Resummed effective Lagrangian for Higgs-mediated flavor-changing neutral current interactions in the *CP*-violating MSSM

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We derive the general resummed effective Lagrangian for Higgs-mediated flavor-changing neutral-current (FCNC) interactions in the minimal supersymmetric standard model, without resorting to particular assumptions that rely on the squark mass or the quark-Yukawa structure of the theory. In our derivation we also include the possibility of explicit CP violation through the Cabibbo-Kobayashi-Maskawa mixing matrix and soft supersymmetry-breaking mass terms. The advantages of our resummed FCNC effective Lagrangian are explicitly demonstrated within the context of phenomenologically motivated scenarios. We obtain new testable predictions in the large $\tan\beta$ regime of the theory for CP-conserving and CP-violating observables related to the K- and B-meson systems, such as $\Delta M_{K,B}$, ϵ_K, ϵ'/ϵ, $\mathcal{B}(B_{d,s}\to\ell^+\ell^-)$ and their associated leptonic CP asymmetries. Finally, based on our resummed FCNC effective Lagrangian, we can identify configurations in the soft supersymmetry-breaking parameter space, for which a kind of a Glashow-Iliopoulos-Maiani-cancellation mechanism becomes operative and hence all Higgs-mediated, $\tan\beta$-enhanced effects on K- and B-meson FCNC observables vanish.

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I. INTRODUCTION

The appearance of too large flavor-changing neutral-current (FCNC) interactions of Higgs bosons to fermionic matter is a generic feature of $SU(2)_L\times U(1)_Y$ theories with two and more Higgs doublets. Unless there is a symmetry to forbid these Higgs-mediated FCNC interactions to occur in the bare Lagrangian of the model [1], their unsuppressed existence will inevitably lead to predictions for rare processes in the kaon and B-meson systems that violate experimental limits by several orders of magnitude [1,2]. In the minimal realization of softly broken supersymmetry (SUSY), the minimal supersymmetric standard model (MSSM), the holomorphicity of the superpotential prevents the occurrence of Higgs-boson FCNCs by coupling the one Higgs-doublet superfield $\hat{H}_1$ to the down-quark sector, and the other one $\hat{H}_2$ to the up-quark sector. However, the above holomorphic property of the superpotential is violated by finite radiative (threshold) corrections due to soft SUSY-breaking interactions [3,4]. As a consequence, Higgs-mediated FCNCs reappear at the one-loop level, but are naturally suppressed for low and intermediate values of $\tan\beta=\langle\hat{H}_2\rangle/\langle\hat{H}_1\rangle$, i.e. for $\tan\beta\lesssim 20$. For larger values of $\tan\beta$, e.g., $\tan\beta\gtrsim 30$, the FCNCs partially overcome the loop suppression factor $1/(16\pi^2)$ and become phenomenologically relevant [5,6], especially for the K- and B-meson systems.

Recently, the topic of Higgs-boson FCNCs in the large-$\tan\beta$ limit of the MSSM has received much attention [5–16]. Several approaches have been devised to implement the nonholomorphic finite radiative corrections into the phenomenological analysis of FCNC processes, such as $K^0\overline{K}^0$ and $B^0\overline{B}^0$ mixings, $B\to X_s\gamma$ and $B_s\to\ell^+\ell^-$. In most cases, however, the suggested approaches to threshold radiative effects involve certain explicit or implicit assumptions pertinent to the squark-mass and the quark-Yukawa structures of the theory, such as the dominance of the top quark in the FCNC transition amplitudes. We term the latter assumption the t-quark dominance hypothesis. On the other hand, some of the approaches neglect higher-order terms in the resummation of threshold corrections to d-quark Yukawa couplings, which become important in the large-$\tan\beta$ regime of the theory.

In this paper we derive the effective Lagrangian that properly takes into account the resummation of higher-order threshold effects on Higgs-boson FCNC interactions to down quarks. To accomplish this in Sec. II, we avoid the imposition of particular assumptions on the structure of the soft squark masses and the quark-Yukawa couplings of the theory. Moreover, we do not rely on specific kinematic approximations to the transition amplitudes, such as the aforementioned t-quark dominance hypothesis in the FCNC matrix elements. In our derivation of the effective Lagrangian, we also consider the possibility of CP violation through two sources: (i) the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [17] and (ii) the soft SUSY-violating mass terms. As we explicitly demonstrate in Sec. III, our resummed FCNC effective Lagrangian gives rise to new testable predictions for CP-conserving as well as CP-violating observables related to the K- and B-meson systems. In the same section, we qualitatively discuss the implications of $\tan\beta$-enhanced Higgs-mediated interactions for the direct CP-violation parameter ϵ'/ϵ in the kaon system. Section IV is devoted to our numerical analysis of a number of K- and B-meson observables, such as ΔM_K, ϵ_K, $\Delta M_{B_{d,s}}$, $\mathcal{B}(B_{d,s}\to\ell^+\ell^-)$ and their associated leptonic CP asymmetries. In particular, based on our resummed FCNC effective Lagrangian, we are able to identify configurations in the soft SUSY-breaking parameter space, for which a kind of a Glashow-Iliopoulos-Maiani-mechanism cancellation (GIM) [18] becomes operative in the Higgs–d-quark sector. As a result, all Higgs-mediated, $\tan\beta$-enhanced effects on K- and B-meson FCNC observables vanish. Finally, Sec. V summarizes our conclusions.

II. RESUMMED FCNC EFFECTIVE LAGRANGIAN

In this section, we derive the general form for the effective Lagrangian of Higgs-mediated FCNC interactions in the CP-violating MSSM. For this purpose, we also consistently resum the $\tan\beta$-enhanced radiative effects on the d-quark Yukawa couplings [7]. First, we analyze a simple soft SUSY-breaking model based on the assumption of minimal flavor violation [6,10,13,14], where the CKM matrix is the only source of flavor and CP violations. We find that even within this minimal framework, the usually neglected c-quark contribution to Higgs-mediated FCNC interactions may be competitive to the t-quark one in certain regions of the parameter space. After having gained some insight from the above considerations, we then extend our resummed effective Lagrangian approach to more general cases that include a nonuniversal or hierarchical squark sector as well as CP violation originating from the CKM matrix and the soft SUSY-breaking parameters.

Before discussing the most general case, let us first consider the following simple form for the effective Yukawa Lagrangian governing the Higgs-mediated FCNC interactions in the quark sector [5,6]:

$$-\mathcal{L}_Y=\overline{d_R^0}\mathbf{h_d}[\Phi_1^{0*}+\Phi_2^{0*}(\hat{\mathbf{E}}_\mathbf{g}+\hat{\mathbf{E}}_\mathbf{u}\mathbf{h}_\mathbf{u}^\dagger\mathbf{h_u})]d_L^0 + \Phi_2^0\overline{u_R^0}\mathbf{h_u}u_L^0+\text{H.c.}, \qquad (2.1)$$

where $\Phi_{1,2}^0$ are the electrically neutral dynamical degrees of freedom of the two Higgs doublets[1] and the superscript "0" on the d- and u-type quarks denotes fields in the interaction basis. In Eq. (2.1), $\mathbf{h_d}$ and $\mathbf{h_u}$ are 3×3-dimensional down- and up-quark Yukawa matrices, and [4–6]

$$\hat{\mathbf{E}}_\mathbf{g}=\mathbf{1}\frac{2\alpha_s}{3\pi}m_{\tilde g}^*\mu^* I(m_{\tilde d_L}^2,m_{\tilde d_R}^2,|m_{\tilde g}|^2), \qquad (2.2)$$

$$\hat{\mathbf{E}}_\mathbf{u}=\mathbf{1}\frac{1}{16\pi^2}\mu^* A_U^* I(m_{\tilde u_L}^2,m_{\tilde u_R}^2,|\mu|^2) \qquad (2.3)$$

are finite nonholomorphic radiative effects induced by the diagrams shown in Fig. 1. In the above, the loop integral $I(x,y,z)$ is given by

$$I(x,y,z)=\frac{xy\ln(x/y)+yz\ln(y/z)+xz\ln(z/x)}{(x-y)(y-z)(x-z)}, \qquad (2.4)$$

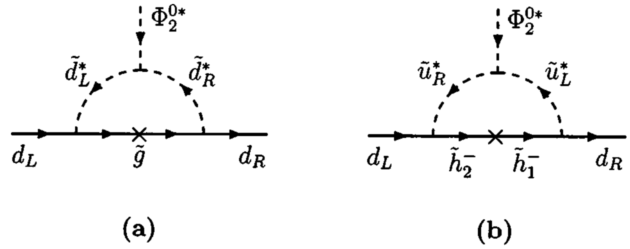

FIG. 1. Nonholomorphic radiative effects on the d-quark Yukawa couplings induced by (a) gluinos $\tilde g$ and (b) charged Higgsinos $\tilde h_{1,2}^-$.

with $I(x,x,x)=1/(2x)$. To keep things simple in the beginning, we assume that in Eqs. (2.2) and (2.3), the bilinear soft SUSY-breaking masses of the squarks, $m_{\tilde u_{L,R}}^2$, $m_{\tilde d_{L,R}}^2$, and the trilinear soft Yukawa couplings $A_U=A_{u,d}$ are flavor-diagonal and universal at the soft SUSY-breaking scale $M_{\rm SUSY}$. We also neglect the left-right mixing terms $\tilde u_L$-$\tilde u_R$ and $\tilde d_L$-$\tilde d_R$ in the squark mass matrices. The consequences of relaxing the above assumptions will be discussed later on.

From Eq. (2.1), we can easily write down the effective Lagrangian relevant to the effective d- and u-type quark masses:

$$-\mathcal{L}_{\rm mass}=\frac{v_1}{\sqrt2}\overline{d_R^0}\mathbf{h_d}[\mathbf{1}+\tan\beta(\hat{\mathbf{E}}_\mathbf{g}+\hat{\mathbf{E}}_\mathbf{u}\mathbf{h}_\mathbf{u}^\dagger\mathbf{h_u})]d_L^0 + \frac{v_2}{\sqrt2}\overline{u_R^0}\mathbf{h_u}u_L^0+\text{H.c.} \qquad (2.5)$$

Our next step is to redefine the quark fields as follows:

$$u_L^0=\mathcal{U}_\mathbf{L}^\mathbf{Q}u_L,\quad d_L^0=\mathcal{U}_\mathbf{L}^\mathbf{Q}\mathbf{V}d_L,\quad u_R^0=\mathcal{U}_\mathbf{R}^\mathbf{u}u_R,\quad d_R^0=\mathcal{U}_\mathbf{R}^\mathbf{d}d_R, \qquad (2.6)$$

where $\mathcal{U}_\mathbf{L}^\mathbf{Q}$, $\mathcal{U}_\mathbf{R}^\mathbf{u}$, $\mathcal{U}_\mathbf{R}^\mathbf{d}$ and $\mathbf{V}$ are 3-by-3 unitary matrices that relate the weak to mass eigenstates of quarks. Evidently, $\mathbf{V}$ is by construction the physical CKM matrix. Substituting Eq. (2.6) into Eq. (2.5) yields

$$\begin{aligned}-\mathcal{L}_{\rm mass}&=\frac{v_1}{\sqrt2}\overline{d}_R\mathcal{U}_\mathbf{R}^{\mathbf{d}\dagger}\mathbf{h_d}\mathcal{U}_\mathbf{L}^\mathbf{Q}[\mathbf{1}+\tan\beta(\hat{\mathbf{E}}_\mathbf{g}+\hat{\mathbf{E}}_\mathbf{u}|\hat{\mathbf{h}}_\mathbf{u}|^2)]\mathbf{V}d_L + \frac{v_2}{\sqrt2}\overline{u}_R\hat{\mathbf{h}}_\mathbf{u}u_L+\text{H.c.}\\ &=\overline{d}_R\hat{\mathbf{M}}_\mathbf{d}d_L+\overline{u}_R\hat{\mathbf{M}}_\mathbf{u}u_L+\text{H.c.},\end{aligned} \qquad (2.7)$$

where $\hat{\mathbf{M}}_\mathbf{d}$ and $\hat{\mathbf{M}}_\mathbf{u}$ are the physical d- and u-quark mass matrices, respectively. Consistency of Eq. (2.7) implies

$$\hat{\mathbf{M}}_\mathbf{u}=\frac{v_2}{\sqrt2}\hat{\mathbf{h}}_\mathbf{u}, \qquad (2.8)$$

$$\mathcal{U}_\mathbf{R}^{\mathbf{d}\dagger}\mathbf{h_d}\mathcal{U}_\mathbf{L}^\mathbf{Q}=\frac{\sqrt2}{v_1}\hat{\mathbf{M}}_\mathbf{d}\mathbf{V}^\dagger\hat{\mathbf{R}}^{-1}, \qquad (2.9)$$

[1]Throughout the paper, we follow the notation and the CP-phase conventions of [19].

with

$$\hat{\mathbf{R}} = \mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta + \hat{\mathbf{E}}_{\mathbf{u}} \tan\beta |\hat{\mathbf{h}}_{\mathbf{u}}|^2. \tag{2.10}$$

Notice that Eq. (2.9) plays the role of a redefining (renormalization) condition for the d-quark Yukawa couplings, in the process of resumming higher-order radiative corrections. Observe also that the matrix $\hat{\mathbf{R}}$ cannot be zero, as this would result in massless d quarks.

With the help of Eqs. (2.6) and (2.9), we can now express our original Yukawa Lagrangian (2.1) in terms of the mass eigenstates $d_{L,R}$ and $u_{L,R}$ in a resummed form:

$$\begin{aligned} -\mathcal{L}_Y &= \frac{\sqrt{2}}{v_1} \overline{d}_R \hat{\mathbf{M}}_{\mathbf{d}} \mathbf{V}^\dagger \hat{\mathbf{R}}^{-1} [\Phi_1^{0*} + \Phi_2^{0*} (\hat{\mathbf{E}}_{\mathbf{g}} + \hat{\mathbf{E}}_{\mathbf{u}} |\hat{\mathbf{h}}_{\mathbf{u}}|^2)] \mathbf{V} d_L \\ &\quad + \Phi_2^0 \overline{u}_R \hat{\mathbf{h}}_{\mathbf{u}} u_L + \text{H.c.} \\ &= \frac{\sqrt{2}}{v_2} (\tan\beta \Phi_1^{0*} - \Phi_2^{0*}) \overline{d}_R \hat{\mathbf{M}}_{\mathbf{d}} \mathbf{V}^\dagger \hat{\mathbf{R}}^{-1} \mathbf{V} d_L \\ &\quad + \frac{\sqrt{2}}{v_2} \Phi_2^{0*} \overline{d}_R \hat{\mathbf{M}}_{\mathbf{d}} d_L + \Phi_2^0 \overline{u}_R \hat{\mathbf{h}}_{\mathbf{u}} u_L + \text{H.c.} \end{aligned} \tag{2.11}$$

In deriving the last equality in Eq. (2.11), we have employed the relation $\hat{\mathbf{R}}^{-1}(\hat{\mathbf{E}}_{\mathbf{g}} + \hat{\mathbf{E}}_{\mathbf{u}} |\hat{\mathbf{h}}_{\mathbf{u}}|^2) = (\mathbf{1} - \hat{\mathbf{R}}^{-1})/\tan\beta$.

