Heavy to light meson exclusive semileptonic decays in effective field theory of heavy quarks

W. Y. Wang
Department of Physics, Tsinghua University, Beijing 100084, China

Y. L. Wu and M. Zhong
Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

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We present a general study of exclusive semileptonic decays of heavy (B,D,B_s) to light (π,ρ,K,K^*) mesons in the framework of the effective field theory of heavy quarks. The transition matrix elements of these decays can be systematically characterized by a set of wave functions which are independent of the heavy quark mass except for the implicit scale dependence. Form factors for all these decays are calculated consistently within the effective theory framework using the light cone sum rule method at the leading order of the $1/m_Q$ expansion. The branching ratios of these decays are evaluated, and the heavy and light flavor symmetry breaking effects are investigated. We also give a comparison of our results and the predictions from other approaches, among which are the relations proposed recently in the framework of large energy effective theory.

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I. INTRODUCTION

Heavy meson decays, both inclusive and exclusive, have long been an interesting subject in both experimental and theoretical studies. These decays play their special role in extracting the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements and probing new physics beyond the standard model. The inclusive decays of heavy mesons are more difficult to measure but theoretically cleaner than exclusive decays. On the other hand, exclusive decays are cleaner in experimental measurements but more difficult in theoretical calculations as they require the knowledge of form factors, which contain long distance ingredients and have to be estimated via nonperturbative methods such as sum rules, lattice simulations, or phenomenological models.

The heavy to light exclusive decays can be grouped as semileptonic decays and rare decays. In addition to using lattice calculations [1–7] and quark models [8–12], these decays have been analyzed by using sum rules in full QCD [13–21]. Since the heavy meson $B_{(s)}$ or $D_{(s)}$ contains one heavy quark and one light quark, it is expected that the heavy quark symmetry (HQS) and the effective field theory of heavy quarks may help to improve our understanding of heavy to light decays. Recently, some work has been done in this direction [22–25]. In particular, in Refs. [24], [25] the $B\to\pi(\rho)l\nu$ decays were investigated within the framework of heavy quark effective field theory (HQEFT) [26–28], and the relevant form factors were calculated at the leading order of heavy quark expansion, from which $|V_{ub}|$ was extracted. We note that HQEFT differs from the usual heavy quark effective theory (HQET) because of the explicit consideration of the antiquark contributions in the HQEFT Lagrangian. Since these antiquark effects contribute only to $1/m_Q$ corrections in heavy quark expansion (HQE), at the leading order of the $1/m_Q$ expansion HQEFT should be completely equivalent to the usual HQET.

This paper will provide a more general discussion of exclusive heavy to light decays within the framework of effective theory. We shall apply heavy quark expansion and light cone sum rule (LCSR) techniques to more heavy (B,D,B_s) to light (π,ρ,K,K^*) exclusive semileptonic decays; namely, we extend the study to D decays, and also extend it to decays into kaon mesons. We know that the reliability of HQE is different for B and D mesons, and that SU(3) symmetry breaking effects arise in the kaon systems. So, through a consistent study of the decays of both bottom and charm mesons with the final mesons including both nonstrange and strange ones, we aim at a general view of the applicability of the combination of HQEFT and LCSR methods in studying heavy to light semileptonic decays.

The paper is organized as follows. In Sec. II, we first formulate the heavy to light transition matrix elements in the framework of HQEFT, and then present the light cone sum rules for the heavy flavor independent wave functions. Some of the analytic formulas are found to be similar to those presented in Refs. [24], [25] and so have a general meaning. In reviewing them, we pay our main attention to the relations and differences among different decay channels. Section III contains our numerical analysis of the heavy to light transition form factors. The numerical results are compared with data from other approaches. Based on the results of Sec. III, branching ratios are evaluated and discussed in Sec. IV. Finally, a short summary is presented in Sec. V.

II. WAVE FUNCTIONS AND LIGHT CONE SUM RULES

For convenience of discussion, we denote in this paper the light pseudoscalar and vector mesons as P and V, respectively, and use M to represent the heavy mesons B, B_s, and D. Then the decay matrix elements and form factors can be written in a general form as follows:

$$\langle P(p)|\bar{q}\gamma^\mu Q|M(p+q)\rangle = 2f_+(q^2)p^\mu + [f_+(q^2)+f_-(q^2)]q^\mu, \quad (2.1)$$

$$\langle V(p,\epsilon^*)|\bar{q}\gamma^\mu(1-\gamma^5)Q|M(p+q)\rangle$$
$$= -i(m_M+m_V)A_1(q^2)\epsilon^{*\mu}$$
$$+i\frac{A_2(q^2)}{m_M+m_V}[\epsilon^*\cdot(p+q)]\times(2p+q)^\mu$$
$$+i\frac{A_3(q^2)}{m_M+m_V}[\epsilon^*\cdot(p+q)]q^\mu$$
$$+\frac{2V(q^2)}{m_M+m_V}\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha(p+q)_\beta p_\gamma, \tag{2.2}$$

where q in the currents (not to be confused with the lepton pair momentum) represents light quarks (u,d, or s), and Q denotes any heavy quark (b or c). $m_M, m_{P(V)}$ are the heavy and light pseudoscalar (vector) meson masses, respectively. In HQEFT, the leading order matrix elements in the $1/m_Q$ expansion can be simply expressed as the following trace formulas [22–25]:

$$\langle P(p)|\bar{q}\Gamma Q_v^+|M_v\rangle = -\mathrm{Tr}[\pi(v,p)\Gamma\mathcal{M}_v], \tag{2.3}$$

$$\langle V(p,\epsilon^*)|\bar{q}\Gamma Q_v^+|M_v\rangle = -i\mathrm{Tr}[\Omega(v,p)\Gamma\mathcal{M}_v], \tag{2.4}$$

with

$$\pi(v,p) = \gamma^5[A(v\cdot p)+\hat{\not p}B(v\cdot p)], \tag{2.5}$$

$$\Omega(v,p) = L_1(v\cdot p)\not\epsilon^* + L_2(v\cdot p)(v\cdot\epsilon^*) + [L_3(v\cdot p)\not\epsilon^* + L_4(v\cdot p)(v\cdot\epsilon^*)]\hat{\not p}, \tag{2.6}$$

where $\hat{p}^\mu = p^\mu/v\cdot p$. Q_v^+ in Eqs. (2.3), (2.4) is the effective heavy quark field variable introduced in HQEFT [26,27], which carries only the residual momentum $k^\mu = p_Q^\mu - m_Q v^\mu$ with v^μ the heavy meson's velocity. Correspondingly, M_v is the effective heavy meson state. It is related to the heavy meson state M in Eqs. (2.1) and (2.2) by the normalization of the hadronic matrix elements [27]:

$$\frac{1}{\sqrt{m_M}}\langle\pi(\rho)|\bar{q}\Gamma Q|M\rangle = \frac{1}{\sqrt{\bar{\Lambda}_M}}\{\langle\pi(\rho)|\bar{q}\Gamma Q_v^+|M_v\rangle + O(1/m_Q)\} \tag{2.7}$$

with $\bar{\Lambda}_M = m_M - m_Q$. $\mathcal{M}_v$ in Eqs. (2.3), (2.4) is the heavy pseudoscalar spin wave function in HQEFT [27],

$$\mathcal{M}_v = -\sqrt{\bar{\Lambda}}\frac{1+\not v}{2}\gamma^5, \tag{2.8}$$

which exhibits a manifest heavy flavor symmetry. Here $\bar{\Lambda} = \lim_{M_{Q\to\infty}}\bar{\Lambda}_M$ is the heavy flavor independent binding energy. Generally the functions A, B, and L_i ($i=1,2,3,4$) also depend on the energy scale μ, but for simplicity here we do not write this explicitly. The μ dependence will be considered in the numerical analysis in Sec. III.

The relations between the form factors defined in Eqs. (2.1), (2.2) and the universal wave functions defined in Eqs. (2.3)–(2.6) can be derived straightforwardly. One has

$$f_\pm(q^2) = \frac{1}{m_M}\sqrt{m_M\bar{\Lambda}/\bar{\Lambda}_M}\left\{A(v\cdot p)\pm B(v\cdot p)\frac{m_M}{v\cdot p}\right\}+\cdots, \tag{2.9}$$

$$A_1(q^2) = \frac{2}{m_M+m_V}\sqrt{m_M\bar{\Lambda}/\bar{\Lambda}_M}\{L_1(v\cdot p)+L_3(v\cdot p)\} +\cdots, \tag{2.10}$$

$$A_2(q^2) = 2(m_M+m_V)\sqrt{m_M\bar{\Lambda}/\bar{\Lambda}_M}\times\left\{\frac{L_2(v\cdot p)}{2m_M^2}+\frac{L_3(v\cdot p)-L_4(v\cdot p)}{2m_M(v\cdot p)}\right\}+\cdots, \tag{2.11}$$

$$A_3(q^2) = 2(m_M+m_V)\sqrt{m_M\bar{\Lambda}/\bar{\Lambda}_M}\times\left\{\frac{L_2(v\cdot p)}{2m_M^2}-\frac{L_3(v\cdot p)-L_4(v\cdot p)}{2m_M(v\cdot p)}\right\}+\cdots, \tag{2.12}$$

$$V(q^2) = \sqrt{m_M\bar{\Lambda}/\bar{\Lambda}_M}\frac{m_M+m_V}{m_M(v\cdot p)}L_3(v\cdot p)+\cdots. \tag{2.13}$$

The ellipses in Eqs. (2.9)–(2.13) denote higher order contributions. Since all considerations in this paper are made at the leading order of HQE, it is also reasonable to omit the overall factor $\sqrt{\bar{\Lambda}/\bar{\Lambda}_M}$ in Eqs. (2.9)–(2.13).

