Alternatives to quintessence model building

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(Received 31 August 2002; published 24 January 2003)

We discuss the issue of toy model building for the dark energy component of the universe. Specifically, we consider two generic toy models recently proposed as alternatives to quintessence models, respectively known as Cardassian expansion and the Chaplygin gas. We show that the former is entirely equivalent to a class of quintessence models. We determine the observational constraints on the latter, coming from recent supernovae results and from the shape of the matter power spectrum. As expected, these restrict the model to a behavior that closely matches that of a standard cosmological constant Λ.

DOI: 10.1103/PhysRevD.67.023511 PACS number(s): 98.80.Cq

I. INTRODUCTION

Currently available observations, especially from high-z type Ia supernovae combined with cosmic microwave background (CMB) results [1], suggest that about one-third of the critical energy density of the Universe is in the form of ordinary matter (including classical dark matter), while the remaining two-thirds are in an unclustered form which is commonly called dark energy. Among other effects, this unknown component produces a recent accelerated expansion, a behavior which standard decelerating Friedmann models are unable to match—though see [2]. The cosmological constant Λ is arguably the simplest candidate for this dark energy, although it is well known that theoretical predictions for its value are many orders of magnitude off from observationally acceptable values.

Noteworthy among the many proposed alternatives are time varying scalar fields, dubbed quintessence [3,4]. Quintessence models typically involve a scalar field function and in some cases more than one. However, quintessence models often suffer from a major problem that also afflicts the cosmological constant: fine-tuning. This is often referred to as the ''why now?'' problem: why is the cosmological constant (or a quintessence field) so small, and why does it become dominant over the matter content of the Universe right about the present day? There are so-called ''tracking'' models where one obtains that quintessence energy density is reasonably independent of initial conditions, but on the other hand one does have to tweak some parameters in the scalar field potential in order to obtain the desired behavior, so this cannot really be claimed as a satisfactory solution to the problem.

On the other hand, given that one has yet to see a scalar field in action, it is clear that all such toy models are not much better justified than a classical cosmological constant (despite some claims to the contrary). This is further compounded by the fact that, given some time dependence for the scale factor and energy density, one is always able to *construct* a potential for a quintessence-type model which reproduces them (see for example [5]). One is therefore reminded of Occam's razor and can legitimately ask if observational data provide any strong justification for them, as compared to the conceptually simpler cosmological constant.

Here we make a contribution to this ongoing discussion by studying two particularly illuminating such toy models. Cardassian expansion [6,7] has recently been suggested as a model for an accelerating flat Universe without any use of a cosmological constant or vacuum energy whatsoever but solely depending on a purely matter driven acceleration. This has been accomplished by the use of a modified version of the Friedmann equation where an additional *empirical* term has been added. Unfortunately, as we show here, the Cardassian model does not bring anything particularly new since, for most practical purposes, it reduces to a class of quintessence models. An example of an alternative to quintessence is the so-called Chaplygin gas [8], which obeys a physically exotic equation of state (although it can be motivated, at some level, within the context of higher dimensions brane theory). In this case we will show that, as expected, current observations restrict the model parameters to a gravitational behavior that is very similar to that of a cosmological constant.

II. QUINTESSENCE MODELS

Quintessence is a generic name given to a fluid component, other than ordinary pressureless matter and radiation,

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parametrized by an equation of state of the form $p_i=\omega_i\rho_i$ (in fundamental units) where ω_i is related to a scalar field function. Given that our chances of distinguishing between several scalar field models yielding various different time varying ω_i seem rather bleak [9,10], we will for the time being take ω_i to be constant. We shall start by reviewing some basic properties of this class of models.

A. Basic dynamical properties

The total energy density appearing in the Friedmann equation can be expanded into a sum of quintessential fluid components, therefore turning

$$\mathrm{H}^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}, \tag{1}$$

where a is the scale factor, $\mathrm{H}=\dot{a}/a$, the dot stands for time derivative and k, the curvature term, into

$$\mathrm{H}^2=\sum_i \Omega_i\frac{\rho_i}{\rho_{i0}}+\left(1-\sum_i \Omega_i\right)a^{-2}. \tag{2}$$

Here $\Omega_i=\rho_{i0}/\rho_{c0}$ is the ith density parameter (ρ_c being the critical density) and the subscript 0 refers to present time. We have also taken $a_0=\mathrm{H}_0=1$.

Now, an adiabatic flow is usually assumed for each individual fluid component, so that $d(\rho_i a^3)+p_i d(a^3)=0$. This in turn implies, for each fluid

$$\rho_i=\rho_{i0}a^{-3(1+\omega_i)}=\rho_{i0}(1+z)^{3(1+\omega_i)}, \tag{3}$$

where $z=1/a-1$ is the usual redshift parameter. Of course, for $\omega_i=0$ and $\omega_i=1/3$ we retrieve the usual relations for dust and radiation. Inserting this adiabatic condition into Eq. (2) we finally obtain

$$\mathrm{H}^2=\sum_i \Omega_i a^{-3(1+\omega_i)}+\left(1-\sum_i \Omega_i\right)a^{-2}. \tag{4}$$

Notice also how for $\omega_i=-1$ a constant Ω_i term appears, formally acting as a Λ cosmological constant.

