# Varying constants, black holes, and quantum gravity

S. Carlip*
*Department of Physics, University of California, Davis, California 95616*
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Tentative observations and theoretical considerations have recently led to renewed interest in models of fundamental physics in which certain ''constants'' vary in time. Assuming fixed black hole mass and the standard form of the Bekenstein-Hawking entropy, Davies, Davis and Lineweaver have argued that the laws of black hole thermodynamics disfavor models in which the fundamental electric charge $e$ changes. I show that with these assumptions, similar considerations severely constrain ''varying speed of light'' models, unless we are prepared to abandon cherished assumptions about quantum gravity. Relaxation of these assumptions permits sensible theories of quantum gravity with ''varying constants,'' but also eliminates the thermodynamic constraints, though the black hole mass spectrum may still provide some restrictions on the range of allowable models.

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## I. INTRODUCTION

The idea that the fundamental ''constants'' of our Universe may vary in time dates back at least to Dirac's large number hypothesis [1,2]. Until recently, physically interesting variations seemed to be excluded by observation. Over the past several years, however, Webb *et al.* [3,4] have reported evidence that the fine structure constant $\alpha$ may have been slightly smaller in the early Universe. While this claim is still far from being established, the possibility, along with work on cosmological implications of ''varying constants,'' has inspired renewed interest in models in which either the elementary charge $e$ or the speed of light $c$ is dynamical; for a sample of this work, see [5–15].

In a Brief Communication [1], Davies, Davis and Lineweaver contend that black hole thermodynamics favors models with a varying speed of light. Their basic argument is simple. The Bekenstein-Hawking entropy of a charged black hole of mass $M$ and charge $Q$ is

$$S/k=\frac{\pi G}{\hbar c}[M+\sqrt{M^2-Q^2/G}]^2. \tag{1.1}$$

Suppose $\alpha$ is indeed increasing in time, as Webb *et al.* suggest. If this variation comes from an increase in $e$, the resulting increase in $Q$ will cause the entropy of such a black hole to decrease, apparently violating the generalized second law of thermodynamics. If, on the other hand, the change comes from a decrease in $c$, the entropy will increase with time, as it should.

This is an intriguing argument, but it requires several key assumptions:

(1) The Bekenstein-Hawking formula, and in particular Eq. (1.1), remains a good approximation for black hole entropy in a theory with ''varying constants.''

(2) Planck's constant $\hbar$ and Newton's constant $G$ remain constant.

(3) It is sufficient to look at the entropy of the black hole alone, and not its environment.

(4) The black hole mass $M$ remains constant as $\alpha$ varies.

While these assumptions seem plausible, they need not be correct. Indeed, there are particular models in which each is violated [16,17], and a full analysis would require a much more specific and detailed theory. Still, these assumptions offer an interesting ''phenomenological'' starting point for investigating the broader question of whether, and to what extent, black hole quantum mechanics can constrain theories of ''varying constants.''

In the first part of this paper, I show that the implications of assumptions (1)–(4) are far more radical than Davies *et al.* suggest, and lead to predictions unpalatable enough to militate against ''varying speed of light'' models. I then discuss the options that become available when one relaxes these assumptions. The resulting loopholes are wide enough to allow sensible quantum theories of gravity with ''varying constants''—string theories with time-dependent compactification radii are useful examples—though, of course, the thermodynamic constraints of Ref. [1] are then lost. Still, while black hole quantum mechanics will not constrain *all* models of ''varying constants,'' it may still narrow the range of models we must consider.

## II. VARYING CONSTANTS AND THE BLACK HOLE MASS SPECTRUM

Consider a Reissner-Nordström black hole with charge $Q=qe$ and mass $M=\sqrt{r}M_P$, where $M_P=(\hbar c/G)^{1/2}$ is the Planck mass. Quantization of charge requires that $q$ be an integer. In simple models of black hole thermodynamics (see [18] for a review), $r$ is an integer, or a fixed constant multiple of an integer, as well. More elaborate approaches to quantum gravity lead to more complicated black hole spectra: for instance, neutral black holes in loop quantum gravity have [19]

$$r=\frac{\gamma}{4}\sum_i \sqrt{p_i(p_i+2)} \tag{2.1}$$

where the $p_i$ are arbitrary integers and $\gamma$ is a constant of order unity, while the string theoretical black holes of Ref. [20] depend on four pairs of integers $(p_i,\overline{p_i})$ that count branes wrapped around various compactified dimensions, with