Note that the form factors introduced in Eqs. (2.1), (2.2) are heavy flavor dependent. But the functions A, B, and L_i ($i=1,2,3,4$) in Eqs. (2.5), (2.6) are leading order wave functions in the $1/m_Q$ expansion, so they should be (at least explicitly) independent of the heavy quark mass. An advantage of using the effective field theory of heavy quarks is that it enables one to formulate wave functions conveniently in such a heavy quark mass independent way. When the wave functions A, B, and L_i ($i=1,2,3$) are estimated to a relatively precise extent, then the form factors f_+, f_-, A_i ($i=1,2,3$), and V for different decay channels can be easily obtained by adopting the relevant parameters such as the masses and binding energies of the mesons. In other words, Eqs. (2.9)–(2.13) show the relations among different decays. From this point of view, considering the whole group of heavy to light semileptonic decays, one can say that to a certain order of the $1/m_Q$ expansion, the HQS and HQE simplify the theoretical analysis and reduce the number of independent functions, although it is well known that for an individual decay the number of independent functions does not decrease.

In light cone sum rule analysis, the wave functions are explored from the study of appropriate two point correlation functions. For example, for decays into pseudoscalar and vector light mesons, using the interpolating current $\overline{Q}i\gamma^5 q$ for the pseudoscalar heavy mesons, one may consider the functions

$$P^\mu(p,q)=i\int d^4x\, e^{iq\cdot x}\langle P(p)|T\{\overline{q}(x)\gamma^\mu Q(x),\overline{Q}(0)\times i\gamma^5 q(0)\}|0\rangle, \tag{2.14}$$

$$V^\mu(p,q)=i\int d^4x\, e^{-ip_M\cdot x}\langle V(p,\epsilon^*)|T\{\overline{q}(0)\times\gamma^\mu(1-\gamma^5)Q(0),\overline{Q}(x)i\gamma^5 q(x)\}|0\rangle. \tag{2.15}$$

In applying the sum rule method, from phenomenological considerations, a complete set of states with heavy meson quantum numbers are inserted into the above two point functions, i.e., between the two currents. For insertion of the ground states of heavy mesons, one obtains meson pole contributions, while for insertion of higher resonances, the results are generally written in the form of integrals over physical densities $\rho_P(v\cdot p,s)$ and $\rho_V(v\cdot p,s)$. In the resulting formulas, the matrix elements can be expanded in powers of $1/m_Q$ in the effective theory. In this paper we consider only the leading order contributions in the heavy quark expansion. As in Refs. [24], [25], we have

$$P^\mu(p,q)=2iF\frac{Av^\mu+B\hat{p}^\mu}{2\overline{\Lambda}_M-2v\cdot k}+\int_{s_0}^\infty ds\frac{\rho_P(v\cdot p,s)}{s-2v\cdot k}+\text{subtractions}, \tag{2.16}$$

$$V^\mu(p,q)=\frac{2F}{2\overline{\Lambda}_M-2v\cdot k}\left\{(L_1+L_3)\epsilon^{*\mu}-L_2 v^\mu(\epsilon^*\cdot v)-(L_3-L_4)p^\mu\frac{\epsilon^*\cdot v}{v\cdot p}-i\frac{L_3}{v\cdot p}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu p_\alpha v_\beta\right\}+\int_{s_0}^\infty ds\frac{\rho_V(v\cdot p,s)}{s-2v\cdot k}+\text{subtractions}, \tag{2.17}$$

where F is the scaled decay constant of the heavy meson at the leading order of $1/m_Q$ [28].

Note that when studying the $B\to\rho$ decay in Ref. [25] there is an overall factor $m_B\overline{\Lambda}/m_b\Lambda_B$ multiplying the meson pole contribution. Although such a factor can be obtained in the effective field theory, it can be written in the form $1+O(1/m_Q)$. Since we want Eqs. (2.16) and (2.17) to represent only the leading order contribution in heavy quark expansion, it is consistent to leave out the factor $m_M\overline{\Lambda}/m_Q\Lambda_M$ here. This change may enlarge the $B\to\rho l\nu$ decay form factors obtained in [25] by the same ratio, as can be seen in the next section.

Heavy quark expansion can be performed on the correlators (2.14),(2.15) in the frame-work of the effective theory, after which the field variables and meson states can be replaced by their counterparts in the effective theory, and those correlators turn into

$$P^\mu(p,q)=i\int d^4x\, e^{i(q-m_Q v)\cdot x}\langle P(p)|T\overline{q}(x)\gamma^\mu Q_v^+(x),\overline{Q}_v^+(0)i\gamma^5 q(0)|0\rangle+O(1/m_Q), \tag{2.18}$$

$$V^\mu(p,q)=i\int d^4x\, e^{-ip_M\cdot x+im_Q v\cdot x}\langle V(p,\epsilon^*)|\times T\{\overline{q}(0)\gamma^\mu(1-\gamma^5)Q_v^+(0),\overline{Q}_v^+(x)i\gamma^5 q(x)\}|0\rangle+O(1/m_Q). \tag{2.19}$$

We will include for light pseudoscalar and vector mesons the distribution amplitudes up to the same order as in Refs. [24], [25]. In other words, we consider the π meson distribution amplitudes up to twist 4 and ρ meson distribution amplitudes up to twist 2. These π, ρ distribution amplitudes are defined by

$$\langle\pi(p)|\overline{u}(x)\gamma^\mu\gamma^5 d(0)|0\rangle=-ip^\mu f_\pi\int_0^1 du\, e^{iup\cdot x}[\phi_\pi(u)+x^2 g_1(u)]+f_\pi\left(x^\mu-\frac{x^2p^\mu}{x\cdot p}\right)\int_0^1 du\, e^{iup\cdot x}g_2(u),$$

$$\langle\pi(p)|\overline{u}(x)i\gamma^5 d(0)|0\rangle=\frac{f_\pi m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{iup\cdot x}\phi_p(u),$$

$$\langle\pi(p)|\overline{u}(x)\sigma_{\mu\nu}\gamma^5 d(0)|0\rangle=i(p_\mu x_\nu-p_\nu x_\mu)\frac{f_\pi m_\pi^2}{6(m_u+m_d)}\int_0^1 du\, e^{iup\cdot x}\phi_\sigma(u),$$

$$\langle\rho(p,\epsilon^*)|\overline{u}(0)\sigma_{\mu\nu}d(x)|0\rangle=-if_\rho^\perp(\epsilon^*_\mu p_\nu-\epsilon^*_\nu p_\mu)\int_0^1 du\, e^{iup\cdot x}\phi_\perp(u),$$

$$\langle\rho(p,\epsilon^*)|\overline{u}(0)\gamma_\mu d(x)|0\rangle=f_\rho m_\rho p_\mu\frac{\epsilon^*\cdot x}{p\cdot x}\int_0^1 du\, e^{iup\cdot x}\phi_\parallel(u)+f_\rho m_\rho\left(\epsilon^*_\mu-p_\mu\frac{\epsilon^*\cdot x}{p\cdot x}\right)\int_0^1 du\, e^{iup\cdot x}g_\perp^{(v)}(u),$$

$$\langle\rho(p,\epsilon^*)|\overline{u}(0)\gamma_\mu\gamma_5 d(x)|0\rangle = \frac{1}{4}f_\rho m_\rho \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha x^\beta \int_0^1 du\, e^{iup\cdot x} g_\perp^{(a)}(u). \tag{2.20}$$

For K and K^* mesons, we need only replace the π and ρ mesons and d quark in the left-hand side (LHS) of the above equations by K and K^* mesons and an s quark, and at the same time change the quantities and distribution amplitudes related to π and ρ mesons to those related to K and K^* mesons in the RHS of the equations. For decays into kaon mesons, the light flavor SU(3) breaking effects may show up via both the light meson related quantities and the kaon meson distribution amplitudes.

