Constraining supersymmetric models from B_d-$\overline{B}_d$ mixing and the $B_d \to J/\psi K_S$ asymmetry

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(Received 2 August 2002; published 29 January 2003)

We analyze the chargino contributions to B_d-$\overline{B}_d$ mixing and CP asymmetry of the $B_d \to J/\psi K_S$ decay, in the framework of the mass insertion approximation. We derive model independent bounds on the relevant mass insertions. Moreover, we study these contributions in supersymmetric models with minimal flavor violation, Hermitian flavor structure, and small CP violating phases and universal strength Yukawa couplings. We show that, in supersymmetric models with large flavor mixing, the observed values of $\sin 2\beta$ may be entirely due to the chargino–up-squark loops.

DOI: 10.1103/PhysRevD.67.015008 PACS number(s): 12.60.Jv, 11.30.Er, 13.25.Hw

I. INTRODUCTION

Since its discovery in 1964 in K-meson decays, the origin of CP violation has remained an open question in particle physics. In the standard model (SM), the phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix provides an explanation of the CP violating effect in these decays. Although the SM is able to account for the observed CP violation in the kaon system and the recent measurement of the (time-dependent) CP asymmetry in $B_d \to J/\psi K_S$ decays, new CP violating sources are necessarily required to describe the observed baryon asymmetry [1]. Moreover, it is expected that with B factories the B system will represent an ideal framework for crucial tests of CP violation in the SM and probing new physics effects at low energy.

It is a common feature for any new physics beyond the SM to possess additional CP violating phases in addition to the δ_{CKM} phase. In supersymmetric (SUSY) models, the soft SUSY breaking terms contain several parameters that may be complex, as also may be the SUSY preserving μ parameter. These new phases have significant implications for the electric dipole moment (EDM) of the electron, neutron, and mercury atom [2]. It was shown that the EDM can be suppressed in SUSY models with small CP phases [3,4] or in SUSY models with flavor off-diagonal CP violation [3,5].

The idea of having small CP phases ($\lesssim 10^{-2}$) as an approximate CP symmetry at low energy could be an interesting possibility if supported by a mechanism of CP symmetry restoration at high energy scale. However, this mechanism might also imply that the δ_{CKM} phase is small [6]. The large asymmetry of the B-meson decay $a_{J/\psi K_S}$ observed by BaBar and Belle experiments [7] are in agreement with SM predictions for large δ_{CKM}, and thus the idea of small phases might be disfavored.

However, in Ref. [8] it was shown that in the framework of the minimal supersymmetric standard model (MSSM) with nonuniversal soft terms and large flavor mixing in the Yukawa coupling, supersymmetry can give the leading contribution to $a_{J/\psi K_S}$ with simultaneous account of the experimental results in the K system. Thus the supersymmetric models with small CP violating phases at high energy scales are still phenomenologically viable. An alternative possibility for suppressing the EDMs is that SUSY CP phases have a flavor off-diagonal character as in the SM [5,9]. Such models would allow for phases of order $\mathcal{O}(1)$ which may have significant effects in B physics [5].

A useful tool for analyzing SUSY contributions to flavor changing neutral current (FCNC) processes is provided, as known, by the mass insertion method [10]. One chooses a basis for the fermion and sfermion states where all the couplings of these particles to neutral gauginos are flavor diagonal, leaving all the sources of FC inside the off-diagonal terms of the sfermion mass matrix. These terms are denoted by $(\Delta^q_{AB})^{ij}$, where as usual $A,B=(L,R)$ and $i,j=1,3$ indicate chiral and flavor indices, respectively, and $q=u,d$. The sfermion propagator is then expanded as a series of $(\delta^q_{AB})_{ij} = (\Delta^q_{AB})^{ij}/\tilde{m}^2$, where $\tilde{m}^2$ is an average sfermion mass. This method allows one to parametrize, in a model independent way, the main sources of flavor violations in SUSY models. In this framework, the gluino and chargino contributions to the K system have been analyzed in Refs. [10] and [11], respectively. These analyses showed that the bounds on the imaginary parts of mass insertions, coming from gluino exchanges to ε_K and ε'/ε, are very severe [10], while the corresponding ones from chargino exchanges are less constrained [11]. In particular, in order to saturate ε_K from the gluino contributions one should have [10] $\sqrt{|\mathrm{Im}(\delta^d_{12})^2_{LL}|} \sim 10^{-3}$ or $\sqrt{|\mathrm{Im}(\delta^d_{12})^2_{LR}|} \sim 10^{-4}$, and $\sqrt{|\mathrm{Im}(\delta^d_{12})^2_{LR}|} \sim 10^{-5}$ from ε'/ε, while chargino contributions require [11] $\mathrm{Im}(\delta^u_{12})^2_{LL} \sim 10^{-2}$, for average squark masses of the order of 500 GeV and gluino masses of the same order.

Recently, in the framework of the mass insertion approximation, gluino contributions to the B_d-$\overline{B}_d$ mixing and CP asymmetry in the decay $B_d \to J/\psi K_S$ were analyzed by including next-to-leading order (NLO) QCD corrections [12] (see also Ref. [13]). However, an analogous study for chargino contributions to these processes is still missing.

This kind of analysis would be interesting for the following reasons. First, it would provide a new set of upper bounds on the mass insertion parameters, namely, $(\delta^u_{ij})_{AB}$ in the up-squark sector, which are complementary to the ones obtained from gluino exchanges [which constrain only $(\delta^d_{ij})_{AB}$]. Second, upper bounds on $(\delta^u_{ij})_{AB}$ would be very useful in order to perform easy tests on SUSY models which receive from chargino exchanges the main contributions to B_d-$\bar{B}_d$ mixing and CP asymmetry. Indeed, in many SUSY scenarios the gluino exchanges are always subleading.

In this paper we focus on the dominant chargino contributions to the B_d-$\bar{B}_d$ mixing and CP asymmetry $a_{J/\psi K_S}$. We use the mass insertion method and derive the corresponding bounds on the relevant mass insertion parameters. We perform this analysis at NLO accuracy in QCD by using the results available in Ref. [12]. As an application of our analysis, we also provide a comparative study for supersymmetric models with minimal flavor violation, Hermitian flavor structure with small CP violating phases and universal strength of Yukawa couplings. We show that in all these scenarios, by comparing $(\delta^u_{ij})_{AB}$ and $(\delta^d_{ij})_{AB}$ with their corresponding upper bounds, the chargino contributions are dominant over the gluino ones.

The paper is organized as follows. In Sec. II we present the supersymmetric contributions to B_d-$\bar{B}_d$ mixing and CP asymmetry $a_{J/\psi K_S}$. We start with a brief review on gluino contributions and then we present our results for the chargino ones, both in the mass insertion approach. In Sec. III we derive model independent bounds on the relevant mass insertions involved in the B_d-$\bar{B}_d$ mixing and $a_{J/\psi K_S}$. In Sec. IV we generalize these results by including the case of a light top-squark (stop) right. Section V is devoted to the study of the supersymmetric contribution to $a_{J/\psi K_S}$ in three different supersymmetric models. We show that the observed values of $\sin 2\beta$ may be entirely due to the chargino–up-squark loops in some classes of these models. Our conclusions are presented in Sec. VI.

II. SUPERSYMMETRIC CONTRIBUTIONS TO $\Delta B=2$ TRANSITIONS

We start this section by summarizing the main results on B_d-$\bar{B}_d$ mixing and CP asymmetry $a_{J/\psi K_S}$, then we will consider the relevant SUSY contributions to the effective Hamiltonian for $\Delta B=2$ transitions, given by the chargino and gluino box diagram exchanges.

In the B_d and $\bar{B}_d$ systems, the flavor eigenstates are given by $B_d=(\bar{b}d)$ and $\bar{B}_d=(b\bar{d})$. It is customary to denote the corresponding mass eigenstates by $B_H=pB_d+q\bar{B}_d$ and $B_L=pB_d-q\bar{B}_d$ where the indices H and L refer to heavy and light mass eigenstates respectively, and $p=(1+\bar{\varepsilon}_B)/\sqrt{2(1+|\bar{\varepsilon}_B|)}$, $q=(1-\bar{\varepsilon}_B)/\sqrt{2(1+|\bar{\varepsilon}_B|)}$ where $\bar{\varepsilon}_B$ is the corresponding CP violating parameter in the B_d-$\bar{B}_d$ system, analogous to $\bar{\varepsilon}$ in the kaon system [14]. Then the strength of B_d-$\bar{B}_d$ mixing is described by the mass difference

$$\Delta M_{B_d}=M_{B_H}-M_{B_L}, \tag{1}$$

whose present experimental value is $\Delta M_{B_d}=0.484\pm0.010\ (\mathrm{ps})^{-1}$ [14].