A physical restriction on ω_i stems from the square of the sound speed velocity of each fluid component, given by $\partial p_i/\partial\rho_i$ for constant entropy. The speed at which information is carried by the fluid must necessarily be smaller or equal to unity, so that $\omega_i\leqslant 1$ in the present case. We emphasize that this constraint is only valid for constant ω_i since in general $\partial p_i/\partial\rho_i$ will involve an additional term due to a possible dependence of ω_i on ρ_i.

The dynamics of a collection of two or more different fluids can easily be studied if we interpret the Friedmann equation as an energy integral of motion of a one-dimensional fictitious particle moving with an a coordinate. Consider Eq. (2) rewritten in the following form:

$$\dot{a}^2-\sum_i \Omega_i a^{-(1+3\omega_i)}=\left(1-\sum_i \Omega_i\right), \tag{5}$$

and compare it to the standard equation of motion, $E=K+V$, of the fictitious particle. This allows us to relate the curvature term to the mechanical energy of the system, $\dot{a}^2$ to its kinetic energy and the remaining term to the potential felt by the particle. We can push forward this Newtonian analogy by calculating the force $-dV/da$ felt by the particle and see the Raychaudhuri equation emerge. Each fluid then contributes with a partial force of

$$-dV_i/da=-(1+3\omega_i)\Omega_i a^{-(2+3\omega_i)}, \tag{6}$$

so that fluids with $-1/3<\omega<1$ decelerate expansion while fluids with $\omega<-1/3$ accelerate it.

B. The Cardassian model in a quintessence framework

Consider the Friedmann equation for a flat matter dominated Universe, i.e., $\mathrm{H}^2=A\rho_m$, where $A=8\pi G/3$. Since $\rho_m\propto a^{-3}$ we have $\dot{a}^2=A\rho_{m0}a^{-1}$, hence a decreasing function of the scale factor. Now, the easiest way of making this decreasing function of the scale factor a growing one is by empirically adding some other function that counters the a^{-1} decreasing term. Arguably, the simplest function one can add is a power law of the scale factor, a^m with $m>0$; let us choose $m=2-3n$. So now we have the following form for the time derivative of the scale factor:

$$\dot{a}^2=A\rho_{m0}a^{-1}+Ba^{2-3n}, \tag{7}$$

which we can easily show to be rewritable as

$$\mathrm{H}^2=A\rho_m+B'\rho_m^n, \tag{8}$$

where $B'=B/\rho_{m0}^n$. This is precisely the Cardassian model which consists of a modification to the Friedmann equation motivated by theories with extra dimensions.

However, since $\rho_m/\rho_{m0}=a^{-3}$ we can immediately rewrite Eq. (8) in the form

$$\mathrm{H}^2=\Omega_m a^{-3}+\Omega_\omega a^{-3n}, \tag{9}$$

where we have defined $A=\Omega_m/\rho_{m0}$, $B'=\Omega_\omega/\rho_{m0}^n$ and $n=1+\omega$ (note that $\Omega_m+\Omega_\omega=1$).

Notice the equivalence between Eqs. (8) and (9). We can interpret the Cardassian empirical term in the modified Friedmann equation as the superposition of a quintessential fluid with $\omega=n-1$, to a background of dust. Note that since $m>0$ then $n<2/3$, implying $\omega<-1/3$.

We thus see that any Cardassian model can readily be expressed as a quintessence model with constant ω and is therefore, for most practical purposes, indistinguishable from it. We say ''for most practical purposes'' due to the fact that Cardassian models do not specify the behavior of cosmological density fluctuations on scales larger than the horizon, although it is assumed that Newtonian gravity holds on small scales. Cardassian models are therefore incomplete, effective toy models which describe the *average universe*, which is what one needs for most practical purposes. On the other hand, if one wanted to calculate, e.g. the CMB anisotropy on large angular scales, one would need to go beyond this sim-