The standard procedure of the light cone sum rule method is to calculate the correlation functions in the deep Euclidean region by using QCD or effective theories, and then equate the results with the phenomenological representations. In searching for reasonable and stable results, the quark-hadron duality and Borel transformation are generally applied to both sides of the equations. Unlike the previous LCSR analysis in the QCD framework [13–16], in the current study, we instead calculate the correlation functions in HQEFT, i.e., we adopt the Feynman rules in effective theory. In particular, we use $[(1+v)/2]\int_0^\infty dt\,\delta(x-y-vt)$ for the contraction of the effective heavy quark fields $\overline{Q}_v^+(x)$ and $Q_v^+(y)$. As one can see in Refs. [24], [25], the theoretical calculations can often become simpler in the framework of the effective theory than in QCD. Adopting procedures similar to those in Refs. [24], [25], we get

$$A(y) = -\frac{f_{\pi(K)}}{4F}\int_0^{s_0} ds\, e^{(2\bar{\Lambda}_M - s/T)}\left[\frac{1}{y^2}\frac{\partial}{\partial u}g_2(u) - \frac{\mu_{\pi(K)}}{y}\phi_p(u) - \frac{\mu_{\pi(K)}}{6y}\frac{\partial}{\partial u}\phi_\sigma(u)\right]_{u=1-s/2y}, \tag{2.21}$$

$$B(y) = -\frac{f_{\pi(K)}}{4F}\int_0^{s_0} ds\, e^{(2\bar{\Lambda}_M - s)/T}\left[-\phi_{\pi(K)}(u) + \frac{1}{y^2}\frac{\partial^2}{\partial u^2}g_1(u) - \frac{1}{y^2}\frac{\partial}{\partial u}g_2(u) + \frac{\mu_{\pi(K)}}{6\xi}\frac{\partial}{\partial u}\phi_\sigma(u)\right]_{u=1-s/2y}, \tag{2.22}$$

$$L_1(y) = \frac{1}{4F}\int_0^{s_0} ds\, e^{(2\bar{\Lambda}_M - s)/T}\frac{1}{y}f_V m_V\left[g_\perp^{(v)}(u) - \frac{1}{4}\left(\frac{\partial}{\partial u}g_\perp^{(a)}(u)\right)\right]_{u=s/2y}, \tag{2.23}$$

$$L_2(y) = 0, \tag{2.24}$$

$$L_3(y) = \frac{1}{4F}\int_0^{s_0} ds\, e^{(2\bar{\Lambda}_M - s)/T}\left[\frac{1}{4y}f_V m_V\left(\frac{\partial}{\partial u}g_\perp^{(a)}(u)\right) + f_V^\perp \phi_\perp(u)\right]_{u=s/2y}, \tag{2.25}$$

$$L_4(y) = \frac{1}{4F}\int_0^{s_0} ds\, e^{(2\bar{\Lambda}_M - s)/T}\frac{1}{y}f_V m_V\left[\phi_\parallel(u) - g_\perp^{(v)}(u) + \frac{1}{4}\left(\frac{\partial}{\partial u}g_\perp^{(a)}(u)\right)\right]_{u=s/2y}. \tag{2.26}$$

$L_2(y)$ equals zero in the present approximation since no twist 2 distribution amplitudes contribute to it. As a consequence, one can see from Eqs. (2.11), (2.12) that A_2 and A_3 have the same absolute value but opposite signs at the leading order.

Before proceeding, we would like to address that, although the formulas (2.21)–(2.26) are explicitly independent of the heavy quark mass, it does not mean that the values of these functions for decays of different heavy mesons are necessarily the same. This is because one needs to take into account different energy scales for different heavy mesons in calculating the corresponding functions. It is this difference that makes the distribution amplitudes and other light meson parameters change their values for decays of different heavy mesons. There is also an exponent $e^{2\bar{\Lambda}_M/T}$ in each formula, which indicates the dependence on the binding energy of the heavy meson. Since $\bar{\Lambda}_B - \bar{\Lambda}_D$ is small, this exponent only introduces a slight difference among the wave functions for B and D decays. On the other hand, there are light meson distribution amplitudes, light meson masses, and other light meson parameters in Eqs. (2.21)–(2.26). Consequently, the resulting numerical values for the universal wave functions may be different for different light mesons. These heavy and light flavor symmetry breaking effects will be explored numerically in the following sections.

III. NUMERICAL ANALYSIS OF THE FORM FACTORS

The light cone distribution amplitudes embody the nonperturbative contributions, and they are of crucial importance for the precision that light cone sum rules can reach. The study of these distribution amplitudes constitutes an important and difficult project. They have been studied by several groups. The asymptotic form and the scale dependence of these functions are given by perturbative QCD [29,30].

For light pseudoscalar mesons, the leading twist distribution amplitude is generally written as an expansion in terms of the Gegenbauer polynomials $C_n^{3/2}(x)$ as follows:

$$\phi_{\pi(K)}(u,\mu) = 6u(1-u)\left[1 + \sum_{n=1}^{4} a_n^{\pi(K)}(\mu)C_n^{3/2}(2u-1)\right]. \tag{3.1}$$

One should perform the sum rule analysis at an appropriate energy scale μ. In the processes of $B_{(s)}$ and D decays, the scales can be set by the typical virtualities of the heavy quarks, for example, $\mu_b=\sqrt{m_B^2-m_b^2}\approx 2.4$ GeV and $\mu_c=\sqrt{m_D^2-m_c^2}\approx 1.3$ GeV, respectively [31].

For the π meson, we use [31]

$$a_2^\pi(\mu_c)=0.41,\quad a_4^\pi(\mu_c)=0.23,\quad a_1^\pi=a_3^\pi=0, \tag{3.2}$$

while for the kaon distribution amplitude ϕ_K, when the light flavor SU(3) breaking effects are considered, we may use [16]

$$a_1^K(\mu_c)=0.17,\quad a_2^K(\mu_c)=0.21,\quad a_3^K(\mu_c)=0.07,$$
$$a_4^K(\mu_c)=0.08, \tag{3.3}$$

where the nonvanishing values of the coefficients a_1, a_3 imply asymmetric momentum distributions for the s and u,d quarks inside the K meson. Since the Gegenbauer moments a_i renormalize multiplicatively, the values of $a_i(\mu_b)$ can be obtained from Eqs. (3.2) and (3.3) through the renormalization group evolution.

We neglect the SU(3) breaking effects in the twist 3 and 4 distribution amplitudes included in this paper. This is justified by the analysis in Ref. [32], which indicates that these breaking effects would influence the light cone sum rules very slightly. Therefore we take for both π and K the following twist 3 and 4 distribution amplitudes [15,31,33]:

$$\phi_p(u)=1+\frac{1}{2}B_2[3(2u-1)^2-1]+\frac{1}{8}B_4[35(2u-1)^4-30(2u-1)^2+3],$$

$$\phi_\sigma(u)=6u(1-u)\left\{1+\frac{3}{2}C_2[5(2u-1)^2-1]+\frac{15}{8}C_4[21(2u-1)^4-14(2u-1)^2+1]\right\},$$

$$g_1(u)=\frac{5}{2}\delta^2u^2(1-u)^2+\frac{1}{2}\epsilon\delta^2\left\{u(1-u)\times\left[2+13u(1-u)+10u^3\log u\left(2-3u+\frac{6}{5}u^2\right)\right]+10(1-u)^3\log\left[(1-u)\left(2-3(1-u)+\frac{6}{5}(1-u)^2\right)\right]\right\},$$

$$g_2(u)=\frac{10}{3}\delta^2u(1-u)(2u-1), \tag{3.4}$$

where

$$B_2(\mu_b)=0.29,\quad B_4(\mu_b)=0.58,\quad B_2(\mu_c)=0.41,$$
$$B_4(\mu_c)=0.925,$$
$$C_2(\mu_b)=0.059,\quad C_4(\mu_b)=0.034,$$
$$C_2(\mu_c)=0.087,\quad C_4(\mu_c)=0.054,$$
$$\delta^2(\mu_b)=0.17\ \text{GeV}^2,\quad \delta^2(\mu_c)=0.19\ \text{GeV}^2,$$
$$\epsilon(\mu_b)=0.36,\quad \epsilon(\mu_c)=0.45. \tag{3.5}$$

In addition to in these distribution amplitudes, the SU(3) breaking effects also emerge in the light meson constants in the coefficients of the distribution amplitudes. We take $f_\pi=0.132$ GeV, $f_K=0.16$ GeV, and $\mu_\pi=m_\pi^2/(m_u+m_d)$ with $\mu_\pi(1\ \text{GeV})=1.65$ GeV. For $\mu_K=m_K^2/(m_s+m_{u,d})$, we use the advocation in Ref. [32] to rely on chiral perturbation theory in the SU(3) limit and so use $\mu_K=\mu_\pi$. For the quantities relevant to heavy hadrons, here we use the data evaluated in Ref. [28]. In particular, there we obtained $\bar{\Lambda}=0.53\pm 0.08$ GeV; $F=0.30\pm 0.06\ \text{GeV}^{3/2}$.

For decays into light vector mesons, the leading twist distribution functions $\phi_\perp$ and $\phi_\parallel$ can also be expanded in Gegenbauer polynomials $C_n^{3/2}(x)$ with the coefficients running with the scale and described by the renormalization group method. Explicitly we have

$$\phi_{\perp(\parallel)}(u,\mu)=6u(1-u)\left[1+\sum_{n=2,4,\ldots}a_n^{\perp(\parallel)}(\mu)C_n^{3/2}(2u-1)\right],$$

$$a_n^{\perp(\parallel)}(\mu)=a_n^{\perp(\parallel)}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{(\gamma_n^{\perp(\parallel)}-\gamma_0^{\perp(\parallel)})/(2\beta_0)}, \tag{3.6}$$

where $\beta_0=11-(2/3)n_f$, and $r_n^\perp$, $r_n^\parallel$ are the one loop anomalous dimensions [34,35]. The coefficients $a_n^\perp, a_n^\parallel$ have been studied extensively in Ref. [13]. Here we use the values for ρ and K^* mesons presented in that paper, where the SU(3) breaking effects are included for K^* meson.