It is very illuminating to see how the FCNC part of Eq. (2.11) compares with the literature, e.g. with that obtained in Ref. [6]. To this end, let us first assume that $\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta \neq 0$ and decompose $\hat{\mathbf{R}}^{-1}$ as follows:

$$\hat{\mathbf{R}}^{-1} = \frac{\mathbf{1}}{\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta} + \chi_{\text{FC}}, \tag{2.12}$$

where χ_{FC} is the diagonal matrix

$$\chi_{\text{FC}} = -\frac{\hat{\mathbf{E}}_{\mathbf{u}} |\hat{\mathbf{h}}_{\mathbf{u}}|^2 \tan\beta}{(\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta) \hat{\mathbf{R}}}. \tag{2.13}$$

Making use of the above linear decomposition of $\hat{\mathbf{R}}^{-1}$ and the unitarity of the CKM matrix in Eq. (2.11), the FCNC part of our resummed effective Lagrangian reads

$$\begin{aligned} -\mathcal{L}_{\text{FCNC}} &= \frac{\sqrt{2}}{v_2} (\tan\beta \Phi_1^{0*} - \Phi_2^{0*}) \overline{d}_{iR} m_{d_i} (V_{ti}^* \chi_{\text{FC}}^{(t)} V_{tj} \\ &\quad + V_{ci}^* \chi_{\text{FC}}^{(c)} V_{cj} + V_{ui}^* \chi_{\text{FC}}^{(u)} V_{uj}) d_{jL}, \end{aligned} \tag{2.14}$$

where $\chi_{\text{FC}}^{(u,c,t)}$ are the diagonal entries of χ_{FC} and summation over $i,j = d,s,b$ is understood. The term proportional to $\chi_{\text{FC}}^{(t)}$ gives the top quark contribution, which is the result of [6] and subsequent articles [10,13,14]. However, we should remark here that the frequently used top-quark dominance approximation cannot be justified from considerations based only on minimal flavor-violation models [6,10,13,14]. In fact, the other terms in Eq. (2.14) and especially the one proportional to $\chi_{\text{FC}}^{(c)}$ due to the charm-quark contribution become rather important in the limit $\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta \to 0$. In this limit, the singularity in $\chi_{\text{FC}}^{(t)}$ is canceled against the singularities of $\chi_{\text{FC}}^{(c)}$ and $\chi_{\text{FC}}^{(u)}$ as a result of the unitarity of $\mathbf{V}$. In this context, we should note that the limit $\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta \to 0$ is not attainable before the theory itself reaches a nonperturbative regime. Requiring that all d-quark Yukawa couplings are perturbative, we can estimate the lower bound, $|\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta| \gtrsim 2.5 \times 10^{-2}$, for $\tan\beta = 50$.[2] Although $|\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta|$ must not vanish in perturbation theory, it can be sufficiently close to zero, so that the c-quark contribution becomes competitive with the t-quark one.

So far, we have assumed that the radiatively-induced matrices $\hat{\mathbf{E}}_{\mathbf{g}}$ and $\hat{\mathbf{E}}_{\mathbf{u}}$ in the effective Yukawa Lagrangian (2.1) are proportional to the unity matrix. However, this assumption of flavor universality is rather specific. It gets generally invalidated by the mixing of the squark generations, the soft trilinear Yukawa couplings and renormalization-group (RG) running of the soft SUSY-breaking parameters from the unification to the low-energy scale. In this respect, the minimal-flavor-violation hypothesis, although better motivated, should also be viewed as a particular way from minimally departing from universality.

Nevertheless, given that threshold radiative effects on the up sector are negligible, especially for large values of $\tan\beta$, it is straightforward to derive the general resummed form for the Higgs-mediated FCNC effective Lagrangian. Starting from general nondiagonal matrices $\tilde{\mathbf{E}}_{\mathbf{g}}$ and $\tilde{\mathbf{E}}_{\mathbf{u}}$ in (2.1) and following steps very analogous to those from Eq. (2.1) to Eq. (2.11), we arrive at the same form as in Eq. (2.11) for the resummed FCNC effective Lagrangian, but with $\hat{\mathbf{E}}_{\mathbf{g}}$, $\hat{\mathbf{E}}_{\mathbf{u}}$ and $\hat{\mathbf{R}}$ replaced by

$$\mathbf{E}_{\mathbf{g}} = \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \tilde{\mathbf{E}}_{\mathbf{g}} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}, \quad \mathbf{E}_{\mathbf{u}} = \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \tilde{\mathbf{E}}_{\mathbf{u}} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}, \tag{2.15}$$

$$\mathbf{R} = \mathbf{1} + \tan\beta (\mathbf{E}_{\mathbf{g}} + \mathbf{E}_{\mathbf{u}} |\hat{\mathbf{h}}_{\mathbf{u}}|^2 + \cdots), \tag{2.16}$$

where the ellipses in Eq. (2.16) denote additional (generically subdominant) threshold effects [20]. Notice that the unitary matrix $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ in Eq. (2.15), which is only constrained by the relation $|\hat{\mathbf{h}}_{\mathbf{u}}|^2 = \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger} \mathbf{h}_{\mathbf{u}}^\dagger \mathbf{h}_{\mathbf{u}} \mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$, introduces additional nontrivial model dependence in the matrices $\mathbf{E}_{\mathbf{g}}$, $\mathbf{E}_{\mathbf{u}}$ and $\mathbf{R}$. In other words, the presence of $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ reflects the fact that the 3×3-dimensional matrices $\mathbf{h}_{\mathbf{u}}^\dagger \mathbf{h}_{\mathbf{u}}$, $m_{\tilde{u}_L}^2$ and $m_{\tilde{d}_L}^2$ cannot be diagonalized simultaneously, without generating FCNC couplings in other interactions in the MSSM Lagrangian, e.g., in the $\tilde{W}_3$-u_L-$\tilde{u}_L$ and $\tilde{W}^-$-u_L-$\tilde{d}_L$ couplings. Moreover, even in minimal flavor-violating scenarios, the (3×3)-dimensional matrix $\mathbf{R}$ may generally contain additional radiative effects

[2]To obtain this lower limit, we simply take the trace of the square of Eq. (2.9) and demand that $\text{Tr}|\hat{\mathbf{h}}_{\mathbf{d}}|^2 < 3$, or equivalently $\text{Tr}[(\hat{\mathbf{R}}^{-1})^\dagger V |\hat{\mathbf{M}}_{\mathbf{d}}|^2 V^\dagger \hat{\mathbf{R}}^{-1}] < 3 v_1^2/2$. The latter implies that $|\mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta| \gtrsim (m_s \tan\beta)/(\sqrt{3} m_t) = 5 \times 10^{-4} \tan\beta$. Finally, it is amusing to notice that if $\Im m \hat{\mathbf{E}}_{\mathbf{g}} = 0$ and $\Re e \hat{\mathbf{E}}_{\mathbf{g}} < 0$, a perturbative upper bound on $\tan\beta$, $\tan\beta \lesssim 1/|\hat{\mathbf{E}}_{\mathbf{g}}|$, may be derived beyond the tree level.

proportional to $\mathcal{U}_L^{Q\dagger}\mathbf{h}_d^\dagger\mathbf{h}_d\mathcal{U}_L^Q$ induced by RG running of the squark masses from the unification to the soft SUSY-breaking scale M_{SUSY}. These contributions can be resummed individually by taking appropriately the Hermitian square of the modified Eq. (2.9) and solving for $\mathcal{U}_L^{Q\dagger}\mathbf{h}_d^\dagger\mathbf{h}_d\mathcal{U}_L^Q$. This last step may involve the use of iterative or other numerical methods.

In the general case of a nonuniversal squark sector, the resummed FCNC couplings of the Higgs bosons to down-type quarks can always be parametrized in terms of a well-defined set of parameters at the electroweak scale. In the weak basis, in which $\mathcal{U}_L^Q=\mathbf{1}$, the set of input parameters consists of: (i) the soft squark mass matrices $m^2_{\tilde{u}_{L,R}}$, $m^2_{\tilde{d}_{L,R}}$ and soft Yukawa-coupling matrices $A_{u,d}$; (ii) the u- and d-quark masses; (iii) the CKM mixing matrix $\mathbf{V}$.

In our last step in deriving the resummed FCNC effective Lagrangian, we express the Higgs fields $\Phi^0_{1,2}$ in terms of their mass eigenstates $H_{1,2,3}$ and the neutral would-be Goldstone boson G^0 in the presence of CP violation [21]. Following the conventions of [19], we relate the weak to mass eigenstates through the linear transformations:

$$\Phi_1^0=\frac{1}{\sqrt{2}}[O_{1i}H_i+i(\cos\beta G^0-\sin\beta\ \ O_{3i}H_i)],$$
$$\Phi_2^0=\frac{1}{\sqrt{2}}[O_{2i}H_i+i(\sin\beta G^0+\cos\beta O_{3i}H_i)], \tag{2.17}$$

where O_{ij} is a 3-by-3 orthogonal matrix that accounts for CP-violating Higgs-mixing effects [22]. If we substitute the weak Higgs fields $\Phi^0_{1,2}$ by virtue of Eq. (2.17) into Eq. (2.11), we obtain the general resummed effective Lagrangian for the diagonal as well as off-diagonal Higgs interactions to d quarks,

$$\mathcal{L}_{H_i\bar{d}d'}=-\frac{g_w}{2M_W}\sum_{i=1}^{3}H_i\bar{d}(\hat{\mathbf{M}}_{\mathbf{d}}\mathbf{g}^L_{H_i\bar{d}d'}P_L+\mathbf{g}^R_{H_i\bar{d}d'}\hat{\mathbf{M}}_{\mathbf{d}'}P_R)d', \tag{2.18}$$

where $P_{L(R)}=[1-(+)\gamma_5]/2$ and

$$\mathbf{g}^L_{H_i\bar{d}d'}=\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\frac{O_{1i}}{\cos\beta}+(\mathbf{1}-\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V})\frac{O_{2i}}{\sin\beta}-i\left(\mathbf{1}-\frac{1}{\cos^2\beta}\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\right)\frac{O_{3i}}{\tan\beta},$$
$$\mathbf{g}^R_{H_i\bar{d}d'}=(\mathbf{g}^L_{H_i\bar{d}d'})^\dagger. \tag{2.19}$$

The 3×3-dimensional matrix $\mathbf{R}$ in Eq. (2.19), which resums all $\tan\beta$-enhanced finite radiative effects, is given by Eq. (2.16). Equation (2.18), along with Eq. (2.19), constitutes the major result of the present paper, which will be extensively used in our phenomenological discussions in Sec. III.

Finally, let us summarize the most important properties of the resummed effective Lagrangian (2.18):

(i) The FCNC interactions in Eq. (2.18) are described by $\tan^2\beta$-enhanced terms that are proportional to O_{1i} and O_{3i} and to $\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}$ in Eq. (2.19). These $\tan^2\beta$-enhanced FCNC terms properly take into account resummation,[3] nonuniversality in the squark sector and CP-violating effects.

(ii) The resummation matrix $\mathbf{R}$ controls the strength of the Higgs-mediated FCNC effects. For instance, if $\mathbf{R}$ is proportional to unity, then a kind of a GIM-cancellation mechanism [18] becomes operative and the Higgs-boson contributions to all FCNC observables vanish identically in this case. Furthermore, as well as the top quark, the other two lighter up-type quarks can give significant contributions to FCNC transition amplitudes, which are naturally included in Eq. (2.18) through the resummation matrix $\mathbf{R}$.

(iii) In the CP-invariant limit of the theory, the effective couplings $\mathbf{g}^{L,R}_{H_i\bar{d}d'}$ are either pure real or pure imaginary numbers. Moreover, in the limit $\mathbf{V}\to\mathbf{1}$, the effective Lagrangian (2.18) of the diagonal Higgs couplings to down quarks is in excellent agreement with the one presented in [19,23].

(iv) If $M_{H^+}\sim M_{H_{2,3}}\gg M_{H_1}$, one can show that $O_{11}\approx\cos\beta$, $O_{21}\approx\sin\beta$ and $O_{31}\approx0$. In this case, the H_1 coupling to d quarks becomes SM like and so H_1-mediated FCNC effects are getting suppressed. Instead, the FCNC couplings of the heavy $H_{2,3}$ bosons to d-type quarks retain their $\tan\beta$-enhanced strength in the above kinematic region.

(v) The one-loop resummed effective Lagrangian (2.18) captures the major bulk of the one-loop radiative effects [24] for large values of $\tan\beta$, e.g. for $\tan\beta\gtrsim40$, and for a soft SUSY-breaking scale M_{SUSY} much higher than the electroweak scale [7]. In addition, Eq. (2.18) is only valid in the limit in which the four-momentum of the d quarks and Higgs bosons in the external legs is much smaller than M_{SUSY}. This last condition is automatically satisfied in our computations of low-energy FCNC observables.

In the next section, we will study in detail the phenomenological consequences of the $\tan\beta$-enhanced FCNC effects mediated by Higgs bosons on rare processes and CP asymmetries related to the K- and B-meson systems.

III. APPLICATIONS TO *K*- AND *B*-MESON SYSTEMS

We shall now analyze the impact of our resummed effective Lagrangian (2.18) for Higgs-mediated FCNC interactions on representative K- and B-meson observables. For comprehensive reviews on K- and B-meson physics, we refer the reader to [25–27].

[3]In addition to the nonholomorphic contributions we have been considering here, there are in general holomorphic radiative effects on the Φ_1^0 coupling to d quarks which have an analogous matrix structure, i.e. $\varepsilon_{\mathbf{g}}+\varepsilon_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2$. These additional holomorphic terms are generally small, typically of order 10^{-2}, and only slightly modify the form of the matrix $\mathbf{R}$ in Eq. (2.19) to: $\mathbf{R}=\mathbf{1}+(\mathbf{1}+\varepsilon_{\mathbf{g}}+\varepsilon_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2)^{-1}(\mathbf{E}_{\mathbf{g}}+\mathbf{E}_{\mathbf{u}}|\hat{\mathbf{h}}_{\mathbf{u}}|^2)\tan\beta$. Obviously, such a modification is beyond the one-loop order of our resummation. Therefore, these additional small holomorphic terms can be safely neglected.

A. ΔM_K, ϵ_K and ϵ'/ϵ

Our starting point is the effective Hamiltonian for the $\Delta S=2$ interactions,

$$H_{\rm eff}^{\Delta S=2}=\frac{G_F^2}{16\pi^2}M_W^2\sum_i C_i(\mu)Q_i(\mu), \tag{3.1}$$

where $G_F=1.16639\times 10^{-5}$ GeV^{-2} is the Fermi constant, and $C_i(\mu)$ are the scale-dependent Wilson coefficients associated to the $\Delta S=2$ quark-dependent operators Q_i. Note that the CKM matrix elements in Eq. (3.1) have been absorbed into the Wilson coefficients. The $\Delta S=2$ operators Q_i may be summarized as follows:

$$\begin{aligned}
Q_1^{\rm VLL}&=(\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d),\\
Q_1^{\rm VRR}&=(\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma^\mu P_R d),\\
Q_1^{\rm LR}&=(\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d),\\
Q_2^{\rm LR}&=(\bar{s}P_L d)(\bar{s}P_R d),\\
Q_1^{\rm SLL}&=(\bar{s}P_L d)(\bar{s}P_L d),\\
Q_1^{\rm SRR}&=(\bar{s}P_R d)(\bar{s}P_R d),\\
Q_2^{\rm SLL}&=(\bar{s}\sigma_{\mu\nu}P_L d)(\bar{s}\sigma^{\mu\nu}P_L d),\\
Q_2^{\rm SRR}&=(\bar{s}\sigma_{\mu\nu}P_R d)(\bar{s}\sigma^{\mu\nu}P_R d),
\end{aligned} \tag{3.2}$$

with $\sigma_{\mu\nu}=\frac{1}{2}[\gamma_\mu,\gamma_\nu]$. Here, much of our discussion and notation follows Ref. [28].

