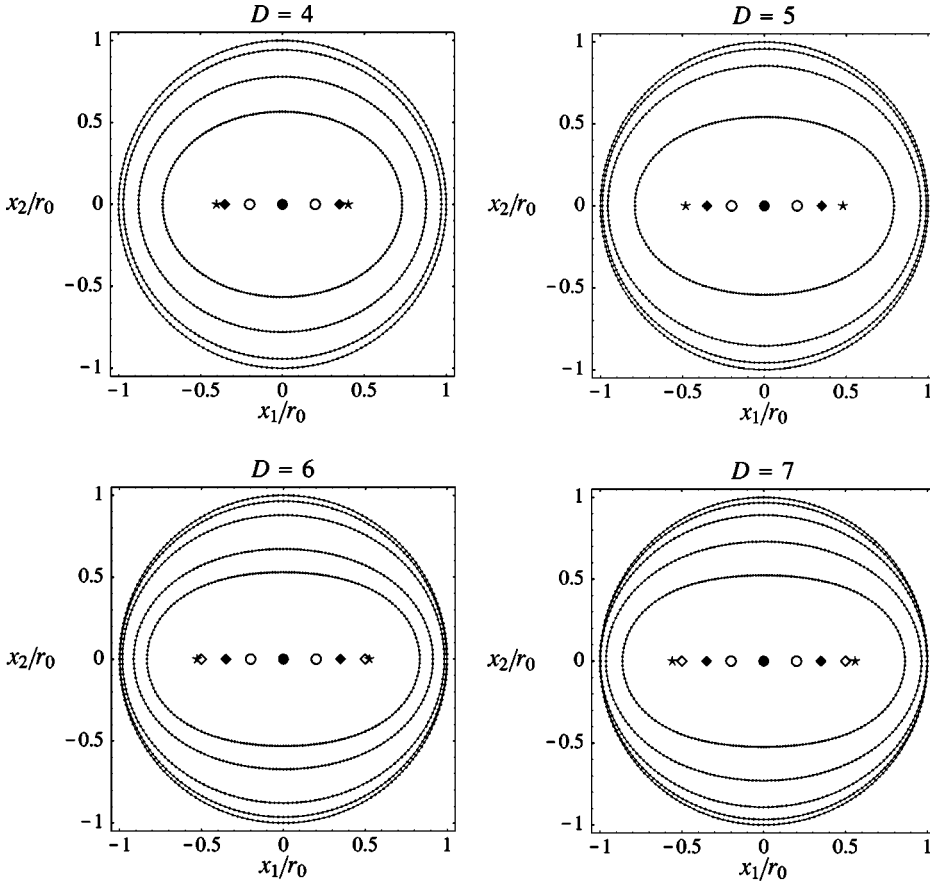


FIG. 1. The shape of the apparent horizon  $\mathcal{C}$  on the  $(x_1, x_2)$  plane in the collision plane  $u=v=0$  for  $D=4, \dots, 7$ . Incoming particles are located on the horizontal line  $x_2=0$ . Values of  $b/r_0$  are 0 (●), 0.4 (○), 0.7 (◆), 1.0 (◇, shown only in  $D=6, 7$ ), and  $b_{\max}/r_0$  (★). As the distance  $b$  between two particles increases, the radius of  $\mathcal{C}$  decreases.

$$\begin{aligned} & \left( 1 + \frac{g'^2}{g^2} \right) h_{,\tilde{r}\tilde{r}} + \frac{1}{\tilde{r}^2} h_{,\tilde{\theta}\tilde{\theta}} - \frac{2}{\tilde{r}} \frac{g'}{g} h_{,\tilde{\theta}\tilde{r}} \\ & + \frac{1}{\tilde{r}} \left( D-3 - \frac{g''}{g} + 2 \frac{g'^2}{g^2} - (D-4) \cot \tilde{\theta} \frac{g'}{g} \right) h_{,\tilde{r}} \\ & + \frac{D-4}{\tilde{r}^2} \cot \tilde{\theta} h_{,\tilde{\theta}} = 0 \end{aligned} \quad (15)$$

and the boundary  $\mathcal{C}$  is given by  $\tilde{r}=1$ . We solve Eq. (15) using the finite differential method with  $(50 \times 100)$  grids. There is a coordinate singularity and we cannot write down the finite difference equation of Eq. (15) at  $\tilde{r}=0$ . For  $\tilde{r}=0$ , we use the Laplace equation in the  $(x_1, \rho)$  coordinate,

$$h_{,x_1 x_1} + (D-3) h_{,\rho \rho} = 0 \quad \text{at } \rho \rightarrow 0, \quad (16)$$

where  $\rho$  is the distance from the  $x_1$  axis. We relate the value at the grid points around  $\tilde{r}=0$  using Eq. (16).