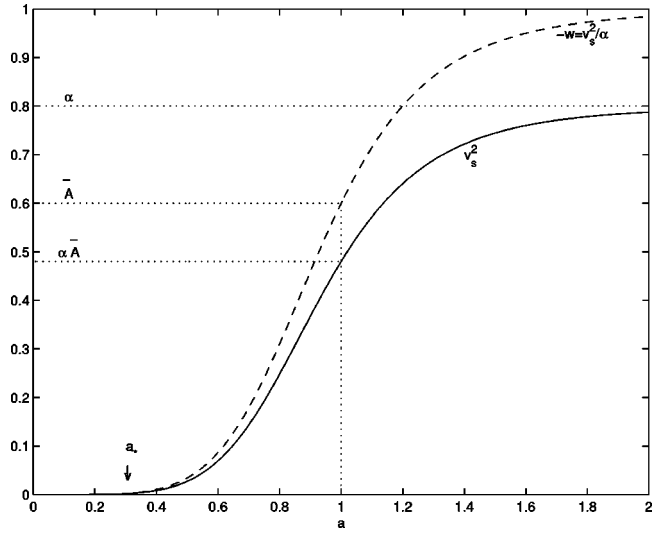

FIG. 1. Sound speed and $\omega = p/\rho$ evolution for the case of a generalized Chaplygin gas—we have taken $\alpha = 0.8$ and $\bar{A} = 0.6$, but this behavior is generic. Notice the phase transition from dust ($\omega = 0$) to a cosmological constant ($\omega = -1$). The accelerating regime begins at $a = a_*$, and the present epoch corresponds to $a = a_0 = 1$.

plified procedure. It is also worth emphasizing that one cannot meaningfully claim that one model is much better justified than the other [6], at least at this stage, since both are no more than toy models.

III. THE CHAPLYGIN GAS MODEL

There are various ways in which one can explain the transition from a matter dominated Universe to one with an accelerated expansion using fluid components with more exotic state equations. Examples include brane-inspired models [11] and vacuum metamorphosis [12]; another is the generalized Chaplygin gas [8] obeying an equation of state of the form

$$p = -\frac{A}{\rho^{\alpha}}, \tag{10}$$

where A is a positive constant and $\alpha \geqslant 0$. The original form of the Chaplygin gas [8,13] where $\alpha = 1$ can be motivated by considering a d-brane in a space-time of $d+2$ dimensions; no such motivation exists for $\alpha \neq 1$. In this case its Nambu-Goto action can be seen as describing a Newtonian fluid with an equation of state given by Eq. (10). We should note that as of now the Chaplygin gas is the only fluid known to admit a supersymmetric generalization. At a toy model level, one can trivially generalize this to a different dependence on the density, as described in [8,14]. We emphasize that, at this simple toy-model level, this should be seen as a generalization simply for the purpose of phenomenological analysis.

As discussed in the above references, the Chaplygin gas can be described in terms of a scalar field with a given potential. As for the quintessence case, we are able to retrieve an integral of motion from energy-momentum conservation,

$$d[a^{3(1+\alpha)}(\rho^{1+\alpha} - A)] = 0, \tag{11}$$

and from it obtain the equivalent expression to Eq. (3), namely

$$\rho = \rho_0[\bar{A} + (1-\bar{A})a^{-3(1+\alpha)}]^{1/1+\alpha}, \tag{12}$$

where $\bar{A} = A/\rho_0^{1+\alpha}$. This quantity can be related to the Chaplygin gas sound speed which we can easily show to be $v_s^2 = \alpha A/\rho^{1+\alpha}$ so that, at present, we have $v_{s0}^2 = \alpha\bar{A}$.

Again, restrictions must be imposed on the values of α since we expect this velocity to be limited by the speed of light. In Fig. 1 we find the sound speed evolution with the scale factor for a generalized Chaplygin gas. We note that it is bounded by α when $a \to \infty$; hence $\alpha\bar{A} \leqslant \alpha \leqslant 1$ implying $\alpha \leqslant 1$ and $\bar{A} \leqslant 1$. If we assume the generalized Chaplygin gas to be a good approximation of the Universe for all times then these later constraints apply. On the other hand, if we chose to make no assumptions about the future behavior of the Universe and treat the Chaplygin gas only as an *effective* model in the sense of effectively reproducing the Universe today without assuming anything beyond, then only $\alpha\bar{A} \leqslant 1$ applies. We will take this approach and use supernovae results in order to constrain the values of α, in the phenomenological spirit discussed above.

One of the most interesting characteristics of the Chaplygin gas resides in its ability to mimic the cosmological con-