The functions $g_\perp^{(v)}$ and $g_\perp^{(a)}$ describe the transverse polarizations of quarks in the longitudinally polarized mesons. They receive contributions from both twist 2 and twist 3. In this paper we will include only the twist 2 contributions, which are related to the longitudinal distribution $\phi_\parallel(u,\mu)$ by Wandzura-Wilczek type relations [36,37]:

$$g_\perp^{(v),\text{twist 2}}(u,\mu)=\frac{1}{2}\left[\int_0^u dv\,\frac{\phi_\parallel(v,\mu)}{1-v}+\int_u^1 dv\,\frac{\phi_\parallel(v,\mu)}{v}\right],$$

$$g_\perp^{(a),\text{twist 2}}(u,\mu)=2\left[(1-u)\int_0^u dv\,\frac{\phi_\parallel(v,\mu)}{1-v}+u\int_u^1 dv\,\frac{\phi_\parallel(v,\mu)}{v}\right]. \tag{3.7}$$

The quatities f_ρ and f_{K^*} are the decay constants of vector mesons, and $f_\rho^\perp$ and $f_{K^*}^\perp$ are couplings defined via

$$\langle 0|\bar{u}\sigma_{\mu\nu}q|V(p,\epsilon)\rangle=i(\epsilon_\mu p_\nu-\epsilon_\nu p_\nu)f_V^\perp \tag{3.8}$$

with the light quark $q=d$ or s corresponding to the vector meson $V=\rho$ or K^*. In the calculations, we use for these couplings [13,36,38,39]

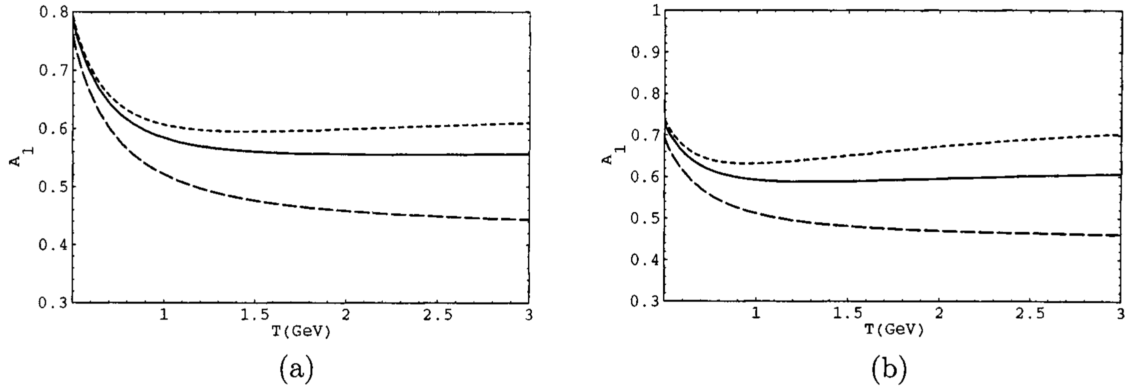

FIG. 1. The form factors $A_1^{D\to\rho}$ (a) and $A_1^{D\to K^*}$ (b) as functions of the Borel parameter T for different values of the continuum threshold s_0. The dashed, solid, and dotted curves correspond to $s_0=1.5$, 2, and 2.5 GeV, respectively. The case considered here is at the momentum transfer $q^2=0$ GeV^2.

$$f_\rho=195\pm 7 \text{ MeV},\quad f_\rho^\perp=160\pm 10 \text{ MeV},$$

$$f_{K^*}=226\pm 28 \text{ MeV},\quad f_{K^*}^\perp=185\pm 10 \text{ MeV}. \tag{3.9}$$

From Eqs. (2.9)–(2.13) and (2.21)–(2.26), the data for form factors can be obtained by using the distribution amplitudes and meson quantities presented above. As an example, Fig. 1 shows the variation of $A_1^{D\to\rho}$ and $A_1^{D\to K^*}$ with respect to the Borel parameter T at the fixed values of $s_0=1.5$, 2, and 2.5 GeV. According to the light cone sum rule criterion that both the higher resonance contributions and the contributions from higher twist distribution amplitudes should not be too large (say, larger than 30%), our interesting regions of T are around $T=2$ GeV for $B_{(s)}$ decays, and around $T=1$ GeV for D decays. We studied the variations of form factors in all the decays studied with respect to the Borel parameter T. According to our detailed study, for all form factors there exist reliable regions of s_0 and T that well satisfy the requirement of stability in the sum rule analysis. We note that in the original calculation (the first reference in [25]) we used a wrong sign for the contribution of $g_\perp^{(a)}$. In later study (the second reference in [25], or an Erratum to be published) it was found that the correction of this sign greatly enlarges the value of V, and at the same time also improves the stability of the $V(T)$ curves in the sum rule window. The threshold energies for all form factors in these decays are found to vary from 0.4 to 3.5 GeV. With the ranges of T and s_0 determined, all the form factors as functions of the momentum transfer q^2 can be derived from Eqs. (2.9)–(2.13) and (2.21)–(2.26).

TABLE I. Results for the form factor f_+ of heavy to light pseudoscalar meson decays. s_0 is the threshold at which the parameters $F(0),a_F,b_F$ are fitted.

		$F(0)$	a_F	b_F	s_0 (GeV)
$B\to\pi l\nu$	f_+	0.35 ± 0.06	1.31 ± 0.15	0.35 ± 0.18	2.3 ± 0.6
$B_s\to K l\nu$	f_+	0.47 ± 0.09	1.12 ± 0.25	0.34 ± 0.19	2.7 ± 0.8
$D\to\pi l\nu$	f_+	0.67 ± 0.19	1.30 ± 0.30	0.68 ± 0.38	0.8 ± 0.4
$D\to K l\nu$	f_+	0.67 ± 0.20	1.65 ± 0.43	1.28 ± 0.52	0.8 ± 0.4

It is known that when the final mesons are light pseudoscalars, the light cone expansion and the sum rule method will break down at large momentum transfer (numerically as q^2 approaches m_b^2) [15]. As a result, the curves of wave functions calculated from light cone sum rules may become unstable in the large q^2 region. Thus in this region we have to rely on other approximations such as the single or double pole approximation. Here we use for the $B\to\pi$ transition

$$f_+(q^2)=\frac{f_{B^*}g_{B^*B\pi}}{2m_{B^*}(1-q^2/m_{B^*}^2)} \tag{3.10}$$

for the large q^2 regions of B decays into light pseudoscalar mesons. Similar monopole approximation formulas are applied to other heavy mesons decaying into light pseudoscalar mesons. In our numerical calculations, we take $f_{B^*}=0.16$

TABLE II. Results for the form factors of heavy to light vector meson decays. s_0 is the threshold at which the parameters $F(0),a_F,b_F$ are fitted.

		$F(0)$	a_F	b_F	s_0 (GeV)
$B\to\rho l\nu$	A_1	0.29 ± 0.05	0.35 ± 0.15	-0.24 ± 0.12	2.1 ± 0.6
	A_2	0.28 ± 0.04	1.09 ± 0.13	0.20 ± 0.18	
	A_3	-0.28 ± 0.04	1.09 ± 0.13	0.20 ± 0.18	
	V	0.35 ± 0.06	1.32 ± 0.12	0.46 ± 0.10	
$B_s\to K^* l\nu$	A_1	0.28 ± 0.05	0.82 ± 0.06	0.05 ± 0.14	2.5 ± 0.5
	A_2	0.28 ± 0.04	1.48 ± 0.05	0.62 ± 0.08	
	A_3	-0.28 ± 0.04	1.48 ± 0.05	0.62 ± 0.08	
	V	0.35 ± 0.05	1.73 ± 0.02	0.95 ± 0.35	
$D\to\rho l\nu$	A_1	0.57 ± 0.08	0.60 ± 0.20	0.51 ± 1.32	2.0 ± 0.5
	A_2	0.52 ± 0.07	0.66 ± 0.20	-2.03 ± 1.52	
	A_3	-0.52 ± 0.07	0.66 ± 0.20	-2.03 ± 1.52	
	V	0.72 ± 0.10	0.95 ± 0.06	2.60 ± 3.62	
$D\to K^* l\nu$	A_1	0.59 ± 0.10	0.58 ± 0.12	0.11 ± 0.28	2.0 ± 0.5
	A_2	0.55 ± 0.08	0.84 ± 0.31	-1.29 ± 1.12	
	A_3	-0.55 ± 0.08	0.84 ± 0.31	-1.29 ± 1.12	
	V	0.80 ± 0.10	0.86 ± 0.65	2.71 ± 1.83	

± 0.03 GeV, $g_{B^*B\pi}=29\pm 3$ [15], $f_{D^*}g_{D^*D\pi}=2.7\pm 0.8$ GeV, $f_{D_s^*}g_{D_s^*DK}=3.1\pm 0.6$ GeV [16], and $f_{B^*}g_{B^*B_sK}=3.88 \pm 0.31$ GeV [40].