Once  $h$  converges, we proceed to the second step to determine the modified boundary  $\mathcal{C}$ . We calculate  $\delta(\theta) = \nabla \Psi_1 \cdot \nabla \Psi_2 - 4$ , and determine the modified boundary  $\mathcal{C}$  as

$$r = g(\theta) + \epsilon \times \delta(\theta). \quad (17)$$

If  $\epsilon$  is sufficiently small,  $\mathcal{C}$  converges. In our calculation, we took the value of  $\epsilon$  as typically  $10^{-3} - 10^{-5}$  times the horizon radius of the system. Smaller  $\epsilon$  is required to converge the

calculation for the larger impact parameter  $b$ . The maximal impact parameter  $b_{\max}$  is obtained when  $\partial g(\theta, b)/\partial b$  becomes  $-\infty$ . For  $D=10$  and  $11$ , the numerical instability occurs at the neighborhood of  $b=b_{\max}$ . We obtained  $b_{\max}$  by extrapolating  $g(\pi/2, b)$  by fitting the numerical data.

To evaluate numerical errors, we compare the numerical solution with the analytic solution given by Eardley and Giddings for the  $D=4$  case. The numerical error is less than 0.02%. For  $D \geq 5$ , we evaluate the error by comparing the solution with the result of the calculation with  $(100 \times 200)$  grids. The error is about 0.04% for  $D=6$ , 0.07% for  $D=8$ , 0.25% for  $D=10$ , and 0.4% for  $D=11$ .

### III. NUMERICAL RESULTS AND CONDITION FOR HORIZON FORMATION

Figure 1 shows the shape of  $\mathcal{C}$  on the  $(x_1, x_2)$  plane for  $D=4, \dots, 7$ . For  $b=0$ , the apparent horizon consists of two  $(D-2)$ -dimensional flat disks whose radius  $r_0$  is

$$r_0 = \left( \frac{8\pi\mu G_D}{\Omega_{D-3}} \right)^{1/(D-3)}. \quad (18)$$

The radius  $g(\theta)$  of  $\mathcal{C}$  takes the maximum value  $r_{\max}$  at  $\theta=0, \pi$  and the minimum value  $r_{\min}$  at  $\theta=\pi/2$ . As the impact parameter  $b$  increases, the radius of the  $(D-3)$ -surface  $\mathcal{C}$  becomes smaller. The shape of  $\mathcal{C}$  at  $b=b_{\max}$  deviates from a sphere and the ratio  $r_{\max}/r_{\min}$  becomes large as  $D$  increases and a more prolate-shaped horizon can exist for larger  $D$ .

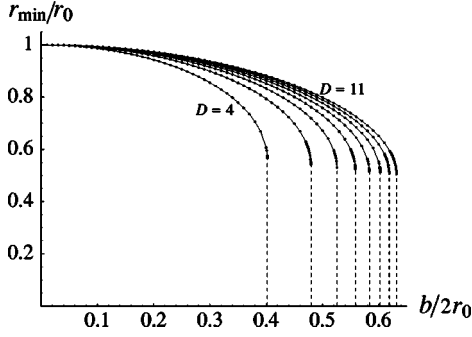


FIG. 2. The relation between the impact parameter  $b$  and the minimum radius  $r_{\min}$  of  $\mathcal{C}$  for  $D=4, \dots, 11$ . The value of  $b_{\max}/2r_0$  grows as  $D$  increases.

Figure 2 shows the relation between  $b$  and  $r_{\min}$  for each  $D$ . The value of  $b_{\max}/r_0$  ranges between 0.8 and 1.3 and becomes large as  $D$  increases. Because  $r_h(2\mu) \approx r_0$ ,  $b_{\max}/r_h(2\mu)$  is also  $O(1)$  and increases with  $D$ . This behavior of  $b_{\max}/r_h(2\mu)$  is different from the case of the head-on collision [4], where the ratio (distance of two particles)/ $r_h(2\mu)$  at the horizon formation decreases with  $D$ . Our numerical results for  $b_{\max}/2r_0$  and  $F(D)$  are summarized in Table I.

