p-p' branes in a pp-wave background

Anindya Biswas,* Alok Kumar,† and Kamal L. Panigrahi‡
Institute of Physics, Bhubaneswar 751 005, India

(Received 9 August 2002; revised manuscript received 27 August 2002; published 19 December 2002)

We present several supergravity solutions corresponding to both Dp and Dp-Dp' systems, in Neveu-Schwarz–Neveu-Schwarz and Ramond-Ramond pp-wave background originating from $AdS_3\times S^3\times R^4$. The D$p$-brane solutions, $p=1,\ldots,5$ are fully localized, whereas Dp-Dp' solutions are localized along common transverse directions. We also discuss the supersymmetry properties of these solutions and the worldsheet construction for the p-p' system.

DOI: 10.1103/PhysRevD.66.126002 PACS number(s): 11.25.Sq

I. INTRODUCTION

pp waves [1–16] are known to be an interesting class of supersymmetric solutions of type IIB supergravity, with wide applications to gauge theories. For these applications, such solutions are considered in the "Penrose" limit of strings in an $AdS_p\times S^p$ background [5]. In several cases, pp waves are also known to be maximally supersymmetric solutions of supergravities in various dimensions [17–19], and are known to give rise to solvable string theories from the worldsheet point of view as well. The particular case of $AdS_5\times S^5$ is of special interest, with applications to $N=4$, $D=4$ gauge theories in the limit of large conformal dimensions and R charges [7]. Interestingly, the D-branes of string theories also have an appropriate representation in such gauge theories, in terms of operators corresponding to "giant gravitons" and "defects" [14,15].

In this paper, we continue the search for explicit D-brane supergravity solutions in string theories in a pp-wave background. Our study will be mainly concentrated on the Neveu-Schwarz–Neveu-Schwarz (NS-NS) and Ramond-Ramond (RR) pp waves arising out of $AdS_3\times S^3$ geometry [10]. We, however, also present branes in other pp waves such as the ones in "little" string theories, etc. An explicit supergravity solution for D-branes, along with the open string spectrum, has been studied in [20–26]. Our new result includes a set of solutions corresponding to brane systems of the type Dp-Dp' in both type IIA and IIB theories, as well as the explicit worldsheet construction in these cases. We would like to mention that Dp-branes from the worldsheet point of view have already been obtained in the NS-NS pp-wave background earlier [35]. Our result gives the realization of such D-branes from the supergravity point of view. We also find it interesting to note that, unlike the case of the $AdS_5\times S^5$ pp wave, in our case we are able to obtain several "localized" D-brane solutions. Dp-branes in RR pp-wave background are obtained by applying a set of S and T duality transformations on the brane solutions in the NS-NS background. Such background pp-wave configurations have already been discussed in the context of D7-branes in [21]. The worldsheet construction of these branes then follows from the results in [21], and will also be discussed below. We point out, however, that there is a crucial difference between the solutions presented in this paper and those in [21]. In the present case, only light-cone directions of the pp wave are along the branes; the remaining four are always transverse to them. On the other hand, for the solutions in [21] the directions transverse to the brane are flat.

The rest of the paper is organized as follows. In Sec. II, new (supersymmetric) Dp- as well as (Dp-Dp')-branes are presented in both NS-NS and RR pp-wave backgrounds. Supersymmetry properties of these branes are examined in detail in Sec. III and it is shown that they preserve some amount of unbroken supersymmetry. The open string constructions of branes is discussed in Sec. IV. Section V is devoted to branes in the "little string theory" background. We conclude in Sec. VI with some general remarks.