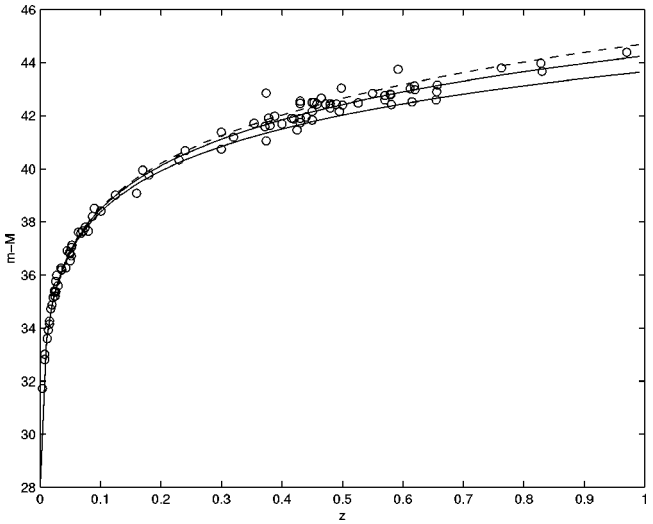

FIG. 2. The Hubble diagram for two fluid models of matter and Chaplygin gas. A flat Universe with $h = 0.652$ was assumed. Small circles depict our dataset of 92 supernova measurements (error bars not displayed). Three limiting cases are shown. The lower solid line corresponds to the $\bar{A} = 0$ case (matter only), the upper solid line is the $\bar{A} = 1$, $\Omega_m = 0.3$ case (ordinary and dark matter plus a cosmological constant) and the dashed line corresponds to the $\bar{A} = 1$, $\Omega_m = \Omega_b = 0.04$ case (baryonic matter only, plus a cosmological constant). General $(\bar{A}, \alpha)$ models for $\Omega_m = 0.3$ lie between the solid curves while for $\Omega_m = \Omega_b = 0.04$ the region extends to the dashed curve. The matter plus cosmological constant provides a good fit so that good Chaplygin models should approach it. This means that high $\bar{A}$ models are preferred. On the other hand, the pure baryonic case is significantly disfavored.

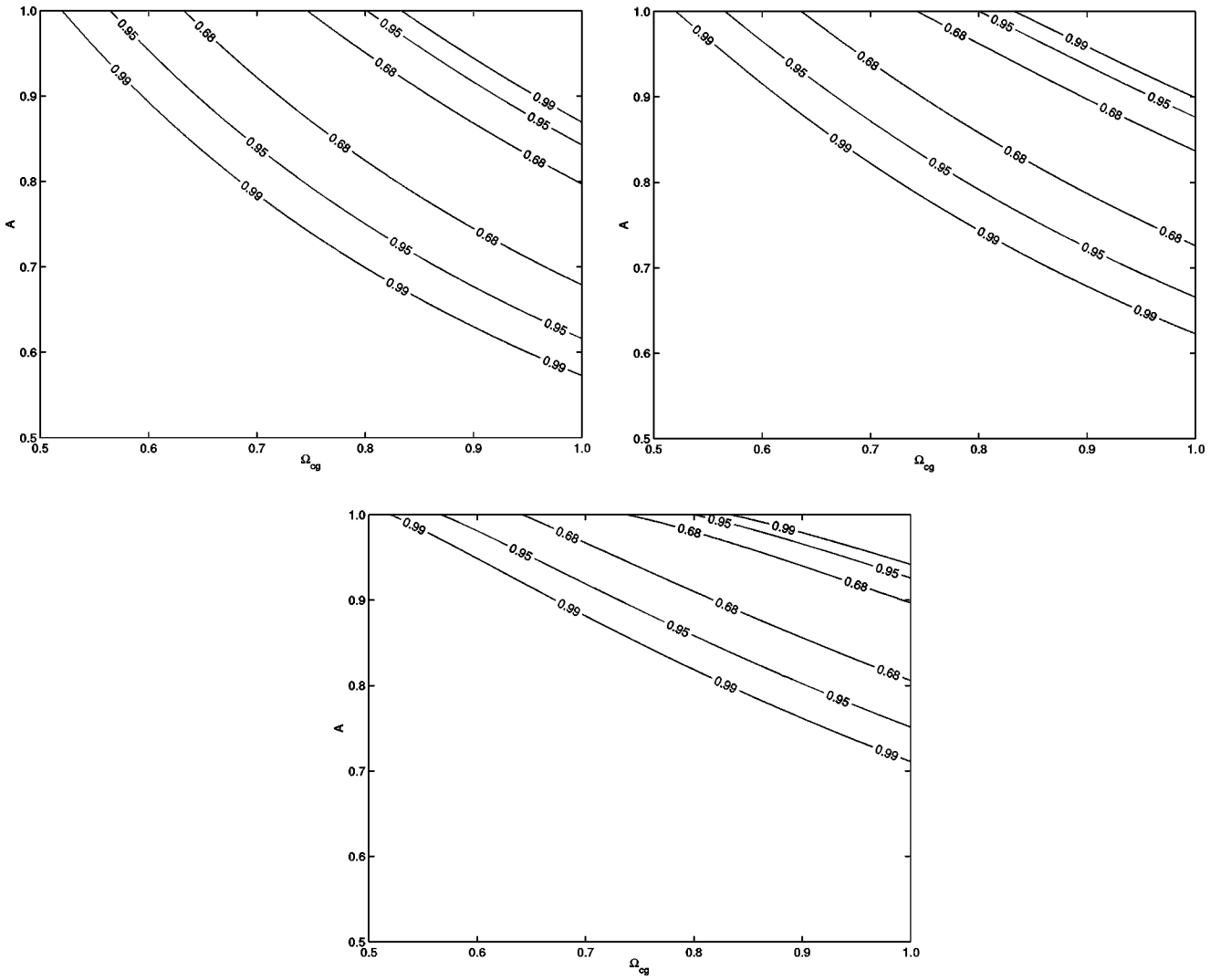

FIG. 3. Assuming a flat Universe where only nonrelativistic matter and a generalized Chaplygin gas are present, we show the confidence regions resulting from a standard χ^2 analysis fit in the $(\Omega_{cg},\overline{A})$ plane, using the 92 available supernovae from the combined SCP and HzST results for $\alpha=0.5$ (top left), $\alpha=1$ (top right) and $\alpha=2$ (bottom). The Hubble parameter H_0 has been removed by integration using standard procedures.