*Email address: carlip@dirac.ucdavis.edu

 

$$r=\gamma'\left(\frac{p_1\bar{p}_1p_2\bar{p}_2p_3\bar{p}_3}{p_4\bar{p}_4}\right)^{1/2}\left(\sum_{i=1}^{4}\left[\sqrt{\frac{p_i}{\bar{p}_i}}+\sqrt{\frac{\bar{p}_i}{p_i}}\right]\right)^2. \tag{2.2}$$

The minisuperspace model of Barvinsky *et al.* [21] has

$$r=\frac{1}{4}\frac{(2n+1)^2}{2n+1+q^2\alpha}+q^2\alpha, \tag{2.3}$$

where $n$ and $q$ are integers, and, as above, $Q=qe$. In each of these examples, though, and in virtually all other models that have been considered, $r$ is still discrete. Indeed, it is hard to see how to reconcile a finite Bekenstein-Hawking entropy with a continuous black hole mass spectrum.

Given the quantum numbers $q$ and $r$, the entropy (1.1) becomes

$$S/k=\pi[\sqrt{r}+\sqrt{r-q^2\alpha}]^2 \tag{2.4}$$

(see also [22]). Clearly, an increase in $\alpha$ will lead to a decrease in the entropy, and an apparent violation of the generalized second law of thermodynamics, unless $q$ or $r$ also evolve. But if $q$ and $r$ are discrete, they can change only in finite jumps [except, of course, through changes in parameters like $\gamma$ in Eq. (2.1) or $\gamma'$ in Eq. (2.2), which may evolve continuously in models like string theory with time-dependent moduli]. This quantization suggests a discrete evolution for $\alpha$ as well, an ingredient not easily incorporated in many current models. But even without such a feature, the discrete nature of $r$ presents a serious problem for a number of models with varying $c$, one already evident for uncharged black holes.

Following Davies *et al.*, I assume for now that $M$ remains constant. The speed of light then enters $r$ through the Planck mass.[1] It is then easy to see that given a black hole with "mass quantum number" $r$, a change $\Delta c$ requires a jump

$$\Delta r=(\Delta x)r \quad \text{with} \quad \Delta x=-\frac{\Delta c}{c}\left(1+\frac{\Delta c}{c}\right)^{-1}. \tag{2.5}$$

Note that a positive $\Delta x$ corresponds to a decrease in $c$ and an increase in $\alpha$. In some models, Planck's constant $\hbar$ also varies with $c$ [11]. Such a variation would lead to a few minor changes—$r$ and $\Delta r$ in Eq. (2.5) would be replaced by $r^\beta$ and $\Delta(r^\beta)$ for some exponent $\beta$—but the qualitative considerations below would not be affected. (Coule [23] has discussed related problems in cosmology coming from such a variation of $M_P$, and Banks *et al.* [24] have pointed out serious fine tuning issues.)

The first thing to notice about Eq. (2.5) is that for a given $r$ or a given $\Delta c$, it may not have any solutions. That is, suppose we are given a quantum theory of gravity that determines a spectrum $\mathcal{S}$ of possible values for $r$. Then for any particular $r\in\mathcal{S}$ and any fixed $\Delta c$, $r+\Delta r=(1+\Delta x)r$ may not lie in $\mathcal{S}$. This situation can have two interpretations, which I will expand upon below: either many black hole masses are forbidden, even for allowed quantum numbers $r\in\mathcal{S}$, or else the allowed changes $\Delta c$ are sharply constrained by the presence of black holes.

For illustration, let us begin with a model in which the allowed values of $r$, and thus $\Delta r$, are integers. For Eq. (2.5) to have any solutions, $\Delta x$ must be rational: $\Delta x=p/N$ and $r=nN$, where $p$, $n$, and $N$ are integers, with $p$ and $N$ relatively prime. It follows that $\Delta c/c=-p/(p+N)$. But the total variation $\Delta\alpha/\alpha$ claimed by Webb *et al.* is only on the order of $10^{-5}$, and larger variations in $\alpha$ are excluded by other observations [25], so $\Delta c/c$ should be no larger than about $10^{-5}$ at each discrete step. We must thus require $N\gtrsim 10^5$.