Now we investigate the condition for horizon formation in the two-particle system. Because  $b_{\max}/r_h(2\mu) \sim O(1)$ , the hoop conjecture can be applicable to estimate the maximal impact parameter in high-energy collisions in the higher-dimensional spacetime. But the hoop conjecture cannot explain the increase of  $b_{\max}/r_h(2\mu)$  with the increase of  $D$ . Figure 3 is the plot of  $b_{\max}(D)$ . We found that the relation  $b_{\max} \approx 1.5r_h(\mu)$  well describes the behavior of  $b_{\max}(D)$  obtained by numerical calculation. As  $r_h(\mu)$  is an effective gravitational radius of each incoming particle, the horizon formation occurs when there is an overlap of more than one-half of  $r_h(\mu)$  between the regions with radius  $r_h(\mu)$  around each particle. Because  $r_h(\mu)$  is proportional to  $\mu^{1/(D-3)}$ , the ratio  $r_h(\mu)/r_h(2\mu) = 2^{-1/(D-3)}$  increases with  $D$ . This leads to the increase of  $b_{\max}/r_h(2\mu)$  with  $D$ .

The behavior  $b_{\max}/r_h(2\mu) \propto 2^{-1/(D-3)}$  can be derived from the more fundamental principle, the  $(D-3)$ -dimensional volume conjecture. In the present system, the condition (4) can be written as

$$\mathcal{H}_D \equiv \left[ \frac{V_{D-3}}{\Omega_{D-3} r_h^{D-3}(M)} \right]^{1/(D-3)} \leq 1. \quad (19)$$

This condition reduces to the hoop conjecture (1) for the  $D=4$  case. As the characteristic length of the system is  $b$  in the  $x_1$  direction and  $r_h(\mu)$  in other directions, the characteristic  $(D-3)$ -volume becomes  $V_{D-3} \sim b r_h^{D-4}(\mu)$  and  $\mathcal{H}_D$  be-

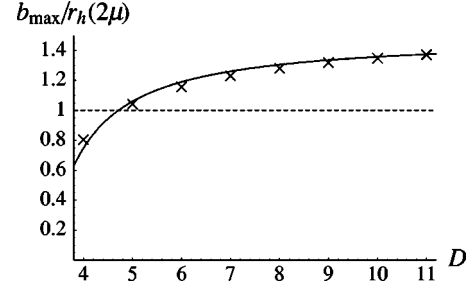


FIG. 3. The value of  $b_{\max}$  (crosses) as a function of the space-time dimension  $D$ . The dotted line  $r_h(2\mu)$  comes from the hoop conjecture. Although the ratio  $b_{\max}/r_h(2\mu)$  takes the value around unity, the hoop conjecture does not explain the increase of  $b_{\max}/r_h(2\mu)$  with  $D$ . The solid line  $b_{\max}/r_h(2\mu) = 1.5r_h(\mu)/r_h(2\mu) \sim 2^{-1/(D-3)}$  gives a good fit of  $b_{\max}$ .

comes  $\sim [b r_h^{D-4}(\mu)/r_h^{D-3}(2\mu)]^{1/(D-3)}$ . By setting  $\mathcal{H}_D \sim 1$ , the maximal impact parameter is given by

$$\frac{b_{\max}}{r_h(2\mu)} \sim \left( \frac{r_h(2\mu)}{r_h(\mu)} \right)^{D-4} \propto 2^{-1/(D-3)}. \quad (20)$$

Although this is a rough-order estimation, the calculation indicates that the volume conjecture provides a better condition than the hoop conjecture.

To test the validity of the two conjectures further, we consider how much energy is trapped by the black hole. This is related to the definition of the mass in the system. In our previous paper [8], we discussed the existence of the gravitational wave energy that does not contribute to the horizon formation in the system with motion and found that the hoop conjecture with Hawking's quasilocal mass provides a good condition for the horizon formation in four-dimensional gravity. In the present system, we calculate the quantity

$$M_{\text{A.H.}} \equiv \frac{(D-2)\Omega_{D-2}}{16\pi G_D} \left( \frac{A_{D-2}}{\Omega_{D-2}} \right)^{(D-3)/(D-2)}, \quad (21)$$

where  $A_{D-2}$  is the  $(D-2)$ -dimensional area of the apparent horizon. By the area theorem,  $M_{\text{A.H.}}$  provides the lower bound of the final mass of the black hole and coincides with Hawking's quasilocal mass on the horizon. Therefore,  $M_{\text{A.H.}}$  can become an indicator of the energy trapped by the horizon. Figure 4 shows the behavior of  $M_{\text{A.H.}}/2\mu$  as a function of  $b$  for each  $D$ . We find that the value of  $M_{\text{A.H.}}/2\mu$  at  $b = b_{\max}$  decreases as  $D$  increases. In the higher-dimensional spacetime, the amount of "junk" energy increases because the gravitational field distributes in the space of the extra dimensions and only a small portion of the total energy of the system can contribute to the horizon formation. This junk energy will be radiated away rapidly after the formation of

TABLE I. The value of  $b_{\max}/2r_0$  and  $F(D)$  for  $D=4, \dots, 11$ .