As a good approximation, for the behavior of the form factors in the whole kinematically accessible region, we use the parametrization

$$F(q^2)=\frac{F(0)}{1-a_F q^2/m_B^2+b_F(q^2/m_B^2)^2}, \tag{3.11}$$

where $F(q^2)$ can be any of the form factors f_+, f_-, A_i ($i=1,2,3$), and V. Thus each form factor will be parametrized by three parameters $F(0)$, a_F, and b_F that need to be fitted. For the form factor f_+, as mentioned above, since the light cone sum rules are most suitable for describing the low q^2 region of the form factors and the very high q^2 region is hard to reach by this approach, we shall use the light cone sum rule results in the small q^2 region and the monopole approximation (3.10) in the large q^2 region to fit the three parameters in Eq. (3.11). For decays into vector mesons, we use only the light cone sum rule predictions in fitting these parameters. This is because the kinematically allowed ranges of B for vector meson decays are small compared with the ranges of B for pseudoscalar meson decays, and therefore the sum rules are expected to yield reasonable values for most allowed regions of q^2 in the vector meson cases.

At certain values of a suitable threshold s_0, we can fix the parameters for each form factor. Our numerical results for those parameters are presented in Table I and II. For the form factor f_-, since it is irrelevant to the decay rates when the lepton masses are neglected, we do not present values in these tables. In this paper we calculate them directly from sum rules, and the results at large recoil regions are used in the next section in comparing with the form factor relations derived based on the large mass and large energy expansion.

The values of $F(0)$ for $B\to\rho$ decay presented in Table II are slightly different from those given in Ref. [25]. The reason is that the overall factor $m_b\overline{\Lambda}_B/m_B\overline{\Lambda}$ in Eq. (3.13) of [25] is missed in the current calculation, as mentioned in the previous section. Uncertainties in the form factors could arise from the meson constants, the light cone distribution amplitudes, and the variation of thresholds s_0. We notice that the latter comprises the largest uncertainty, which may be 15% or so. Variations of other imput parameters within their allowed ranges which we have discussed would increase the uncertainties to about 20%. So, including uncertainties from higher twist amplitudes and other systematic uncertainties in light cone sum rule method, we quote an uncertainty of about 25%.

TABLE III. Comparison of the results for semileptonic decay form factors (at $q^2=0$) from different approaches (QM, quark model; Lat, lattice calculation; SR, sum rules in QCD framework; E791, data extracted from experimental measurements).

	Reference	$f_+(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
$B\to\pi(\rho)l\nu$	This work	0.35 ± 0.06	0.29 ± 0.05	0.28 ± 0.04	0.35 ± 0.06
	QM [8]	0.29	0.26	0.24	0.31
	QM [9]	0.27	0.26	0.24	0.35
	Lat [2]	$0.50(14)^{+7}_{-5}$	$0.16(4)^{+22}_{-16}$	$0.72(35)^{+10}_{-7}$	$0.61(23)^{+9}_{-6}$
	Lat [5]	0.28 ± 0.04			
	SR [17,18]	0.305	0.26 ± 0.04	0.22 ± 0.03	0.34 ± 0.05
	SR [19]	0.28 ± 0.05			
$B_s\to K(K^*)l\nu$	This work	0.47 ± 0.09	0.28 ± 0.05	0.28 ± 0.04	0.35 ± 0.05
	QM [8]	0.31	0.29	0.26	0.38
$D\to\pi(\rho)l\nu$	This work	0.67 ± 0.19	0.57 ± 0.08	0.52 ± 0.07	0.72 ± 0.10
	QM [8]	0.69	0.59	0.49	0.90
	QM [9]	0.67	0.58	0.42	0.93
	QM [10]	0.69	0.78	0.92	1.23
	Lat [2]	$0.68(13)^{+10}_{-7}$	$0.59(7)^{+8}_{-6}$	$0.83(20)^{+12}_{-8}$	$1.31(25)^{+18}_{-13}$
	Lat [5]	$0.64(5)^{+00}_{-07}$			
	Lat [6]	0.65 ± 0.10	0.65 ± 0.07	0.55 ± 0.10	1.1 ± 0.2
	SR [19]	0.65 ± 0.11			
	SR [21]	0.50 ± 0.15	0.5 ± 0.2	0.4 ± 0.2	1.0 ± 0.2
$D\to K(K^*)l\nu$	This work	0.67 ± 0.20	0.59 ± 0.10	0.55 ± 0.08	0.80 ± 0.10
	E791 [42]		0.58 ± 0.03	0.41 ± 0.06	1.06 ± 0.09
	QM [8]	0.78	0.66	0.49	1.03
	QM [9]	0.78	0.66	0.43	1.04
	Lat [2]	$0.71(12)^{+10}_{-7}$	$0.61(6)^{+9}_{-7}$	$0.83(20)^{+12}_{-8}$	$1.34(24)^{+19}_{-14}$
	Lat [6]	0.73 ± 0.07	0.70 ± 0.07	0.6 ± 0.1	1.2 ± 0.2
	SR [19]	0.76 ± 0.03			
	SR [21]	0.60 ± 0.15	0.50 ± 0.15	0.60 ± 0.15	1.10 ± 0.25

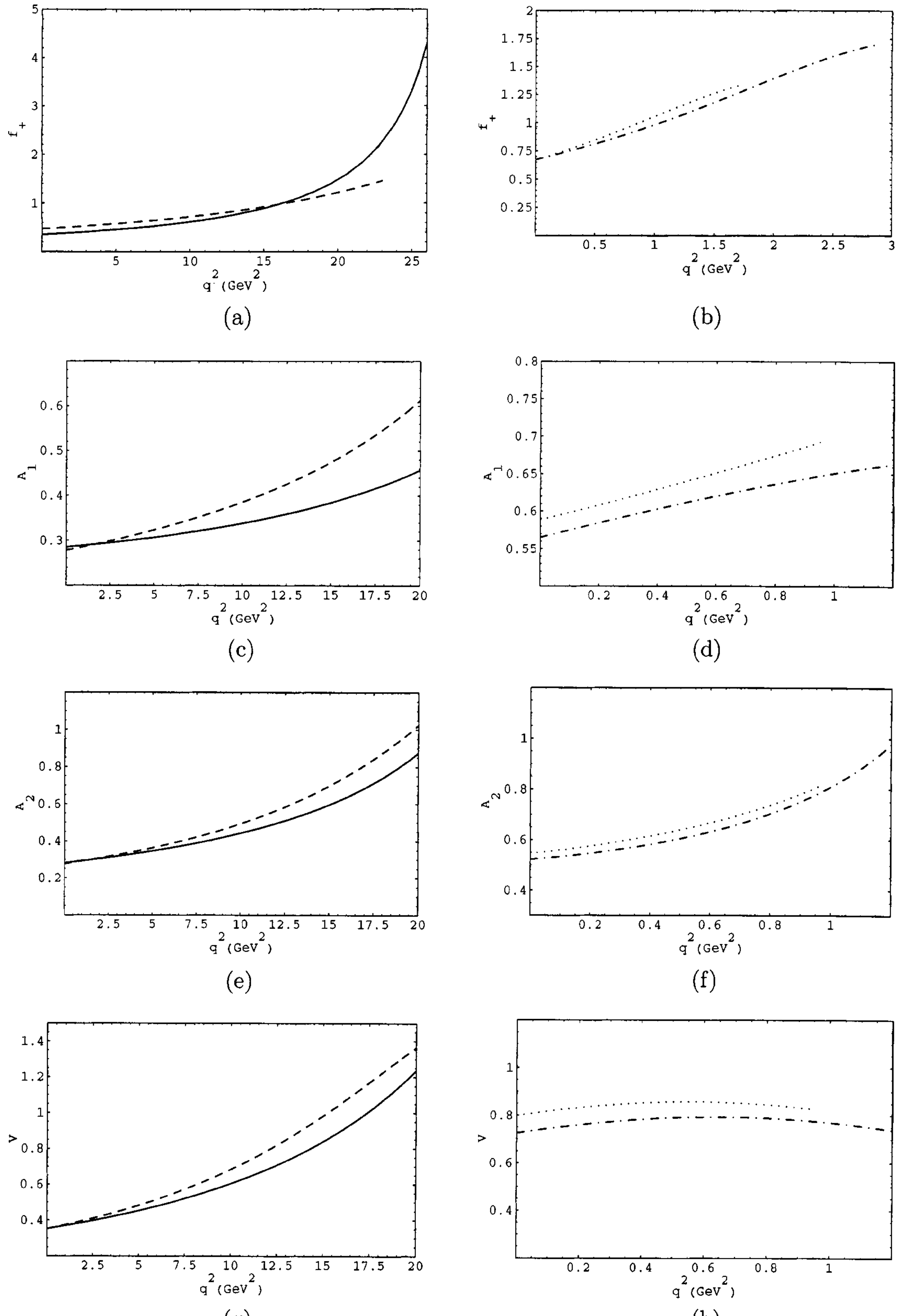

FIG. 2. Results for the heavy to light decay form factors from light cone sum rule study. The solid, dashed, dot-dashed, and dotted curves correspond to $B\to\pi(\rho)$, $B_s\to K(K^*)$, $D\to\pi(\rho)$, and $D\to K(K^*)$ decays, respectively. $A_3(q^2)$ is not shown here as one has $A_3(q^2)=-A_2(q^2)$ at the leading order considered.

