Cosmological consequences of a Chaplygin gas dark energy

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A combination of recent observational results has given rise to what is currently known as the dark energy problem. Although several possible candidates have been extensively discussed in the literature, the nature of this dark energy component is not well understood at present. In this paper we investigate some cosmological implications of another dark energy candidate: an exotic fluid known as the Chaplygin gas, which is characterized by an equation of state $p = -A/\rho$, where A is a positive constant. By assuming a flat scenario driven by nonrelativistic matter plus a Chaplygin gas dark energy we study the influence of such a component on the statistical properties of gravitational lenses. A comparison between the predicted age of the universe and the latest age estimates of globular clusters is also included and the results briefly discussed. In general, we find that the behavior of this class of models may be interpreted as an intermediary case between the standard model and the cold dark matter model with a cosmological constant scenarios.

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I. INTRODUCTION

From a large amount of observational evidence, the currently favored cosmological model is flat, accelerated and composed of ~1/3 matter (baryonic + dark) and ~2/3 of a negative-pressure dark component, usually named dark energy or ''quintessence.'' The nature of such an unclustered dark energy component, however, is not very well understood at present, giving rise to many theoretical speculations.

Certainly, the most extensively studied explanation for this dark energy problem is the vacuum energy density or cosmological constant (Λ), although other interesting possibilities are also given in the current literature. Some examples are as follows: a very light scalar field ϕ, whose effective potential $V(\phi)$ leads to an accelerated phase at the late stages of the Universe [1], an *X*-matter component [2], which is simply characterized by an equation of state $p_x = \omega_x \rho_x$, where $-1 \leq \omega_x < 0$ and that includes, as a particular case, models with a cosmological constant [cold dark matter model with a cosmological constant (ΛCDM)], a vacuum decaying energy density or a time varying Λ term whose present value of the cosmological constant (Λ_o) is a remnant of the primordial inflationary or deflationary stage [3], geometrical effects from extra dimensions [4] or still an exotic fluid, the so-called Chaplygin gas, whose equation of state is given by

$$p = -A/\rho, \tag{1}$$

where A is a positive constant [5].

All the above-mentioned candidates for quintessence have interesting features that make them at some level compatible with the recent obervational facts (see, for example, [6–9]). Although most of these scenarios have been extensively explored in the recent literature, in the case of a Chaplygin gas-type dark energy, however, only a few analyses have focused attention on its cosmological consequences. From a theoretical viewpoint, an interesting connection between the Chaplygin gas equation of state and string theory has been identified [10–12]. As explained in [13,14], a Chaplygin gas-type equation of state is associated with the parametrization invariant Nambu-Goto d-brane action in a $d+2$ spacetime. In the light-cone parametrization, such an action reduces itself to the action of a Newtonian fluid which obeys Eq. (1) so that the Chaplygin gas corresponds effectively to a gas of d-branes in a $d+2$ spacetime. Moreover, the Chaplygin gas is the only gas known to admit supersymmetric generalization [11].

From the observational viewpoint, it has been argued that the Chaplygin gas (CG) may unify the cold dark matter and the dark energy scenarios [12]. The reason for such a belief is the general behavior of the Chaplygin gas equation of state: it can behave as cold dark matter at small scales and as a negative-pressure dark energy component at large scales. Recently, Fabris *et al.* [15] analyzed a cold dark matter plus a Chaplygin gas scenario in the light of type Ia supernovae data (SNe Ia). As a general result, they found a universe completely dominated by the Chaplygin gas as the best fit model. More recently, Avelino *et al.* [16] used a larger sample of SNe Ia and the shape of the matter power spectrum to show that such data restrict the model to a behavior that closely matches that of a ΛCDM model while Bento

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et al. [17] showed that the location of the cosmic microwave background (CMB) peaks imposes tight constraints on the free parameters of the model.

The aim of this paper is to explore some other observational consequences of a Chaplygin gas dark energy. We mainly focus our attention on the constraints from statistical properties of gravitationally lensed quasars in Eq. (1). We also investigate other observational quantities like the deceleration parameter, the acceleration redshift and the expanding age of the universe. To obtain such results we assume a flat model driven by nonrelativistic matter plus a Chaplygin gas dark energy component (from now on CGCDM).