The model thus forbids black holes with masses less than some minimum value $M_0=\sqrt{N}M_P\gtrsim 350M_P$, and it requires that *all* black hole masses be multiples of $M_0$ by the square root of an integer. Note that this estimate of $M_0$ is conservative: I have assumed not only that the change in $\alpha$ is as large as that reported by Webb *et al.*, but also that it all came in a single jump. It is easy to see that if $\Delta c$ is spread over $k$ equal steps, $M_0$ increases by a factor of $\sqrt{k}$. If, for instance, jumps occur at a characteristic electromagnetic time scale $\tau=\hbar/m_ec^2$, one finds $M_0\sim 10^{16}g$, excluding standard primordial black holes.

The existence of a change $\Delta c$ thus restricts the allowed black hole masses. Conversely, the existence of black holes limits the possible changes in $c$: given a collection of black holes with mass quantum numbers $\{r_i\}$, $\Delta c/c$ is restricted to be of the form $-p/(p+N)$, where $p$ and $N$ are relatively prime and $N$ is a common divisor of the $r_i$. If the greatest common divisor of the $r_i$ is less than about $10^5$, no variation in $c$ is compatible with observation. Note also that the restriction on $\Delta c/c$ is time dependent: a jump in $c$ causes a corresponding shift $r_i\rightarrow(1+\Delta x)r_i$, leading to a new condition on any future jump in $c$.

Integral quantization of $r$ may well be too simple, but the model demonstrates the key features present for more complicated spectra. Equation (2.5) can be satisfied in two ways:

(1) In any region containing a black hole of mass $\sqrt{r}M_P$, allow only a sharply limited set of changes in $c$ (choose $\Delta c/c$ as a function of $r$ so that $r+\Delta r$ is in the spectrum).

(2) Or, for fixed $\Delta c/c$, allow only a sharply limited set of black holes [restrict $r$ to those values for which both $r$ and $(1+\Delta x)r$ are in the spectrum].

Alternative (1) requires a mysterious new local coupling between $c(t)$ and the black hole mass. One could imagine treating the restriction as a boundary value problem for $c(x,t)$, though this would require boundary conditions that evolve as black hole masses change: the global evolution of $c(t)$ would be affected each time a black hole captured an electron or emitted a quantum of Hawking radiation, and boundary conditions would multiply as new black holes form and vanish as old black holes evaporate. It is not obvious that such boundary conditions are consistent, but if they are, a new problem will appear—if $c$ is permitted to jump by dif-

[1] Duff [22] has criticized models involving variations of dimensionful parameters. I take "varying $c$" as shorthand for "variation of all dimensionless parameters, such as $m_e/M_P$, that depend on $c$." The disentangling of dimensionful parameters is still somewhat ambiguous, but given a standard choice of conventions, this prescription leads to a well-defined model.

ferent discrete steps in different regions, it becomes difficult to maintain the observed large-scale homogeneity of the fundamental constants.

Alternative (2) drastically reduces the number of permitted states of a black hole. The allowed states will typically be no less sparse than they were in our illustrative example. For models such as loop quantum gravity, mass spacings can become very small at high masses [26], and Eq. (2.5) will have many *approximate* solutions, but only for very peculiar spectra will many black hole masses give *exact* solutions (see below). Note that the excluded states occur in all mass ranges; this is not merely a Planck scale effect that can be blamed on our ignorance of quantum gravity. Apart from the difficulty of implementing such a restriction in existing models, such a reduction in the number of allowed states would seem to invalidate any statistical mechanical interpretation of the Bekenstein-Hawking entropy as a measure of the number of accessible microscopic states.

With either alternative, one is likely to have a problem with ''small'' black holes, those large enough that semiclassical approximations should be reliable but small enough that the selection rule (2.5) is highly restrictive. For the case of integer spacing, we saw that there was a large mass gap between the Planck scale and the first allowed black hole. Other spectra may not have a completely empty gap, but for spectra depending on more than one integer, mass spacings are generally much wider at low masses than at high masses [26], so one expects a paucity of admissible low-mass black holes.

## III. SEARCHING FOR LOOPHOLES

At a minimum, these considerations severely constrain ''varying speed of light'' models that satisfy assumptions (1)–(4) of the Introduction. It may well be that the observations of Webb *et al.* are wrong, and that $\alpha$ is actually constant. But until the observational situation is more settled, it is worth investigating other ways out of this dilemma.