stant when $\overline{A}=1$ and ordinary matter when $\overline{A}=0$. We note that the accelerating regime begins after a_* given by

$$a_*=[(1-\overline{A})/2\overline{A}]^{1/3(1+\alpha)}. \tag{13}$$

Thus, for $a\gg a_*$ we are able to expand the energy density in Eq. (12) into $\rho\approx B+Ca^{-3(1+\alpha)}$ where B and C are constants. What this tells us is that the Chaplygin gas in the accelerating regime describes a kind of mixture between a cosmological constant and a type of matter known as stiff matter obeying the equation of state $p=\alpha\rho$. Note that this is again very similar to the quintessence results, though not quite identical: a Chaplygin cosmology can be interpreted as an interpolation from a dust to a de Sitter universe. This we can see most clearly in Fig. 1 where we have also drawn the evolution of $\omega=p/\rho$ for a generalized Chaplygin gas which resembles a phase transition studied in [12].

IV. CONSTRAINTS FROM OBSERVATIONAL RESULTS

We have shown the Cardassian model to be formally equivalent to a quintessence model if we take $\omega=n-1$. Therefore, it is possible to use recent constraints on ω to place limits on the Cardassian model parameter n. Several papers using supernova data have constrained the ω and the dark energy density, namely [15–18]. Some even introduced a spatial (or temporal) dependency on $\omega\equiv\omega(z)$ and tried to ascertain the first expansion coefficients [19]. However, this work is still severely impaired by the small data set available today.

For a flat Universe, these analyses determined with a 95% confidence level that $\omega<-0.45$ which includes the Λ cosmological constant. On the other hand, CMB restrictions [20] place ω much closer to a Λ scenario, in fact, with a 68% confidence level ω should be smaller than -0.85. Without further observational data, no significantly different analysis

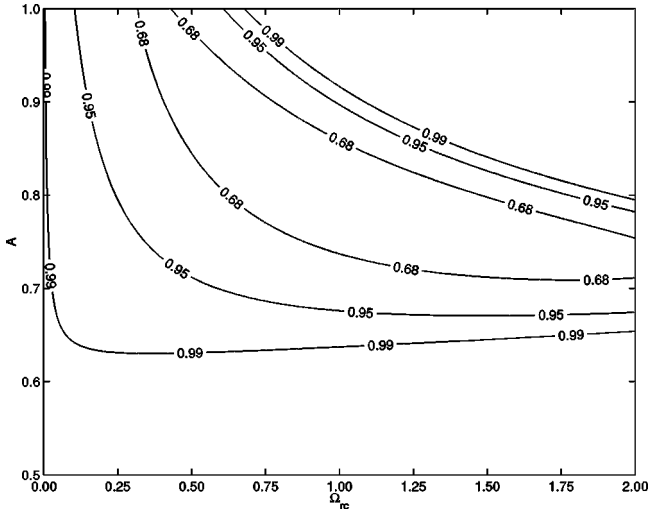

FIG. 4. Assuming there is only baryonic matter and a pure Chaplygin gas, we plot the confidence regions resulting from a χ^2 fit in the $(\Omega_{cg},\overline{A})$ plane, using the 92 supernovae from the combined SCP and HzST results. A value of $\Omega_b=0.04$ was assumed. Note that the Hubble parameter H_0 has been removed by integration using standard procedures.

from the work by previous authors can be done. Therefore, we concentrate upon the addition of a Chaplygin gas to a standard model.

We should also point out that a conceptually very different possibility for a redshift-dependent equation of state is that of an inhomogeneous universe, for example one permeated by a network of light domain walls, which divides it into regions with different cosmological parameters [21]. Since these regions tend to become more and more different as the universe evolves, it turns out to be quite difficult to exclude this type of scenario, or even distinguish it from the standard one. In particular, the presently available supernovae data still allow for significant variations in the matter and vacuum densities [22], whereas the CMB constraints are somewhat more restrictive [23]. Note that in this type of model the equation of state changes abruptly as the line of sight goes across a domain wall separating two such regions. In this sense, this model is similar, for observational purposes, to vacuum metamorphosis [12].

A. Type Ia supernovae

Following the release of the results from the Supernova Cosmology Project (SCP) [24] and the High-z Supernova Search Team (HzST) [25] there has been a surging interest in the study of the energy content of the Universe using type Ia supernovae. Here we will use the combined observational results from both groups, using a procedure first described in [26], to produce constraints on the parameters of the Chaplygin gas—much simpler estimates were done in [27].