$D$	4	5	6	7	8	9	10	11
$b_{\max}/2r_0$	0.402	0.480	0.526	0.559	0.583	0.603	0.619	0.632
$F(D)$	0.647	1.084	1.341	1.515	1.642	1.741	1.819	1.883

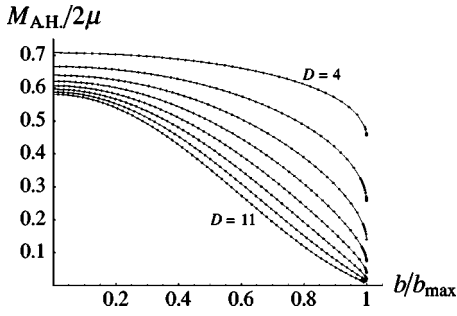


FIG. 4. The relation between the horizon mass  $M_{A.H.}/2\mu$  and the impact parameter  $b$  for  $D=4, \dots, 11$ .  $M_{A.H.}/2\mu$  becomes small as  $D$  increases. The black hole in the higher-dimensional spacetime can trap a small amount of the energy.

the black hole. This strongly suggests that we should use the quasilocal mass to check the hoop and the volume conjectures.

To test the validity of the two conjectures with the quasilocal mass, we must calculate  $M(S)$ ,  $C(S)$ , and  $V_{D-3}(S)$  for all surfaces  $S$ , and then take the minimum value of  $C(S)/2\pi r_h(M(S))$  and  $V_{D-3}(S)/\Omega_{D-3} r_h^{D-3}(M(S))$ . In the present system, calculation of the quasilocal mass is difficult because we treat the black hole on two connected null hyperplanes, while ordinary quasilocal mass is defined in a spacelike hypersurface. Hence we calculate the value of  $H_D^{A.H.} \equiv C(S_{A.H.})/2\pi r_h(M_{A.H.})$  and  $\mathcal{H}_D^{A.H.} \equiv [V_{D-3}(S_{A.H.})/\Omega_{D-3} r_h^{D-3}(M_{A.H.})]^{1/(D-3)}$  on the apparent horizon  $S_{A.H.}$ . In calculating  $H_D^{A.H.}$ , we must specify the hoop for the surface  $S_{A.H.}$ . The hoop is defined as the longest closed geodesic on the surface. In our system, the hoop is the length of the curve  $C$  on the  $(x_1, x_2)$  plane shown in Fig. 1. Figure 5 shows the numerical results of  $H_D^{A.H.}(b)$  for each  $D$ .  $H_D^{A.H.}(b)$  monotonically increases with  $b$  for all  $D$  and it can be a parameter to judge the existence of the horizon. The value at horizon formation  $H_D^{A.H.}(b_{max})$  ranges between 1.4 and 1.6 and increases with  $D$ . The deviation of the value  $H_D$  from unity is due to the definition of the mass and we expect that this value approaches close to unity if we use the appropriate mass of the system and the hoop conjecture holds. The increase of  $H_D^{A.H.}(b_{max})$  with  $D$  reflects the fact that a more prolate-shaped horizon can form for larger  $D$  and  $b_{max}/r_h(2\mu)$  increases with  $D$ .

As for  $(D-3)$ -volume, a general definition does not exist yet. Ida and Nakao [5] selected some  $(D-3)$ -surfaces by

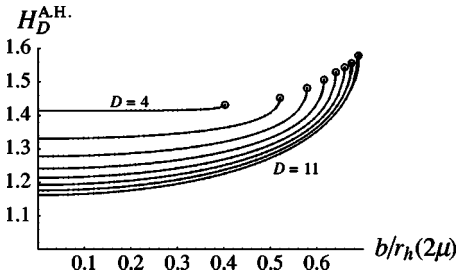


FIG. 5. The value of  $H_D^{A.H.}$  as a function of  $b/r_h(2\mu)$  for  $D=4, \dots, 11$ . The circles show the values at  $b=b_{max}$ .  $H_D^{A.H.}(b_{max})$  becomes large as  $D$  increases.