It now proves convenient to decompose both the K^0-$\bar{K}^0$ mass difference ΔM_K and the known CP-violating mixing parameter ϵ_K into a SM and a SUSY contribution:

$$\begin{aligned}
\Delta M_K&=M_{K_L}-M_{K_S}=\Delta M_K^{\rm SM}+\Delta M_K^{\rm SUSY},\\
\epsilon_K&=\epsilon_K^{\rm SM}+\epsilon_K^{\rm SUSY}.
\end{aligned} \tag{3.3}$$

To a good approximation, one has

$$\Delta M_K^{\rm SM,\,SUSY}=2\Re e\langle\bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SM,\,SUSY}, \tag{3.4}$$

$$\epsilon_K^{\rm SM,\,SUSY}=\frac{\exp(i\pi/4)}{\sqrt{2}\Delta M_K}\Im m\langle\bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SM,\,SUSY}. \tag{3.5}$$

The SUSY contribution to the matrix element $\langle\bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}$ may be written down as [28,29]

$$\begin{aligned}
&\langle\bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}\\
&=\frac{G_F^2}{12\pi^2}M_W^2 m_K F_K^2\eta_2\hat{B}_K[\bar{P}_1^{\rm VLL}(C_1^{\rm VLL}+C_1^{\rm VRR})\\
&\quad+\bar{P}_1^{\rm LR}C_1^{\rm LR}+\bar{P}_2^{\rm LR}C_2^{\rm LR}+\bar{P}_1^{\rm SLL}(C_1^{\rm SLL}+C_1^{\rm SRR})\\
&\quad+\bar{P}_2^{\rm SLL}(C_2^{\rm SLL}+C_2^{\rm SRR})],
\end{aligned} \tag{3.6}$$

where $m_K=498$ MeV, $F_K=160$ MeV and the $\bar{P}$'s are the next-to-leading order (NLO) QCD factors that include the relevant hadronic matrix elements [28–32]. At the scale $\mu=2$ GeV, they are given by [28]

$$\begin{aligned}
&\bar{P}_1^{\rm VLL}=0.25,\quad \bar{P}_1^{\rm LR}=-18.6,\quad \bar{P}_2^{\rm LR}=30.6,\\
&\bar{P}_1^{\rm SLL}=-9.3,\quad \bar{P}_2^{\rm SLL}=-16.6.
\end{aligned} \tag{3.7}$$

On obtaining Eq. (3.7), we have used the numerical values: $\eta_2=0.57$ and $\hat{B}_K=0.85\pm 0.15$.

From studies in the CP-conserving MSSM with minimal flavor violation [10,14,29], it is known that for large values of $\tan\beta\gtrsim 40$, the dominant contribution to $\langle\bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}$ comes from Higgs-mediated two-loop double penguin (DP) diagrams proportional to $C_2^{\rm LR}$. Within the framework of our large-$\tan\beta$-resummed FCNC effective Lagrangian (2.18) that includes CP violation, the Wilson coefficients due to DP graphs are found to be

$$\begin{aligned}
C_1^{\rm SLL\,(DP)}&=-\frac{16\pi^2 m_s^2}{\sqrt{2}G_F M_W^2}\sum_{i=1}^3\frac{g^L_{H_i\bar{s}d}g^L_{H_i\bar{s}d}}{M_{H_i}^2},\\
C_1^{\rm SRR\,(DP)}&=-\frac{16\pi^2 m_d^2}{\sqrt{2}G_F M_W^2}\sum_{i=1}^3\frac{g^R_{H_i\bar{s}d}g^R_{H_i\bar{s}d}}{M_{H_i}^2},\\
C_2^{\rm LR\,(DP)}&=-\frac{32\pi^2 m_s m_d}{\sqrt{2}G_F M_W^2}\sum_{i=1}^3\frac{g^L_{H_i\bar{s}d}g^R_{H_i\bar{s}d}}{M_{H_i}^2},
\end{aligned} \tag{3.8}$$

where the $\tan^2\beta$-enhanced couplings $g^{L,R}_{H_i\bar{s}d}$ may be evaluated from Eq. (2.19). Note that the DP Wilson coefficients in Eq. (3.8) exhibit a $\tan^4\beta$ dependence and, although being two-loop suppressed, they become very significant for large values of $\tan\beta\gtrsim 40$.

In addition to the aforementioned DP contributions due to Higgs-boson exchange graphs, there exist relevant one-loop contributions to $\langle\bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}$ at large $\tan\beta$: (i) the t-$H^\pm$ box contribution to $C_2^{\rm LR}$ of the two-Higgs-doublet-model (2HDM) type, and (ii) the one-loop chargino-top-squark box diagram contributing to $C_1^{\rm SLL}$. The first contribution (i) becomes significant, up to 10%, only in the kinematic region $M_{H^\pm}\approx m_t$. In this case, to a good approximation, $C_1^{\rm SLL}$ may be given by [10,29]

$$C_2^{\rm LR\,(2HDM)}\approx -\frac{2m_s m_d}{M_W^2}(V_{ts}^* V_{td})^2\tan^2\beta. \tag{3.9}$$

Note that the light-quark masses contained in Eqs. (3.8) and (3.9) are running and are evaluated at the top-quark mass scale, i.e. $m_s(m_t)\simeq 61$ MeV, $m_d(m_t)\simeq 4$ MeV. The second contribution (ii) becomes non-negligible only for small values of the μ parameter [10,29], i.e. for $|\mu|\lesssim 200$ GeV.

In view of the above discussion, the kinematic parameter range of interest to us is

$$M_{\rm SUSY},\mu \gg m_t, \quad \tan\beta\gtrsim 40, \tag{3.10}$$

including the case $M_{H^\pm}\approx m_t$, for which the Higgs-mediated DP effects can dominate the K^0-$\bar{K}^0$ transition amplitude. Thus, taking also into account the subdominant 2HDM contribution (3.9), formula (3.6) simplifies to

$$\begin{aligned}\langle \bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY} &= 4.6\times 10^{-11}\ \ {\rm GeV}\left(\frac{F_K}{160\ \ {\rm MeV}}\right)^2\left(\frac{\eta_2}{0.57}\right)\left(\frac{\hat{B}_K}{0.85}\right)\\ &\quad\times[30.6(C_2^{\rm LR\,(DP)}+C_2^{\rm LR\,(2HDM)})-9.3(C_1^{\rm SLL\,(DP)}\\ &\quad+C_1^{\rm SRR\,(DP)})].\end{aligned} \tag{3.11}$$

Observe that the Wilson coefficients C_1 and C_2 contribute with opposite signs to $\langle \bar{K}^0|H_{\rm eff}^{\Delta S=2}|K^0\rangle_{\rm SUSY}$. Based on Eq. (3.11), we will give numerical estimates of ΔM_K and ϵ_K in Sec. IV.

We now turn our attention to the computation of the direct CP-violation parameter ϵ'/ϵ in the kaon system, induced by CP-violating Higgs-mediated FCNC interactions. In the SM, unlike the Z-penguin graphs, the Higgs-penguin contribution to $K\to\pi\pi$, which is proportional to the operator

$$Q_H=(\bar{s}P_L d)\sum_{q=u,d,s}(\bar{q}q), \tag{3.12}$$

has a suppressed Wilson coefficient proportional to m_q^2/M_H^2, where the SM Higgs-boson mass M_H is subject into the experimental bound: $M_H\gtrsim 114$ GeV. One might even think of the possibility that the operator Q_H in Eq. (3.12), which has enhanced Wilson coefficients for $q=c,b$, mixes with the gluonic and electroweak penguin diagrams, as well as with the other basis operators in Eq. (3.2). However, as was already pointed out in [33], this is not the case, and so the SM-Higgs penguin effects remain negligible.

The situation changes drastically in the MSSM with explicit CP violation, since the Higgs-boson FCNC couplings to down-type quarks are substantially enhanced by $\tan^2\beta$-dependent terms, for large values of $\tan\beta$. Furthermore, besides the CKM phase, the presence of complex soft SUSY-breaking masses with large CP-violating phases may further increase the Higgs-boson FCNC effects on ϵ'/ϵ. In fact, we note that soft CP-odd phases could even be the only source [3] to account for direct CP violation.

To reliably estimate the new SUSY effect on ϵ'/ϵ due to Higgs-boson FCNC interactions, we normalize each individual contribution with respect to the dominant SM contribution arising from the operator $Q_6=\Sigma_{q=u,d,s}(\bar{s}P_R q)(\bar{q}P_L d)$ [34,35], with Wilson coefficient y_6, viz.

$$\frac{\epsilon'}{\epsilon}=\left(\frac{\epsilon'}{\epsilon}\right)_6(\Omega_{\rm SM}+\Omega_{\rm SUSY}+\Omega_{\rm SUSY}^{\rm Higgs}). \tag{3.13}$$

In the MSSM with minimal flavor violation, the non-Higgs SUSY contribution $\Omega_{\rm SUSY}$ is small [36]. Sizable contributions may be obtained if one relaxes the assumptions of universality and CP conservation in the squark sector [37,38].

Here, we compute a novel contribution to ϵ'/ϵ, namely the quantity $\Omega_{\rm SUSY}^{\rm Higgs}$ in Eq. (3.13), which entirely originates from Higgs-boson exchange diagrams in the CP-violating MSSM. Based on our resummed FCNC effective Lagrangian (2.18), we obtain in the zero strong-phase approximation

$$\begin{aligned}\Omega_{\rm SUSY}^{\rm Higgs}=&\,2\sum_{i=1}^{3}\sum_{q=u,d,s}\frac{m_s m_q}{M_{H_i}^2}\Bigg[\frac{\Im m(g^L_{H_i\bar{s}d})g^S_{H_i\bar{q}q}}{A^2\lambda^5\eta}\left(\frac{\langle\pi^+\pi^-|(\bar{s}P_L d)(\bar{q}q)|K^0\rangle_0}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}-\frac{1}{|\omega|}\frac{\langle\pi^+\pi^-|(\bar{s}P_L d)(\bar{q}q)|K^0\rangle_2}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}\right)\\ &+\frac{\Re e(g^L_{H_i\bar{s}d})g^P_{H_i\bar{q}q}}{A^2\lambda^5\eta}\left(\frac{\langle\pi^+\pi^-|(\bar{s}P_L d)(\bar{q}\gamma_5 q)|K^0\rangle_0}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}-\frac{1}{|\omega|}\frac{\langle\pi^+\pi^-|(\bar{s}P_L d)(\bar{q}\gamma_5 q)|K^0\rangle_2}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}\right)\Bigg]\\ &+2\sum_{i=1}^{3}\sum_{q=u,d,s}\frac{m_d m_q}{M_{H_i}^2}\Bigg[\frac{\Im m(g^R_{H_i\bar{s}d})g^S_{H_i\bar{q}q}}{A^2\lambda^5\eta}\left(\frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}q)|K^0\rangle_0}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}-\frac{1}{|\omega|}\frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}q)|K^0\rangle_2}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}\right)\\ &+\frac{\Re e(g^R_{H_i\bar{s}d})g^P_{H_i\bar{q}q}}{A^2\lambda^5\eta}\left(\frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}\gamma_5 q)|K^0\rangle_0}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}-\frac{1}{|\omega|}\frac{\langle\pi^+\pi^-|(\bar{s}P_R d)(\bar{q}\gamma_5 q)|K^0\rangle_2}{y_6\langle\pi^+\pi^-|Q_6|K^0\rangle_0}\right)\Bigg],\end{aligned} \tag{3.14}$$

where the subscripts 0, 2 adhered to the hadronic matrix elements denote the total isospin I of the final states, and $A^2\lambda^5\eta$ is a CKM combination in the Wolfenstein parametrization, which has the value $A^2\lambda^5\eta\approx 1.3\times 10^{-4}$ in the SM. Furthermore, for $\Lambda_{\rm QCD}=325$ MeV and $m_s=150$ MeV, the SM Wilson coefficient y_6 and the matrix element of Q_6 take on the values [35] $y_6=-0.089$ and $\langle\pi^+\pi^-|Q_6|K^0\rangle_0=-0.35$ GeV^3, respectively. Also, experimental analyses suggest the value $|\omega|=0.045$, approximately yielding the SM contribution $\Omega_{\rm SM}\sim 1$ to ϵ'/ϵ in Eq. (3.13). Finally, the parameters $g^{S,P}_{H_i\bar{d}d}$ and $g^{S,P}_{H_i\bar{u}u}$ that occur in Eq. (3.14) are the diagonal scalar and pseudoscalar couplings of the H_i bosons to u- and d-type quarks [19], whose strengths are normalized to the SM Higgs-boson coupling. These reduced H_i couplings are given by

$$g^S_{H_i\bar{d}d}=\frac{1}{2}(g^L_{H_i\bar{d}d}+g^R_{H_i\bar{d}d}),\quad g^P_{H_i\bar{d}d}=\frac{i}{2}(g^L_{H_i\bar{d}d}-g^R_{H_i\bar{d}d}), \tag{3.15}$$

$$g^S_{H_i\bar{u}u}=O_{2i}/\sin\beta,\quad g^P_{H_i\bar{u}u}=-O_{3i}\cot\beta, \tag{3.16}$$

where we have neglected the small radiative threshold effects in the up sector.

On the experimental side, the latest world average result for $\Re e(\epsilon'/\epsilon)$ is [39]

$$\Re e\left(\frac{\epsilon'}{\epsilon}\right)=(1.66\pm 0.16)\times 10^{-3}, \tag{3.17}$$

at the 1-σ confidence level (C.L.). In the light of the experimental result (3.17) and the discussion given above, we may conservatively require that

$$|\Omega^{\rm Higgs}_{\rm SUSY}|\lesssim 1. \tag{3.18}$$

The biggest contribution in the sum over quarks in Eq. (3.14) comes from the d quark and exhibits the qualitative scaling behavior

$$\Omega^{\rm Higgs}_{\rm SUSY}\simeq\frac{2m_sm_d}{M_H^2}\,\frac{\tan^3\beta}{|\omega|}\times\mathcal{O}(1). \tag{3.19}$$

For instance, for $\tan\beta=50$ and $M_H=200$ GeV, Eq. (3.19) gives $\Omega^{\rm Higgs}_{\rm SUSY}\simeq 0.1\times\mathcal{O}(1)$. Obviously, such a contribution is, in principle, non-negligible, but very sensitively depends on the actual values of the new hadronic matrix elements:

$$(Q^{L,R}_{S,P})_I=\langle\pi^+\pi^-|(\bar{s}P_{L,R}d)(\bar{q}(1,\gamma_5)q)|K^0\rangle_I, \tag{3.20}$$

with $I=0,2$. A detailed calculation of the hadronic matrix elements $(Q^{L,R}_{S,P})_I$ in Eq. (3.20) will be given elsewhere. In Sec. IV, however, we will present numerical estimates of ΔM_K and ϵ_K within the context of generic soft SUSY-breaking models.

B. ΔM_{B_q}, $B_q\to\ell^+\ell^-$ and associated *CP* asymmetries

We start our discussion of a set of observables in the B-meson system by first analyzing the B^0_q-$\bar{B}^0_q$ mass difference, ΔM_{B_q} with $q=s,d$, in the CP-violating MSSM at large $\tan\beta$. In the applicable limit of equal B-meson lifetimes, ΔM_{B_q} may be written as the modulus of a sum of a SM and a SUSY term:

$$\Delta M_{B_q}=2|\langle\bar{B}^0_q|H^{\Delta B=2}_{\rm eff}|B^0_q\rangle_{\rm SM}+\langle\bar{B}^0_q|H^{\Delta B=2}_{\rm eff}|B^0_q\rangle_{\rm SUSY}|, \tag{3.21}$$

where the effective $\Delta B=2$ Hamiltonian $H^{\Delta B=2}_{\rm eff}$ may be obtained from the $\Delta S=2$ one stated in Eq. (3.1), after making the obvious replacements: $s\to b$ and $d\to q$, with $q=d,s$. Proceeding as in Sec. III A, we arrive at analogous closed expressions for the SUSY contributions to the $\Delta B=2$ transition amplitudes:

$$\langle\bar{B}^0_d|H^{\Delta B=2}_{\rm eff}|B^0_d\rangle_{\rm SUSY}=1711\ \ {\rm ps}^{-1}\left(\frac{\hat{B}^{1/2}_{B_d}F_{B_d}}{230\ \ {\rm MeV}}\right)^2\left(\frac{\eta_B}{0.55}\right)\times[0.88(C_2^{\rm LR\,(DP)}+C_2^{\rm LR\,(2HDM)})-0.52(C_1^{\rm SLL\,(DP)}+C_1^{\rm SRR\,(DP)})], \tag{3.22}$$

$$\langle\bar{B}^0_s|H^{\Delta B=2}_{\rm eff}|B^0_s\rangle_{\rm SUSY}=2310\ \ {\rm ps}^{-1}\left(\frac{\hat{B}^{1/2}_{B_s}F_{B_s}}{265\ \ {\rm MeV}}\right)^2\left(\frac{\eta_B}{0.55}\right)\times[0.88(C_2^{\rm LR\,(DP)}+C_2^{\rm LR\,(2HDM)})-0.52(C_1^{\rm SLL\,(DP)}+C_1^{\rm SRR\,(DP)})].$$

In deriving Eq. (3.22), we have also substituted the values determined in [28–32] for the NLO-QCD factors, along with their hadronic matrix elements at the scale $\mu=4.2$ GeV:

$$\bar{P}_1^{\rm LR}=-0.58,\quad\bar{P}_2^{\rm LR}=0.88,\quad\bar{P}_1^{\rm SLL}=-0.52,\quad\bar{P}_2^{\rm SLL}=-1.1. \tag{3.23}$$

Moreover, the corresponding Wilson coefficients appearing in Eq. (3.22) may be recovered from those in Eqs. (3.8) and (3.9), after performing the quark replacements mentioned above.