II. SUPERGRAVITY SOLUTIONS

A. D-branes in NS-NS pp-wave background

We now start by writing down the D-string solutions in the pp-wave background originating from the Penrose limit of NS-NS $AdS_3\times S^3\times R^4$ [10]. The supergravity solution of a system of N D-strings in such a background is given by

$$ds^2=f_1^{-1/2}\left(2dx^+dx^- -\mu^2\sum_{i=1}^{4} x_i^2(dx^+)^2\right)+f_1^{1/2}\sum_{a=1}^{8}(dx^a)^2,$$

$$e^{2\phi}=f_1,\quad H_{+12}=H_{+34}=2\mu,$$

$$F_{+-a}=\partial_a f_1^{-1},\quad f_1=1+\frac{Ng_s l_s^6}{r^6}, \tag{2.1}$$

with f_1 satisfying the Green's function equation in the eight-dimensional transverse space. We have explicitly verified that the solution presented in Eq. (2.1) satisfies the type IIB field equations (see, e.g., [27,28]). One notices that in this

*Email address: anindyab@iopb.res.in
†Email address: kumar@iopb.res.in
‡Email address: kamal@iopb.res.in

case constant NS-NS three-forms along the pp-wave direction are required precisely to cancel the μ-dependent part of the R_{++} equation of motion.

Starting from the D-string solution in Eq. (2.1), one can write down all the Dp-brane solutions ($p=1,\ldots,5$) in the NS-NS pp-wave background by applying successive T dualities along $x^5,\ldots,x^8$. As is known, this procedure also involves smearing of the brane along these directions. For example, a D3-brane solution has the form

$$ds^2=f_3^{-1/2}\left(2dx^+dx^- - \mu^2\sum_{i=1}^{4} x_i^2(dx^+)^2 + (dx_5)^2+(dx_6)^2\right)+f_3^{1/2}\sum_{a=1,\ldots,4,7,8}(dx_a)^2,$$

$$H_{+12}=H_{+34}=2\mu,$$

$$F_{+-56a}=\partial_a f_3^{-1},\quad e^{2\phi}=1,\quad f_3=1+\frac{Ng_s l_s^4}{r^4}, \tag{2.2}$$

with f_3 being the harmonic function in the transverse space of the D3-brane.

Now we present the supergravity solution of an intersecting (Dp-Dp')-brane system in a pp-wave background. These solutions are described as "branes lying within branes." In particular, for the D1-D5 case, the solution is given by

$$ds^2=(f_1f_5)^{-1/2}\left(2dx^+dx^- - \mu^2\sum_{i=1}^{4} x_i^2(dx^+)^2\right) + \left(\frac{f_1}{f_5}\right)^{1/2}\sum_{a=5}^{8}(dx^a)^2+(f_1f_5)^{1/2}\sum_{i=1}^{4}(dx_i)^2,$$

$$e^{2\phi}=\frac{f_1}{f_5},$$

$$H_{+12}=H_{+34}=2\mu,$$

$$F_{+-i}=\partial_i f_1^{-1},\quad F_{mnp}=\epsilon_{mnpl}\partial_l f_5, \tag{2.3}$$

with

$$f_{1,5}=1+\frac{N_{1,5}g_s l_s^2}{r^2} \tag{2.4}$$

being the solutions of the Green's function equations in directions transverse to (N_1) D1- and (N_5) D5-branes, respectively. Equation (2.3) provides one of the main results of our paper and shows that, as in flat space, intersecting brane solutions are possible in the pp-wave background as well. We have once again checked that the solution presented above does satisfy type IIB field equations of motion.

One can now apply T duality transformations to generate more intersecting brane solutions starting from the one given in Eq. (2.3) [30,31]. Note that the directions $x^5,\ldots,x^8$ are transverse to the D-string in Eq. (2.3), whereas they lie along the longitudinal directions of D5. As a result, one can easily obtain solutions of the type D2-D4 as well as D3-D3$'$ in this pp-wave background. These solutions will give a pp-wave generalization of the intersecting solutions given in [32]. We skip the details of this analysis, however.