This paper is organized in the following way. In Sec. II the field equations and distance formulas are presented. We also derive the expression for the deceleration parameter and discuss the redshift at which the accelerated expansion begins. The predicted age of the Universe in the context of CGCDM models is briefly discussed in Sec. III. We then proceed to analyze the constraints from lensing statistics on these scenarios in Sec. IV. We end the paper by summarizing the main results in the conclusion section.

II. FIELD EQUATIONS, DECELERATION PARAMETER AND DISTANCE FORMULAS

Let us now consider the Friedmann-Robertson-Walker (FRW) line element ($c=1$)

$$ds^2=dt^2-R^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)\right], \quad (2)$$

where $k=0,\ \pm 1$ is the curvature parameter of the spatial section, r, θ, and ϕ are dimensionless comoving coordinates, and $R(t)$ is the scale factor. Since the two components (nonrelativistic matter and Chaplygin gas) are separately conserved, we use the energy conservation law together with Eq. (1) to find the following expression for the Chaplygin gas density:

$$\rho_{CG}=\sqrt{A+B\left(\frac{R_o}{R}\right)^6}, \quad (3)$$

or, equivalently,

$$\rho_{CG}=\rho_{CG_o}\sqrt{A_s+(1-A_s)\left(\frac{R_o}{R}\right)^6}, \quad (4)$$

where the subscript o denotes present day quantities, $B=\rho^2_{CG_o}-A$ and $A_s=A/\rho^2_{CG_o}$ is a quantity related with the sound speed for the Chaplygin gas today. As can be seen from Eq. (3), the Chaplygin gas interpolates between nonrelativistic matter $[\rho_{CG}(R\to 0)\simeq\sqrt{B}/R^3]$ and negative-pressure dark component regimes $[\rho_{CG}(R\to\infty)\simeq\sqrt{A}]$.

The Friedmann's equation for the kind of models we are considering is

$$\frac{\dot{R}}{R}=H_o\left[\Omega_{\rm m}\left(\frac{R_o}{R}\right)^3+\Omega_{CG}\sqrt{A_s+(1-A_s)\left(\frac{R_o}{R}\right)^6}\right]^{1/2}. \quad (5)$$

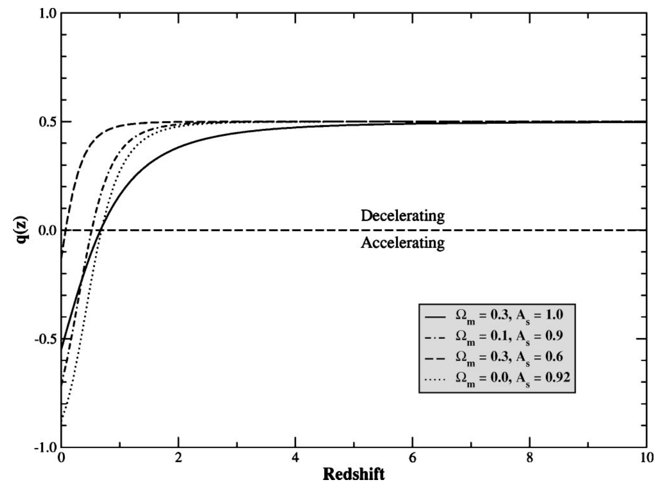

FIG. 1. Deceleration parameter as a function of redshift for some selected values of $\Omega_{\rm m}$ and A_s. The horizontal line labeled "decelerating/accelerating" ($q_o=0$) divides models with a decelerating or accelerating expansion at a given redshift.

In the above equation, an overdot denotes a derivative with respect to time, $H_o=100$ h km s^{-1}Mpc^{-1} is the present day value of the Hubble parameter, and $\Omega_{\rm m}$ and Ω_{CG} are, respectively, the matter and the Chaplygin gas density parameters.

The deceleration parameter, usually defined as $q_o=-R\ddot{R}/\dot{R}^2|_{t_o}$, now takes the following form:

$$q_o=\frac{\frac{3}{2}[\Omega_{\rm m}+\Omega_{CG}(1-A_s)]}{\Omega_{\rm m}+\Omega_{CG}}-1. \quad (6)$$

As one may check, for $A_s=0$ and $A_s=1$, the above expressions reduce to the standard and ΛCDM models, respectively.