TABLE IV. Comparison of measurements and theoretical predictions for form factor ratios and $A_1(0)$.

	R_V	R_2	$A_1(0)$
This work	1.36 ± 0.39	0.93 ± 0.29	0.59 ± 0.10
BEATRICE [41]	$1.45\pm0.23\pm0.07$	$1.00\pm0.15\pm0.03$	
E791 [42]	$1.84\pm0.11\pm0.08$	$0.71\pm0.08\pm0.09$	0.58 ± 0.03
E687 [43]	$1.74\pm0.27\pm0.28$	$0.78\pm0.18\pm0.10$	
E653 [44]	$2.00^{+0.34}_{-0.32}\pm0.16$	$0.82^{+0.22}_{-0.23}\pm0.11$	
E691 [45]	$2.0\pm0.6\pm0.3$	$0.0\pm0.5\pm0.2$	

In addition to the SU(3) symmetry breaking effects arising from relevant light meson parameters, it is found in our investigation that SU(3) symmetry breaking effects considered in the Gegenbauer polynomial moments a_i ($i=1,2,3,4$) cause changes of $F(0)$ by no more than 15%.

The form factors as functions of q^2 are shown in Fig. 2. Table III is a comparison of the form factor values at $q^2=0$ obtained in this work and those obtained from other approaches, including quark models, lattice calculations, and also sum rules in the QCD framework.

The form factor ratios $R_V\equiv V(0)/A_1(0)$, $R_2\equiv A_2(0)/A_1(0)$ for $D\to K^*l\nu$ decay have been measured by several groups. We present in Table IV a comparison of our results for these ratios with the measurements. Our result for R_V agrees well with the latest measurement [41] but is smaller than other measurements. R_2 obtained in this work is also in good agreement with the BEATRICE measurement [41].

With the consideration that in heavy to light decays the final light meson usually carries a large recoil momentum, the large energy effective theory (LEET) [46–50] has been proposed to study heavy to light transitions in the region of large energy release. In that framework and neglecting QCD short distance corrections, it is quoted that there are only three form factors needed to describe all the pseudoscalar to pseudoscalar or vector transition matrix elements in the leading order of the heavy quark mass and large energy expansion. In particular, taking into account contributions of second order in the ratio of the light meson mass to the large recoil energy, a recent work [50] derived interesting relations concerning form factors for semileptonic B decays to light pseudoscalar and vector mesons:

$$f_+(q^2)=\frac{m_M}{2E_F}\left(1+\frac{m_P^2}{m_M^2}\right)f_0(q^2), \tag{3.12}$$

$$\frac{m_M}{m_M+m_V}\,\frac{\sqrt{E_F^2-m_V^2}}{E_F}V(q^2)$$

$$=\frac{m_M+m_V}{2E_F}A_1(q^2)$$

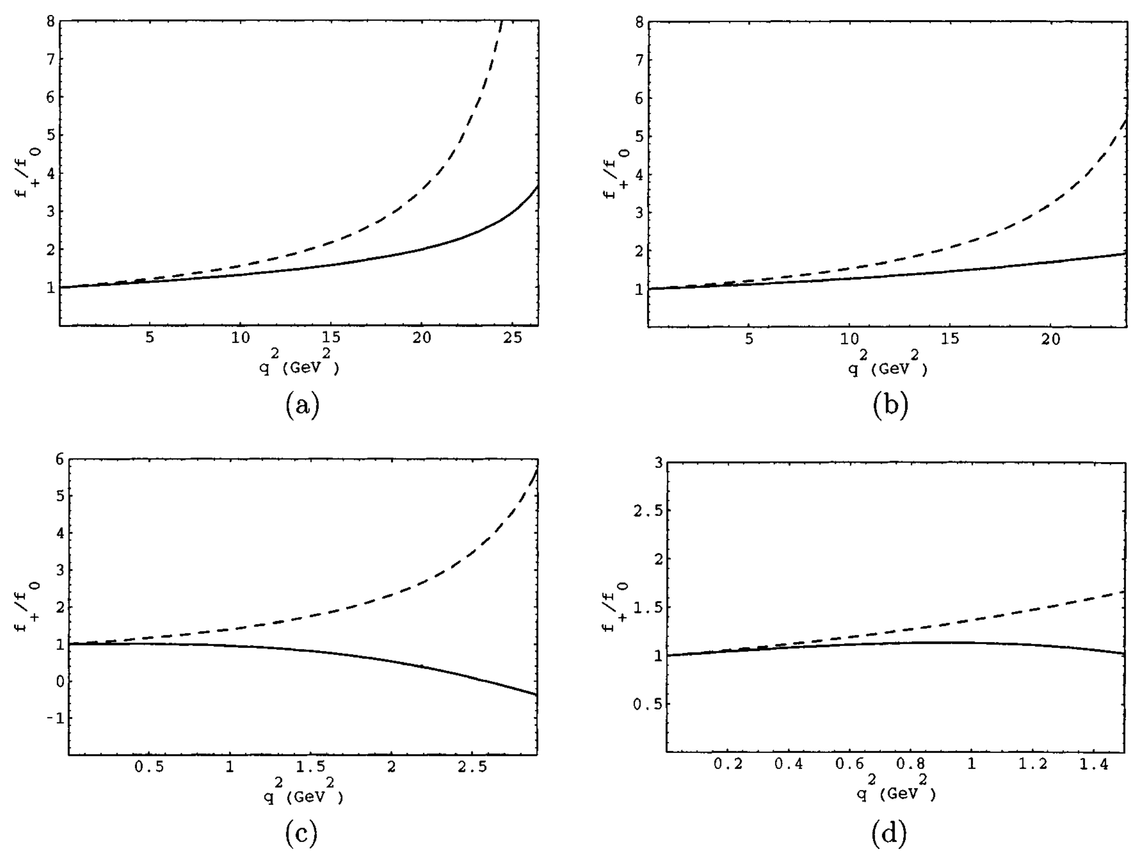

FIG. 3. Results for the ratio $f_+(q^2)/f_0(q^2)$. (a), (b), (c), and (d) are for $B\to\pi l\nu$, $B_s\to Kl\nu$, $D\to\pi l\nu$, and $D\to Kl\nu$, respectively. Solid lines are our sum rule results, while the dashed lines are produced from the LEET relation (3.17).

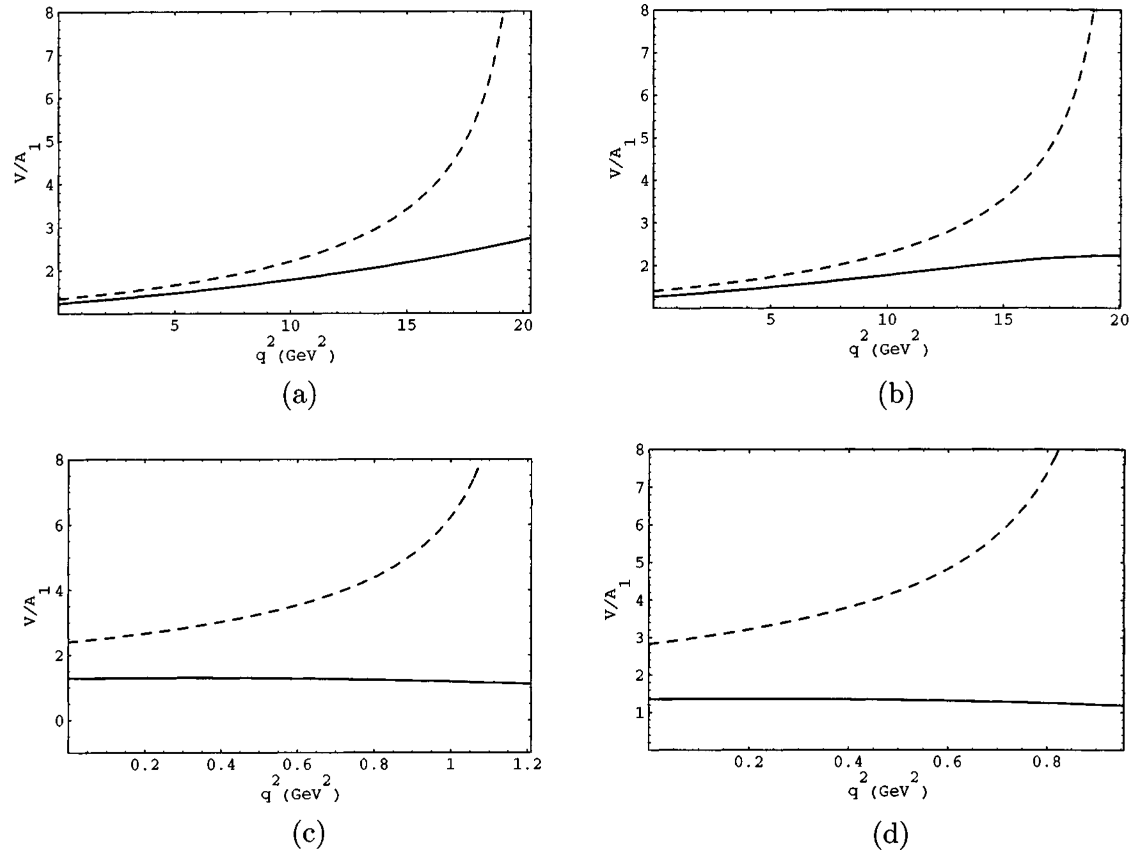

FIG. 4. Results for the ratio $V(q^2)/A_1(q^2)$. (a), (b), (c), and (d) are for $B\to\rho l\nu$, $B_s\to K^*l\nu$, $D\to\rho l\nu$, and $D\to K^*l\nu$, respectively. Solid lines are our sum rule results, while the dashed lines are produced from the LEET relation (3.18).