The CP asymmetry of the B_d and $\bar{B}_d$ meson decay to the CP eigenstate ψK_S is given by

$$\begin{aligned} a_{\psi K_S}(t) &= \frac{\Gamma(B^0_d(t)\to\psi K_S)-\Gamma(\bar{B}^0_d(t)\to\psi K_S)}{\Gamma(B^0_d(t)\to\psi K_S)+\Gamma(\bar{B}^0_d(t)\to\psi K_S)} \\ &= -a_{\psi K_S}\sin(\Delta m_{B_d} t). \end{aligned} \tag{2}$$

The most recent measurements of this asymmetry are given by [7]

$$a_{\psi K_S}=\begin{cases} 0.59\pm0.14\pm0.05 & (\text{BaBar}),\\ 0.99\pm0.14\pm0.06 & (\text{Belle}),\end{cases} \tag{3}$$

where the second and third numbers correspond to statistic and systematic errors, respectively, and so the present world average is given by $a_{\psi K_S}=0.79\pm12$. These results show that there is a large CP asymmetry in the B-meson system. This implies that either the CP is not an approximate symmetry in nature and that the CKM mechanism is the dominant source of CP violation [5] or CP is an approximate symmetry with large flavor structure beyond the standard CKM matrix [8]. Generally, ΔM_{B_d} and $a_{\psi K_S}$ can be calculated via

$$\Delta M_{B_d}=2|\langle B^0_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}^0_d\rangle|, \tag{4}$$

$$a_{\psi K_S}=\sin 2\beta_{\text{eff}} \quad \text{and} \quad \beta_{\text{eff}}=\frac{1}{2}\arg\langle B^0_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}^0_d\rangle, \tag{5}$$

where $H^{\Delta B=2}_{\text{eff}}$ is the effective Hamiltonian responsible for the $\Delta B=2$ transitions. In the framework of the standard model, $a_{\psi K_S}$ can be easily related to one of the inner angles of the unitarity triangles and parametrized by the V_{CKM} elements as follows:

$$a^{\text{SM}}_{\psi K_S}=\sin 2\beta, \quad \beta=\arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right). \tag{6}$$

In supersymmetric theories the effective Hamiltonian for $\Delta B=2$ transitions can be generated, in addition to the W box diagrams of the SM, through other box diagrams mediated by charged Higgs boson, neutralino, photino, gluino, and chargino exchanges. The Higgs boson contributions are suppressed by the quark masses and can be neglected. The neutralino and photino exchange diagrams are also very suppressed compared to the gluino and chargino ones, due to their electroweak neutral couplings to fermion and sfermions. Thus, the dominant SUSY contribution to the off-diagonal entry in the B-meson mass matrix $\mathcal{M}_{12}(B_d)=\langle B^0_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}^0_d\rangle$ is given by

$$\mathcal{M}_{12}(B_d)=\mathcal{M}^{\text{SM}}_{12}(B_d)+\mathcal{M}^{\tilde{g}}_{12}(B_d)+\mathcal{M}^{\tilde{\chi}^+}_{12}(B_d), \tag{7}$$

where $\mathcal{M}_{12}^{\rm SM}(B_d)$, $\mathcal{M}_{12}^{\tilde{g}}(B_d)$, and $\mathcal{M}_{12}^{\tilde{\chi}^+}$ indicate the SM, gluino, and chargino contributions, respectively. The SM contribution is known at NLO accuracy in QCD [14] (as well as the leading SUSY contributions [12]) and it is given by

$$\mathcal{M}_{12}^{\rm SM}(B_d)=\frac{G_F^2}{12\pi^2}\,\eta_B\hat{B}_{B_d}f_{B_d}^2M_{B_d}M_W^2(V_{td}V_{tb}^*)^2S_0(x_t), \tag{8}$$

where f_{B_d} is the B-meson decay constant, $\hat{B}_{B_d}$ is the renormalization group invariant B parameter (for its definition and numerical value, see Ref. [14] and references therein), and $\eta=0.55\pm0.01$. The function $S_0(x_t)$, connected to the $\Delta B=2$ box diagram with W exchange, is given by

$$S_0(x_t)=\frac{4x_t-11x_t^2+x_t^3}{4(1-x_t)^2}-\frac{3x_t^3\ln x_t}{2(1-x_t)^3}, \tag{9}$$

where $x_t=M_t^2/M_W^2$.

The effect of supersymmetry can be simply described by a dimensionless parameter r_d^2 and a phase $2\theta_d$ defined as follows:

$$r_d^2e^{2i\theta_d}=\frac{\mathcal{M}_{12}(B_d)}{\mathcal{M}_{12}^{\rm SM}(B_d)}, \tag{10}$$

where $\Delta M_{B_d}=2|\mathcal{M}_{12}^{\rm SM}(B_d)|r_d^2$. Thus, in the presence of SUSY contributions, the CP asymmetry $B_d\to\psi K_S$ is modified, and now we have

$$a_{\psi K_S}=\sin 2\beta_{\rm eff}=\sin(2\beta+2\theta_d). \tag{11}$$

Therefore, the measurement of $a_{\psi K_S}$ would not determine $\sin 2\beta$ but rather $\sin 2\beta_{\rm eff}$, where

$$2\theta_d=\arg\left(1+\frac{\mathcal{M}_{12}^{\rm SUSY}(B_d)}{\mathcal{M}_{12}^{\rm SM}(B_d)}\right), \tag{12}$$

and $\mathcal{M}_{12}^{\rm SUSY}(B_d)=\mathcal{M}_{12}^{\tilde{g}}(B_d)+\mathcal{M}_{12}^{\tilde{\chi}^+}(B_d)$.

A. Gluino contributions

The most general effective Hamiltonian for $\Delta B=2$ processes induced by gluino and chargino exchanges through $\Delta B=2$ box diagrams can be expressed as

$$H_{\rm eff}^{\Delta B=2}=\sum_{i=1}^{5}C_i(\mu)Q_i(\mu)+\sum_{i=1}^{3}\tilde{C}_i(\mu)\tilde{Q}_i(\mu)+\text{H.c.}, \tag{13}$$

where $C_i(\mu),\tilde{C}_i(\mu)$ and $Q_i(\mu),\tilde{Q}_i(\mu)$ are the Wilson coefficients and operators, respectively, renormalized at the scale μ, with

$$Q_1=\overline{d}_L^{\alpha}\gamma_\mu b_L^{\alpha}\overline{d}_L^{\beta}\gamma_\mu b_L^{\beta},\quad Q_2=\overline{d}_R^{\alpha}b_L^{\alpha}\overline{d}_R^{\beta}b_L^{\beta},\quad Q_3=\overline{d}_R^{\alpha}b_L^{\beta}\overline{d}_R^{\beta}b_L^{\alpha},$$
$$Q_4=\overline{d}_R^{\alpha}b_L^{\alpha}\overline{d}_L^{\beta}b_R^{\beta},\quad Q_5=\overline{d}_R^{\alpha}b_L^{\beta}\overline{d}_L^{\beta}b_R^{\alpha}. \tag{14}$$

In addition, the operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging $L\leftrightarrow R$.

Now we summarize the main results for gluino contributions to the above Wilson coefficients at SUSY scale, in the framework of the mass insertion approximation. As we will show in the next section, in order to connect the Wilson coefficients at SUSY scale with the corresponding ones at the low energy scale $\mu\simeq\mathcal{O}(m_b)$, the renormalization group equations for QCD corrections must be solved. In the case of the gluino exchange all the above operators give significant contributions and the corresponding Wilson coefficients are given by [10,12]

$$\begin{aligned}
C_1(M_S)&=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}[24xf_6(x)+66\tilde{f}_6(x)](\delta_{13}^d)_{LL}^2,\\
C_2(M_S)&=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}204xf_6(x)(\delta_{13}^d)_{RL}^2,\\
C_3(M_S)&=\frac{\alpha_s^2}{216m_{\tilde{q}}^2}36xf_6(x)(\delta_{13}^d)_{RL}^2,\\
C_4(M_S)&=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}\{[504xf_6(x)-72\tilde{f}_6(x)]\\
&\quad\times(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR}-132\tilde{f}_6(x)\\
&\quad\times(\delta_{13}^d)_{LR}(\delta_{13}^d)_{RL}\},\\
C_5(M_S)&=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}\{[24xf_6(x)+120\tilde{f}_6(x)]\\
&\quad\times(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR}-180\tilde{f}_6(x)\\
&\quad\times(\delta_{13}^d)_{LR}(\delta_{13}^d)_{RL}\},
\end{aligned} \tag{15}$$

where $x=m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and $\tilde{m}^2$ is an average squark mass. The expression for the functions $f_6(x)$ and $\tilde{f}_6(x)$ can be found in Ref. [12]. The Wilson coefficients $\tilde{C}_{1-3}$ are simply obtained by interchanging $L\leftrightarrow R$ in the mass insertions appearing in C_{1-3}.