Figure 1 shows the behavior of the deceleration parameter as a function of redshift for selected values of $\Omega_{\rm m}$ and A_s. The best fit ΛCDM case is also showed for the sake of comparison ($A_s=1$). Note that the value of A_s determines the acceleration redshift z_a. At late times, a CGCDM model with $\Omega_{\rm m}=0.1$ and $A_s=0.9$ accelerates faster than a ΛCDM scenario with $\Omega_{\rm m}=0.3$. In such a model the accelerated expansion begins at $z_a\simeq 0.51$. For the best fit model found in Ref. [13], i.e., $\Omega_{\rm m}=0$ and $A_s=0.92$, the universe is strongly accelerated today with the accelerated phase beginning at $z_a\simeq 0.68$ whereas for $A_s=1$ and $A_s=0.6$ and $\Omega_{\rm m}=0.3$ we find, respectively, $z_a\simeq 0.67$ and $z_a\simeq 0.07$.

From Eqs. (2) and (5), it is straightforward to show that the comoving distance $r_1(z)$ to a light source located at $r=r_1$ and $t=t_1$ and observed at $r=0$ and $t=t_o$ can be written as

$$r_1(z)=\frac{1}{R_oH_o}\int_{x'}^{1}\frac{dx}{x^2 f(x,\Omega_{\rm m},A_s)}, \quad (7)$$

where $x'=R(t)/R_o=(1+z)^{-1}$ is a convenient integration variable and the dimensionless function $f(x,\Omega_{\rm m},A_s)$ is given by

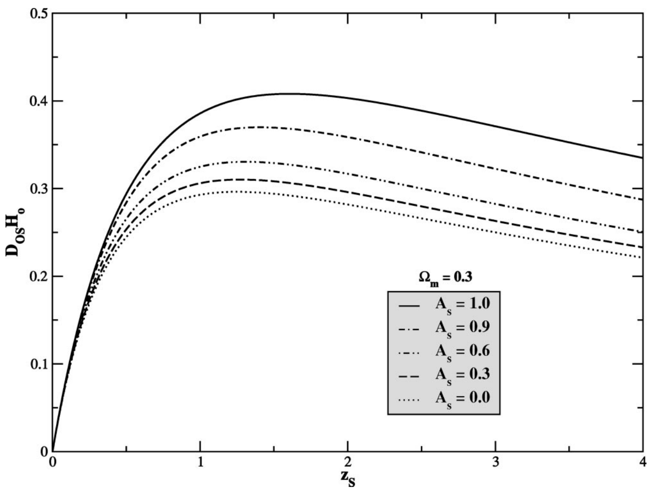

FIG. 2. Dimensionless angular diameter distance as a function of the source redshift (z_S) for some selected values of A_s. In all curves the value of the matter density parameter has been fixed ($\Omega_m = 0.3$).

$$f(x,\Omega_m,A_s)=\left[\frac{\Omega_m}{x^3}+(1-\Omega_m)\sqrt{A_s+\frac{(1-A_s)}{x^6}}\right]^{1/2}. \quad (8)$$

In order to derive the constraints from lensing statistics in Sec. IV we shall deal with the concept of angular diameter distance. For the class of models here investigated, the angular diameter distance, $D_{LS}(z_L,z_S)=R_o r_1(z_L,z_S)/(1+z_S)$, between two objects, for example a lens at z_L and a source (galaxy) at z_S, reads

$$D_{LS}(z_L,z_S)=\frac{H_o^{-1}}{(1+z_S)}\int_{x'_S}^{x'_L}\frac{dx}{x^2 f(x,\Omega_m,A_s)}. \quad (9)$$

In Fig. 2 we show the dimensionless angular diameter distance between an observer O and the source S ($D_{OS}H_o$) as a function of the source redshift (z_S) for $\Omega_m = 0.3$ and selected values of A_s. As physically expected, the larger the value of A_s the larger the distance that is predicted between two redshifts. This result shows that, for the value of Ω_m considered, the behavior of this class of CGCDM models may be interpreted as an intermediary case between the ΛCDM ($A_s=1$) and the Einstein–de Sitter ($A_s=0$) scenarios. This particular feature of CGCDM models may be important for the lensing statistics analysis because, as is well known, the large distances predicted by ΛCDM models make the lensing constraints on the vacuum energy contribution very restrictive (see, for instance, [18]). In this concern, we expect that the constraints from this particular test will be weaker for CGCDM scenarios than for their ΛCDM counterparts. It is worth mentioning that the behavior of CGCDM cosmology can be very different from that one present by ΛCDM scenarios and general quintessence cosmologies. For example, as shown in Ref. [19], the trajectories of the statefinder parameters [20] in CGCDM scenarios differ considerably from the one presented by quintessence or ΛCDM models. As commented in [19], the statefinder diagnostic combined with future supernovae observations (as, for example, the SNAP mission) may be able to discriminate between CGCDM and general quintessence cosmologies. More recently, an analysis for the location of the CMB peaks showed that CGCDM models and ΛCDM have very different predictions for large values of the parameter A_s [17].