As usual, the parameter fit is based upon the luminosity distance d_L defined through $\mathcal{F}=\mathcal{L}/4\pi {d_L}^2$ where $\mathcal{L}$ stands for the intrinsic luminosity of the source and $\mathcal{F}$ for the measured flux. From the Friedmann metric [28] it follows that the luminosity distance, for a flat geometry, as a function of redshift is given by

$$d_L = d_{\rm H}(1+z)\int_0^z \frac{dz'}{{\rm E}(z')}, \tag{14}$$

where $d_{\rm H}$ is the Hubble distance ($c/{\rm H}_0=1$ in geometrized units) and

$${\rm E}^2(z)=\Omega_m x^3+\Omega_{cg}\left(\overline{A}+\frac{(1-\overline{A})}{x^{-3(1+\alpha)}}\right)^{1/1+\alpha} \tag{15}$$

where $x=1+z$ for the case of a mixture of two fluids: one of ordinary matter and the other of generalized Chaplygin gas. The apparent magnitude m of a supernova (a parameter more often used than the measured flux $\mathcal{F}$ to which it is related) at a given magnitude is then given by

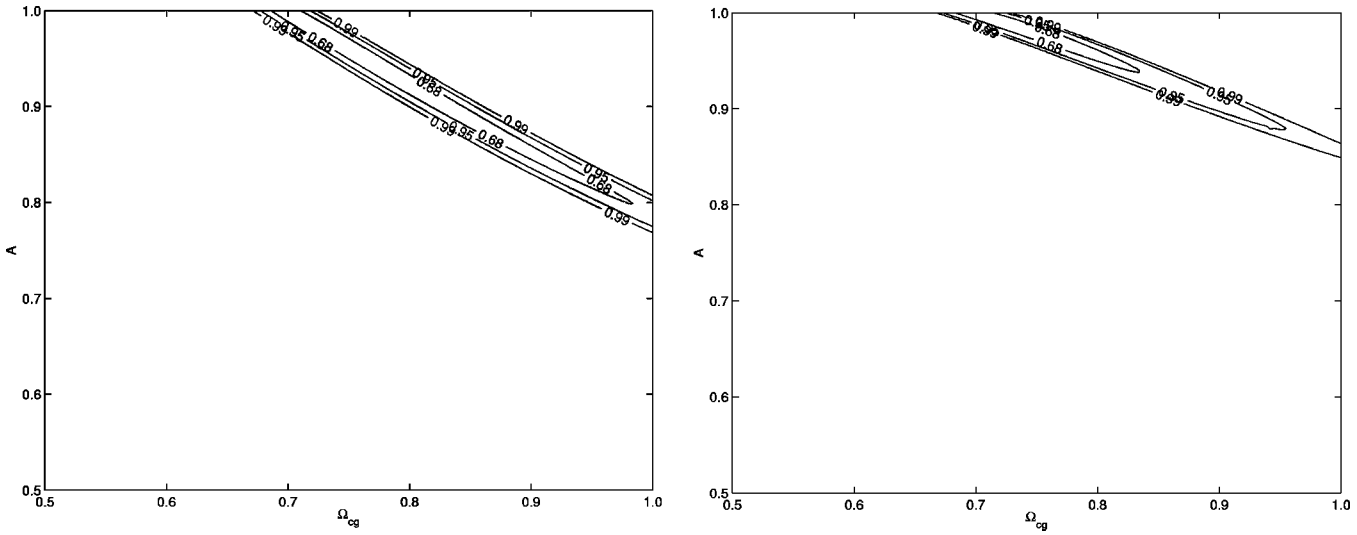

FIG. 5. A forecast of the supernovae constraints on the Chaplygin gas model, with $\alpha=0.5$ (left) and $\alpha=1$ (right), for a fiducial model $\Omega_m=1-\Omega_{cg}=0.3$. Again the Hubble parameter H_0 has been removed by integration using standard procedures.

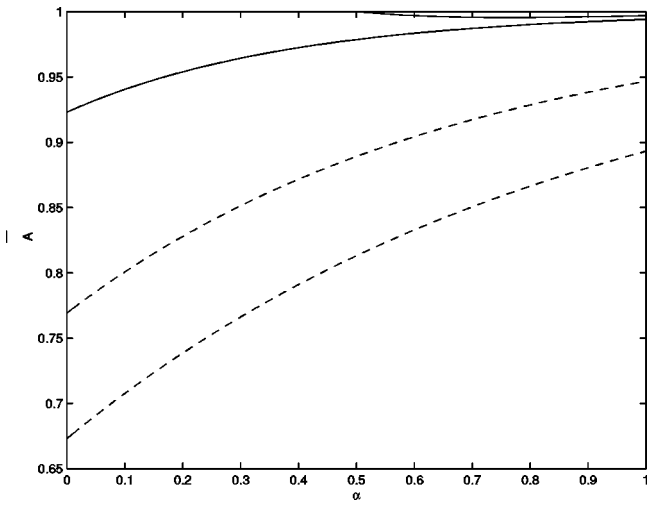

FIG. 6. 2dF constraints on $\bar{A}$ as a function of α, for $\Omega_m=0.3$ (solid) and $\Omega_m=0.04$ (dashed). In both cases the allowed region is enclosed by the two respective lines. A value of $h=0.65$ was assumed for the Hubble parameter.