B. Chargino contributions

Here we present our results for the chargino contributions to the effective Hamiltonian in Eq. (13) in the mass insertion approximation. The leading diagrams are illustrated in Fig. 1, where the cross in the middle of the squark propagator represents a single mass insertion. As we will explain in more detail below, the dominant chargino exchange can significantly affect only the operators Q_1 and $\tilde{Q}_3$. We recall here that in the case of K-$\bar{K}$ mixing the relevant chargino exchange affects only the operator Q_1 [11], as in the SM.

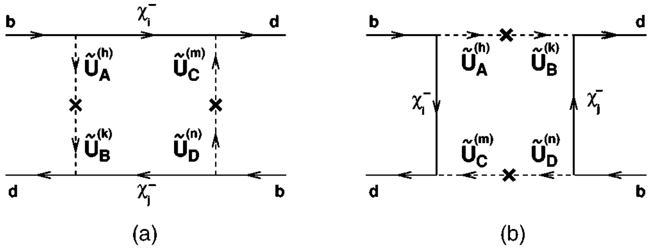

FIG. 1. The leading chargino–up-squark contribution to the B_d-$\overline{B}_d$ mixing.

In the framework of the mass insertion approximation, one chooses a basis (super-CKM basis) where the couplings of the fermions and sfermions to neutral gauginos are flavor diagonal. In this basis, the interacting Lagrangian involving charginos is given by

$$\mathcal{L}_{q\tilde{q}\tilde{\chi}^+} = -g\sum_k \sum_{a,b} [V_{k1}K^*_{ba}\overline{d^a_L}(\tilde{\chi}^+)^*\tilde{u}^b_L - U^*_{k2}(Y^{\rm diag}_d\cdot K^+)_{ab}\overline{d^a_R}(\tilde{\chi}^+)^*\tilde{u}^b_L - V^*_{k2}(K\cdot Y^{\rm diag}_u)_{ab}\overline{d^a_L}(\tilde{\chi}^+)^*\tilde{u}^b_R], \qquad (16)$$

where $Y^{\rm diag}_{u,d}$ are the diagonal Yukawa matrices, and K is the usual CKM matrix. The indices a, b, and k label flavor and chargino mass eigenstates, respectively, and V,U are the chargino mixing matrices defined by

$$U^* M_{\tilde{\chi}^+} V^{-1} = \mathrm{diag}(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}) \quad \text{and}$$

$$M_{\tilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix}. \qquad (17)$$

As one can see from Eq. (16), the Higgsino couplings are suppressed by Yukawa couplings of the light quarks, and therefore they are negligible, except for the stop–bottom quark interaction which is directly enhanced by the top Yukawa coupling (Y_t). The other vertex involving the down and stop could also be enhanced by Y_t, but one would pay the price of a λ^3 suppression, where λ is the Cabibbo mixing. Since in our analysis we adopt the approximation of retaining only terms proportional to order λ, we will neglect the effect of this vertex. Moreover, we also set to zero the Higgsino contributions proportional to Yukawa couplings of light quarks with the exception of the bottom Yukawa coupling Y_b, since its effect could be enhanced by large $\tan\beta$. In this respect, it is clear that the chargino contribution to the Wilson coefficients C_4 and C_5 is negligible. Furthermore, due to the color structure of chargino box diagrams there is no contribution to C_2 or $\tilde{C}_2$. However, as we will show in the next section, chargino contributions to $\tilde{C}_2$ will always be induced at low energy by QCD corrections through the mixing with $\tilde{C}_3$.

Now we calculate the relevant Wilson coefficients $C^{\chi}_{1,3}(M_S)$ at SUSY scale M_S, by using the mass insertion approximation. As mentioned in this case the flavor mixing is displayed by the nondiagonal entries of the sfermion mass matrices. Denoting by Δ^q_{AB} the off-diagonal terms in the sfermion ($\tilde{q}=\tilde{u},\tilde{d}$) mass matrices for the up and down squarks, respectively, where A,B indicate chirality couplings to fermions $A,B=(L,R)$, the A-B squark propagator can be expanded as

$$\langle \tilde{q}^a_A \tilde{q}^{b*}_B\rangle = i(k^2\mathbf{1}-\tilde{m}^2\mathbf{1}-\Delta^q_{AB})^{-1}_{ab} \simeq \frac{i\delta_{ab}}{k^2-\tilde{m}^2} + \frac{i(\Delta^q_{AB})_{ab}}{(k^2-\tilde{m}^2)^2} + \mathcal{O}(\Delta^2), \qquad (18)$$

where $q=u,d$ selects the up or down sector, respectively, $a,b=(1,2,3)$ are flavor indices, $\mathbf{1}$ is the unit matrix, and $\tilde{m}$ is the average squark mass. As we will see in the following, it is convenient to parametrize this expansion in terms of the dimensionless quantity $(\delta^q_{AB})_{ab} \equiv (\Delta^q_{AB})_{ab}/\tilde{m}^2$. At the first order in the mass insertion approximation, we find for the Wilson coefficients $C^{\chi}_{1,3}(M_S)$ the following result:

$$\begin{aligned} C^{\chi}_1(M_S) = \frac{g^4}{768\pi^2\tilde{m}^2}\sum_{i,j} \{&|V_{i1}|^2|V_{j1}|^2[(\delta^u_{LL})^2_{31} \\ &+2\lambda(\delta^u_{LL})_{31}(\delta^u_{LL})_{32}] \\ &-2Y_t|V_{i1}|^2V_{j1}V^*_{j2}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{31} \\ &+\lambda(\delta^u_{LL})_{32}(\delta^u_{RL})_{31}+\lambda(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}] \\ &+Y^2_tV_{i1}V^*_{i2}V_{j1}V^*_{j2}[(\delta^u_{RL})^2_{31} \\ &+2\lambda(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]\}L_2(x_i,x_j), \end{aligned} \qquad (19)$$

$$\begin{aligned} \tilde{C}^{\chi}_3(M_S) = \frac{g^4Y^2_b}{192\pi^2\tilde{m}^2}\sum_{i,j} &U_{i2}U_{j2}V_{j1}V_{i1}[(\delta^u_{LL})^2_{31} \\ &+2\lambda(\delta^u_{LL})_{31}(\delta^u_{LL})_{32}]L_0(x_i,x_j), \end{aligned} \qquad (20)$$

TABLE I. Numerical values for the coefficients x_i (with $i=1,2,3$) in Eq. (23) for some representative values of the SUSY scale M_S, and evaluated at the low energy scale $\mu=m_b$.

M_S	$x_1(\mu)$	$x_2(\mu)$	$x_3(\mu)$
200	0.844	−0.327	0.571
400	0.827	−0.367	0.536
600	0.817	−0.389	0.518
800	0.810	−0.404	0.506

TABLE II. Upper bounds on $\sqrt{|\mathrm{Re}[(\delta^u_{LL})_{31}]^2|}$ from ΔM_{B_d} (assuming zero CKM and SUSY phases), for $\mu=200$ GeV and $\tan\beta=5$, and for some values of $\tilde{m}$ and M_2 (in GeV).

	m			
$M_2 m$	300	500	700	900
150	1.3×10^{-1}	1.7×10^{-1}	2.2×10^{-1}	2.8×10^{-1}
250	1.9×10^{-1}	2.3×10^{-1}	2.7×10^{-1}	3.2×10^{-1}
350	2.7×10^{-1}	2.8×10^{-1}	3.3×10^{-1}	3.7×10^{-1}
450	3.6×10^{-1}	3.6×10^{-1}	3.9×10^{-1}	4.3×10^{-1}

where $x_i = m^2_{\tilde{\chi}^+_i}/\tilde{m}^2$, and the functions $L_0(x,y)$ and $L_2(x,y)$ are given by

$$L_0(x,y) = \sqrt{xy}\left(\frac{xh_0(x)-yh_0(y)}{x-y}\right),$$

$$h_0(x) = \frac{-11+7x-2x^2}{(1-x)^3} - \frac{6\ln x}{(1-x)^4},$$

$$L_2(x,y) = \frac{xh_2(x)-yh_2(y)}{x-y}, \tag{21}$$

$$h_2(x) = \frac{2+5x-x^2}{(1-x)^3} + \frac{6x\ln x}{(1-x)^4}.$$

III. CONSTRAINTS FROM ΔM_{B_d} AND sin 2β

In this section we present our numerical results for the bounds on $(\delta^u_{AB})_{ij}$ which come from ΔM_{B_d} and the CP violating parameter $\sin 2\beta$. We start with the chargino contribution, which is found to be the dominant SUSY source in various models [5,8,15]. We also provide analytical expressions for ΔM_{B_d} and $\sin 2\beta$ as functions of the mass insertions in the Wilson coefficients $C_i(M_S)$ of Eqs. (19),(20). In our calculation we take into account NLO QCD corrections in both Wilson coefficients and hadronic matrix elements given in [12].