B. p-p' branes in RR pp-wave background

In this section, we will present the Dp as well as the (Dp-Dp')-branes in a RR pp wave of $AdS_3\times S^3\times R^4$. These backgrounds can be obtained from the solutions given in the last subsection by applying S and T duality transformations in several steps. For example, from the D3-brane solution in the NS-NS pp-wave background (2.2), one gets a D3-brane in the RR pp-wave background under the S duality transformation. Now applying T duality along the directions (x^5,x^6), we can generate a D-string solution. On the other hand, by applying T duality along two transverse directions (x^7,x^8) of the D3-brane, one gets a D5-brane lying along ($x^+,x^-,x^5,\ldots,x^8$) directions. The supergravity solution of a system of N D-strings is then given explicitly by

$$ds^2=f_1^{-1/2}\left(2dx^+dx^- - \mu^2\sum_{i=1}^{4} x_i^2(dx^+)^2\right) + f_1^{1/2}\sum_{a=1}^{8}(dx^a)^2,$$

$$e^{2\phi}=f_1,\quad F_{+1256}=F_{+3456}=2\mu,$$

$$F_{+-a}=\partial_a f_1^{-1},\quad f_1=1+\frac{Ng_s l_s^6}{r^6}, \tag{2.5}$$

with f_1 satisfying the Green's function in eight-dimensional transverse space. One notices that the solution has a constant five-form field strength.

The supergravity solution of the D5-brane is given by

$$ds^2=f_5^{-1/2}\left(2dx^+dx^- - \mu^2\sum_{i=1}^{4} x_i^2(dx^+)^2+\sum_{a=5}^{8}(dx^a)^2\right) + f_5^{1/2}\sum_{i=1}^{4}(dx^i)^2,$$

$$e^{2\phi}=f_5^{-1},\quad F_{+1256}=F_{+3456}=F_{+1278}=F_{+3478}=2\mu,$$

$$F_{mnp}=\epsilon_{mnpq}\partial_q f_5,\quad f_5=1+\frac{Ng_s l_s^2}{r^2}, \tag{2.6}$$

with f_5 satisfying the Green's function in the tranverse directions ($x^1,\ldots,x^4$). Now we will present the (Dp-Dp')-brane solutions in the RR pp-wave background. In particular, to write down the supergravity solution of a (D1-D5) system, we made an ansatz which combines the D-string of Eq. (2.5) and the D5-brane given in Eq. (2.6). The final configuration is as follows:

$$ds^2=(f_1f_5)^{-1/2}\left(2dx^+dx^- - \mu^2\sum_{i=1}^{4} x_i^2(dx^+)^2\right) + \left(\frac{f_1}{f_5}\right)^{1/2}\sum_{a=5}^{8}(dx^a)^2+(f_1f_5)^{1/2}\sum_{i=1}^{4}(dx_i)^2,$$

$$e^{2\phi}=\frac{f_1}{f_5},$$

$$F_{+1256}=F_{+3456}=F_{+1278}=F_{+3478}=2\mu,$$

$$F_{+-i}=\partial_i f_1^{-1},\quad F_{mnp}=\epsilon_{mnpl}\partial_l f_5, \tag{2.7}$$

with f_1 and f_5 as defined in Eq. (2.4). One can check that the solution presented above do satisfy the type IIB field equations. Once again, more p-p' branes can be obtained from the D1-D5 solution in Eq. (2.7) by applying T dualities.

One may also attempt to find a (D1-D5) solution by taking a decoupling limit, followed by Penrose scaling, of the solution presented in [29] in a similar way as for the D5-brane solution in [21]. The starting solution along which one would take the Penrose limit is as follows:

$$ds^2=\frac{1}{(H_1'H_5')^{1/2}}\left[\frac{r^2}{R_1^2}(-dt^2+dx^2)\right]+\left(\frac{H_1'}{H_5'}\right)^{1/2}\frac{R_1^2}{r^2}dr^2 + \left(\frac{H_1'}{H_5'}\right)^{1/2}(d\psi^2+\sin^2\psi d\Omega_2^2) + (H_1'H_5')^{1/2}(dy^2+y^2d\Omega_3^2), \tag{2.8}$$

where

$$H_1=1+\frac{R_1^2}{x^2},\quad H_5=1+\frac{R_5^2}{x^2},\quad H_1'=1+\frac{R'^2_1}{y^2},$$

$$H_5'=1+\frac{R'^2_5}{y^2}. \tag{2.9}$$

One notices, however, that different terms in the metric above come with different powers of H_1', leading to difficulty in choosing a "null geodesic" to define an appropriate Penrose limit and find brane solutions.