$$m = M + 5\log d_L + 25, \tag{16}$$

M being the absolute magnitude of the supernova (related to its intrinsic luminosity $\mathcal{L}$). Following Wang [26], we use results from both the SCP and the HzST even though their published data sets differ in their presentation. We define the distance modulus to be

$$\mu_0 = 5\log d_L + 25, \tag{17}$$

as presented in the HzST comprising 50 supernovae. Comparatively, SCP published its measured effective rest-frame B-magnitude $m_b^{\rm eff}$ for 60 supernovae which relate to the HzST results through

$$m_b^{\rm eff} = M_b + \mu_0, \tag{18}$$

where M_b is the peak B-band absolute magnitude of a standard supernova. The published results of the SCP and the HzST groups have 18 common supernovae, 16 of which are from the Calán-Tololo Survey [29]. If we calculate M_b by comparing results from these 18 supernovae (using the results from the HzST estimated by means of the MLCS method), we are able to get

$$M_b = m_b^{\rm eff} - \mu_0 = -19.33 \pm 0.25. \tag{19}$$

Hence, assuming the value $M_b=-19.33$ for the absolute magnitude of the supernovae, we can convert SCP results to distance modulus through Eq. (18). We then add 42 of these supernovae to the data set from HzST leaving out the 18 already present, thus making a total of 92 supernovae used.

Our dataset is displayed in Fig. 2, together with the ranges which the models under discussion can take within the diagram. Note that for each region a larger $\bar{A}$ corresponds to a larger value of $m-M$ at high redshift. The same is true for large values of α, though this effect is subdominant relative to that of $\bar{A}$. This reflects the fact that the luminosity distance is fairly insensitive to time variations of w [10].

Results for the constraints imposed on the values of the Chaplygin gas density and its $\bar{A}$ parameter, for $\alpha=0.5,1,2$ are shown in Fig. 3 where we have performed a χ^2 analysis using the combined SCP and HzST data set of the 92 measured supernovae, assuming a flat universe filled only by ordinary matter and a Chaplygin gas: $\Omega_m+\Omega_{cg}=1$. We should point out, however, that there are potential *caveats* to the chi-squared analysis: see for example [30]. Here we have also integrated the Hubble parameter so as to remove its uncertainties from the results.

In analogy with quintessence models, where a large amount of dark energy with a less negative ω plays the same role as a smaller amount of energy with a more negative ω [24], it is perceptible that the Chaplygin gas models with a higher value of α need a greater $\bar{A}$ to reproduce models with an inferior α, even though all α values indicate roughly the same Chaplygin gas density.

The $\bar{A}$ parameter is restricted with a 95% confidence level to be in the region $0.66<\bar{A}<1$ in the case of a pure Chaplygin gas form, to $0.62<\bar{A}<1$ for $\alpha=0.5$ and to $0.75<\bar{A}<1$ when $\alpha=2$. Notice, however, that in the latter case the limitation $\alpha\bar{A}<1$ implies sound velocities for $\bar{A}>0.5$ greater than the speed of light. Hence the case $\alpha=2$ is in fact completely ruled out. Similar arguments can be used to obtain sound velocity restrictions on the values of $\bar{A}$ and α (more specifically, their product): it is often the case that the values given a higher probability by the χ^2 analysis are physically unreasonable.

We should point out, however, that since one of the strongest claims of the Chaplygin gas is that of a unified explanation for the dark matter and dark energy, one might expect that the only components of the universe would be the Chaplygin gas and standard baryonic matter. In this case, since $\Omega_b=1-\Omega_{cg}\sim 0.04$, we see that the supernovae data strongly exclude a Λ-like behavior, that is the case $\bar{A}=1$.

Therefore, we lift the flatness restriction [modifying the luminosity distance definition (14) accordingly] and assume the energy content to be comprised of a pure Chaplygin gas and baryonic matter with a present day density of $\Omega_b=0.04$. The results of this analysis are depicted in Fig. 4. A large degeneracy is clearly evident. However, the plot does seem to indicate that $\bar{A}$ should be around ~ 0.8 and we can say with a 95% confidence level that $\bar{A}>0.69$. Note that for lower values of the Chaplygin gas density we are forced to approach the cosmological constant $\bar{A}\to 1$. Note also that for the case of a plane geometry the acceptable values for $\bar{A}$ already exclude a Λ scenario.