$$=\frac{m_M}{m_M+m_V}\left(1-\frac{2m_V^2}{m_M^2}\right)A_2(q^2)+\frac{m_V}{E_F}A_0(q^2), \tag{3.13}$$

where E_F is the on-shell energy of the final light meson:

$$E_F=\frac{m_M^2+m_{P(V)}^2-q^2}{2m_M}. \tag{3.14}$$

$f_0(q^2)$ and $A_0(q^2)$ are form factors defined in the way of Ref. [50], and they can be represented by the form factors defined in this paper as follows:

$$f_0(q^2)=\frac{q^2}{m_M^2-m_P^2}f_-(q^2)+f_+(q^2), \tag{3.15}$$

TABLE V. Decay widths Γ (in units of $|V_{qQ}|^2\ \mathrm{ps}^{-1}$) and branching ratios (BR) for heavy to light meson decays. In deriving the branching ratios we used $|V_{ub}|=0.0037$, $|V_{cd}|=0.22$, $|V_{cs}|=0.97$ and the lifetimes of heavy measons $\tau_{B^0}=1.56\pm0.06$ ps, $\tau_{D^0}=0.4126\pm0.0028$ ps, $\tau_{B_S}=1.493\pm0.062$ ps.

	Γ ($\lvert V_{qQ}\rvert^2$ ps^{-1})	BR	BR(measurements)
$B\to\pi l\nu$	10.2 ± 1.5	$(2.1\pm0.5)\times10^{-4}$	$(1.8\pm0.6)\times10^{-4}$
$B_s\to Kl\nu$	14.3 ± 2.8	$(2.9\pm0.7)\times10^{-4}$	
$B\to\rho l\nu$	15.2 ± 4.2	$(3.2\pm1.0)\times10^{-4}$	$(2.6^{+0.6}_{-0.7})\times10^{-4}$
$B_s\to K^*l\nu$	20.2 ± 4.3	$(4.1\pm1.0)\times10^{-4}$	
$D\to\pi l\nu$	0.152 ± 0.042	$(3.0\pm0.9)\times10^{-3}$	$(3.7\pm0.6)\times10^{-3}$
$D\to Kl\nu$	0.101 ± 0.030	$(3.9\pm1.2)\times10^{-2}$	$(3.47\pm0.17)\times10^{-2}$
$D\to\rho l\nu$	0.071 ± 0.015	$(1.4\pm0.3)\times10^{-3}$	
$D\to K^*l\nu$	0.050 ± 0.014	$(2.0\pm0.5)\times10^{-2}$	$(2.02\pm0.33)\times10^{-2}$

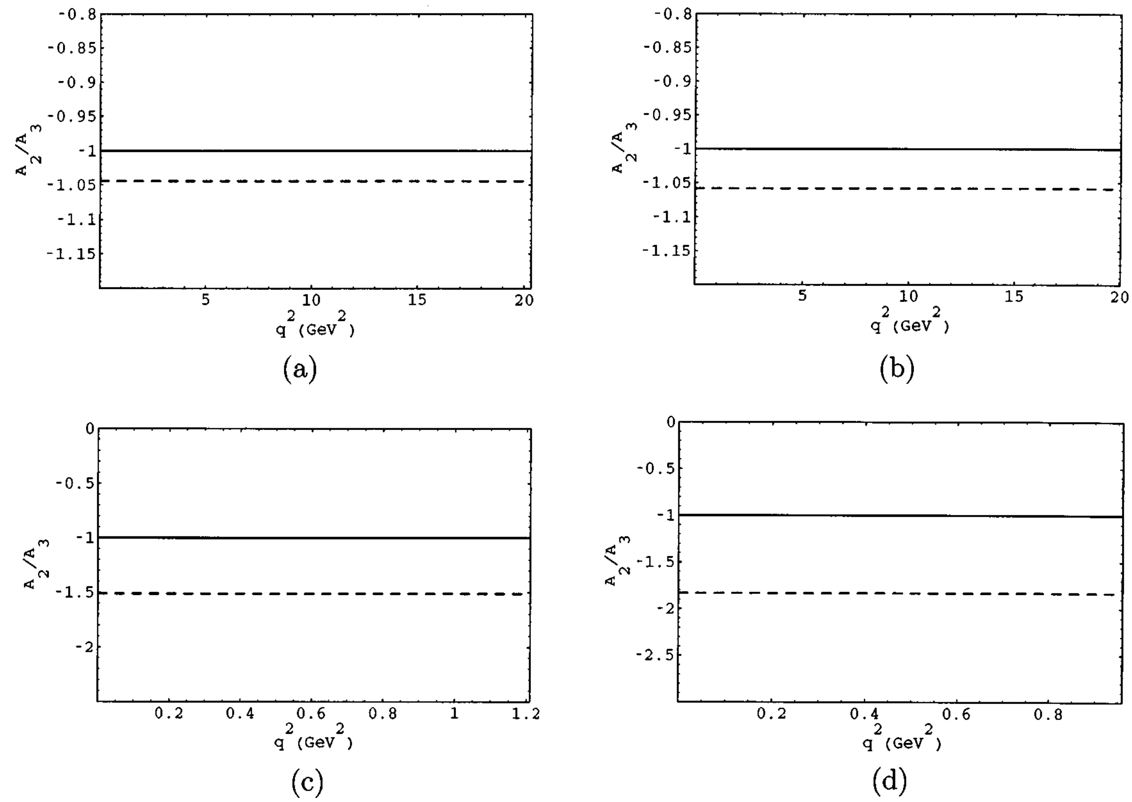

FIG. 5. Results for the ratio $A_2(q^2)/A_3(q^2)$. (a), (b), (c), and (d) are for $B\rightarrow\rho l\nu$, $B_s\rightarrow K^* l\nu$, $D\rightarrow\rho l\nu$, and $D\rightarrow K^* l\nu$, respectively. Solid lines are our sum rule results, while the dashed lines are produced from the LEET relation (3.19).

$$A_0(q^2)=\frac{1}{2m_V}\left[(m_M+m_V)A_1(q^2)-(m_M-m_V)A_2(q^2)-\frac{q^2}{m_M+m_V}A_3(q^2)\right]. \tag{3.16}$$

As a result, the relations (3.12), (3.13) lead to

$$f_+(q^2)=\frac{m_M^2+m_P^2}{m_M^2+m_P^2-q^2}f_0(q^2), \tag{3.17}$$

$$V(q^2)=\frac{(m_M+m_V)^2}{2m_M\sqrt{E_F^2-m_V^2}}A_1(q^2), \tag{3.18}$$

$$A_2(q^2)=\frac{m_M^2}{2m_V^2-m_M^2}A_3(q^2), \tag{3.19}$$

where in Eq. (3.19) terms containing m_V^4 have been discarded because they can be attributed to higher order contributions in LEET. Note that Eq. (3.19) implies the relation $A_2(q^2)=-A_3(q^2)$ at the leading order of LEET, which is consistent with our result in this work (see Sec. II).

One can now make a numerical comparison between our results for form factors and the LEET predictions. In Figs. 3–5, we show the form factor ratios $f_+(q^2)/f_0(q^2)$, $V(q^2)/A_1(q^2)$, and $A_2(q^2)/A_3(q^2)$ derived from our sum rule calculations and from the relations (3.17)–(3.19). In Fig. 3, the ratio $f_+(q^2)/f_0(q^2)$ from our calculation and that from the LEET relation (3.17) are close to each other in regions near the maximum recoil point, but they quickly split from each other as q^2 increases. Similar variations can be observed for the $B_{(s)}$ decay ratio $V(q^2)/A_1(q^2)$ in Figs. 4(a) and 4(b). For D decays to light vector mesons, however, in Figs. 4(c) and 4(d) a large difference exists between our results and the predictions of relation (3.18). In Figs. 5(a) and 5(b) (i.e., for $B_{(s)}$ decays), the ratios $A_2(q^2)/A_3(q^2)$ of our results and the LEET prediction are compatible in the whole kinematically allowed region. In Figs. 5(c) and 5(d), our results for the D decay ratio $A_2(q^2)/A_3(q^3)$ and the prediction of relation (3.19) are again incompatible.