In order to connect $C_i(M_S)$ at the SUSY scale M_S with the corresponding low energy ones $C_i(\mu)$ [where $\mu \simeq \mathcal{O}(m_b)$], one has to solve the renormalization group equations (RGEs) for the Wilson coefficients corresponding to the effective Hamiltonian in Eq. (13). Then, $C_i(\mu)$ will be related to $C_i(M_S)$ by [12]

$$C_r(\mu) = \sum_i \sum_s \left(b_i^{(r,s)} + \eta c_i^{(r,s)}\right)\eta^{a_i} C_s(M_S), \tag{22}$$

where $M_S > m_t$ and $\eta = \alpha_S(M_S)/\alpha_S(\mu)$. The values of the coefficients $b_i^{(r,s)}$, $c_i^{(r,s)}$, and a_i appearing in Eq. (22) can be found in Ref. [12]. In our analysis the SUSY scale, where SUSY particles are simultaneously integrated out, is identified with the average squark mass $\tilde{m}$. By using the NLO results of [12], we obtain, for the relevant chargino contributions

$$C_1(\mu) = x_1(\mu)C_1(M_S), \quad C_2(\mu) = x_2(\mu)C_3(M_S),$$

$$C_3(\mu) = x_3(\mu)C_3(M_S), \tag{23}$$

while for the other coefficients $C_i(\mu) = 0$ $(i = 4,5)$. Numerical values for $x_i(\mu)$, evaluated at $\mu = m_b$, are shown in Table I for some representative values of M_S. Notice that the coefficients $b_i^{(23)}$ and $c_i^{(23)}$ are different from zero and so the contribution to $C_2(\mu)$ is radiatively generated at NLO by the off-diagonal mixing with $C_3(M_S)$. For the coefficients $\tilde{C}_{1-3}$ we have the same results as in Eq. (23) and in Table I, since the corresponding $\tilde{b}_i^{(r,s)}$ and $\tilde{c}_i^{(r,s)}$ coefficients in Eq. (22) are the same as the ones for the evolution of C_{1-3} [12].

The off-diagonal matrix elements of the operators Q_i are given by [12]

$$\langle B_d|Q_1|\overline{B}_d\rangle = \frac{1}{3} m_{B_d} f^2_{B_d} B_1(\mu),$$

$$\langle B_d|Q_2|\overline{B}_d\rangle = -\frac{5}{24}\left(\frac{m_{B_d}}{m_b(\mu)+m_d(\mu)}\right)^2 m_{B_d} f^2_{B_d} B_2(\mu),$$

$$\langle B_d|Q_3|\overline{B}_d\rangle = \frac{1}{24}\left(\frac{m_{B_d}}{m_b(\mu)+m_d(\mu)}\right)^2 m_{B_d} f^2_{B_d} B_3(\mu), \tag{24}$$

$$\langle B_d|Q_4|\overline{B}_d\rangle = \frac{1}{4}\left(\frac{m_{B_d}}{m_b(\mu)+m_d(\mu)}\right)^2 m_{B_d} f^2_{B_d} B_4(\mu),$$

$$\langle B_d|Q_5|\overline{B}_d\rangle = \frac{1}{12}\left(\frac{m_{B_d}}{m_b(\mu)+m_d(\mu)}\right)^2 m_{B_d} f^2_{B_d} B_4(\mu).$$

The value of B_1 has been extensively studied on the lattice [16], but for the other B_i parameters, they have been recently calculated on the lattice by the collaboration in Ref. [17]. In our analysis we will use the central values reported in [12], namely, $B_1(\mu) = 0.87$, $B_2(\mu) = 0.82$,$B_3(\mu) = 1.02$,$B_4(\mu) = 1.16$, and $B_5 = 1.91$. The same results of Eq. (24) are also valid for the corresponding operators $\tilde{Q}_i$, with the same values for the B_i parameters, since strong interactions preserve parity.

TABLE III. Upper bounds on mass insertions as in Table II, for $M_2 = \mu = 200$ GeV and $\tan\beta = 5$.

m	$\sqrt{\|\mathrm{Re}[(\delta^u_{LL})_{31}]^2\|}$	$\sqrt{\|\mathrm{Re}[(\delta^u_{RL})_{31}]^2\|}$	$\sqrt{\|\mathrm{Re}[(\delta^u_{LL})_{31}(\delta^u_{LL})_{32}]\|}$
200	1.4×10^{-1}	4.7×10^{-1}	2.1×10^{-1}
400	1.8×10^{-1}	9.0×10^{-1}	2.7×10^{-1}
600	2.2×10^{-1}	1.5	3.4×10^{-1}
800	2.7×10^{-1}	2.3	4.1×10^{-1}

TABLE IV. Upper bounds on mass insertions as in Table II, for $M_2 = \mu = 200$ GeV and $\tan\beta = 5$.

m	$\sqrt{\|\mathrm{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{31}]\|}$	$\sqrt{\|\mathrm{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]\|}$	$\sqrt{\|\mathrm{Re}[(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]\|}$
200	1.8×10^{-1}	4.0×10^{-1}	7.1×10^{-1}
400	3.0×10^{-1}	6.3×10^{-1}	1.3
600	4.5×10^{-1}	9.5×10^{-1}	2.3
800	6.3×10^{-1}	1.3	3.5

TABLE V. Upper bounds on $\sqrt{|\mathrm{Im}[(\delta^u_{LL})_{31}]^2|}$ from $\sin 2\beta = 0.79$ (assuming a zero CKM phase), for $\mu = 200$ GeV and $\tan\beta = 5$, and for some values of $\tilde{m}$ and M_2 (in GeV).

	m			
$M_2 m$	300	500	700	900
150	1.5×10^{-1}	2.0×10^{-1}	2.6×10^{-1}	3.1×10^{-1}
250	2.2×10^{-1}	2.6×10^{-1}	3.1×10^{-1}	3.6×10^{-1}
350	3.0×10^{-1}	3.3×10^{-1}	3.7×10^{-1}	4.2×10^{-1}
450	4.0×10^{-1}	4.1×10^{-1}	4.4×10^{-1}	4.8×10^{-1}

Now we start our analysis, by discussing first the dominant chargino contribution to ΔM_{B_d}. Using Eqs. (4),(19),(20), and (23),(24), we obtain for ΔM_{B_d} the following result:

$$\Delta M_{B_d} = \frac{g^4 m_{B_d} f^2_{B_d}}{(48\pi)^2 \tilde{m}^2} |R|, \tag{25}$$

$$\begin{aligned} R = {} & [(\delta^u_{LL})^2_{31} + 2\lambda(\delta^u_{LL})_{31}(\delta^u_{LL})_{32}]\{2A_1 x_1(\mu)B_1(\mu) \\ & + A_4 X(\mu)[x_3(\mu)B_3(\mu) - 5x_2(\mu)B_2(\mu)]\} \\ & + \{(\delta^u_{LL})_{31}(\delta^u_{RL})_{31} + \lambda[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32} \\ & + (\delta^u_{LL})_{32}(\delta^u_{RL})_{31}]\} 2A_2 x_1(\mu)B_1(\mu) + [(\delta^u_{RL})^2_{31} \\ & + 2\lambda(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}] 2A_3 x_1(\mu)B_1(\mu) \end{aligned} \tag{26}$$

where $X(\mu) = \{m_{B_d}/[m_b(\mu) + m_d(\mu)]\}^2$, and the expressions for A_i are given by

$$A_1 = \sum_{i,j} |V_{i1}|^2 |V_{j1}|^2 L_2(x_i, x_j),$$

$$A_2 = Y_t \sum_{i,j} |V_{i1}|^2 V_{j1} V^*_{j2} L_2(x_i, x_j),$$

$$A_3 = Y_t^2 \sum_{i,j} V_{i1} V^*_{i2} V_{j1} V^*_{j2} L_2(x_i, x_j),$$

$$A_4 = Y_b^2 \sum_{i,j} U_{i2} U_{j2} V_{j1} V_{i1} L_0(x_i, x_j), \tag{27}$$

where the definition of the quantities appearing in Eq. (27) can be found in Sec. II. Notice that the renormalization scheme dependence in Eq. (25) [for μ varying in the range $\mu \simeq (m_b/2, 2m_b)$] is strongly reduced due to the NLO QCD accuracy.