Finally we have tried to ascertain how future SNAP results will improve our results. Assuming a flat Universe filled with ordinary matter and a Chaplygin gas portrayed by $\Omega_m=1-\Omega_{cg}=0.3$ we repeat the χ^2 analysis as detailed above, and show the corresponding results in Fig. 5.

Obviously the constraints are now much tighter, though it is also clear that there exists a fundamental degeneracy in the

$(\Omega_{cg}, \overline{A})$ plane. Again we notice that for the case $\Omega_m = 0.3$ the model is restricted to be very similar to a standard cosmological constant $\overline{A} \sim 1$, while for $\Omega_m = \Omega_b \sim 0.04$ the Λ-like case is excluded outright.

B. The matter power spectrum

The length scale of the co-moving horizon at the epoch of equality between matter and radiation is directly related to the shape of the power spectrum of matter perturbations. It is straightforward to show that this scale is proportional to the scale factor at that epoch

$$\frac{\mathrm{H}_{eq}^{-1}}{a_{eq}} \propto a_{eq}. \tag{20}$$

In the standard model it is a simple matter to show that $a_{eq} \propto (\Omega_m h^2)^{-1}$. The introduction of a generalized Chaplygin gas naturally implies a change in this expression which becomes

$$a_{eq} \propto ((\Omega_m + (1-\Omega_m)(1-\overline{A})^{1/1+\alpha}) h^2)^{-1}, \tag{21}$$

where we have only assumed a flat universe. Note that since the Chaplygin gas behaves as cold dark matter (CDM) except near the present time (see Fig. 1) the shape of the power spectrum goes unmodified by the evolution of density perturbations deep in the matter era, except for an overall amplitude change. Indeed, the shape of the power spectrum remains the same as for CDM except for the slight modification that Eq. (21) introduces. It is common to express the wave number k in units of $\Omega_m h^2$ or Γh, where Γ is known as the shape parameter (here we ignore the small correction due to the dependence on the baryon density). In the case where a generalized Chaplygin gas is also present, we can show by using Eqs. (20) and (21) that the shape parameter reduces to

$$\Gamma = (\Omega_m + (1-\Omega_m)(1-\overline{A})^{1/1+\alpha}) h. \tag{22}$$

Recent work by [31] using data from the 2dF Galaxy Redshift Survey (2dFGRS) has constrained this parameter to be of the order of $\sim 0.2 \pm 0.03$, in agreement with preliminary Sloan Digital Sky Survey (SDSS) results [32]. Therefore, this allows us to obtain simple bounds on $\overline{A}$ as a function of α, h and Ω_m. Figure 6 shows two such bounds, for the "extreme" cases $\Omega_m = 0.3$ and $\Omega_m = 0.04$; in both cases we have assumed a Hubble parameter $h = 0.65$. We see that $\overline{A} \geqslant 0.65$ for all generalized Chaplygin models, a result indicative of the preference for Λ-type scenarios. In the best-motivated case $\alpha = 1$, the constraint is much tighter, $\overline{A} \geqslant 0.85$.

V. RESULTS AND COMMENTS

We have studied some cosmological implications of two dark energy models presented as an alternative to the now standard cosmological constant scenario, and discussed how these toy models relate to the more familiar quintessence paradigm. We have shown that the Cardassian model is, for most cosmological purposes, a standard quintessence model, whereas the Chaplygin gas does in principle have distinguishing characteristics.

On the other hand, by using supernova and density perturbation growth constraints, we have shown that any Chaplygin gas type component must have a behavior that is very close to that of a "standard" cosmological constant Λ, assuming that the density of "normal" (clustered) matter is $\Omega_m \sim 0.3$. Of course, this result is also known to apply for the standard quintessence models [20]. Indeed, if by an independent method we were able to determine the total matter content of the Universe (including dark matter) to be around ~ 0.3, then in the context of this model we would in fact *require* a cosmological constant so as to account for the current observational results (see Fig. 3).

Conversely, the (arguably best-motivated) case where the matter content is entirely baryonic ($\Omega_b \sim 0.04$), so that the Chaplygin gas provides both the dark matter and the dark energy, is the one where the differences with respect to the standard case would be maximal. In this case, the Λ-like limit of this model is already strongly disfavored by observations. It should also be noted that future observations may be able to provide fairly tight constraints on the exponent α, in particular if the degeneracies discussed above are broken.

We thus conclude that the potential relevance of the Chaplygin model, in the sense of yielding observational consequences that are significantly different from those of the simpler cosmological constant, is strongly dependent on the total matter content of the Universe.

ACKNOWLEDGMENTS

We are grateful to Irit Maor and Alessandro Melchiorri for enlightening discussions of this topic. C.M. is funded by FCT (Portugal), under Grant No. FMRH/BPD/1600/2000. Additional support for this project came from grant CERN/FIS/43737/2001.

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