Another observable, which is enhanced at large $\tan\beta$, is the pure leptonic decay of B mesons [5,6,8–15], $\bar{B}^0_q\to\ell^+\ell^-$, with $\ell=\mu,\ \tau$. Neglecting contributions proportional to the lighter quark masses $m_{d,s}$, the relevant effective Hamiltonian for $\Delta B=1$ FCNC transitions, such as $b\to q\ell^+\ell^-$ with $q=d,s$, is given by

$$H^{\Delta B=1}_{\rm eff}=-2\sqrt{2}G_FV_{tb}V^*_{tq}(C_S\mathcal{O}_S+C_P\mathcal{O}_P+C_{10}\mathcal{O}_{10}), \tag{3.24}$$

where

$$\mathcal{O}_S = \frac{e^2}{16\pi^2} m_b (\bar{q} P_R b)(\bar{\ell}\ell),$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2} m_b (\bar{q} P_R b)(\bar{\ell}\gamma_5 \ell), \tag{3.25}$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \gamma_5 \ell).$$

Employing our resummed FCNC effective Lagrangian (2.18), it is not difficult to compute the Wilson coefficients C_S and C_P in the region of large values of $\tan\beta$:[4]

$$C_S = \frac{2\pi m_\ell}{\alpha_{\rm em}} \frac{1}{V_{tb}V_{tq}^*} \sum_{i=1}^{3} \frac{g^R_{H_i\bar{q}b} g^S_{H_i\bar{\ell}\ell}}{M^2_{H_i}}, \tag{3.26}$$

$$C_P = i\frac{2\pi m_\ell}{\alpha_{\rm em}} \frac{1}{V_{tb}V_{tq}^*} \sum_{i=1}^{3} \frac{g^R_{H_i\bar{q}b} g^P_{H_i\bar{\ell}\ell}}{M^2_{H_i}},$$

while $C_{10} = -4.221$ is the leading SM contribution. In analogy to Eq. (3.15), the reduced scalar and pseudoscalar Higgs couplings to charged leptons $g^{S,P}_{H_i\bar{\ell}\ell}$ in Eq. (3.26) are given by

$$g^S_{H_i\bar{\ell}\ell} = \frac{O_{1i}}{\cos\beta}, \quad g^P_{H_i\bar{\ell}\ell} = -\tan\beta O_{3i}, \tag{3.27}$$

where nonholomorphic vertex effects on the leptonic sector have been omitted as being negligibly small.

With the approximations mentioned above, the branching ratio for the $\bar{B}^0_q$ meson decay to $\ell^+\ell^-$ acquires the simple form [9]

$$\mathcal{B}(\bar{B}^0_q \to \ell^+\ell^-) = \frac{G_F^2 \alpha_{\rm em}^2}{16\pi^3} M_{B_q} \tau_{B_q} |V_{tb}V_{tq}^*|^2 \sqrt{1-\frac{4m_\ell^2}{M^2_{B_q}}} \Bigg[\left(1-\frac{4m_\ell^2}{M^2_{B_q}}\right) \times |F^q_S|^2 + |F^q_P + 2m_\ell F^q_A|^2 \Bigg], \tag{3.28}$$

where τ_{B_q} is the total lifetime of the B_q meson and

$$F^q_{S,P} = -\frac{i}{2} M^2_{B_q} F_{B_q} \frac{m_b}{m_b + m_q} C_{S,P}, \quad F^q_A = -\frac{i}{2} F_{B_q} C_{10}. \tag{3.29}$$

In our numerical estimates, we ignore the contribution from C_{10}, as being subdominant in the region of large $\tan\beta$, i.e. for $\tan\beta \gtrsim 40$, where all Higgs-particle masses are well below the TeV scale. The SM predictions as well as the current experimental bounds pertinent to $\mathcal{B}(\bar{B}^0_d \to \ell^+\ell^-)$ can be read off from Table I in [41].

In the CP-violating MSSM, an equally important class of observables related to $\mathcal{B}(\bar{B}^0_{d,s} \to \ell^+\ell^-)$ [16] is the one probing possible CP asymmetries that can take place in the same leptonic B-meson decays. The leptonic CP asymmetries may shed even light on the CP nature of possible new-physics effects, as the SM prediction for these observables turns out to be the dismally small order 10^{-3} [42]. This SM result is a consequence of the fact that the CP-violating phase in B^0-$\bar{B}^0$-mixing parameter q/p is opposite to the one entering the ratio of the amplitudes $\bar{\mathcal{A}}_{L(R)}(\bar{B}^0_{d,s} \to l^+_{L(R)} l^-_{L(R)}) / \mathcal{A}_{L(R)}(B^0_{d,s} \to l^+_{L(R)} l^-_{L(R)})$, such that the net CP-violating effect on the observable parameter $\lambda_{L(R)} = (q/p)(\bar{\mathcal{A}}_{L(R)}/\mathcal{A}_{L(R)})$ almost cancels out.

There are two possible time-dependent CP asymmetries associated with leptonic B-meson decays that are physically allowed:

$$\mathcal{A}_{CP}^{(B^0_q \to l^+_L l^-_L)} = \frac{\int_0^\infty dt\, \Gamma(B^0_q(t) \to l^+_L l^-_L) - \int_0^\infty dt\, \Gamma(\bar{B}^0_q(t) \to l^+_R l^-_R)}{\int_0^\infty dt\, \Gamma(B^0_q(t) \to l^+_L l^-_L) + \int_0^\infty dt\, \Gamma(\bar{B}^0_q(t) \to l^+_R l^-_R)} \tag{3.30}$$

and $\mathcal{A}_{CP}^{(B^0_q \to l^+_R l^-_R)}$, $L \leftrightarrow R$. Under the assumption that q/p is a pure phase, one finds [42]

$$\mathcal{A}_{CP}^{(B^0_q \to l^+_L l^-_L)} = -\frac{2x_q \Im m \lambda_q}{(2+x_q^2) + x_q^2|\lambda_q|^2},$$
$$\mathcal{A}_{CP}^{(B^0_q \to l^+_R l^-_R)} = -\frac{2x_q \Im m \lambda_q}{(2+x_q^2)|\lambda_q|^2 + x_q^2}, \tag{3.31}$$

where $x_q = \Delta M_{B_q}/\Gamma_{B_q}$ and

$$\lambda_q = \frac{M^{q*}_{12}}{|M^q_{12}|} \left(\frac{V_{tb}V^*_{tq}}{V^*_{tb}V_{tq}} \right) \frac{\beta_l C_S + C_P + 2m_l C_{10}/(m_b M_{B_q})}{\beta_l C^*_S - C^*_P - 2m_l C_{10}/(m_b M_{B_q})}. \tag{3.32}$$

In Eq. (3.32), $\beta_l = (1-4m_l^2/M^2_{B_q})^{1/2}$, M^q_{12} is the dispersive part of the B^0_q-$\bar{B}^0_q$ matrix element, and $C_{S,P}$ are Wilson coefficients given in Eq. (3.26). The maximal value that the lep-

[4]Our approach to Higgs-mediated FCNC effects presented here may be extended to consistently account for charged-lepton flavor violation in B-meson decays, such as $B_{s,d} \to \ell^+\ell'^-$ [40], where the effective off-diagonal Higgs-lepton-lepton couplings $g^{S,P}_{H_i\bar{\ell}\ell'}$ can be derived by following a methodology very analogous to the one described in Sec. II.

tonic CP asymmetries in Eq. (3.31) can reach is $\mathcal{A}_{CP}^{\max} = 1/\sqrt{2+x_q^2}$ and is obtained for $\Im m\lambda_q = |\lambda_q|^2$. From current experimental data [43], one may extract the values $x_d = 0.76$ and $x_s \gtrsim 19$ at the 95% C.L., which leads to $\mathcal{A}_{CP}^{\max}(B_s) \approx 5\%$ and $\mathcal{A}_{CP}^{\max}(B_d) \approx 62\%$.

Within the framework of our resummed FCNC effective Lagrangian, we also improve earlier calculations [42] of the CP asymmetries by including B_q^0-$\overline{B}_q^0$ mixing effects through $M_{12}^{q*}/|M_{12}^q|$ in Eq. (3.32). According to our standard approach of splitting the amplitude into a SM and a MSSM part, we obtain for the SM part

$$M_{12}^{q*(\mathrm{SM})} = \frac{G_F^2 M_W^2}{12\pi^2} M_{B_q} \eta_B \hat{B}_{B_q} F_{B_q}^2 (V_{tq} V_{tb}^*)^2 S_{tt}, \tag{3.33}$$

where $S_{tt} \approx 2.38$ is the value of the dominant m_t-dependent loop function for a top-pole mass $m_t = 175$ GeV. The SUSY contribution to $M_{12}^{q*(\mathrm{SUSY})} = \langle \overline{B}_d^0 | H_{\mathrm{eff}}^{\Delta B=2} | B_d^0 \rangle_{\mathrm{SUSY}}$ may be obtained from Eq. (3.22).

In the next section, we will present numerical estimates for the K- and B-meson FCNC observables, based on the analytic expressions derived above.

IV. NUMERICAL ESTIMATES

In this section, we shall numerically analyze the impact of the $\tan^2\beta$-enhanced FCNC interactions on a number of K- and B-meson observables which were discussed in detail in Secs. III A and III B, such as ΔM_K, ϵ_K, ΔM_{B_d}, ΔM_{B_s}, $B_d \to \tau^+\tau^-$, $B_s \to \mu^+\mu^-$ and their associated leptonic CP asymmetries. For our illustrations, we consider two generic low-energy soft SUSY-breaking scenarios, (A) and (B).

In scenario (A), the squark masses are taken to be universal and $\hat{\mathbf{E}}_\mathbf{g}$ and $\hat{\mathbf{E}}_\mathbf{u}$ are proportional to the unity matrix at the soft SUSY-breaking scale M_{SUSY}. The CP-conserving version of this scenario has frequently been discussed in the literature within the context of minimal flavor-violation models, see e.g. [6].

In scenario (B) we assume the existence of a mass hierarchy between the first two generations of squarks and the third generation, namely the first two generations are degenerate and can be much heavier than the third one. In addition, although not mandatory, we assume for simplicity that the model-dependent unitary matrix $\boldsymbol{\mathcal{U}}_\mathbf{L}^\mathbf{Q}$ in Eq. (2.15) is such that $\mathbf{E}_\mathbf{g}$ and $\mathbf{E}_\mathbf{u}$ become diagonal matrices in this scenario. Clearly, in the limit in which all squarks are degenerate, scenario (B) coincides with (A).

In Fig. 2 we give a schematic representation of the generic mass spectrum that will be assumed in our numerical analysis. More explicitly, we fix the charged Higgs-boson M_{H^+} to the value 200 GeV. Since the effect of the gaugino-Higgsino mixing [15,16] on the resummation matrix $\mathbf{R}$ can be significantly reduced for $m_{\widetilde{W}} \ll \mu$, we ignore this contribution by considering the relatively low value $m_{\widetilde{W}} \approx 2m_t \ll M_{\mathrm{SUSY}}$ in our numerical estimates, with $m_t = 175$ GeV. As can be seen from Fig. 2, the third-generation soft squark mass $m_{\tilde{t}}$, the μ-parameter, the gluino mass $m_{\tilde{g}}$ and the trilinear soft Yukawa coupling A_U are set, for simplicity reasons, to the common soft SUSY-breaking scale M_{SUSY}, which is typically taken to be 1 TeV.

The soft squark masses of the other two generations are assumed to be equal to $m_{\tilde{q}}$ in our generic framework. To account for a possible hierarchical difference between the mass scales $m_{\tilde{t}}$ and $m_{\tilde{q}}$, we introduce the so-called hierarchy factor ρ, such that $m_{\tilde{q}} = \rho m_{\tilde{t}} = \rho M_{\mathrm{SUSY}}$. As has been mentioned above, models of minimal flavor violation correspond to scenario (A) with $\rho = 1$. As we will see in detail in Secs. IV A and IV B, the predictions for the K- and B-meson FCNC observables crucially depend on the values of the hierarchy factor ρ. Equally important modifications in the predictions are obtained for different values of the soft CP-violating phases $\phi_g = \arg(m_{\tilde{g}})$ and $\phi_{A_U} = \arg(A_U)$. In addition, the K- and B-meson FCNC observables exhibit a nontrivial dependence on the CKM phase δ_{CKM}, which is varied independently in our figures.

Although we primarily use $\tan\beta = 50$ and $M_{H^+} = 0.2$ TeV as inputs in our numerical analysis, approximate predictions for other values of the input parameters may be obtained by rescaling the numerical estimates by a factor

$$x_{\mathcal{O}} = \left(\frac{\tan\beta}{50}\right)^n \times \left(\frac{0.2\ \ \mathrm{TeV}}{M_{H^+}}\right)^k, \tag{4.1}$$

where the integers n and k depend on the FCNC observable $\mathcal{O}$ under study. Such a rescaling proves to be fairly accurate for $\tan\beta \gtrsim 40$ and $M_{H^+} \gtrsim 150$ GeV, which is the kinematic region of our interest.

A. ΔM_K and $|\epsilon_K|$

The SM effects on ΔM_K and $|\epsilon_K|$ were extensively discussed in the literature [44], so we will not dwell upon this issue here as well. Instead, we assume that the SM explains well the experimental results for the above two observables [43]:

$$\Delta M_K^{\mathrm{exp}} = (3.490 \pm 0.006) \times 10^{-12}\ \ \mathrm{MeV}, \tag{4.2}$$

$$|\epsilon_K^{\mathrm{exp}}| = (2.282 \pm 0.017) \times 10^{-3}. \tag{4.3}$$

Given the significant uncertainties in the calculation of hadronic matrix elements, however, our approach will be to constrain the soft SUSY-breaking parameters by conservatively requiring that $\Delta M_K^{\mathrm{SUSY}}$ and $|\epsilon_K^{\mathrm{SUSY}}|$ do not exceed in size the SM predictions.[5]

To start with, we display in Fig. 3 numerical values for the Higgs-boson DP effects on ΔM_K and $|\epsilon_K|$ as functions of the gluino phase $\arg m_{\tilde{g}}$, where the hierarchy factor ρ and the

[5]Both ΔM_K and $|\epsilon_K|$ place important constraints on the ρ-η plane of the unitarity triangle. The so-derived limits can be used to constrain new physics. In this case, a global fit of all the relevant FCNC observables to the unitarity triangle might be more appropriate. We intend to address this issue in a future work.

phase $\phi_{A_U}=\arg(A_{t,b})$ of the soft SUSY-breaking trilinear Yukawa couplings assume the discrete values $(\rho,\phi_{A_U})=(1,0°),(10,0°),(1,90°),(10,90°),(1,180°),(10,180°)$. According to our CP-phase conventions [19], μ is always taken to be positive, while the CKM phase $\delta_{\rm CKM}$ is chosen to its maximal value 90°. The subdominant one-loop 2HDM contribution coming from $W^{\pm}$-$H^{\mp}$ box graphs [cf. Eq. (3.9)] has also been indicated by an arrow in Fig. 3. Predictions for M_{H^+} and $\tan\beta$ values other than those shown in Fig. 2 may be approximately obtained by multiplying the numerical estimates by a factor $x_{\mathcal{O}}=(\tan\beta/50)^4\times(0.2\ {\rm TeV}/M_{H^+})^2$. We observe in Fig. 3 that the resulting values for $\Delta M_K^{\rm SUSY}$ can exceed the experimental error in Eq. (4.2) by one order of magnitude, for $\rho=10$ and $|\phi_{A_U}|,|\phi_{\tilde{g}}|\gtrsim 90°$. For the same inputs, $|\epsilon_K^{\rm SUSY}|$ takes on values comparable to the experimentally measured one (4.3). Here, we should stress the fact that universal squark-mass scenarios corresponding to $\rho=1$ can still predict sizeable effects on ϵ_K. This nonzero result should be contrasted with the one of the gluino-squark box contributions to ΔM_K and $|\epsilon_K|$ [45] which do vanish in the limit of strictly degenerate squarks due to a SUSY-GIM-cancellation mechanism.