As is customary in this kind of analysis [10], in order to find conservative upper bounds on mass insertions, the SM contribution to ΔM_{B_d} is set to zero. Moreover, since we are analyzing ΔM_{B_d}, which is a CP conserving quantity, we keep the squark mass matrices real. Upper bounds are then obtained by requiring that the contribution of the real part of each independent combination of mass insertions in Eq. (25) does not exceed the experimental central value $\Delta M_{B_d} < 0.484\ (\mathrm{ps})^{-1}$.

TABLE VI. Upper bounds on mass insertions as in Table V, for $M_2 = \mu = 200$ GeV and $\tan\beta = 5$.

m	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{LL})_{31}]^2\rvert}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{RL})_{31}]^2\rvert}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{LL})_{31}(\delta^u_{LL})_{32}]\rvert}$
200	1.6×10^{-1}	5.4×10^{-1}	2.4×10^{-1}
400	2.0×10^{-1}	1.0	3.0×10^{-1}
600	2.5×10^{-1}	1.7	3.8×10^{-1}
800	3.1×10^{-1}	2.7	4.6×10^{-1}

These constraints depend on the relevant MSSM low energy parameters, in particular,[1] by $\tilde{m}$, M_2, μ, and $\tan\beta$. Notice that with respect to the gluino mediated FCNC processes, which are parametrized by $\tilde{m}, M_3$, the chargino mediated ones contain two free parameters more.

In Tables II and III, we present our results for upper bounds on mass insertions coming from ΔM_{B_d}, given for some representative values of $\tilde{m}$ and M_2 and for fixed values of $\mu = 200$ GeV and $\tan\beta = 5$. In Table II we provide constraints on $\sqrt{|\mathrm{Re}[(\delta^u_{LL})_{31}]^2|}$ for several combinations of $\tilde{m}$ and M_2. We find that these bounds are almost insensitive to μ and $\tan\beta$ in the ranges of 200–500 GeV and 3–40 respectively. This can be simply understood by noticing that the contributions to $\Delta B = 2$ transitions mediated by LL interactions are mainly given by the weak gaugino component of chargino. Therefore, the corresponding bounds are more sensitive to M_2 instead of μ and $\tan\beta$, since these last two parameters contribute to the Higgsino components of the chargino. The only term in Eq. (25) that is quite sensitive to $\tan\beta$ is A_4, because it is proportional to the bottom Yukawa coupling squared. However, $(\delta^u_{LL})_{31}$, in addition to A_4, receives contributions also from the A_1 term. This term is larger than A_4 and almost insensitive to $\tan\beta$, leaving the bounds on $(\delta^u_{LL})_{31}$ almost independent of $\tan\beta$.

In Tables III and IV we give our results for the real parts of the other mass insertions (and also for $\sqrt{|\mathrm{Re}[(\delta^u_{LL})_{31}]^2|}$) which are less constrained, for several values of $\tilde{m}$ and evaluated at $M_2 = \mu = 200$ GeV and $\tan\beta = 5$. For larger values of μ and M_2, these bounds become clearly less stringent due to the decoupling. Notice that they are also quite insensitive to $\tan\beta$, since no mass insertion in Eq. (25) receives leading contributions from bottom Yukawa couplings. It is also worth mentioning that the bounds on the mass insertion $(\delta^u_{LL})_{32}(\delta^u_{RL})_{31}$ are identical to the bounds of $(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}$, since they have the same coefficients in C_1^χ as can be seen from Eq. (26). Therefore, here we just present the bounds of one of them.

[1]With abuse of notation, we used here the same symbol μ for the renormalization scale of Wilson coefficients and the Higgs mixing parameter of the MSSM.

TABLE VII. Upper bounds on mass insertions as in Table V, for $M_2=\mu=200$ GeV and $\tan\beta=5$.

m	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{31}]\rvert}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]\rvert}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]\rvert}$
200	2.1×10^{-1}	4.5×10^{-1}	8.0×10^{-1}
400	3.4×10^{-1}	7.2×10^{-1}	1.5
600	5.1×10^{-1}	1.1	2.5
800	7.2×10^{-1}	1.5	4.0

In analogy to the procedure used for obtaining bounds from ΔM_{B_d}, the imaginary parts will be constrained by switching off the SM CKM phase and imposing that the contribution of the SUSY phases to $\sin 2\beta$ does not exceed its experimental central value $(\sin 2\beta)^{\mathrm{expt}}=0.79$. In particular we obtain

$$(\tan 2\beta)^{\mathrm{expt}}<\left(\frac{g^4 m_{B_d} f^2_{B_d}}{(48\pi)^2\tilde{m}^2\Delta M_{B_d}}\mathrm{Im}[R]\right) \tag{28}$$

where R is defined in Eq. (26).

In Tables V–VIII. we present our numerical results for the bounds on imaginary parts of mass insertions. Clearly, due to the procedure used in our analysis, these bounds turn out to be just proportional to the corresponding ones in Tables II–IV, and therefore the same considerations about μ and $\tan\beta$ dependence hold for these bounds as well.

Next we consider the upper bounds on the relevant mass insertions in the down-squark sector, mediated by gluino exchange. In Ref. [12] the maximum allowed values for the real and imaginary parts of the mass insertions $(\delta^d_{LL})_{13}$ and $(\delta^d_{LR})_{13}$ are given by taking into account the NLO QCD corrections. However, in that analysis the SM contributions to ΔM_{B_d} and $\sin 2\beta$ are assumed not vanishing. In order to compare our bounds on up-squark mass insertions with the corresponding ones in the down-squark sector, we should use for these last ones the same strategy adopted above. Therefore, in order to find conservative upper bounds on down-squark mass insertions, we will impose that the pure gluino contribution does not exceed the experimental values on ΔM_{B_d} and $\sin 2\beta$, setting to zero the SM contribution. In these results we include the NLO QCD corrections for the Wilson coefficients given in Eq. (22).

The upper bounds on the real parts of relevant combinations of mass insertions $(\delta^d_{AB})_{13}$ [with $A,B=(L,R)$] from the gluino contribution to ΔM_{B_d} are presented in Table VIII. In Table IX we show the corresponding bounds for the imaginary parts obtained from the gluino contribution to CP asymmetry $a_{J/\psi K_S}$, again assuming zero SM contribution. The upper bounds on the other mass insertions in which $L\leftrightarrow R$ are not shown here, since they turn out to be exactly the same as the corresponding ones in Tables VIII and IX.

IV. LIGHT STOP SCENARIO

In this section we will provide analytical and numerical results for the bounds on mass insertions, in the particular case in which one of the eigenvalues of the up-squark mass matrix is much lighter than the other (almost degenerate) ones. This scenario appears in the specific model that we will analyze in Sec. V C, where the mass of the stop right $(m^2_{\tilde{t}_R})$ is lighter than the other diagonal terms in the up-squark mass matrix. Then the analytical results for the Wilson coefficients provided in Sec. III will be generalized by including this effect. In our case, this modification will affect only the expression for the Wilson coefficient $C_1^{\chi}(M_S)$ in Eq. (19), since the stop right does not contribute to $C_3^{\chi}(M_S)$ at $\mathcal{O}(\lambda)$ order, as can be seen from Eq. (20).

By taking the mass of the stop right different from the average squark mass, we obtain the following result:[2]

$$\begin{aligned}C_1^{\chi}(M_S)=\frac{g^4}{768\pi^2\tilde{m}^2}\sum_{i,j}\{&|V_{i1}|^2|V_{j1}|^2[(\delta^u_{LL})^2_{31}\\&+2\lambda(\delta^u_{LL})_{31}(\delta^u_{LL})_{32}]L_2(x_i,x_j)\\&-2Y_t|V_{i1}|^2V_{j1}V^*_{j2}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{31}\\&+\lambda(\delta^u_{LL})_{32}(\delta^u_{RL})_{31}\end{aligned}$$

[2]We have used the same method introduced in Ref. [18], but our results are presented in a different way.