III. SUPERSYMMETRY ANALYSIS

A. NS-NS pp wave

In this section we present the supersymmetry of the solutions described earlier in Sec. II A. The supersymmetry variation of dilatino and gravitino fields of type IIB supergravity in ten dimensions, in the string frame, is given by [33,34,23]

$$\delta\lambda_\pm=\frac{1}{2}\left(\Gamma^\mu\partial_\mu\phi\mp\frac{1}{12}\Gamma^{\mu\nu\rho}H_{\mu\nu\rho}\right)\epsilon_\pm + \frac{1}{2}e^\phi\left(\pm\Gamma^M F_M^{(1)}+\frac{1}{12}\Gamma^{\mu\nu\rho}F^{(3)}_{\mu\nu\rho}\right)\epsilon_\mp, \tag{3.1}$$

$$\delta\Psi^\pm_\mu=\left[\partial_\mu+\frac{1}{4}\left(w_{\mu\hat a\hat b}\mp\frac{1}{2}H_{\mu\hat a\hat b}\right)\Gamma^{\hat a\hat b}\right]\epsilon_\pm + \frac{1}{8}e^\phi\left[\mp\Gamma^\mu F^{(1)}_\mu-\frac{1}{3!}\Gamma^{\mu\nu\rho}F^{(3)}_{\mu\nu\rho} \mp\frac{1}{2.5!}\Gamma^{\mu\nu\rho\alpha\beta}F^{(5)}_{\mu\nu\rho\alpha\beta}\right]\Gamma_\mu\epsilon_\mp, \tag{3.2}$$

where we have used (μ,ν,ρ) to describe the ten-dimensional space-time indices, and carets represent the corresponding tangent space indices. Solving the above two equations for the solution describing a D-string as given in Eq. (2.1), we get several conditions. First, the dilatino variation gives

$$\Gamma^{\hat a}\epsilon_\pm-\Gamma^{\hat + \hat - \hat a}\epsilon_\mp=0, \tag{3.3}$$

$$(\Gamma^{\hat + \hat 1\hat 2}+\Gamma^{\hat + \hat 3\hat 4})\epsilon_\mp=0. \tag{3.4}$$

In fact, both the conditions (3.3) and (3.4) are required for satisfying the dilatino variation condition. Gravitino variation gives the following conditions on the spinors:

$$\delta\psi^\pm_+\equiv\partial_+\epsilon_\pm\mp\frac{\mu}{2}f_1^{-1/2}(\Gamma^{\hat 1\hat 2}+\Gamma^{\hat 3\hat 4})\epsilon_\pm=0,$$

$$\delta\psi^\pm_-\equiv\partial_-\epsilon_\pm=0,$$

$$\delta\psi^\pm_a\equiv\partial_a\epsilon_\pm=-\frac{1}{8}\frac{f_{1,a}}{f_1}\epsilon_\pm,\quad \delta\psi^\pm_i\equiv\partial_i\epsilon_\pm=-\frac{1}{8}\frac{f_{1,i}}{f_1}\epsilon_\pm. \tag{3.5}$$

In writing the above set of equations, we have also imposed the necessary condition

$$\Gamma^{\hat +}\epsilon_\pm=0, \tag{3.6}$$

in addition to Eq. (3.3). Further, by using

$$(1-\Gamma^{\hat 1\hat 2\hat 3\hat 4})\epsilon_\pm=0, \tag{3.7}$$

all the supersymmetry conditions are solved by spinors: $\epsilon_\pm=\exp(-\frac{1}{8}\ln f_1)\epsilon^0_\pm$, with $\epsilon^0_\pm$ being a constant spinor. The D-string solution in Eq. (2.1) therefore preserves 1/8 supersymmetry. All other Dp-branes $(p=1,\ldots,5)$, obtained by applying T dualities as discussed above, will also preserve the same amount of supersymmetry.