In our case of Higgs-mediated FCNC observables, however, the situation is slightly different. As we have discussed in Sec. II, the size of the FCNC effects is encoded in the flavor structure of the 3-by-3 resummation matrix **R**. Since **R** is diagonal for the scenarios (A) and (B) under consideration, we can expand the $\tan\beta$-enhanced FCNC terms $(\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V})_{dd'}$ in Eq. (2.18) as follows:

$$(\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V})_{dd'}=V_{ud}^*R_u^{-1}V_{ud'}+V_{cd}^*R_c^{-1}V_{cd'}+V_{td}^*R_t^{-1}V_{td'}\,, \tag{4.4}$$

where d and d' collectively denote all down-type quarks, with $d\neq d'$. For the parameters adopted in Fig. 2, the quantities $R_{u,c,t}^{-1}$ can be simplified further to[6]

$$R_u^{-1}\approx\left[1+\left(\frac{\alpha_s}{3\pi\rho^2}e^{-i\phi_{\tilde{g}}}+\frac{|h_u|^2}{32\pi^2\rho^2}e^{-i\phi_{A_U}}\right)\tan\beta\right]^{-1},$$

$$R_c^{-1}\approx\left[1+\left(\frac{\alpha_s}{3\pi\rho^2}e^{-i\phi_{\tilde{g}}}+\frac{|h_c|^2}{32\pi^2\rho^2}e^{-i\phi_{A_U}}\right)\tan\beta\right]^{-1},$$

$$R_t^{-1}=\left[1+\left(\frac{\alpha_s}{3\pi}e^{-i\phi_{\tilde{g}}}+\frac{|h_t|^2}{32\pi^2}e^{-i\phi_{A_U}}\right)\tan\beta\right]^{-1}. \tag{4.5}$$

Then, from Eq. (4.5), it is easy to see that the off-diagonal elements of $\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}$ increase if $\phi_{A_U},\phi_{\tilde{g}}=\pm\pi$ and so the effective couplings $g^{L,R}_{H_i\bar{d}d'}$, thereby giving rise to enhanced predictions. This is a very generic feature which is reflected in Fig. 3 and, as we will see in Sec. IV B, also holds true for our numerical estimates of B-meson FCNC observables.

Neglecting the small Yukawa couplings of the first two generations and making use of the unitarity of **V**, we find for $d\neq d'$

$$(\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V})_{dd'}\propto V_{td}^*V_{td'}\left[\frac{\alpha_s}{3\pi}e^{-i\phi_{\tilde{g}}}\left(\frac{1}{\rho^2}-1\right)-\frac{|h_t|^2}{32\pi^2}e^{-i\phi_{A_U}}\right]\tan\beta. \tag{4.6}$$

If $\rho=1$, the dominant FCNC effect originates from the second term in the square brackets of Eq. (4.6), provided $|1+(\alpha_s/3\pi)e^{-i\phi_{\tilde{g}}}\tan\beta|\gg 5\times10^{-4}\tan\beta$ (see also footnote 2). If $\rho\gg1$, then gluino corrections become dominant; they are larger by a factor $(a_s/3\pi)/(1/32\pi^2)\simeq3.6$. However, between the low and high ρ regime, there is an intermediate value of ρ, where $(\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V})_{dd'}$ does exactly vanish for $d\neq d'$, and so the effective couplings $g^{L,R}_{H_i\bar{d}d'}$. In this case, one has $R_u^{-1}=R_c^{-1}=R_t^{-1}$ in Eq. (4.5), implying that **R** is proportional to the unity matrix. Then, it is $(\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V})_{dd'}=0$, as a result of a GIM-cancellation mechanism due to the unitarity of the CKM matrix. We call such a point in the parameter space *GIM operative point*. The ρ value, for which the GIM-

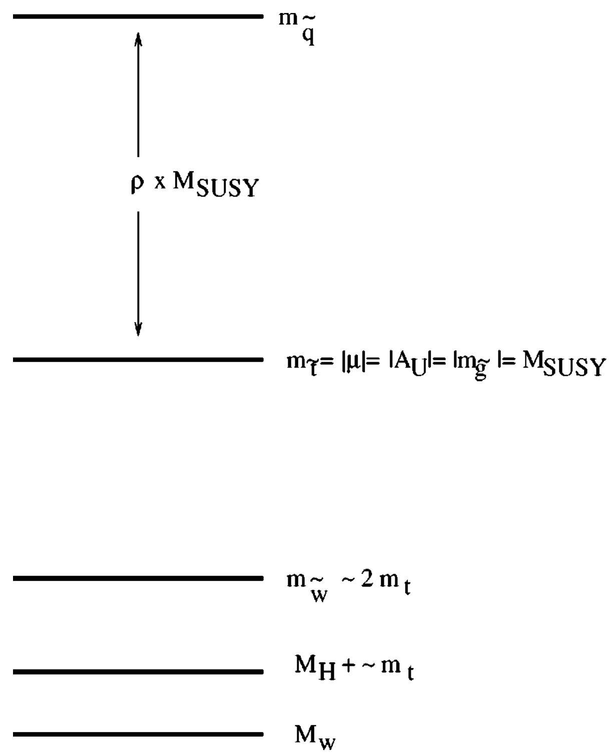

FIG. 2. Schematic representation of the SUSY mass spectrum considered in our numerical analysis, where $m_{\tilde{q}}$ and $m_{\tilde{t}}$ denote the masses of the first two and third generations of squarks, respectively. The hierarchy factor ρ, the phase $\phi_{\tilde{g}}$ of the gluino mass, and the phases $\phi_{A_{t,b}}$ of the soft SUSY breaking trilinear couplings, with $\phi_{A_t}=\phi_{A_b}=\phi_{A_U}$, are varied independently (see also discussion in the text).

[6]For $\rho\gtrsim10$, the first two equations in Eq. (4.5) may be better approximated by replacing $\rho\to\rho/\sqrt{2}$.

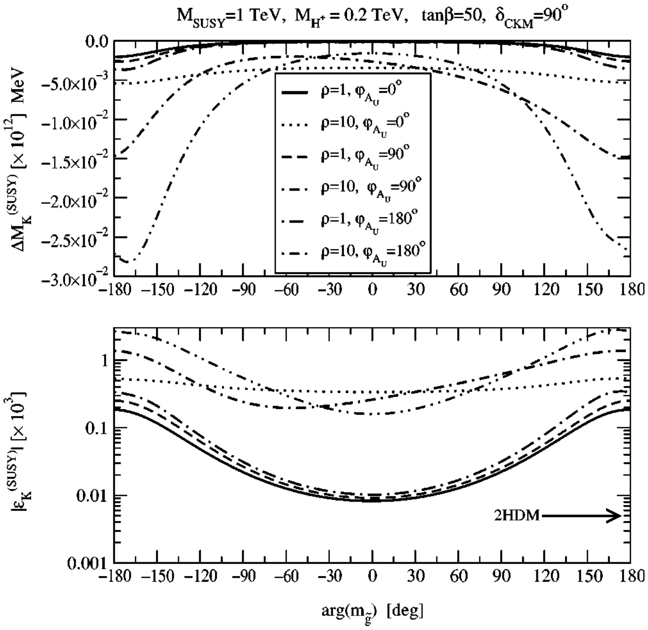

FIG. 3. SUSY Higgs-DP contributions to ϵ_K and ΔM_K given in units of 10^{-12} MeV and 10^{-3}, respectively, as functions of the gluino phase $\arg(m_{\tilde{g}})$, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$ and $\delta_{\rm CKM}=90°$. As is shown above, the different curves are obtained for selected values of ρ and ϕ_{A_U}. The size of the 2HDM effect alone on ϵ_K is indicated by an arrow.

cancellation mechanism becomes fully operative, may easily be determined from Eq. (4.6), i.e.

$$\rho^2_{\rm GIM}=\left(1+\frac{3|h_t|^2}{32\pi\alpha_s}e^{i(\phi_{\tilde{g}}-\phi_{A_U})}\right)^{-1}. \tag{4.7}$$

For the MSSM parameter space under study, there is always a GIM-operative value for the hierarchy factor ρ, iff $\phi_{A_U}-\phi_{\tilde{g}}=0$ or $\pm\pi$. For $\phi_{A_U}-\phi_{\tilde{g}}=0$, we have $\rho_{\rm GIM}<1$, whereas it is $\rho_{\rm GIM}>1$, for $\phi_{A_U}-\phi_{\tilde{g}}=\pi$. In fact, the second case is realized in Fig. 4 for $\rho_{\rm GIM}\approx 1.22$, where $\Delta M_K^{\rm SUSY}$ and $|\epsilon_K^{\rm SUSY}|$ vanish independently of the value of the CKM phase. Here, we should emphasize the fact that the value of $\rho_{\rm GIM}$ does not depend on the FCNC observable under consideration and is in excellent agreement with the one determined by Eq. (4.7). Because of the above flavor-universal property of $\rho_{\rm GIM}$, one may even face the very unusual possibility of discovering SUSY at high-energy colliders, without accompanying such a discovery with any new-physics signal in low-energy K- and B-meson FCNC observables.

It is now instructive to gauge the relative size of the different DP-induced Wilson coefficients in Eq. (3.8). For simplicity, let us take $\rho=1$. Then, each individual DP-induced

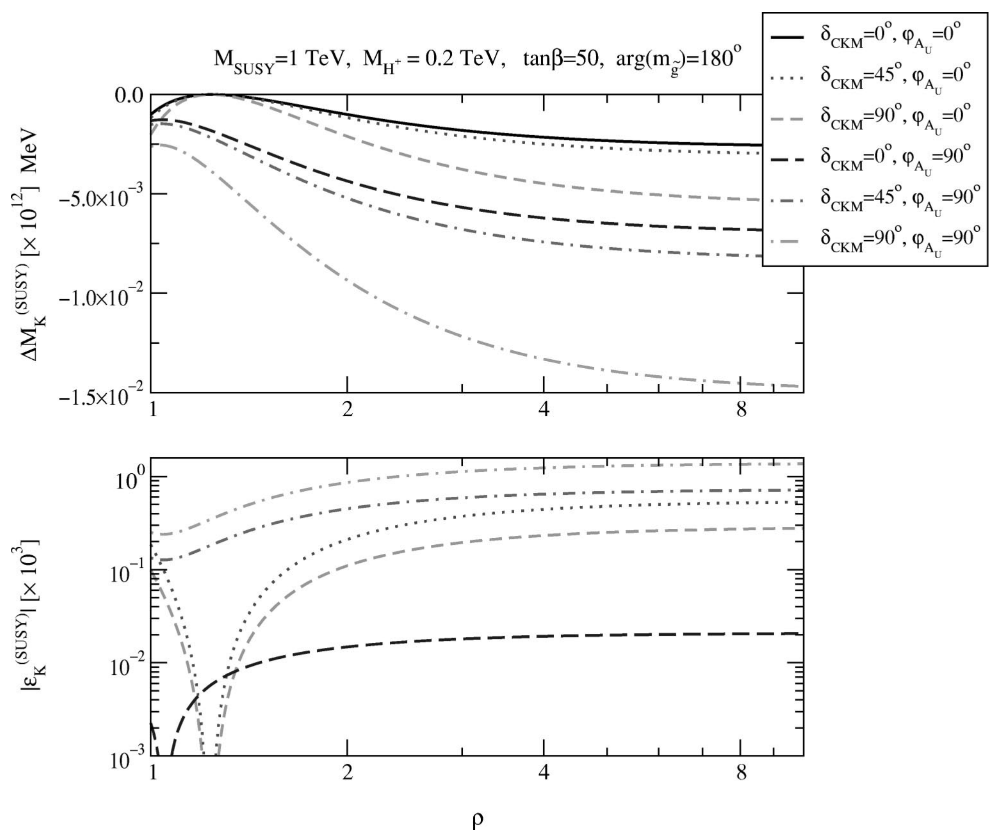

FIG. 4. SUSY Higgs-DP contributions to ϵ_K and ΔM_K given in units of 10^{-12} MeV and 10^{-3}, respectively, as functions of the hierarchy factor ρ, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$, and $\arg m_{\tilde{g}}=180°$, where the values of $\delta_{\rm CKM}$ and ϕ_{A_U} are varied discretely.

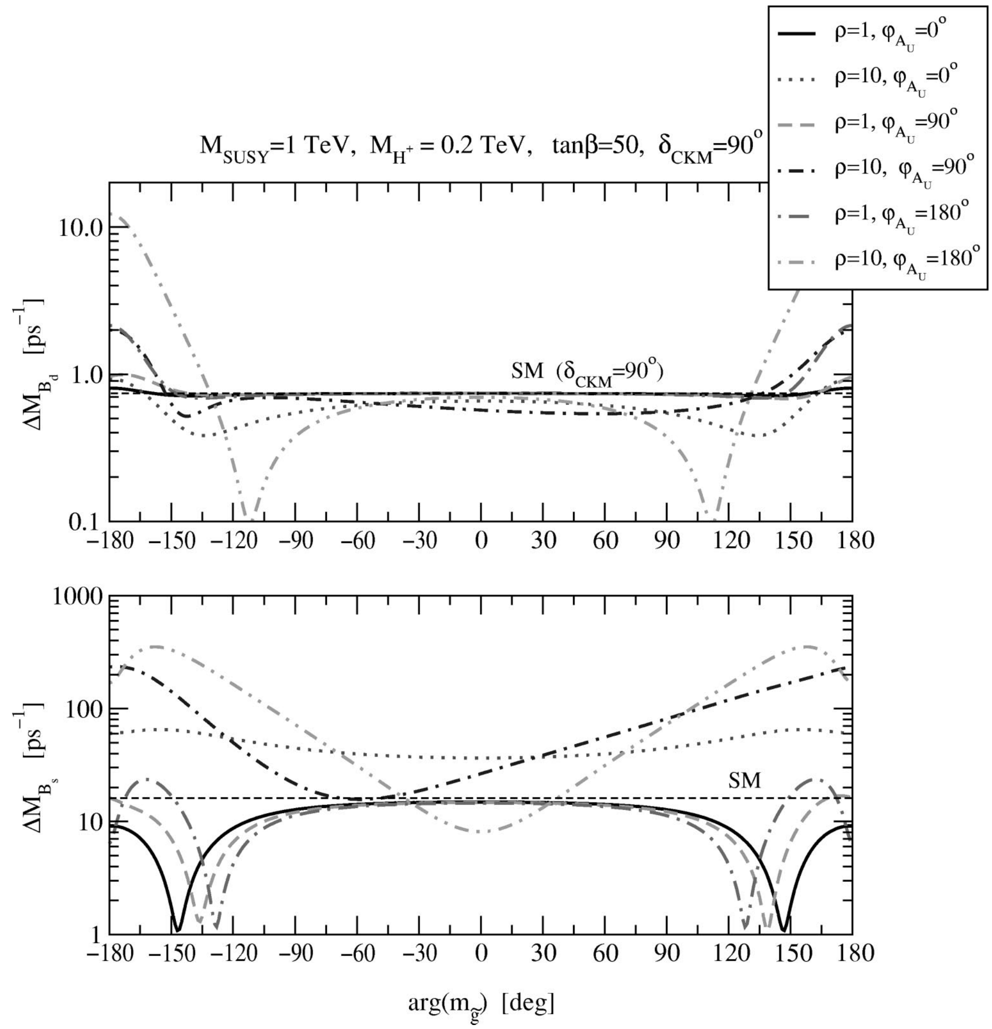

FIG. 5. SM and SUSY Higgs-DP contributions to ΔM_{B_d} and ΔM_{B_s} as functions of the gluino phase $\arg(m_{\tilde{g}})$, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$ and $\delta_{\rm CKM}=90°$, where the hierarchy factor ρ and ϕ_{A_U} are varied independently as shown above. The SM contributions alone for $\delta_{\rm CKM}=90°$ are displayed by horizontal dashed lines.