TABLE VIII. Upper bounds on real parts of combinations of mass insertions $(\delta^d_{AB})_{31}$, with $(A,B)=L,R$, from gluino contributions to ΔM_{B_d} (assuming zero SM contribution), evaluated at $\tilde{m}=400$ GeV and for some values of gluino mass M_3 (in GeV).

M_3	$\sqrt{\lvert\mathrm{Re}[(\delta^d_{LL})^2_{31}]\rvert}$	$\sqrt{\lvert\mathrm{Re}[(\delta^d_{RL})^2_{31}]\rvert}$	$\sqrt{\lvert\mathrm{Re}[(\delta^d_{LL})_{31}(\delta^d_{RR})_{31}]\rvert}$	$\sqrt{\lvert\mathrm{Re}[(\delta^d_{LR})_{31}(\delta^d_{RL})_{31}]\rvert}$
200	4.6×10^{-2}	2.2×10^{-2}	8.4×10^{-3}	1.1×10^{-2}
400	1.0×10^{-1}	2.4×10^{-2}	9.6×10^{-3}	1.9×10^{-2}
600	4.8×10^{-1}	2.9×10^{-2}	1.2×10^{-2}	3.0×10^{-2}
800	2.4×10^{-1}	3.4×10^{-2}	1.4×10^{-2}	4.4×10^{-2}

TABLE IX. Upper bounds on imaginary parts of combinations mass insertions $(\delta^d_{AB})_{31}$, with $(A,B)=L,R$, from gluino contributions to $\sin 2\beta$ (assuming zero SM contribution), evaluated at $\tilde{m}=400$ GeV and for some values of gluino mass M_3 (in GeV).

M_3	$\sqrt{\mathrm{Im}[(\delta^d_{LL})^2_{31}]}$	$\sqrt{\|\mathrm{Im}[(\delta^d_{RL})^2_{31}]\|}$	$\sqrt{\|\mathrm{Im}[(\delta^d_{LL})_{31}(\delta^d_{RR})_{31}]\|}$	$\sqrt{\|\mathrm{Im}[(\delta^d_{LR})_{31}(\delta^d_{RL})_{31}]\|}$
200	5.2×10^{-2}	2.5×10^{-2}	9.6×10^{-3}	1.2×10^{-2}
400	1.2×10^{-1}	2.7×10^{-2}	1.1×10^{-2}	2.2×10^{-2}
600	5.5×10^{-1}	3.3×10^{-2}	1.3×10^{-2}	3.4×10^{-2}
800	2.8×10^{-1}	3.9×10^{-2}	1.6×10^{-2}	5.0×10^{-2}

$$+\lambda(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]R_2(x_i,x_j,z)$$
$$+Y_t^2 V_{i1}V^*_{i2}V_{j1}V^*_{j2}[(\delta^u_{RL})^2_{31}$$
$$+2\lambda(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]\tilde{R}_2(x_i,x_j,z)\}, \qquad (29)$$

where $x_i=m^2_{\tilde{\chi}^+_i}/\tilde{m}^2$, $z=m^2_{\tilde{t}_R}/\tilde{m}^2$ and the functions $R_2(x,y,z)$ and $\tilde{R}_2(x,y,z)$ are given by

$$R_2(x,y,z)=\frac{1}{x-y}[H_2(x,z)-H_2(y,z)],$$

$$\tilde{R}_2(x,y,z)=\frac{1}{x-y}[\tilde{H}_2(x,z)-\tilde{H}_2(y,z)],$$

$$H_2(x,z)=\frac{3}{D_2(x,z)}\{(-1+x)(x-z)(-1+z)$$
$$\times(-1-x-z+3xz)+6x^2(-1+z)^3\log(x)$$
$$-6(-1+x)^3z^2\log(z)\},$$

$$\tilde{H}_2(x,z)=\frac{-6}{\tilde{D}_2(x,z)}\{(-1+x)(x-z)(-1+z)$$
$$\times[x+(-2+x)z]+6x^2(-1+z)^3\log(x)$$
$$-6(-1+x)^2z(-2x+z+z^2)\log(z)\}, \qquad (30)$$

where $D_2(x,z)=(-1+x)^3(x-z)(-1+z)^3$ and $\tilde{D}_2(x,z)=(-1+x)^2(x-z)^2(-1+z)^3$. Notice that in the limit $z\to1$ both the functions $R_2(x,y,z)$ and $\tilde{R}_2(x,y,z)$ tend to $L_2(x,y)$, recovering the result in Eq. (19). Analogously, the expressions for A_2 and A_3 entering in Eq. (26) must be substituted by

$$A_2=Y_t\sum_{i,j}|V_{i1}|^2V_{j1}V^*_{j2}R_2(x_i,x_j,z),$$

$$A_3=Y_t^2\sum_{i,j}V_{i1}V^*_{i2}V_{j1}V^*_{j2}\tilde{R}_2(x_i,x_j,z) \qquad (31)$$

while A_1 and A_4 remain the same. In Tables X and XI we show our results, analogous to the ones in Tables III–VI, for the bounds on the real and imaginary parts on mass insertions, respectively, by taking into account a light stop right mass. We considered two representative cases of $\tilde{m}_{t_R}=100,\ 200$ GeV. Clearly, the light stop right effect does not affect bounds on mass insertions containing LL interactions. From these results we can see that the effect of taking $\tilde{m}_{t_R}<\tilde{m}$ is sizable, in particular, on the bounds of the mass insertions $(\delta^u_{RL})_{31}(\delta^u_{RL})_{3i}$ $(i=1,2)$ which are the most sensitive to a light stop right.

From the results in Tables X and XI, it is remarkable to notice that, in the limit of very heavy squark masses but with fixed right stop and chargino masses, the bounds on $(\delta^u_{RL})_{31}(\delta^u_{RL})_{3i}$ tend to constant values. This is indeed an interesting property which shows a particular nondecoupling effect of supersymmetry when two light right stop run inside the diagrams in Fig. 1. This feature is related to the infrared singularity of the loop function $\tilde{R}_2(x,x,z)$ in the limit $z\to0$. In particular, we find that $\lim_{z\to0}\tilde{R}_2(x,x,z)=f(x)/x$, where $x=m^2_\chi/\tilde{m}^2$, and $f(x)$ is a nonsingular and non-null function in $x=0$. Then, in the limit $\tilde{m}\gg m_\chi$ the rescaling factor $1/\tilde{m}^2$ in C_1^χ will be canceled by the $1/x$ dependence in the loop function and replaced by $1/m^2_\chi$ times a constant factor.

This is a quite interesting result, since it shows that in the case of light right stop and charginos masses, in comparison to the other squark masses, the SUSY contribution (mediated by charginos) to the $\Delta B=2$ processes might not decouple and could be sizable, provided that the mass insertions $(\delta^u_{RL})_{3i}$ are large enough. This effect could be achieved, for instance, in supersymmetric models with nonuniversal soft breaking terms.

V. SPECIFIC SUPERSYMMETRIC MODELS

In this section we focus on three specific supersymmetric models and study the impact of the constraints derived in previous sections on their predictions. We discuss first SUSY models with minimal flavor violation, then we study the ones with Hermitian flavor structure, and finally we consider a SUSY model with small CP violating phases with universal strength of the Yukawa couplings.

A. SUSY models with minimal flavor violation

In supersymmetric models with minimal flavor violation (MFV) the CKM matrix is the only source of flavor violation. In the framework of the MSSM (with R parity conserved) we consider a minimal model, as in the supergravity scenario, where the soft SUSY breaking term is assumed to be universal at grand unification scale, i.e., the soft scalar masses, gaugino masses, and trilinear and bilinear couplings are given by

TABLE X. Upper bounds on real parts of mass insertions as in Tables III and IV, for some values of $\tilde{m}$ and $\tilde{m}_{t_R}$ (in GeV). In the fourth column the first number and the one in parentheses correspond to $i=1$ and $i=2$, respectively. Upper bounds on mass insertions involving only LL interactions are the same as in Tables III and IV.