III. THE AGE OF THE UNIVERSE

The predicted age of the Universe for the class of CGCDM models considered in this paper is given by

$$t_o=\frac{1}{H_o}\int_0^1\frac{dx}{x f(x,\Omega_m,A_s)}. \quad (10)$$

As widely known, a lower bound for this quantity can be estimated in a variety of different ways. For instance, Oswalt *et al.* [21], analyzing the cooling sequence of white dwarf stars found a lower age limit for the galactic disk of 9.5 Gyr. Later on, a value of 15.2±3.7 Gyr was also determined using radioactive dating of thorium and europium abundances in stars [22]. In this connection, the recent age estimate of an extremely metal-poor star in the halo of our Galaxy (based on the detection of the 385.957 nm line of singly ionized ^{238}U) indicated an age of 12.5±3 Gyr [23]. Another important way of estimating a lower limit to the age of the Universe is dating the oldest stars in globular clusters. Such estimates, however, have oscillated considerably since the publication of the statistical parallax measures done by Hipparcos. Initially, some studies implied a lower limit of 9.5 Gyr at 95% confidence level (C.L.) [24]. Nevertheless, subsequent studies [25], using new statistical parallax measures and updating some stellar model parameters, found 13.2 Gyr with a lower limit of 11 Gyr at 95% C.L. as a corrected mean value for age estimates of globular clusters (see also [26]). Such a value implies that the Einstein–de Sitter model is ruled out for $h \geq 0.50$, while the most recent measurements of h point consistently to $h \geq 0.65$ [27,28]. These results are also in accordance with recent estimates based on rather different methods for which the ages of the oldest globular clusters in our Galaxy fall on the interval 13.8–16.3 Gyr [29].

By assuming $t_o = 13\pm 1$ Gyr as a median value for the most recent age estimates of globular clusters and $H_o = 72 \pm 8$ km s^{-1} Mpc^{-1}, in accordance with the final results of the *Hubble Space Telescope* Key Project [28], we find $H_o t_o = 0.95\pm 0.11$, a value that is compatible with the estimates discussed above as well as very close to some determinations based on SNe Ia data [30,31]. In Fig. 3 we show the dimensionless age parameter $H_o t_o$ as a function of Ω_m for some selected values of A_s. Horizontal dashed lines indicate $\pm 2\sigma$ of the age parameter for the values of H_o and t_o considered here. Similarly to the discussion for the angular diameter distance, for a fixed value of Ω_m the predicted age of the Universe is larger for larger values of A_s. If $\Omega_m = 0.2$–0.4, as suggested by dynamical estimates on scales up to about 2h^{-1} Mpc [32], we find $A_s \geq 0.96$ (see also [33]).

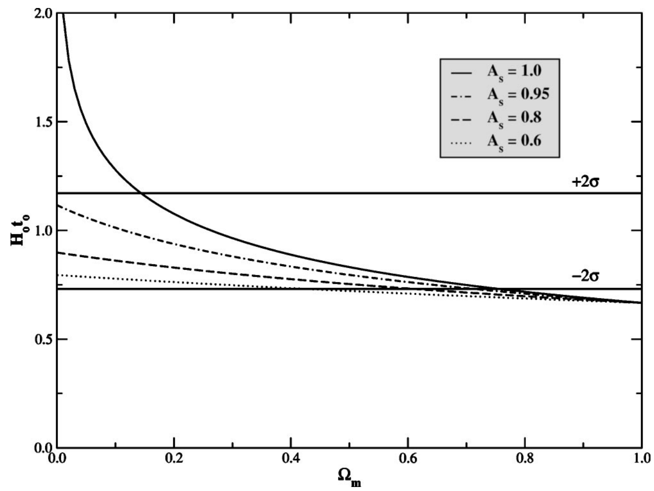

FIG. 3. $H_o t_o$ as a function of the matter density parameter for some values of A_s. Horizontal lines correspond to $\pm 2\sigma$ limits of the age parameter $H_o t_o = 0.95 \pm 0.11$.