Wilson coefficient in Eq. (3.8) may be approximately given by

$$\sum_{i=1}^{3}(g^L_{H_i\bar{s}d})^2 \approx (\chi^{(t)}_{\rm FC})^2(V^*_{ts}V_{td})^2\sum_{i=1}^{3}\frac{O^2_{1i}-O^2_{3i}+2iO_{1i}O_{3i}}{M^2_{H_i}},$$

$$\sum_{i=1}^{3}(g^R_{H_i\bar{s}d})^2 \approx (\chi^{(t)*}_{\rm FC})^2(V^*_{ts}V_{td})^2\sum_{i=1}^{3}\frac{O^2_{1i}-O^2_{3i}-2iO_{1i}O_{3i}}{M^2_{H_i}},$$

$$\sum_{i=1}^{3}(g^L_{H_i\bar{s}d}g^R_{H_i\bar{s}d}) \approx |\chi^{(t)}_{\rm FC}|^2(V^*_{ts}V_{td})^2\sum_{i=1}^{3}\frac{O^2_{1i}+O^2_{3i}}{M^2_{H_i}}, \tag{4.8}$$

where $\chi^{(t)}_{\rm FC}$ is the t-quark dependent entry of the diagonal matrix $\chi_{\rm FC}$ defined in Eq. (2.13). For $M_{H^+}\gtrsim 180$ GeV, CP-violation and Higgs-mixing effects start to decouple from the lightest H_1 sector [19]. Moreover, in the region $\tan\beta\gtrsim 40$, the Φ_2 component in the H_2- and H_3-boson mass eigenstates is suppressed. As a consequence of the latter, we obtain

$$\sum_{i=1}^{3}\frac{O^2_{1i}-O^2_{3i}}{M^2_{H_i}} = (O^2_{11}-O^2_{31})\left(\frac{1}{M^2_{H_1}}-\frac{1}{M^2_{H_{23}}}\right) + \mathcal{O}\left(\frac{M^2_{H_2}-M^2_{H_3}}{M^2_{H_{23}}}\right), \tag{4.9}$$

$$\sum_{i=1}^{3}\frac{O_{1i}O_{3i}}{M^2_{H_i}} = O_{11}O_{31}\left(\frac{1}{M^2_{H_1}}-\frac{1}{M^2_{H_{23}}}\right) + \mathcal{O}\left(\frac{M^2_{H_2}-M^2_{H_3}}{M^2_{H_{23}}}\right), \tag{4.10}$$

where $M^2_{H_{23}}=\frac{1}{2}(M^2_{H_2}+M^2_{H_3})$ and the orthogonality of the O matrix has been used. Since it is $O_{11},O_{31}\ll 1$ in the kine-

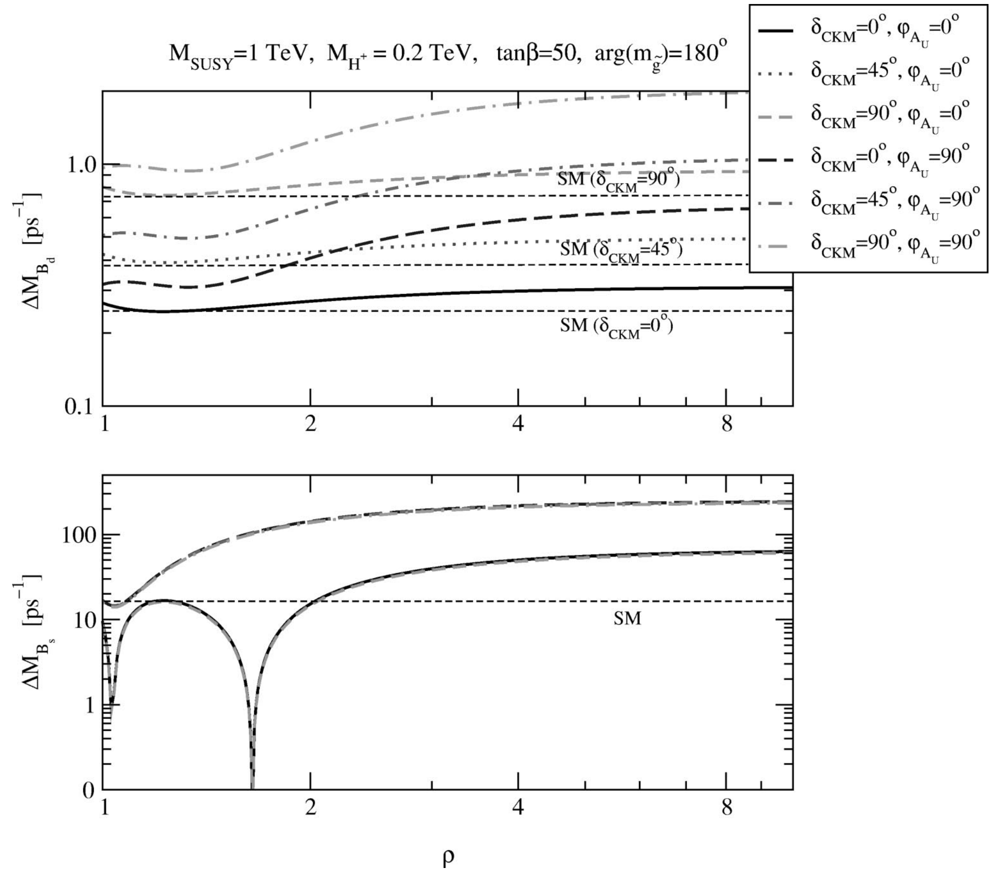

FIG. 6. SM and SUSY Higgs-DP contributions to ΔM_{B_d} and ΔM_{B_s} versus the hierarchy factor ρ, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$, and $\arg m_{\tilde{g}}=180°$, where $\delta_{\rm CKM}$ and ϕ_{A_U} obtain discrete values as shown above. Also shown are the SM effects alone for different choices of the CKM phase $\delta_{\rm CKM}$ (horizontal dashed lines).

matic region of our interest, then on account of Eqs. (4.9) and (4.10) and for maximal CKM phase $\delta_{\rm CKM}=90°$, the dominant contribution to $\Delta M_K^{\rm SUSY}$ and $\epsilon_K^{\rm SUSY}$ comes from the last DP expression in Eq. (4.8), namely from the Wilson coefficient $C_2^{\rm LR(DP)}$ in Eq. (3.8), despite the additional suppression factor $m_d/m_s\approx 1/10$ with respect to $C_1^{\rm SLL\,(DP)}$. If $\delta_{\rm CKM}=0$, $\Delta M_K^{\rm SUSY}$ still receives its largest contribution from $C_2^{\rm LR\,(DP)}$, while $|\epsilon_K^{\rm SUSY}|$ is dominated by the first DP expression in Eq. (4.8), i.e. from $C_1^{\rm SLL\,(DP)}$; the second DP expression in Eq. (4.8) is very suppressed with respect to the first one by two powers of the ratio m_d/m_s. From Fig. 3, we also see that in addition to the CKM phase $\delta_{\rm CKM}$, the soft SUSY-breaking CP phases, such as $\arg(m_{\tilde{g}})$ and $\arg(A_U)$, may also give rise by themselves to enhancements of $|\Delta M_K^{\rm SUSY}|$ and $|\epsilon_K^{\rm SUSY}|$ even up to one order of magnitude. Analogous remarks and observations also hold true for the B-meson FCNC observables which are to be discussed in the next section.

Finally, we should comment on the fact that the 2HDM contribution by itself due to $C_2^{\rm LR\,(2HDM)}$ in Eq. (3.9) can only give rise to the undetectably small numerical values, $|\Delta M_K^{\rm 2HDM}|=5\times 10^{-17}$ MeV and $|\epsilon_K^{\rm 2HDM}|=5\times 10^{-6}$ (indicated by an arrow in Fig. 3), for $\delta_{\rm CKM}=90°$.

B. ΔM_{B_q}, $B_q\to \ell^+\ell^-$ and associated leptonic CP asymmetries

In this section, we will present numerical estimates for a number of B-meson FCNC observables, such as the mass difference ΔM_{B_q}, the branching ratio for $B_q\to\ell^+\ell^-$ and the CP asymmetries associated with the B-meson leptonic decays. The current experimental status of these observables is as follows [43]:

$$\Delta M_{B_d}=0.489\pm 0.008\ \ {\rm ps}^{-1}, \tag{4.11}$$

$$\Delta M_{B_s}>13.1\ \ {\rm ps}^{-1}, \tag{4.12}$$

$$\mathcal{B}(B_s\to\mu^+\mu^-)<2.0\times 10^{-6} \tag{4.13}$$

and [46]

$$\mathcal{B}(B_d\to\tau^+\tau^-)<0.015. \tag{4.14}$$

Future experiments at an upgraded phase of the Tevatron collider may reach higher sensitivity to $\mathcal{B}(B_s\to\mu^+\mu^-)$ up to the 10^{-8} level [11,12,47].

Let us start our discussion by numerically analyzing the B-meson mass differences ΔM_{B_d} and ΔM_{B_s}. As in the case of the K-meson observables, we use the same input values as

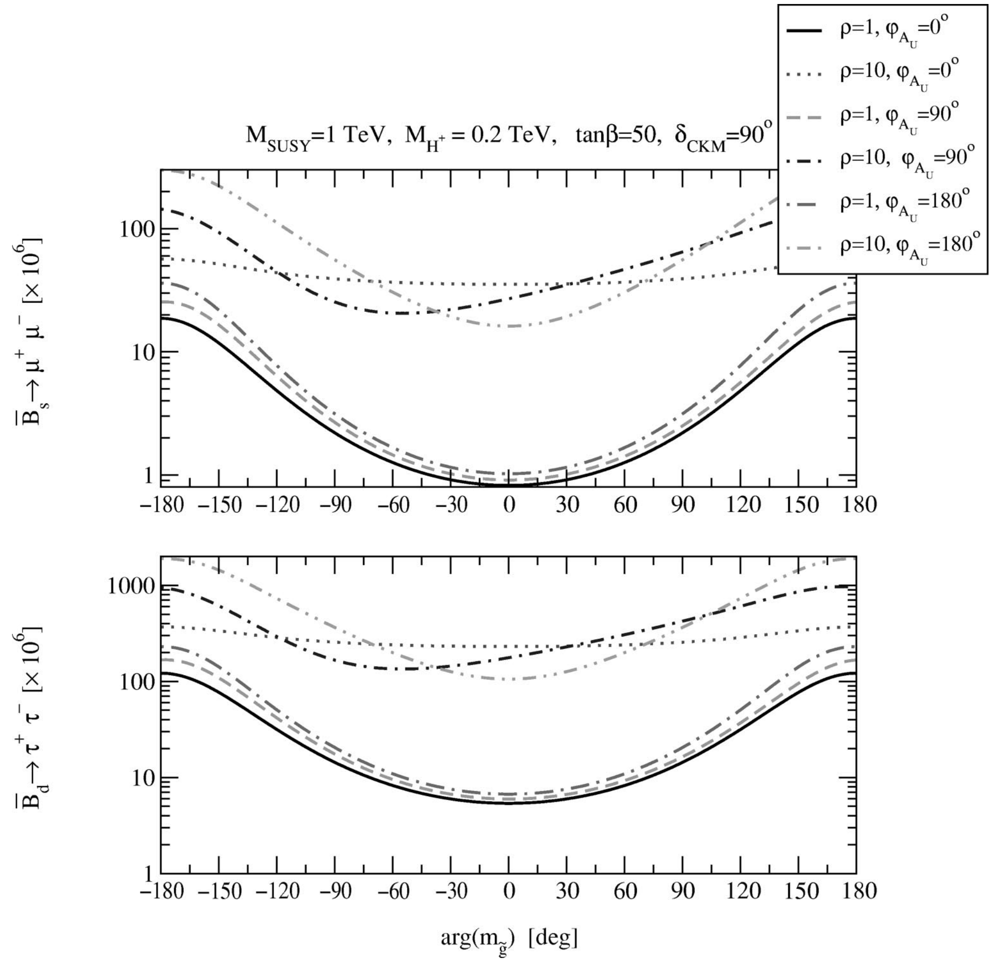

FIG. 7. SUSY Higgs-penguin contributions to $\mathcal{B}(\overline{B}^0_s \to \mu^+\mu^-)$ and $\mathcal{B}(\overline{B}^0_d \to \tau^+\tau^-)$ versus the gluino phase $\arg(m_{\tilde{g}})$, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$, and $\delta_{\rm CKM}=90°$, where ρ and ϕ_{A_U} are varied discretely.

those shown in Fig. 2, i.e. $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV and $\tan\beta=50$. Then, Fig. 5 displays the combined, SM and Higgs-DP, contributions to ΔM_{B_d} and ΔM_{B_s} as functions of the gluino phase $\arg(m_{\tilde{g}})$, for $\delta_{\rm CKM}=90°$ and different choices of hierarchy factor ρ and ϕ_{A_U}. Note that the SM contributions alone for $\delta_{\rm CKM}=90°$ are displayed by horizontal dashed lines. Even though the SM predictions for $\Delta M_{B_{d,s}}$ may adequately describe by themselves the experimental values in Eqs. (4.11) and (4.12), they cannot yet decisively exclude possible new-physics contributions due to the inherent uncertainties in the calculation of hadronic matrix elements such as those induced by SUSY Higgs-mediated FCNC interactions. In particular, we observe in Fig. 5 that SM and Higgs-DP effects may add constructively or destructively to the mass differences $\Delta M_{B_{d,s}}$. Similar features are found in Fig. 6, where the SM and SUSY Higgs-DP contributions to ΔM_{B_d} and ΔM_{B_s} are plotted versus the hierarchy factor ρ, for discrete values of $\delta_{\rm CKM}$ and ϕ_{A_U}. In our SM CKM-phase convention [43], unlike the CKM matrix element V_{ts}, the matrix element V_{td} is very sensitive to $\delta_{\rm CKM}$ values. As a result, the SM predictions for ΔM_{B_d} strongly depend on the selected value of $\delta_{\rm CKM}$, as can be seen from Fig. 6.

In Fig. 7, we exhibit numerical values for the branching ratios $\mathcal{B}(\overline{B}^0_s \to \mu^+\mu^-)$ and $\mathcal{B}(\overline{B}^0_d \to \tau^+\tau^-)$ as functions for the gluino phase $\arg(m_{\tilde{g}})$, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$, and $\delta_{\rm CKM}=90°$, where ρ and ϕ_{A_U} are varied discretely. Since the branching ratios are driven by Higgs-penguin effects in the region of large $\tan\beta$, predictions for other inputs of $\tan\beta$ and M_{H^+} may be easily estimated by rescaling the numerical values by a factor $x_{\mathcal{O}}=(\tan\beta/50)^6\times(0.2\ {\rm TeV}/M_{H^+})^4$, for $\tan\beta\gtrsim 40$. Thus, confronting the predictions for $\mathcal{B}(\overline{B}^0_s \to \mu^+\mu^-)$ with experiment data in Eq. (4.13), combined bounds on the $\tan\beta$-M_{H^+} plane may be obtained for a given set of soft SUSY-breaking parameters. As we see in Fig. 7, these combined bounds become even more restrictive for large gluino phases, $\arg m_{\tilde{g}} \gtrsim 90°$, in agreement with our discussions in Sec. IV A.