$\tilde{m}$	$\tilde{m}_{t_R}$	$\sqrt{\lvert\mathrm{Re}[(\delta^u_{RL})^2_{31}]\rvert}$	$\sqrt{\lvert\mathrm{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{3i}]\rvert}$	$\sqrt{\lvert\mathrm{Re}[(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]\rvert}$
400	100	1.9×10^{-1}	$1.6(3.3)\times10^{-1}$	2.8×10^{-1}
600	100	1.8×10^{-1}	$1.9(4.0)\times10^{-1}$	2.6×10^{-1}
800	100	1.8×10^{-1}	$2.3(4.9)\times10^{-1}$	2.6×10^{-1}
400	200	3.5×10^{-1}	$2.0(4.2)\times10^{-1}$	5.2×10^{-1}
600	200	3.3×10^{-1}	$2.3(5.0)\times10^{-1}$	4.9×10^{-1}
800	200	3.2×10^{-1}	$2.8(5.9)\times10^{-1}$	4.8×10^{-1}

$$m_i^2=m_0^2, \qquad M_a=m_{1/2}e^{-i\alpha_M},$$

$$A_\alpha=A_0e^{-i\alpha_A}, \quad B=B_0e^{-i\alpha_B}. \tag{32}$$

As mentioned in the Introduction, only two of the above phases are independent, and they can be chosen as

$$\phi_A=\arg(A^*M), \quad \phi_B=\arg(B^*M). \tag{33}$$

The main constraints on ϕ_A and ϕ_B are due to the EDMs of the electron, neutron, and mercury atom. The present experimental bound on EDMs implies that $\phi_{A,B}$ should be $\lesssim 10^{-2}$ unless the SUSY masses are unnaturally large [3].

In these scenarios, where SUSY phases $\phi_{A,B}$ are constrained to be very small by EDM bounds, the supersymmetric contributions to CP violating phenomena in K and B mesons do not generate any sizable deviation from the SM prediction. We have to mention that the universal structure for the soft breaking terms, especially the universality of the trilinear couplings, is a very strong assumption. Indeed, in the light of recent work on SUSY breaking in string theories, the soft breaking sector at grand unification theory (GUT) scale is generally found to be nonuniversal [19]. Notice that, even if we start with universal soft breaking terms at the GUT scale, some off-diagonal terms in the squark mass matrices are induced at electroweak (EW) scale by Yukawa interactions through the renormalization group equation evolution. Therefore these off-diagonal entries are suppressed by the smallness of the CKM angles and/or the smallness of the Yukawa couplings.

It is important to stress that, even though one ignores the bounds from the EDMs and allows larger values [of order $\mathcal{O}(1)$] for the SUSY phases $\phi_{A,B}$, this class of models with MFV cannot generate any large contribution to ε_K and ε'/ε. Therefore, the Yukawa couplings remain the main source of CP violation [20].

Here we also found that, within MFV scenarios, the SUSY contributions to ΔM_{B_d} and $a_{J/\psi K_S}$ are negligible. In fact, due to the universality assumption of soft SUSY breaking terms, it turns out that the gluino and chargino contributions are quite suppressed. For instance, for $m_0\sim m_{1/2}\sim A_0 \sim 200$ GeV and $\phi_{A,B}\sim\pi/2$ (which corresponds to $\tilde{m}^2$ and m_g at a SUSY scale of order 500 GeV) we find the following values of the relevant mass insertions: $\mathrm{Im}(\delta^d_{13})_{LL}\sim\mathrm{Re}(\delta^d_{13})_{LL}\sim10^{-4}$ and $\mathrm{Im}(\delta^d_{13})_{LR}\sim\mathrm{Re}(\delta^d_{13})_{LR}\sim10^{-6}$, which are clearly much smaller than the corresponding bounds mentioned in the previous section.[3]

Therefore, we conclude that SUSY models with MFV do not give any genuine contribution to the CP violating and flavor changing processes in K and B systems and this scenario cannot be distinguished from the SM model one.

B. SUSY models with Hermitian flavor structure

As discussed in the Introduction, a possible solution for suppressing the EDMs in SUSY models is to have Hermitian

[3]In our analysis we have taken into account the effect of the CP violating phases in the RGE evolution.

TABLE XI. Upper bounds on imaginary parts of mass insertions as in Tables V and VI, for some values of $\tilde{m}$ and $\tilde{m}_{t_R}$ (in GeV). In the fourth column the first number and the one in parentheses correspond to $i=1$ and $i=2$, respectively. Upper bounds on mass insertions involving only LL interactions are the same as in Tables V and VI.

$\tilde{m}$	$\tilde{m}_{t_R}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{RL})^2_{31}]\rvert}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{3i}]\rvert}$	$\sqrt{\lvert\mathrm{Im}[(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]\rvert}$
400	100	2.1×10^{-1}	$1.8(3.7)\times10^{-1}$	3.1×10^{-1}
600	100	2.0×10^{-1}	$2.2(4.6)\times10^{-1}$	3.0×10^{-1}
800	100	2.0×10^{-1}	$2.6(5.5)\times10^{-1}$	3.0×10^{-1}
400	200	4.0×10^{-1}	$2.2(4.8)\times10^{-1}$	6.0×10^{-1}
600	200	3.7×10^{-1}	$2.7(5.6)\times10^{-1}$	5.6×10^{-1}
800	200	3.6×10^{-1}	$3.1(6.7)\times10^{-1}$	5.4×10^{-1}

flavor structures [5]. In this class of models, the flavor blind quantities, such as the μ terms and gaugino masses, are real while the Yukawa couplings and A terms are Hermitian, i.e., $Y_{u,d}^{\dagger}=Y_{u,d}$ and $A_{u,d}^{\dagger}=A_{u,d}$. It has been shown that these models are free from the EDM constraints and the off-diagonal phases lead to significant contributions to the observed CP violation in the kaon system, in particular to ε'/ε [5].

Let us consider, for instance, the case of Hermitian and hierarchical quark mass matrices with three zeros [21]

$$M_i=\begin{pmatrix} 0 & a_i e^{i\alpha_i} & 0 \\ a_i e^{-i\alpha_i} & A_i & b_i e^{i\beta_i} \\ 0 & b_i e^{-i\beta_i} & B_i \end{pmatrix}, \quad i=u,d, \tag{34}$$

with $A_i=(m_c,m_s)$, $B_i=(m_t-m_u,m_b-m_d)$, $a_i=(\sqrt{m_u m_c},\sqrt{m_d m_s})$, and $b_i=(\sqrt{m_u m_t},\sqrt{m_d m_b})$. The phases α_i and β_i satisfy $\alpha_d-\alpha_u=\pi/2$ and $\beta_d-\beta_u=\pi/2$. These matrices reproduce the correct values for the quark masses and CKM matrix. We also assume the following Hermitian A terms:

$$A_d=A_u=\begin{pmatrix} A_{11} & A_{12}e^{i\varphi_{12}} & A_{13}e^{i\varphi_{13}} \\ A_{12}e^{-i\varphi_{12}} & A_{22} & A_{23}e^{i\varphi_{23}} \\ A_{13}e^{-i\varphi_{13}} & A_{23}e^{-i\varphi_{23}} & A_{33} \end{pmatrix}. \tag{35}$$

Notice that the scenario with nondegenerate A terms is an interesting possibility for enhancing the SUSY contributions to ε_K and ε'/ε [9] and it is also well motivated by many string inspired models. In this case, the mass insertions are given by

$$(\delta_{ij}^q)_{LL}=\frac{1}{m_{\tilde{q}}^2}(V^q M_Q^2 V^{q^\dagger})_{ij}, \tag{36}$$

$$(\delta_{ij}^q)_{LR}=\frac{1}{m_{\tilde{q}}^2}[(V^q Y_q^{A*} V^{q^\dagger})_{ij} v_{1(2)} - \mu Y_i^q \delta_{ij} v_2(1)], \tag{37}$$

where $q\equiv u,d$ and $(Y_q^A)_{ij}=Y_{ij}^q A_{ij}^q$. Since the Yukawa couplings are Hermitian matrices, they are diagonalized by only one unitary transformation.

In this class of models, we find that in most of the parameter space the chargino gives the dominant contribution to B_d-$\bar{B}_d$ mixing and CP asymmetry $a_{J/\psi K_S}$, while the gluino one is subleading. As we emphasized above, in order to have a significant gluino contribution for $\tilde{m}\sim m_g\sim 500$ GeV (i.e., $m_0\sim M_{1/2}\sim 200$ at the GUT scale), the real and imaginary parts of the mass insertion $(\delta_{13}^d)_{LL}$ or $(\delta_{13}^d)_{LR}$ should be of order 10^{-1} and 10^{-2}, respectively. However, with the above hierarchical Yukawa couplings we find that these mass insertions are two orders of magnitude below the required values so that the gluino contributions are very small.