One possibility is to give up quantization of black hole masses. This is not a very comfortable choice: it would require not only that we abandon most existing approaches to quantum gravity (loop quantum gravity, most string theoretical models), but that we give up powerful theoretical ideas like holography [27] that depend on the finiteness of the number of black hole states. Of course, it could be that black holes in ''varying speed of light'' theories are drastically different from those in standard general relativity [16], in which case these issues would have to be rethought. But this would require that we discard even the little we believe we now understand about black hole entropy and quantum gravity.

Another possibility is to look for a spectrum $\mathcal{S}$ of black hole states for which the constraints we have found are unimportant. Such a spectrum exists, but it is physically unrealistic: it requires that $\log M/M_P$ be integrally spaced. To see this, let $\Delta c/c = -\Delta x/(1+\Delta x)$ be an allowed change in $c$ at time $t_0$, and suppose that a black hole with mass quantum number $r_0$ is present. This requires that both $r_0$ and $(1+\Delta x)r_0$ be in $\mathcal{S}$. If we now require that black holes with $r=(1+\Delta x)r_0$ also be allowed at time $t_0$, $(1+\Delta x)^2 r_0$ must also be in $\mathcal{S}$. Continuing this argument, one is led to a spectrum $2\log(M/M_P)=\log r_0+k\log(1+\Delta x)$, where all integers $k$ must occur. Such a spectrum is increasingly sparse at high masses, and would lead to rather odd predictions. For example, if $\Delta x \sim 10^{-5}$, one would not be able to drop the Earth into a Solar-mass black hole: no mass states would be available. Even if jumps in $c$ occur at an electromagnetic time scale $\tau=\hbar/m_e c^2$, so $\Delta x \sim 10^{-42}$, the spectrum of supermassive black holes would still have a spacing of several grams: one could not drop a dime into the black hole at the center of the Milky Way.

A third possibility is that while $c$ varies cosmologically, it remains constant at black hole horizons [28]. This is the case for single static black hole solutions in certain models of ''varying constants,'' essentially as a result of no hair theorems. It is not at all clear, however, that this argument can be extended to dynamical solutions with more than one black hole; such boundary conditions are strong enough that there may be no solutions except those with globally constant $c$. The process of new black hole formation seems especially problematic. While one could imagine a process in which a collapsing star radiates away varying $c$ ''hair,'' it is hard to see why $c$ should freeze to the same value at widely separated black holes formed at very different times, but without such constancy, it is even harder to understand the observed spatial homogeneity of fundamental constants.

## IV. MODIFYING THE INITIAL ASSUMPTIONS

The final possibility is to abandon one of the four assumptions enumerated in the Introduction. This would, of course, invalidate the original argument of Ref. [1] that black hole thermodynamics constrains varying constants. But one may ask whether quantization of the black hole mass spectrum continues to provide any useful constraints.

The first assumption of Ref. [1] was that the Bekenstein-Hawking formula remains a good approximation for the entropy in theories with ''varying constants.'' Such theories necessarily contain at least one new dynamical field, $\alpha$ itself, which could contribute to the entropy. We know that $\alpha$ varies very slowly, if at all, in time, so the standard Bekenstein-Hawking entropy is plausibly a good approximation. But $\alpha$ may vary significantly in *space* near a black hole horizon; indeed, in some models [16], $\alpha$ goes to zero or infinity at the horizon of an isolated static black hole. In such a situation, the thermodynamic argument of Davies *et al.* clearly fails, and black hole thermodynamics imposes no obvious restrictions on ''varying constants.'' The black hole mass spectrum is a different matter, though; as long as black hole solutions exist, quantization of this spectrum will continue to constrain models, independent of purely thermodynamic considerations.

The second starting assumption was that $\hbar$ and $G$ remain constant as $\alpha$ changes. One can, of course, postulate *ad hoc* changes in these parameters in such a way as to save the second law of thermodynamics, although it is not clear how to justify such an assumption from first principles. Again, however, as long as black hole masses are quantized and the Planck mass evolves, the nonthermodynamic constraints on admissible models described above will remain.

The third assumption was that it is sufficient to look at the entropy of a black hole alone, and not its environment. This is probably incorrect; it is argued elsewhere [17] that the change in the Hawking temperature coming from a variation in $\alpha$ induces heat flow, which in turn affects the black hole equilibrium mass, fundamentally altering the entropy balance. Once again, though, quantization of the black hole mass spectrum continues to constrain models, independent of thermodynamic considerations.