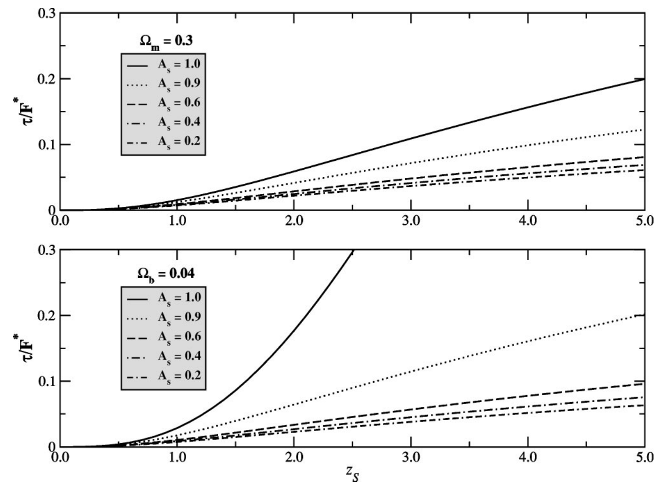

FIG. 4. The normalized optical depth (τ/F^*) as a function of the source redshift (z_S) for some selected values of A_s. Upper panel: CGCDM models with $\Omega_m = 0.3$. Lower panel: a universe dominated by baryonic matter ($\Omega_b = 0.04$) and the Chaplygin gas.

IV. CONSTRAINTS FROM LENSING STATISTICS

In order to constrain the parameters Ω_m and A_s from lensing statistics we work with a sample of 867 ($z > 1$) high luminosity optical quasars which includes five lensed quasars. This sample consists of data from the following optical lens surveys: the HST Snapshot survey [34], the Crampton survey [35], the Yee survey [36], the Surdej survey [37], the NOT Survey [38] and the FKS survey [39].

The differential probability $d\tau$ of a beam having a lensing event in traversing dz_L is [40,41]

$$d\tau = F^*(1+z_L)^3\left(\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\right)^2 \frac{1}{R_0}\frac{dt}{dz_L}dz_L, \tag{11}$$

where

$$\frac{dt}{dz_L} = \frac{H_o^{-1}}{(1+z_L)f(x,\Omega_m,A_s)} \tag{12}$$

and

$$F^* = \frac{16\pi^3}{cH_0^3}\phi_* v_*^4 \Gamma\left(\alpha + \frac{4}{\gamma} + 1\right). \tag{13}$$

In Eq. (11), D_{OL}, D_{OS} and D_{LS} are, respectively, the angular diameter distances from the observer to the lens, from the observer to the source and between the lens and the source. We use the Schechter luminosity function with the lens parameters for E/SO galaxies taken from Madgwick *et al.* [42], i.e., $\phi_* = 0.27h^3 \times 10^{-2}$ Mpc^{-3}, $\alpha = -0.5$, $\gamma = 4$, $v_* = 220$ km/s and $F^* = 0.01$.

The total optical depth is obtained by integrating $d\tau$ along the line of sight from z_O ($z=0$) to z_S. One obtains