Concerning the chargino amplitude to the CP asymmetry $a_{J/\psi K_S}$, we find that the mass insertions $(\delta_{31}^u)_{RL}$ and $(\delta_{31}^u)_{LL}$ give the leading contribution to $a_{J/\psi K_S}$. However, for the representative case of $m_0=m_{1/2}=200$ and $\phi_{ij}\simeq\pi/2$ the values of these mass insertions are given by

$$\sqrt{|\mathrm{Im}[(\delta_{LL}^u)_{31}]^2|}=6\times 10^{-4}, \tag{38}$$

$$\sqrt{|\mathrm{Im}[(\delta_{LL}^u)_{31}]^2|}=4\times 10^{-3}, \tag{39}$$

$$\sqrt{|\mathrm{Im}[(\delta_{LL}^u)_{31}(\delta_{RL}^u)_{32}]|}=1\times 10^{-4}. \tag{40}$$

These results show that, for this class of models also, SUSY contributions cannot give sizable effects in $a_{J/\psi K_S}$. As expected, with hierarchical Yukawa couplings (where the mixing between different generations is very small), the SUSY contributions to the B-$\bar{B}$ mixing and the CP asymmetry of $B_d\to J/\psi K_S$ are subdominant, and the SM should give the dominant contribution.

C. SUSY model with universal strength of Yukawa couplings

Supersymmetric models with small CP violating phases are a possible solution for suppressing the EDMs. In Ref. [8] it was shown that, among this class of models, the ones with universal strength of the Yukawa couplings naturally provide very small CP violating phases. However, due to the large mixing between different generations, it was found that LL mass insertions can give sizable effects to ε_K and ε'/ε by means of gluino and chargino exchanges, respectively. Furthermore, it was also emphasized that in these models the SM contribution to the CP asymmetry $a_{J/\psi K_S}$ might be negligible, leaving the dominant SUSY effect (due to the chargino exchange) to account for the experimental results.

Here we will discuss the different contributions to B_d-$\bar{B}_d$ and $a_{J/\psi K_S}$ in terms of mass insertions and compare the predictions of this model with the corresponding ones of Hermitian flavor structure discussed in the previous subsection. In the framework of universal strength Yukawa couplings, the quark Yukawa couplings can be written as

$$U_{ij}=\frac{\lambda_u}{3}\exp[i\Phi_{ij}^u] \quad \text{and} \quad D_{ij}=\frac{\lambda_d}{3}\exp[i\Phi_{ij}^d], \tag{41}$$

where $\lambda_{u,d}$ are overall real constants, and $\Phi^{u,d}$ are pure phase matrices which are constrained to be very small by the hierarchy of the quark masses [8]. The values of these parameters that lead to the correct quark spectrum and mixing can be found in Ref. [8]. As explained in that paper, an important feature of this model is the presence of a large mixing between the first and third generations. As we will show in the following, this property can account for large SUSY contributions to $a_{J/\psi K_S}$.

In the framework of universal strength Yukawa couplings Eq. (41), due to the large generation mixing, the EDMs impose severe constraints on the parameter space and force the trilinear couplings to take particular patterns as the factorizable matrix form [8], i.e.,

$$A=m_0\begin{pmatrix} a & a & a \\ b & b & b \\ c & c & c \end{pmatrix}. \tag{42}$$

In order to satisfy the bound of the mercury EDM, the phases of the entries a, b, and c should be of order 10^{-2}–10^{-1} [8]. As an illustrative example, we consider $m_0 = m_{1/2} = 200$ GeV, $\phi_a = \phi_c = 0$, $\phi_b = 0.1$, and $a = -1$, $b = -2$, $c = -3$. In this case one finds that at low energy the average squark mass is of order 500 GeV; however, one of the stop masses ($\tilde{t}_R$) is much lighter, $m_{\tilde{t}_R} \simeq 200$ GeV. The gaugino mass M_2 is of order 170 GeV and, from the EW breaking condition, $|\mu|$ turns out to be of the order of 400 GeV. In this case, the relevant mass insertions for the gluino contribution are given by

$$(\delta^d_{LL})_{31} \simeq -0.001 + 0.02i, \quad (\delta^d_{RL})_{31} \simeq 0.00002 + 0.0009i. \tag{43}$$

Regarding the other mass insertions (LR and RR), they are much smaller [$\lesssim \mathcal{O}(10^{-6})$], and so we do not show them. It is clear that, with these values for the down-squark mass insertions, the gluino contribution to $a_{J/\psi K_S}$ is negligible (of the order of 3%).

On the contrary, the relevant up-squark mass insertions for the chargino contribution are given by

$$(\delta^u_{LL})_{31} \simeq 0.001 + 0.05i, \quad (\delta^u_{RL})_{31} \simeq -0.0004 + 0.13i, \tag{44}$$

$$(\delta^u_{LL})_{32} \simeq -0.008 - 0.11i, \quad (\delta^u_{RL})_{32} \simeq 0.01 - 0.28i. \tag{45}$$

Comparing these results with the ones in Tables X and XI, we see that for this model the chargino contribution to the imaginary parts $(\delta^u_{RL})_{31}$ and $(\delta^u_{RL})_{32}$ is of the same order as the corresponding upper bounds. Notice that these imaginary parts are of the same order, so that they might coherently contribute to give a sizable effect on $a_{J/\psi K_S}$. In particular, by using the exact one-loop calculation, we find that the chargino contribution leads to $\sin(2\theta_d) \sim 0.75$. Moreover, as a check on our computations, we have compared our results from the MIA approximation with the corresponding ones obtained by using the full calculation [8]. In this case we find that, by taking into account the effect of a light stop, the MIA predictions are quite compatible with the results of the full computation.

VI. CONCLUSIONS

In this paper we have studied the chargino contributions to B_d-$\bar{B}_d$ mixing and CP asymmetry $a_{J/\psi K_S}$ in the mass insertion approximation. In our analysis we have taken into account the NLO QCD corrections to the effective Hamiltonian for $\Delta B = 2$ transitions $H^{\Delta B=2}_{\rm eff}$. We provided analytical results for the chargino contribution to $H^{\Delta B=2}_{\rm eff}$ in the framework of the mass insertion method, and given the expressions for the B_d-$\bar{B}_d$ and CP asymmetry $a_{J/\psi K_S}$ at NLO in QCD, as a function of mass insertions in the up-squark sector. We have also provided model independent upper bounds on mass insertions by requiring that the pure chargino contribution does not exceed the experimental values of B-$\bar{B}$ mixing and CP asymmetry $a_{J/\psi K_S}$. Since in many SUSY models the chargino contribution is the dominant effect in B-$\bar{B}$ mixing and CP asymmetry $a_{J/\psi K_S}$, our results are particularly useful for a ready check of the viability of these models. Moreover, we generalized our results by including the case of a light right-stop scenario. In this case we found the interesting property that the bounds on mass insertions combinations $(\delta^u_{RL})_{31}(\delta^u_{RL})_{3i}$ are not sensitive to the common squark mass when this is very large in comparison to the chargino and stop right ones.

Finally, we applied these results to a general class of SUSY models that are particularly suitable to solve the SUSY CP problem, namely, the SUSY models with minimal flavor violations, Hermitian flavor structure, and small CP violating phases with universal strength Yukawa couplings. We showed that in SUSY models with minimal flavor violation and with Hermitian (and hierarchical) Yukawa couplings and A terms, the SUSY contributions to the B-$\bar{B}$ mixing and the CP asymmetry $a_{\psi K_S}$ are very small and the SM contribution in these classes of models should give the dominant effect. On the contrary, in the case of SUSY scenarios with large mixing between different generations in the soft terms, the SUSY contributions become significant and can even be the dominant source for saturating the experimental value of $a_{\psi K_S}$. Among this class of models, we investigated a SUSY model with universal strength of Yukawa couplings. In this case, we found that the chargino exchange provides the leading contribution to $a_{\psi K_S}$ through the mass insertions $(\delta^u_{LL})_{31}(\delta^u_{RL})_{3i}$, $i = 1,2$, and $(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}$.

ACKNOWLEDGMENTS

We acknowledge the kind hospitality of the CERN Theory Division where part of this work was done. The work of S.K. was supported by PPARC. E.G. would like to thank Katri Huitu for useful discussions.

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