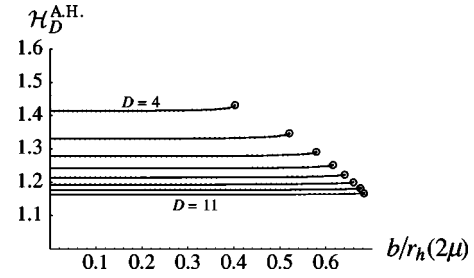


FIG. 6. The value of  $\mathcal{H}_D^{A.H.}$  as a function of  $b/r_h(2\mu)$  for  $D=4, \dots, 11$ . The circles show the values at  $b=b_{max}$ .  $\mathcal{H}_D^{A.H.}(b_{max})$  becomes small as  $D$  increases.

considering symmetries of the horizon and took the maximum value of their  $(D-3)$ -volume. Using the similar method, we adopt the  $(D-3)$ -volume of the  $(D-3)$ -surface  $C$ . Figure 6 shows the numerical results of  $\mathcal{H}_D^{A.H.}(b)$ .  $\mathcal{H}_D^{A.H.}(b)$  is almost constant and slightly increases with  $b$ :

$$\mathcal{H}_D^{A.H.}(b) \approx \mathcal{H}_D^{A.H.}(0) = \left[ \frac{(D-2)\Omega_{D-2}}{2\Omega_{D-3}} \right]^{1/(D-2)}. \quad (22)$$

Thus the isoperimetric inequality  $\mathcal{H}_D^{A.H.}(b) \leq 1$  holds on the horizon in this system. The factor in Eq. (22) contradicts the constancy of  $\mathcal{H}_D^{A.H.}(b_{max})$  with respect to  $D$  that we have expected from Eq. (20). It may be absorbed in the definition of the volume factor, but we do not have the rigorous definition of the volume and we cannot explain the meaning of this factor at this stage. By ignoring this factor, the behavior of  $\mathcal{H}_D^{A.H.}(b)$  for each  $D$  is similar to  $H_D^{A.H.}(b)$  for  $D=4$ . This indicates that  $\mathcal{H}_D$  with the quasilocal mass provides a value that is close to unity at the horizon formation and the volume conjecture holds.

#### IV. SUMMARY AND DISCUSSION

We have investigated black hole formation in the grazing collision of high-energy particles numerically. A black hole is produced when the impact parameter  $b$  satisfies the condition  $b \leq 1.5r_h(\mu)$ . This condition can be derived by using the volume conjecture and suggests that the volume conjecture provides a better condition for horizon formation than the hoop conjecture in the higher-dimensional gravity.

We evaluated the value of  $H_D$  and  $\mathcal{H}_D$  on the apparent horizon and both inequalities  $H_D \leq 1$  and  $\mathcal{H}_D \leq 1$  give fairly good conditions for the horizon formation. This result is in contrast to the horizon formation by the spindle matter distribution [5]; in this case, the gravitational field in the transverse direction of spindle matter is  $(D-1)$ -dimensional and the black hole forms even if the hoop  $C$  is far greater than  $2\pi r_h(M)$  for  $D \geq 5$  and  $H_D \leq 1$  does not give the condition for the horizon formation. But in the system with two point particles, the gravitational field in the middle of two particles is weak if the distance of two particles is larger than  $r_h(M)$  and becomes sufficiently strong to form the horizon if the distance of two particles equals  $r_h(M)$  for any  $D$ . This leads to the condition of the black hole formation  $H_D \sim 1$ . We

therefore expect that the hoop conjecture in the higher-dimensional spacetime holds for the system that consists of two point particles.

The behavior of  $\mathcal{H}_D^{\text{A.H.}}(b_{\text{max}})$  indicates that  $\mathcal{H}_D \lesssim 1$  with the quasilocal mass provides a better condition for the horizon formation compared to the hoop conjecture. But what we have confirmed is the necessary condition for the horizon

formation; if an apparent horizon exists, there is a surface that satisfies  $\mathcal{H}_D \lesssim 1$ . There remains the possibility that even if the apparent horizon does not exist, there is a surface that satisfies the  $\mathcal{H}_D \lesssim 1$ . Hence further investigation is required to confirm whether  $(D-3)$ -volume conjecture provides a sufficient condition for the horizon formation. This is our remaining problem.

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