$$\tau(z_S) = \frac{F^*}{30}[D_{OS}(1+z_L)]^3 \ \ R_o^3. \tag{14}$$

In Fig. 4 we show the normalized optical depth as a function of the source redshift (z_S) for values of $A_s = 0.2$, 0.4, 0.6, 0.9 and 1.0. Two different cases are illustrated: a conventional CGCDM model with the matter density parameter fixed at $\Omega_m = 0.3$ and a Chaplygin gas + baryonic matter model with $\Omega_b = 1 - \Omega_{Cg} = 0.04$. As discussed earlier, the reason for considering the later case is because one of the strongest claims for a Chaplygin gas dark energy is the possibility of a unified explanation for the dark matter and dark energy problems [12]. In this case, one might expect that the only two components of the Universe would be the Chaplygin gas and the baryonic matter. Note that in both cases an increase in A_s at fixed Ω_m tends to increase the optical depth for lensing. For example, for $\Omega_m = 0.3$, the value of τ/F^* for $A_s = 0.2$ at $z_S = 3.0$ is down from the ΛCDM ($A_s = 1$) value by a factor of ~ 2.97, while at the same redshift, τ/F^* for $A_s = 0.4$ is down from that for $A_s = 1.0$ by ~ 2.63. By fixing the value of A_s, for example, $A_s = 0.6$, we observe that the value of τ/F^* is smaller for a universe with $\Omega_m = 0.3$ than for a universe composed only of the Chaplygin gas + baryonic matter ($\Omega_b = 0.04$) by a factor of ~ 1.18. This increase of the optical depth as the value of A_s is increased (at a fixed z_S and Ω_m) is an expected consequence since this model more closely approaches the ΛCDM case as $A_S \rightarrow 1$.

The likelihood function is defined by

$$\mathcal{L} = \prod_{i=1}^{N_U}(1-p_i')\prod_{k=1}^{N_L}p_k' \ \ p_{ck}', \tag{15}$$

where N_L is the number of multiple-imaged lensed quasars, N_U is the number of unlensed quasars, and p_k' and p_{ck}^i are, respectively, the probability of quasar k to be lensed and the configuration probability. These quantities are defined by

$$p_i'(m,z) = p_i \int \frac{d(\Delta\theta)p_c(\Delta\theta)B(m,z,M_f(\Delta\theta),M_2)}{B(m,z,M_0,M_2)} \tag{16}$$

and

$$p'_{ci}=p_{ci}(\Delta\theta)\ \frac{p_i}{p'_i}\ \frac{B(m,z,M_f(\Delta\theta),M_2)}{B(m,z,M_0,M_2)}, \tag{17}$$

where

$$p_c(\Delta\theta)=\frac{1}{\tau(z_S)}\int_0^{z_S}\frac{d^2\tau}{dz_L d(\Delta\theta)}\ dz_L \tag{18}$$

and

$$M_f=M_0(f+1)/(f-1) \quad \text{with} \quad f=10^{0.4\ \Delta m(\theta)}. \tag{19}$$

The magnification bias, $\mathbf{B}(m,z)$, is considered in order to take into account the increase in the apparent brightness of a quasar due to lensing which, in turn, increases the expected number of lenses in flux limited sample. The bias factor for a quasar at redshift z with apparent magnitude m is given by [18,40]

$$\mathbf{B}(m,z)=M_0^2\ B(m,z,M_0,M_2), \tag{20}$$

where

$$B(m,z,M_1,M_2)=2\left(\frac{dN_Q}{dm}\right)^{-1}\int_{M_1}^{M_2}\frac{dM}{M^3}\frac{dN_Q}{dm}\times[m+2.5\log(M),z]. \tag{21}$$

In the above equation $[dN_Q(m,z)/dm]$ is the measure of number of quasars with magnitudes in the interval $(m,m+dm)$ at redshift z. Since we are modeling the lens by a singular isothermal model profile, $M_0=2$, we adopt $M_2=10^4$ in the numerical computation.

For the quasar luminosity function we use Kochanek's "best model" [18],

$$\frac{dN_Q}{dm}(m,z)\propto(10^{-a(m-\bar{m})}+10^{-b(m-\bar{m})})^{-1}, \tag{22}$$

where

$$\bar{m}=\begin{cases} m_o+(z-1) & \text{for } z<1,\\ m_o & \text{for } 1<z\leqslant 3,\\ m_o-0.7(z-3) & \text{for } z>3,\end{cases} \tag{23}$$

and we assume $a=1.07\pm0.07$, $b=0.27\pm0.07$ and $m_o=18.92\pm0.16$ at B magnitude [18].

Because of the selection effects the survey can detect lenses with magnification larger than a certain magnitude M_f given by Eq. (19) which becomes the lower limit in Eq. (22). To obtain selection function corrected probabilities, we follow [18] and divide our sample into two parts, namely, the ground based surveys and the Hubble Space Telescope (HST) survey.