However, there is an additional factor that may crucially affect our predictions for the branching ratios of the decays $\overline{B}^0_s \to \mu^+\mu^-$ and $\overline{B}^0_d \to \tau^+\tau^-$, namely the hierarchy parameter ρ. As we show in Fig. 8, even for the extreme choice of a gluino phase $\arg m_{\tilde{g}}=180°$, $\mathcal{B}(\overline{B}^0_d \to \mu^+\mu^-)$ and $\mathcal{B}(\overline{B}^0_s \to \mu^+\mu^-)$ can get very suppressed for a specific value of ρ in certain soft SUSY-breaking scenarios that can realize a

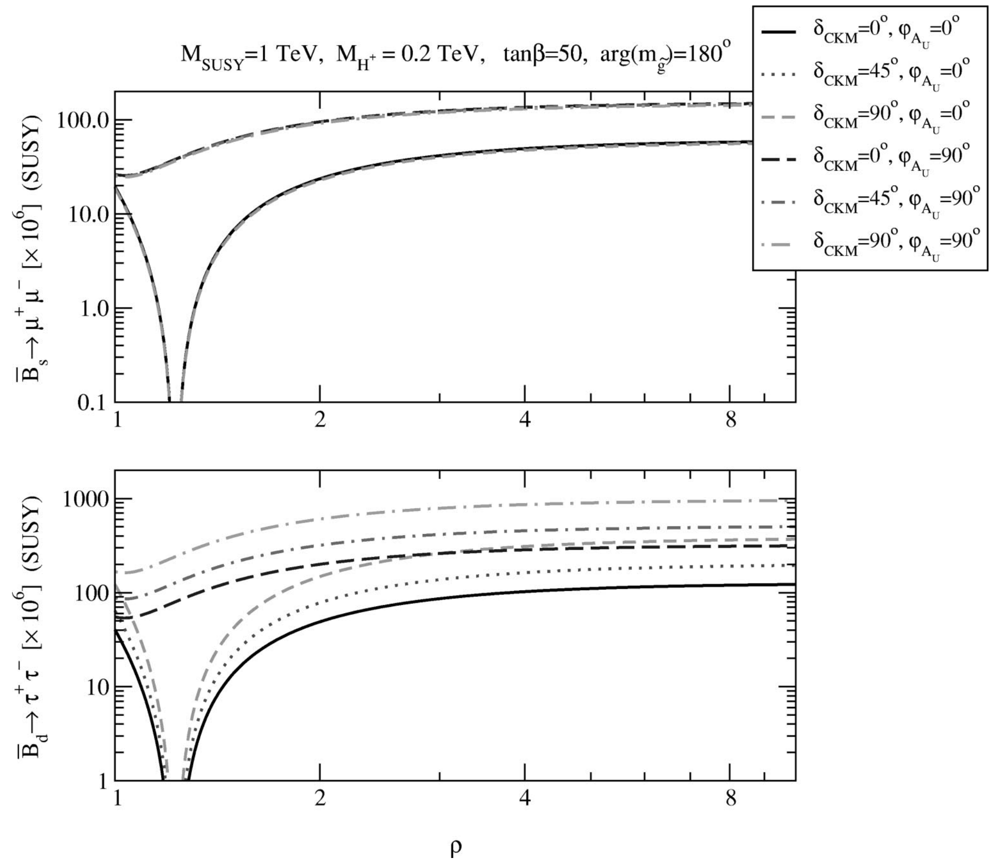

FIG. 8. SUSY Higgs-penguin contributions to $\mathcal{B}(\overline{B}{}^0_s \to \mu^+\mu^-)$ and $\mathcal{B}(\overline{B}{}^0_d \to \tau^+\tau^-)$ as functions of the hierarchy factor ρ, for $M_{\rm SUSY} = 1$ TeV, $M_{H^+} = 0.2$ TeV, $\tan\beta = 50$, and $\arg m_{\tilde g} = 180°$, where $\delta_{\rm CKM}$ and ϕ_{A_U} take discrete values.

GIM-operative point in their parameter space. As we detailed in Sec. IV A, this phenomenon occurs for the universal value of $\rho = \rho_{\rm GIM} = 1.22$, when $\phi_{A_U} - \phi_{\tilde g} = \pm 180°$. As can be seen in Fig. 8, the predicted values for $\overline{B}{}^0_s \to \mu^+\mu^-$ and $\overline{B}{}^0_d \to \tau^+\tau^-$, where $\phi_{A_U} = 0°$, confirm the above observation.

As we have already mentioned, the observables ΔM_{B_s} and $\mathcal{B}(\overline{B}{}^0_s \to \mu^+\mu^-)$ exhibit a different scaling behavior with respect to M_{H^+} and $\tan\beta$, through the scaling factor $x_{\mathcal{O}}$ in Eq. (4.1). Once the above two kinematic parameters are fixed to some input values, the two observables ΔM_{B_s} and $\mathcal{B}(\overline{B}{}^0_s \to \mu^+\mu^-)$ are then rather correlated to each other, since their dependences on the μ parameter and A_U are very similar in the minimal flavor-violating case $\rho = 1$ [14]. As one can see from Figs. 5–8, our numerical analysis agrees well with the above result for $\rho = 1$. However, we also observe that the correlation is practically lost for $\rho > 1$, e.g. close to the GIM-operative points, and/or by the inclusion of CP-violating effects, as ΔM_{B_s} and $\mathcal{B}(\overline{B}{}^0_s \to \mu^+\mu^-)$ have different dependences on the soft CP-odd phases.

Let us now investigate the size of the CP asymmetries in the leptonic B_d-meson decays in the CP-violating MSSM; the corresponding CP asymmetries for the B_s meson are experimentally constrained to be rather small, less than 5% [see also discussion after Eq. (3.32)]. In Fig. 9, we display numerical values for the CP asymmetries $\mathcal{A}_{CP}^{(B^0_d \to \mu^+_L \mu^-_L)}$ and $\mathcal{A}_{CP}^{(B^0_d \to \mu^+_R \mu^-_R)}$ as functions of the gluino phase $\arg(m_{\tilde g})$, for $M_{\rm SUSY} = 1$ TeV, $M_{H^+} = 0.2$ TeV, $\tan\beta = 50$, and $\delta_{\rm CKM} = 90°$. As usual, we independently vary the parameters ρ and ϕ_{A_U} to take on the discrete values $\rho = 1{,}10$ and $\phi_{A_U} = 0°{,}90°$ and $180°$. We find that if the B^0_d-$\overline{B}{}^0_d$ mixing is consistently taken into account, the typical size of $\mathcal{A}_{CP}^{(B^0_d \to \mu^+_L \mu^-_L)}$ and $\mathcal{A}_{CP}^{(B^0_d \to \mu^+_R \mu^-_R)}$ does not exceed 0.7% and 3%, respectively. If B^0_d-$\overline{B}{}^0_d$ mixing is not included, the CP asymmetries can reach slightly higher values up to 1.2% and 6%, respectively. The apparent reason for the smallness of the CP asymmetries is due to the occurrence of an approximate cancellation in the sum $C_S + C_P$ in Eq. (3.32) at large $\tan\beta$, as the muon velocity is $\beta_\mu \approx 1$.

Having gained some insight from the above exercise, one may seek alternative ways to enhance the dimuon asymmetries $\mathcal{A}_{CP}^{(B^0_d \to \mu^+_{L,(R)} \mu^-_{L,(R)})}$. To this end, the first attempt would be to suppress the effect of the B^0_d-$\overline{B}{}^0_d$ mixing by considering smaller $\tan\beta$ values, e.g. $\tan\beta \lesssim 10$. In this intermediate region of $\tan\beta$, the above cancellation in the sum $C_S + C_P$ does not occur due to nontrivial CP-violating Higgs-mixing effects and so the CP asymmetries $\mathcal{A}_{CP}^{(B^0_d \to \mu^+_{L,(R)} \mu^-_{L,(R)})}$ can be significantly increased. To get an idea of the magnitude of the CP asymmetries in this case, we consider the so-called CPX scenario introduced in [48] to maximize CP-violating

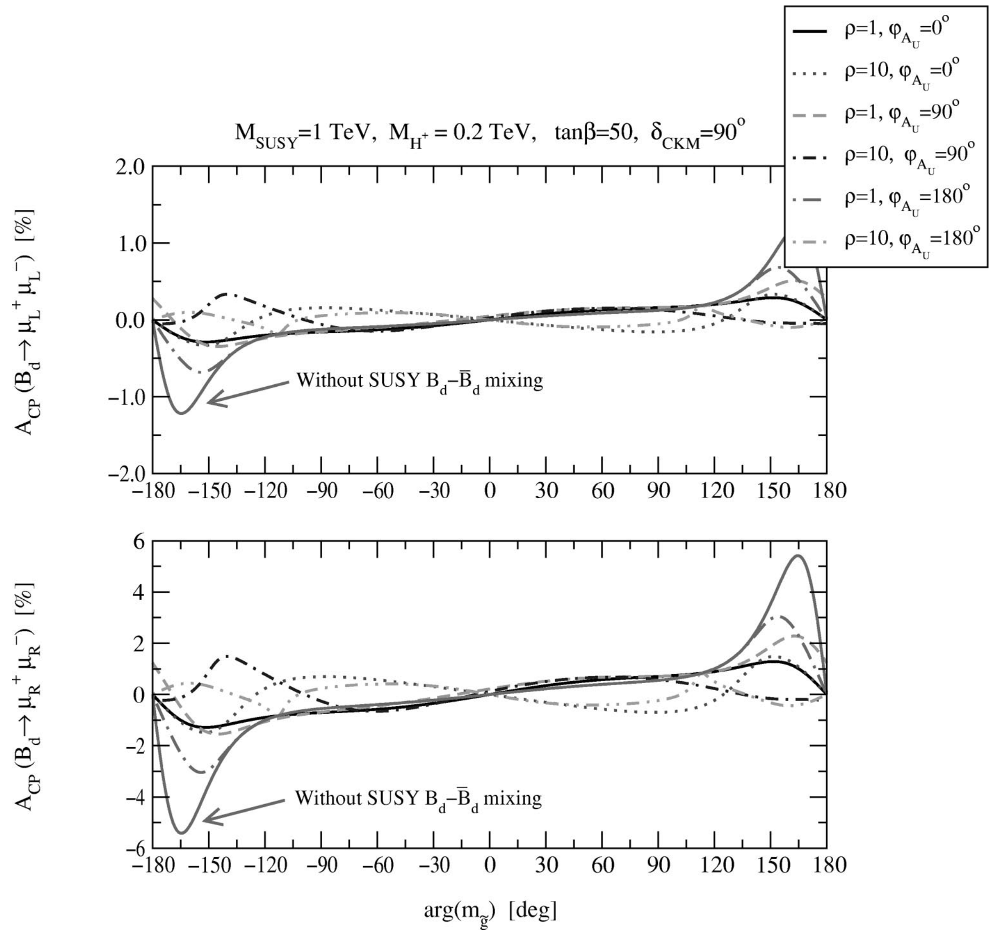

FIG. 9. Numerical values for the CP asymmetries $\mathcal{A}_{CP}^{(B_d^0\to\mu_L^+\mu_L^-)}$ and $\mathcal{A}_{CP}^{(B_d^0\to\mu_R^+\mu_R^-)}$ as functions of the gluino phase $\arg(m_{\tilde g})$, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$, and $\delta_{\rm CKM}=90°$, where ρ and ϕ_{A_U} are varied discretely. Also shown is the prediction for the CP asymmetries without including B_d^0-$\bar{B}_d^0$ mixing.

effects in the lightest Higgs sector of an effective MSSM. In the CPX scenario, the μ-parameter and the soft trilinear Yukawa coupling A_U are set by the relations: $\mu=4M_{\rm SUSY}$ and $A_U=2M_{\rm SUSY}$. Thus, for $M_{H^+}=0.15$ TeV, $M_{\rm SUSY}=1$ TeV, $\tan\beta=7$, $\rho=10$, $\phi_{A_U}=45°$ and $\phi_{\tilde g}=0°$, we find that CP-violating Higgs-penguin effects can give rise to the CP asymmetries:

$$\mathcal{A}_{CP}^{(B_d^0\to\mu_L^+\mu_L^-)}\approx-9\%,\quad \mathcal{A}_{CP}^{(B_d^0\to\mu_R^+\mu_R^-)}\approx-37\% \tag{4.15}$$

where $\mathcal{B}(B_d\to\mu^+\mu^-)=3.6\times10^{-10}$, which is an order of magnitude larger than the SM prediction [41]. Another variant of the CPX scenario of equally phenomenological importance utilizes the parameters: $\tan\beta=10$, $\rho=10$, $\phi_{A_U}=45°$ and $\phi_{\tilde g}=90°$, with $M_{\rm SUSY}=1$ TeV. In this case, we obtain

$$\mathcal{A}_{CP}^{(B_d^0\to\mu_L^+\mu_L^-)}\approx11\%,\quad \mathcal{A}_{CP}^{(B_d^0\to\mu_R^+\mu_R^-)}\approx43\% \tag{4.16}$$

and $\mathcal{B}(B_d\to\mu^+\mu^-)=4.1\times10^{-9}$, which is two orders of magnitude above the SM prediction. The above scenario appears to pass all the experimental constraints, including those deduced from LEP2 analyses of direct Higgs searches [48,49]. Most interestingly, a possible observation of a nonzero CP asymmetry in the leptonic dimuon channel will constitute the harbinger for new physics at B factories. At this stage, it is important to comment on the fact that the numerical values stated in Eqs. (4.15) and (4.16) should be viewed as crude estimates, since they are obtained entirely on the basis of our resummed FCNC effective Lagrangian (2.18) at the intermediate $\tan\beta$ regime. However, in this region of $\tan\beta$, we expect additional one-loop effects to start getting relevant, such as supersymmetric Z-penguin and box diagrams. Even though our initial estimates given above appear to yield rather encouraging results, a complete study of the leptonic B-meson branching ratios and the respective CP asymmetries for all values of $\tan\beta$ would be preferable.

In the case of τ-lepton CP asymmetries, the velocities of the decayed τ-leptons β_τ is roughly 0.5, so one naturally gets an appreciably higher value for the expression $\beta_\tau C_S+C_P\approx0.5C_P$ in Eq. (3.32). As a result, larger values for the τ-lepton CP asymmetries are expected. Indeed, in Fig. 10, we display numerical predictions for $\mathcal{A}_{CP}^{(B_d^0\to\tau_L^+\tau_L^-)}$ and $\mathcal{A}_{CP}^{(B_d^0\to\tau_R^+\tau_R^-)}$ versus the gluino phase $\arg(m_{\tilde g})$, for the same values of the input parameters as in Fig. 9. Then, the CP

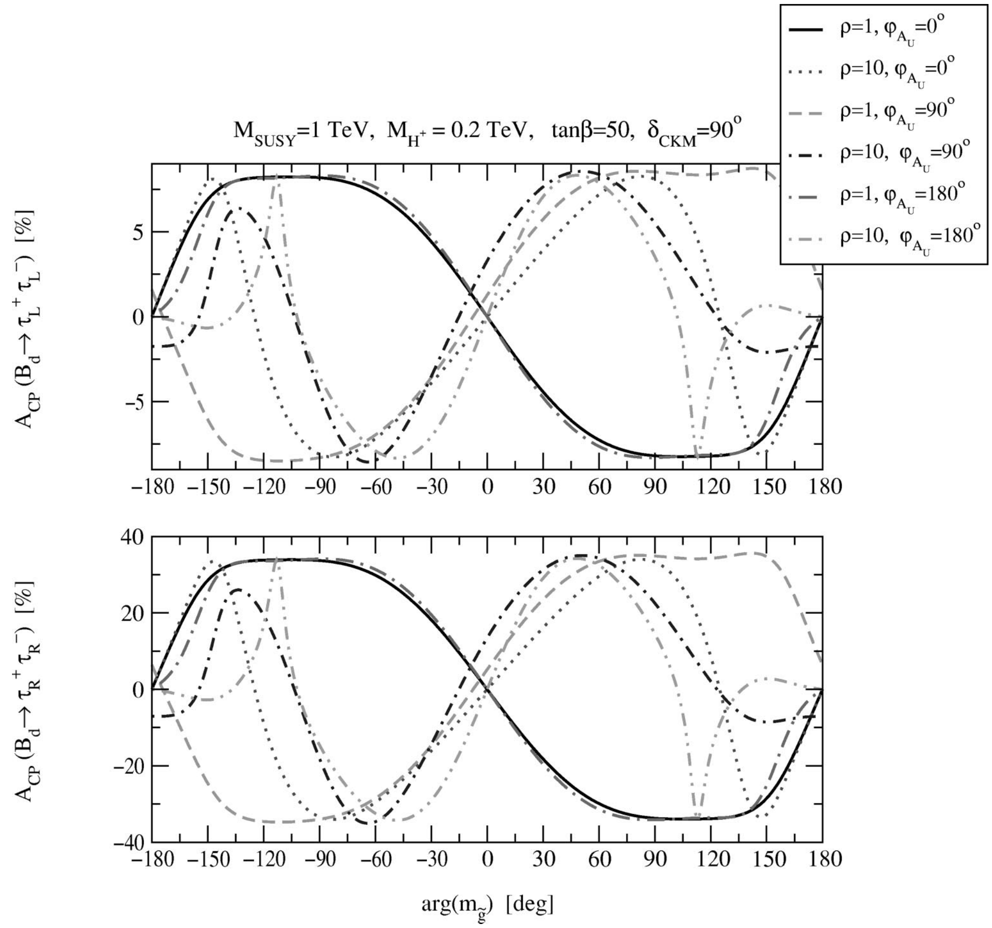

FIG. 10. Numerical estimates of the CP asymmetries $\mathcal{A}_{CP}^{(B_d^0\to\tau_L^+\tau_L^-)}$ and $\mathcal{A}_{CP}^{(B_d^0\to\tau_R^+\tau_R^-)}$ versus the gluino phase $\arg(m_{\tilde{g}})$, for $M_{\rm SUSY}=1$ TeV, $M_{H^+}=0.2$ TeV, $\tan\beta=50$, and $\delta_{\rm CKM}=90°$, where ρ and ϕ_{A_U} take discrete values as shown above.

asymmetries $\mathcal{A}_{CP}^{(B_d^0\to\tau_L^+\tau_L^-)}$ and $\mathcal{A}_{CP}^{(B_d^0\to\tau_R^+\tau_R^-)}$ can be as high as 9% and 36%, respectively.