Next, we will analyze the supersymmetry properties of the intersecting branes. We will concentrate on the (D1-D5) case explicitly. The dilatino variation gives the following conditions on the spinors:

$$\Gamma^{\hat i}\epsilon_\pm-\Gamma^{\hat + \hat - \hat i}\epsilon_\mp=0, \tag{3.8}$$

$$\Gamma^{\hat{i}}\epsilon_{\pm}+\frac{1}{3!}\epsilon_{\hat{i}\hat{j}\hat{k}\hat{l}}\Gamma^{\hat{j}\hat{k}\hat{l}}\epsilon_{\mp}=0, \tag{3.9}$$

$$(\Gamma^{\hat{+}\hat{1}\hat{2}}+\Gamma^{\hat{+}\hat{3}\hat{4}})\epsilon_{\mp}=0. \tag{3.10}$$

One needs to impose all three conditions specified above for the dilatino variation to vanish. On the other hand, the gravitino variation gives

$$\delta\psi_{+}^{\pm}\equiv\partial_{+}\epsilon_{\pm}\mp\frac{\mu}{2}(f_1f_5)^{-1/2}(\Gamma^{\hat{1}\hat{2}}+\Gamma^{\hat{3}\hat{4}})\epsilon_{\pm}=0,$$

$$\delta\psi_{-}^{\pm}\equiv\partial_{-}\epsilon_{\pm}=0,$$

$$\delta\psi_{i}^{\pm}\equiv\partial_{i}\epsilon_{\pm}=-\frac{1}{8}\left[\frac{f_{1,a}}{f_1}\epsilon_{\pm}+\frac{f_{5,a}}{f_5}\right]\epsilon_{\pm}, \quad \delta\psi_{a}^{\pm}\equiv\partial_{a}\epsilon_{\pm}=0. \tag{3.11}$$

In writing down the above gravitino variations we have once again made use of the projection $\Gamma^{\hat{+}}\epsilon_{\pm}=0$. The above set of equations can be solved by imposing

$$(1-\Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}})\epsilon_{\pm}=0 \tag{3.12}$$

in addition to Eq. (3.8) and the solution is given as $\epsilon_{\pm}=\exp[-\frac{1}{8}\ln(f_1f_5)]\epsilon_{\pm}^{0}$. One therefore has 1/8 supersymmetry for the (D1-D5) solution presented in Eq. (2.3).

B. RR *pp* wave

Now we present the supersymmetry of the Dp- as well as of the (Dp-Dp') branes in the RR pp-wave background given in Sec. II B. First we discuss the supersymmetry of the D-string in Eq. (2.5). The dilatino variation (3.1) gives

$$\Gamma^{\hat{a}}\epsilon_{\pm}-\Gamma^{\hat{+}\hat{-}\hat{a}}\epsilon_{\mp}=0. \tag{3.13}$$

Gravitino variation gives the following conditions on the spinors:

$$\delta\psi_{+}^{\pm}\equiv\partial_{+}\epsilon_{\pm}\mp\frac{\mu}{8}f_1^{-1/2}[(\Gamma^{\hat{+}\hat{1}\hat{2}\hat{5}\hat{6}}+\Gamma^{\hat{+}\hat{3}\hat{4}\hat{5}\hat{6}})+(\Gamma^{\hat{+}\hat{1}\hat{2}\hat{7}\hat{8}}+\Gamma^{\hat{+}\hat{3}\hat{4}\hat{7}\hat{8}})]\Gamma^{\hat{-}}\epsilon_{\pm}=0,$$