From Eq. (15) we find that the maximum value of the likelihood function is located at $\Omega_m=0.4$ and $A_s=1.0$. At the 1σ level, however, almost the entire range of A_s is compatible with the observational data for values of $\Omega_m=0-1$. As observed earlier (see Sec. II), this result suggests that a large class of CGCDM scenarios is in accordance with the current gravitational lensing data. For the sake of comparison, we also analyze some possible differences between our best-fit value and the one obtained for general quintessence scenarios with an equation of state $p_x=\omega_x\rho_x$ (XCDM models) [2]. For example, for XCDM models a similar analysis shows that the maximum value of the likelihood function is located at $\Omega_m=0.0$ and $\omega_x=-0.2$ [43]. Such a model corresponds to a decelerated universe with a deceleration parameter $q_o=0.2$ and a total expanding age of $8.1h^{-1}$ Gyr while our best-fit CGCDM model corresponds to an accelerating scenario with $q_o=-0.39$ ($z_a=0.44$) and a total age of the order of $8.7h^{-1}$ Gyr. In Fig. 5(a) the expected number of lensed quasars, $n_L=\Sigma\ p'_i$ (the summation is over a given quasar sample), is displayed as a function of A_s. As indicated in the figure, the horizontal dashed line indicates $n_L=5$, that is the number of lensed quasars in our sample. By this analysis, one finds $A_s=0.9$ ($\Omega_m=0.3$) and $A_s=0.73$ ($\Omega_b=0.04$). In Fig. 5(b) we show the contour for five lensed quasars in the parametric space $A_s-\Omega_m$. The shadowed horizontal region corresponds to the observed range $\Omega_m=0.3\pm0.1$ [32]. As a general result, this analysis provides $\Omega_m\leqslant0.45$ and $A_s\geqslant0.72$. We also observe that the higher the value of Ω_m the higher the value of A_s that is required to fit these data.

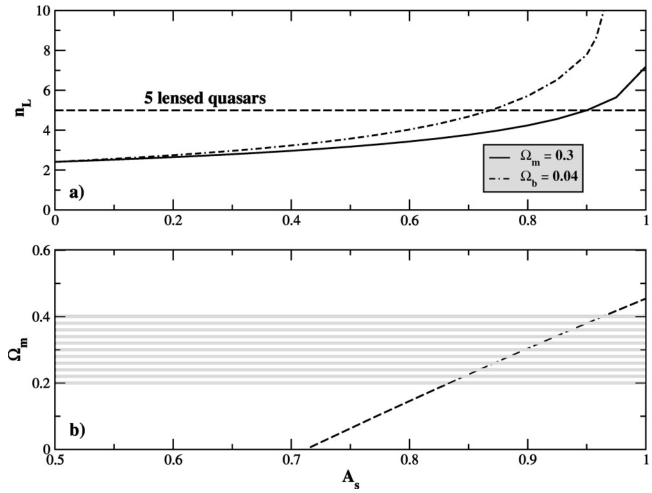

FIG. 5. (a) Predicted number of lensed quasars as a function of A_s for $\Omega_m=0.3$ (solid line) and $\Omega_b=0.04$ (dashed-dotted line) and image separation $\Delta\theta\leqslant4$. (b) Contour for five lensed quasars in the parametric space $A_s-\Omega_m$. The shadowed horizontal region corresponds to the observed range $\Omega_m=0.3\pm0.1$ [32].

V. CONCLUSION

The search for alternative cosmologies is presently in vogue and the leitmotiv is the observational support for an accelerated universe provided by the SNe Ia results. In general, such alternative scenarios contain an unknown negative-pressure dark component that explains the SNe Ia results and reconciles the inflationary flatness prediction ($\Omega_T=1$) with the dynamical estimates of the quantity of matter in the Universe ($\Omega_m\simeq0.3\pm0.1$). In this paper we have focused our attention on another dark energy candidate: the Chaplygin gas. We showed that the predicted age of the Universe in the context of CGCDM models is compatible with the most recent age estimates of globular clusters for values of $\Omega_m \simeq 0.2$ and $A_s \geqslant 0.96$. We also studied the influence of such a component on the statistical properties of gravitational lensing. At the 1σ level we found that a large class of these scenarios is in agreement with the current lensing data with the maximum of the likelihood function [Eq. (15)] located at $\Omega_m = 0.4$ and $A_s = 1.0$. As a general result, the predicted number of lensed quasars requires $\Omega_m \leqslant 0.45$ and $A_s \geqslant 0.72$.

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