As a general view, our results are compatible with the relations (3.17)–(3.19) in appropriate regions for $B_{(s)}$ decays. On the other hand, discrepancies occur in the case of D decays between our results and the predictions of relations (3.17)–(3.19). Nevertheless, this is not unexpected due to the not very large mass of the charm quark. As mentioned in Ref. [48], the $D\rightarrow K^{(*)}$ decay is quite far from the $m_M\rightarrow\infty$ and $E\rightarrow\infty$ limit. Thus one cannot expect relations (3.17)–(3.19) to be valid for D decays.

IV. BRANCHING RATIOS

With the form factors derived in the previous sections, one can extract the values of relevant CKM matrix elements from the experimental measurements of branching ratios. In Refs. [24] and [25], $|V_{ub}|$ is extracted in this way from B

TABLE VI. Theoretical predictions and measurements of the ratios $\Gamma_L/\Gamma_T, \Gamma_+/\Gamma_-$ and the decay rates Γ (in units of $|V_{qQ}|^2\ \mathrm{ps}^{-1}$). The data from BEATRICE, E687, E653, E691, and WA82 are experimental measurements.

	Γ_L/Γ_T	Γ_+/Γ_-	$\Gamma/\lvert V_{qQ}\rvert^2$ (ps^{-1})	Reference
$B\to\rho l\nu$	0.85 ± 0.09	0.04 ± 0.02	15.2 ± 4.2	This work
	0.82		19.1	QM [9]
	0.88 ± 0.08		15.8 ± 2.3	QM [11]
			13 ± 12	Lat [4]
	$0.80^{+0.04}_{-0.03}$		$16.5^{+3.5}_{-2.3}$	Lat [7]
	0.52 ± 0.08		13.5 ± 4.0	SR [20]
	0.06 ± 0.02	0.007 ± 0.004	12 ± 4	SR [21]
$B_s\to K^*l\nu$	0.79 ± 0.08	0.07 ± 0.02	20.2 ± 4.3	This work
$D\to\rho l\nu$	1.17 ± 0.09	0.29 ± 0.13	0.071 ± 0.015	This work
	1.16		0.087	QM [8]
	0.67		0.025	QM [12]
			0.122 ± 0.041	Lat [1]
			0.102 ± 0.047	Lat [4]
	1.31 ± 0.11	0.24 ± 0.03	0.024 ± 0.007	SR [21]
$D\to K^*l\nu$	1.15 ± 0.10	0.32 ± 0.13	0.050 ± 0.014	This work
	$1.09\pm0.10\pm0.02$	$0.28\pm0.05\pm0.02$		BEATRICE [41]
	$1.20\pm0.13\pm0.13$			E687 [43]
	$1.18\pm0.18\pm0.08$			E653 [44]
	$1.8^{+0.6}_{-0.4}\pm0.3$			E691 [45]
	$0.6\pm0.3^{+0.3}_{-0.1}$			WA82 [52]
	1.28		0.063	QM [8]
	1.33		0.058	QM [9]
	1.2 ± 0.3		0.073 ± 0.019	Lat [1]
	1.1 ± 0.2		$0.063^{+0.008}_{-0.017}$	Lat [3]
	0.86 ± 0.06	3.8 ± 1.5		SR [21]

$\to\pi(\rho)l\nu$ decays using the branching ratio measurements. The results obtained there were $|V_{ub}|=(3.4\pm0.5\pm0.5)\times10^{-3}$ and $|V_{ub}|=(3.7\pm0.6\pm0.7)\times10^{-3}$ via the two decays, respectively, where the first (second) errors correspond to the experimental (theoretical) uncertainties.

On the other hand, the decay widths and branching ratios of exclusive semileptonic decays can be predicted if we know the values of relevant CKM matrix elements. When the lepton masses are neglected, we have for decays to pseudoscalar mesons

$$\frac{d\Gamma}{dq^2}=\frac{G_F^2|V_{qQ}|^2}{24\pi^3}\left[\left(\frac{m_M^2+m_P^2-q^2}{2m_M}\right)^2-m_P^2\right]^{3/2}[f_+(q^2)]^2, \tag{4.1}$$

and for decays to vector mesons

$$\frac{d\Gamma}{dq^2}=\frac{G_F^2|V_{qQ}|^2}{192\pi^3m_M^3}\lambda^{1/2}q^2(H_0^2+H_+^2+H_-^2) \tag{4.2}$$

TABLE VII. Rates in unit $|V_{qQ}|^2\ \mathrm{ps}^{-1}$ for semileptonic decays to pseudoscalar mesons from different approaches.

Reference	$B\to\pi l\nu$	$B_s\to Kl\nu$	$D\to\pi l\nu$	$D\to Kl\nu$
This work	10.2 ± 1.5	14.3 ± 2.8	0.152 ± 0.042	0.101 ± 0.030
QM [8]			0.196	0.102
QM [9]	10.0		0.165	0.101
Lat [1]	8 ± 4		0.162 ± 0.041	0.096 ± 0.021
Lat [4]	12 ± 8		0.114 ± 0.073	0.072 ± 0.036
Lat [7]	$8.5^{+3.3}_{-1.4}$			
SR [19]	7.3 ± 2.5		0.13 ± 0.05	0.094 ± 0.036
SR [21]	5.1 ± 1.1		0.080 ± 0.017	0.068 ± 0.014

with $\lambda \equiv (m_M^2 + m_V^2 - q^2)^2 - 4 m_M^2 m_V^2$ and the helicity amplitudes defined as

$$H_{\pm} = (m_M + m_V) A_1(q^2) \mp \frac{\lambda^{1/2}}{m_M + m_V} V(q^2),$$

$$H_0 = \frac{1}{2 m_V \sqrt{q^2}} \left\{ (m_M^2 - m_V^2 - q^2)(m_M + m_V) A_1(q^2) - \frac{\lambda}{m_M + m_V} A_2(q^2) \right\}. \quad (4.3)$$

In this paper we would like to predict the branching ratios from the values of the CKM matrix elements: $|V_{ub}| = 0.0037$, $|V_{cd}| = 0.22$, $|V_{cs}| = 0.97$. Finishing the integration over q^2, we obtain the widths and branching ratios in Table V. In that table we also list the experimental measurements of branching ratios given by Ref. [51].

The largest uncertainty for the branching ratios in Table V is about 30%. So the precision of heavy meson decay measurements expected in the new B factories cannot be well matched unless these theoretical uncertainties can be reduced. This reduction may be reached by consideration of both higher twist contributions and better determination of the meson constants and the higher order contributions in the heavy quark expansion, which should be discussed in future work.

Table VI is a comparison of the results for the ratios $\Gamma_L/\Gamma_T, \Gamma_+/\Gamma_-$ and the total decay rates Γ for heavy to light vector decays, where Γ_L (Γ_T) and Γ_+ ($/\Gamma_-$) represent partial widths for longitudinal (transverse) polarization and positive (negative) helicity, respectively. These widths and ratios have been predicted via other approaches. In particular, the ratios for $D \to K^* l \nu$ decay are available from recent experiments.

Similarly, Table VII presents a comparison of results for heavy to light pseudoscalar decay rates.

V. SUMMARY

In summary, we have studied the exclusive semileptonic decays of heavy to light mesons. The form factors for the decays $B(D,B_s) \to \pi(\rho,K,K^*) l \nu$ have been consistently calculated by using the light cone sum rule method in the effective field theory of heavy quarks. The HQS leads to simplification in studying heavy to light transitions as to a certain order of the $1/m_Q$ expansion the decays of different heavy hadrons such as B and D can be characterized by the same set of wave functions which are explicitly independent of the heavy quark mass, although the HQS does not reduce the number of independent form factors needed for an individual decay. In such calculations, the uncertainties for the form factors are generally about 25%, which, together with the meson constants, may give the branching ratios with a total uncertainty up to 30%. We have also estimated the light flavor SU(3) symmetry breaking effects in these semileptonic decays and found that those effects may influence the form factors up to a total amount of 15%. Our results were compared with data from experiments and from other theoretical approaches. In particular, we checked the compatibility between our results and the form factor relations derived in Ref. [50] using the heavy quark and large recoil expansion. We conclude that the form factors and branching ratios of those heavy to light meson semileptonic decays can be calculated consistently based on the light cone sum rule approach within the framework of HQEFT. Nevertheless, in order to match the expected more precise experimental measurements at B factories in the near future, more accurate calculations for the exclusive heavy to light meson semileptonic decays and needed. The $1/m_Q$ corrections may be important, especially in charm meson decays. This should be investigated in future work.

In this paper we have only considered semileptonic heavy to light decays, but what we used here is a general approach, and similar procedures could be applied straightforwardly to heavy to light rare decays such as $B \to K l l$, $B \to K^* l l$, and $B \to K^* \gamma$. For such rare decays, this framework becomes more interesting because it reduces the number of form factors for an individual decay. We will discuss that in another paper.

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