We conclude this section with some general remarks. In addition to the K- and B-meson observables we have been studying here, there is a large number of other FCNC observables which have to be considered in a combined full-fledged analysis. For example, the decay $B\to X_s\gamma$ [50,51] plays a central role in such a global analysis, because it will enable us to delineate more accurately the CP-conserving or CP-violating soft SUSY-breaking parameter space favored by low-energy FCNC observables. In this context, constraints on CP-violating SUSY phases from the nonobservation of electron and neutron electric dipole moments (EDMs) should also be implemented. Large CP gluino and top squark phases, as the ones considered in our analysis, would require either very heavy squarks with masses larger than 5–6 TeV or the existence of a cancellation mechanism among the different one-, two- and higher-loop EDM contributions [23,52,53]. Especially, it has been shown recently [23] that if the first two generation of squarks are heavier than about 3 TeV, the required degree of cancellations does not exceed the 10% level and hence large CP-violating gluino, gaugino and third-generation phases are still allowed for wide regions of the MSSM parameter space. Finally, in the present numerical analysis, we have concentrated on scenarios that minimally depart from the minimal flavor-violation assumption through the presence of diagonal, but nonuniversal squark masses. In the most general case, however, the squark mass matrices and consequently the resummation matrix $\mathbf{R}$ of the radiative threshold effects may not be diagonal. Such low-energy realizations with off-diagonal soft squark-mass matrices can still be treated exactly within the context of our resummed FCNC effective Lagrangian (2.18), by appropriately considering nontrivial quark-squark CKM-like matrices, such as the 3-by-3 unitary matrix $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ in Eq. (2.9).

V. CONCLUSIONS

We have derived the general form for the effective Lagrangian of Higgs-mediated FCNC interactions to d-type quarks, where large-$\tan\beta$ radiative threshold effects have been resummed consistently [cf. Eqs. (2.18) and (2.19)]. Our resummed FCNC effective Lagrangian is free from pathological singularities, which mainly emanate from the top-quark dominance hypothesis frequently adopted in the literature, and has been appropriately generalized to include effects of nonuniversality in the squark sector, as well as

CP-violation effects originating from the CKM-mixing matrix and the complex soft SUSY-breaking masses. In particular, our resummed effective Lagrangian can be applied to study Higgs-mediated FCNC effects in more general soft SUSY-breaking scenarios, beyond those that have already been discussed within the restricted framework of models with minimal flavor violation. Also, an approach to resumming radiative threshold effects, very analogous to the one developed in Sec. II, can straightforwardly be applied to see-saw SUSY models, so as to properly describe Higgs-mediated lepton-flavor-violating interactions.

Within the context of generic soft SUSY-breaking scenarios, we have analyzed a number of *K*- and *B*-meson observables, such as $\Delta M_{K,B}$, ϵ_K, ϵ'/ϵ, $\mathcal{B}(B_{s,d}\to\ell^+\ell^-)$ and their associated leptonic asymmetries [54], which are enhanced by Higgs-boson FCNC interactions for large values of $\tan\beta$. We have found that the predictions crucially depend on the choice of soft *CP*-violating phases in a given set of soft SUSY-breaking parameters. For example, for certain values of the gluino and top squark phases, the predictions can reach and even exceed the current experimental limits, whereas for other values of the *CP*-odd phases the FCNC effects can be reduced by one or even two orders of magnitude. Most remarkably, we have been able to identify configurations in the soft SUSY-breaking parameter space, such as $\rho_{\rm GIM}$, where a kind of a GIM-cancellation mechanism becomes fully operative [cf. Eq. (4.7)] and, as a result of the latter, all Higgs-mediated, $\tan\beta$-enhanced effects on *K*- and *B*-meson FCNC observables are completely absent.

Based on our resummed effective Lagrangian, one may now carry over the present analysis to a vast number of other *K*- and *B*-meson observables. Evidently, further dedicated studies need be performed in this direction. We expect the obtained predictions to affect other low- and high-energy observables, such as measurements of electron and neutron electric dipole moments and Higgs-boson searches, as well as studies on cosmological electroweak baryogenesis and dark matter. It would be very interesting to determine to which degree the emerging *CP*-violating MSSM framework with *CP*-mixed Higgs bosons mediating $\tan\beta$-enhanced interactions to matter could be potentially responsible for all the present and future *CP*-conserving or *CP*-violating FCNC effects observed in nature.

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[1] S.L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).
[2] T.P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987); G.C. Branco, A.J. Buras, and J.M. Gerard, Nucl. Phys. **B259**, 306 (1985).
[3] T. Banks, Nucl. Phys. **B303**, 172 (1988); E. Ma, Phys. Rev. D **39**, 1922 (1989).
[4] R. Hempfling, Phys. Rev. D **49**, 6168 (1994); L.J. Hall, R. Rattazzi, and U. Sarid, *ibid.* **50**, 7048 (1994); T. Blazek, S. Raby, and S. Pokorski, *ibid.* **52**, 4151 (1995); M. Carena, M. Olechowski, S. Pokorski, and C.E.M. Wagner, Nucl. Phys. **B426**, 269 (1994); S. Heinemeyer, W. Hollik, and G. Weiglein, Phys. Lett. B **440**, 296 (1998); Eur. Phys. J. C **9**, 343 (1999); F. Borzumati, G. Farrar, N. Polonsky, and S. Thomas, Nucl. Phys. **B555**, 53 (1999); J.R. Espinosa and R.J. Zhang, *ibid.* **B586**, 3 (2000).
[5] C.S. Huang and Q.-S. Yan, Phys. Lett. B **442**, 209 (1998); S.R. Choudhury and N. Gaur, *ibid.* **451**, 86 (1999); C.S. Huang, W. Liao, and Q.-S. Yuan, Phys. Rev. D **59**, 011701 (1999); C. Hamzaoui, M. Pospelov, and M. Toharia, *ibid.* **59**, 095005 (1999).
[6] K.S. Babu and C. Kolda, Phys. Rev. Lett. **84**, 228 (2000).
[7] M. Carena, D. Garcia, U. Nierste, and C.E. Wagner, Nucl. Phys. **B577**, 88 (2000).
[8] P.H. Chankowski and Lucja Slawianowska, Phys. Rev. D **63**, 054012 (2001); C.S. Huang, W. Liao, Q.-S. Yuan, and S.-H. Zhu, *ibid.* **63**, 114021 (2001).
[9] C. Bobeth, T. Ewerth, F. Krüger, and J. Urban, Phys. Rev. D **64**, 074014 (2001).
[10] G. Isidori and A. Retico, J. High Energy Phys. **11**, 001 (2001).
[11] A. Dedes, H.K. Dreiner, and U. Nierste, Phys. Rev. Lett. **87**, 251804 (2001); A. Dedes, H.K. Dreiner, U. Nierste, and P. Richardson, hep-ph/0207026.
[12] R. Arnowitt, B. Dutta, T. Kamon, and M. Tanaka, Phys. Lett. B **538**, 121 (2002); S. Baek, P. Ko, and W.Y. Song, hep-ph/0208112.
[13] G. D’Ambrosio, G.F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B645**, 155 (2002).
[14] A.J. Buras, P.H. Chankowski, J. Rosiek, and L. Slawianowska, Phys. Lett. B **546**, 96 (2002).
[15] J. K. Mizukoshi, X. Tata, and Y. Wang, Phys Rev. D **66**, 115003 (2002).
[16] T. Ibrahim and P. Nath, hep-ph/0208142.
[17] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[18] S.L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
[19] M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, Nucl. Phys. **B586**, 92 (2000).
[20] For example, gaugino-Higgsino mixing effects studied in [15] and [16] can be straightforwardly included in our formalism.
[21] A. Pilaftsis, Phys. Rev. D **58**, 096010 (1998); Phys. Lett. B **435**, 88 (1998).
[22] A. Pilaftsis and C.E.M. Wagner, Nucl. Phys. **B553**, 3 (1999); D.A. Demir, Phys. Rev. D **60**, 055006 (1999); S.Y. Choi, M. Drees, and J.S. Lee, Phys. Lett. B **481**, 57 (2000); G.L. Kane and L.-T. Wang, *ibid.* **488**, 383 (2000); T. Ibrahim and P. Nath,

Phys. Rev. D **63**, 035009 (2001); **66**, 015005 (2002); S.Y. Choi, K. Hagiwara, and J.S. Lee, *ibid.* **64**, 032004 (2001); S. Heinemeyer, Eur. Phys. J. C **22**, 521 (2001); M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, Nucl. Phys. **B625**, 345 (2002); M. Boz, Mod. Phys. Lett. A **17**, 215 (2002); S.W. Ham, S.K. Oh, E.J. Yoo, C.M. Kim, and D. Son, hep-ph/0205244.

[23] A. Pilaftsis, Nucl. Phys. **B644**, 263 (2002).

[24] From a Feynman-diagrammatic point of view, resummation of radiative threshold corrections is equivalent to resumming wave-function graphs in the on-shell renormalization scheme. For a detailed discussion on this issue, see H.E. Logan and U. Nierste, Nucl. Phys. **B586**, 39 (2000) and [7].

[25] For reviews on kaon physics, see, E.A. Paschos and U. Türke, Phys. Rep. **178**, 145 (1989); W. Grimus, Fortschr. Phys. **36**, 201 (1988); R. Decker, *ibid.* **37**, 657 (1989); L. Lavoura, Ann. Phys. (N.Y.) **207**, 428 (1991); B. Weinstein and L. Wolfenstein, Rev. Mod. Phys. **65**, 1113 (1993).

[26] For pedagogical introductions to CP violation in the B-meson system, see M. Neubert, Int. J. Mod. Phys. A **11**, 4173 (1996); A.J. Buras, hep-ph/9806471.

[27] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation* (Clarendon, Oxford, UK, 1999), p. 511.

[28] A.J. Buras, S. Jäger, and J. Urban, Nucl. Phys. **B605**, 600 (2001).

[29] A.J. Buras, P.H. Chankowski, J. Rosiek, and L. Slawianowska, Nucl. Phys. **B619**, 434 (2001).

[30] M. Ciuchini *et al.*, J. High Energy Phys. **07**, 013 (2001).

[31] A.J. Buras, M. Misiak, and J. Urban, Nucl. Phys. **B586**, 397 (2000).

[32] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto, and J. Reyes, Nucl. Phys. B (Proc. Suppl.) **106**, 385 (2002).

[33] G. Buchalla, A.J. Buras, and M.K. Harlander, Nucl. Phys. **B337**, 313 (1990).

[34] T. Hambye, G.O. Kohler, E.A. Paschos, P.H. Soldan, and W.A. Bardeen, Phys. Rev. D **58**, 014017 (1998); T. Hambye, G.O. Kohler, E.A. Paschos, and P.H. Soldan, Nucl. Phys. **B564**, 391 (2000).

[35] S. Bosch, A.J. Buras, M. Gorbahn, S. Jäger, M. Jamin, M.E. Lautenbacher, and L. Silvestrini, Nucl. Phys. **B565**, 3 (2000).

[36] E. Gabrielli and G.F. Giudice, Nucl. Phys. **B433**, 3 (1995).

[37] G. Colangelo and G. Isidori, J. High Energy Phys. **09**, 009 (1998); A.J. Buras, G. Colangelo, G. Isidori, A. Romanino, and L. Silvestrini, Nucl. Phys. **B566**, 3 (2000).

[38] A.L. Kagan and M. Neubert, Phys. Rev. Lett. **83**, 4929 (1999); A.J. Buras, P. Gambino, M. Gorbahn, S. Jäger, and L. Silvestrini, Nucl. Phys. **B592**, 55 (2001); C.H. Chen, Phys. Lett. B **541**, 155 (2002).

[39] Y. Nir, hep-ph/0208080.

[40] A. Dedes, J. Ellis, and M. Raidal, Phys. Lett. B **549**, 159 (2002).

[41] See second reference in [11].

[42] C.S. Huang and W. Liao, Phys. Lett. B **525**, 107 (2002); **538**, 301 (2002); P.H. Chankowski and L. Slawianowska, Acta Phys. Pol. B **32**, 1895 (2001).

[43] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[44] For instance, see S. Herrlich and U. Nierste, Phys. Rev. D **52**, 6505 (1995).

[45] See, for example, J.F. Donoghue, H.P. Nilles, and D. Wyler, Phys. Lett. **128B**, 55 (1983); J.M. Gerard, W. Grimus, A. Raychaudhuri, and G. Zoupanos, *ibid.* **140B**, 349 (1984).

[46] Y. Grossman, Z. Ligeti, and E. Nardi, Phys. Rev. D **55**, 2768 (1997).

[47] K. Anikeev *et al.*, hep-ph/0201071.

[48] M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, Phys. Lett. B **495**, 155 (2000).

[49] For a recent review on Higgs phenomenology, see M. Carena and H.E. Haber, hep-ph/0208209.

[50] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991); F.M. Borzumati, Z. Phys. C **63**, 291 (1994).

[51] G. Degrassi, P. Gambino, and G.F. Giudice, J. High Energy Phys. **12**, 009 (2000); M. Carena, D. Garcia, U. Nierste, and C.E. Wagner, Phys. Lett. B **499**, 141 (2001); D.A. Demir and K.A. Olive, Phys. Rev. D **65**, 034007 (2002).

[52] T. Ibrahim and P. Nath, Phys. Rev. D **58**, 111301 (1998); **61**, 093004 (2000); M. Brhlik, G.J. Good, and G.L. Kane, *ibid.* **59**, 115004 (1999); S. Pokorski, J. Rosiek, and C.A. Savoy, Nucl. Phys. **B570**, 81 (2000); E. Accomando, R. Arnowitt, and B. Dutta, Phys. Rev. D **61**, 115003 (2000); A. Bartl, T. Gajdosik, W. Porod, P. Stockinger, and H. Stremnitzer, *ibid.* **60**, 073003 (1999); T. Falk, K.A. Olive, M. Pospelov, and R. Roiban, Nucl. Phys. **B60**, 3 (1999).

[53] D. Chang, W.-Y. Keung, and A. Pilaftsis, Phys. Rev. Lett. **82**, 900 (1999); S.A. Abel, S. Khalil, and O. Lebedev, Nucl. Phys. **B606**, 151 (2001).

[54] Our numerical analysis has been performed with the FORTRAN code FCNC.f, which can be accessed from the web site: http://pilaftsi.home.cern.ch/pilaftsi/