The fourth assumption was that the mass $M$ of a black hole remains constant as $\alpha$ varies. Relaxing this assumption probably provides the most important loophole. For example, in the minisuperspace quantization of Barvinsky *et al.* [21], the black hole mass $M$ depends explicitly on the fine structure constant, and it may be checked from Eqs. (2.3) and (2.4) that the $\alpha$ dependence of the entropy disappears. The cause of this ''miraculous'' cancellation is easy to understand: by construction, the fundamental quantum observables in this model are the charge quantum number and the entropy, while black hole mass is a secondary, derived quantity.

A more compelling example comes from string theory. In the string theoretical models of Ref. [20], the fine structure constant depends on various radii of compactification, and can change as those radii evolve. Nevertheless, the entropy is independent of the compactification radii; in the notation of Eq. (2.2), it is

$$S/k=2\pi\prod_{i=1}^{4}\left(\sqrt{p_i}+\sqrt{\bar{p}_i}\right), \tag{4.1}$$

and depends only on the integers $\{p_i,\bar{p}_i\}$ that count the number of branes. If one reexpresses the entropy (4.1) in terms of mass and charges, the usual Bekenstein-Hawking formula can be recovered, but the mass and charges again depend on the compactification radii in just the right way to ensure that these radii cancel from the entropy.

Again, the physics lying behind this cancellation is not hard to understand. Near-extremal black holes in string theory can be thought of as comprising a collection of weakly coupled branes. These branes are wrapped around compactified spatial dimensions, and their masses depend on the sizes of these dimensions—that is, on the same compactification radii that determine the fine structure constant. The masses and charges therefore depend on moduli such as compactification radii, and can vary if these radii vary. The entropy (4.1) and the mass quantum number (2.2), on the other hand, are fixed by the numbers of branes, and thus decouple from any changes in the moduli [20,29].

A related argument has been advanced by Flambaum [30] in the context of ''phenomenological'' models in which black hole areas are integrally quantized. The Bekenstein-Hawking entropy (1.1) can be rewritten suggestively as

$$S/k=\frac{\pi c^3}{\hbar G}r_+^{\,2}, \quad M=\frac{r_+c^2}{2G}+\frac{Q^2}{2r_+c^2} \tag{4.2}$$

where the event horizon is located at $r=r_+$. In this formulation, it seems natural to guess that a change in $\alpha$ affects the ''electrostatic self-energy'' contribution to the black hole mass while leaving $r_+$, and thus the entropy, fixed. As Dicke noted long ago [31], however, such a variation of electrostatic energy in ordinary matter could lead to violations of the weak equivalence principle.[2] Current limits on such variation are comparable to the reported observations of Webb *et al.*, and planned experiments should give an improvement of several orders of magnitude [25], offering at least an indirect experimental test for such a picture.

## V. CONCLUSIONS

The first conclusion of this analysis is that it can be risky to speculate about the effects of ''varying constants'' without a concrete model. We have seen that a simple set of plausible assumptions leads to drastic and implausible conclusions about the quantum mechanics of black holes, but also that these assumptions are violated in particular models, including, notably, string theory. In particular, this makes the thermodynamic arguments of Ref. [1], which are based on these assumptions, suspect.

Second, though, we have found a new set of criteria that continue to place some constraints on models of ''varying constants'' even when the purely thermodynamic analysis fails [17]. The arguments based on black hole quantum mechanics certainly do not rule out models with varying $e$ or $c$—as I have stressed, they depend on assumptions about black hole masses that do not hold in models such as those coming from string theory. But they limit the spectrum of allowable models, and also provide a simple way to screen out some ''phenomenological'' descriptions that are not based on a detailed theoretical framework.

In particular, suppose the recent claims of observable changes in the fine structure constant are confirmed, for example by precision measurements of the cosmic microwave background [33]. Such an event would confront existing models of ''varying constants'' with a new challenge, demanding detailed, testable predictions of time evolution. But such a radical departure from standard physics would also call upon us to explore a wide range of new, and quite possibly incomplete, ideas. Under those circumstances, the requirements of black hole quantum mechanics—and, in particular, of a sensible mass spectrum—could provide useful constraints on the space of theories to be investigated.

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[2]Bekenstein has recently shown that for a certain class of models with varying $e$, a scalar coupling modifies Coulomb's law in a manner that compensates for position dependence of electrostatic self-energy [32], but in such models it seems rather unlikely that Eq. (1.1) will continue to hold even semiclassically.

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