$$\delta\psi_{-}^{\pm}\equiv\partial_{-}\epsilon_{\pm}=0,$$

$$\delta\psi_{a}^{\pm}\equiv\partial_{a}\epsilon_{\pm}=-\frac{1}{8}\frac{f_{1,i}}{f_1}\epsilon_{\pm},$$

$$\delta\psi_{i}^{\pm}\equiv\partial_{i}\epsilon_{\pm}=-\frac{1}{8}\frac{f_{1,a}}{f_1}\epsilon_{\pm}, \tag{3.14}$$

where we have once again imposed the necessary condition $\Gamma^{\hat{+}}\epsilon_{\pm}=0$, in addition to Eq. (3.13). Further, by using the condition

$$(1-\Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}})=0, \tag{3.15}$$

all the supersymmetry conditions are satisfied, thus preserving 1/8 unbroken supersymmetry.

Next, we present the supersymmetry property of the D1-D5 solution written in Eq. (2.7). The dilatino variation (3.1) gives the following conditions on spinors:

$$\Gamma^{\hat{i}}\epsilon_{\pm}-\Gamma^{\hat{+}\hat{-}\hat{i}}\epsilon_{\mp}=0, \tag{3.16}$$

$$\Gamma^{\hat{i}}\epsilon_{\pm}+\frac{1}{3!}\epsilon_{\hat{i}\hat{j}\hat{k}\hat{l}}\Gamma^{\hat{j}\hat{k}\hat{l}}\epsilon_{\mp}=0. \tag{3.17}$$

On the other hand, the gravitino variation (3.2) gives the following conditions:

$$\delta\psi_{+}^{\pm}\equiv\partial_{+}\epsilon_{\pm}\mp\frac{\mu}{8}(f_1f_5)^{-1/2}[(\Gamma^{\hat{+}\hat{1}\hat{2}\hat{5}\hat{6}}+\Gamma^{\hat{+}\hat{3}\hat{4}\hat{5}\hat{6}})+(\Gamma^{\hat{+}\hat{1}\hat{2}\hat{7}\hat{8}}+\Gamma^{\hat{+}\hat{3}\hat{4}\hat{7}\hat{8}})]\Gamma^{\hat{-}}\epsilon_{\pm}=0,$$

$$\delta\psi_{-}^{\pm}\equiv\partial_{-}\epsilon_{\pm}=0, \quad \delta\psi_{a}^{\pm}\equiv\partial_{a}\epsilon_{\pm}=0,$$

$$\delta\psi_{i}^{\pm}\equiv\partial_{i}\epsilon_{\pm}=-\frac{1}{8}\left[\frac{f_{1,i}}{f_1}+\frac{f_{5,i}}{f_5}\right]\epsilon_{\pm}, \tag{3.18}$$

where we have once again used the necessary condition $\Gamma^{\hat{+}}\epsilon_{\pm}=0$ along with the ones in Eqs. (3.16) and (3.17). The above set of equations can be solved by imposing further

$$(1-\Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}})\epsilon_{\pm}=0. \tag{3.19}$$

One therefore has 1/8 supersymmetry for the D1-D5 system described in Eq. (2.7) as well.

IV. WORLDSHEET CONSTRUCTION OF *p-p'* BRANES

A. NS-NS *pp* wave

In this section, we will discuss the (D1-D5)-brane system, constructed earlier in the paper, from the point of view of first quantized string theory in the Green-Schwarz formalism, in the light-cone gauge. In the present case, in the flat directions $x_{\alpha}(\alpha=5,\ldots,8)$, we have the Dirichlet boundary condition at one end and the Neumann boundary condition at the other end of the open string. Along $x_i(i=1,\ldots,4)$ directions, one has the usual Dirichlet boundary condition. The relevant classical action to be studied in our case (after imposing the light-cone gauge conditions on fermions and bosons [10]) is as follows [35]:

$$L=L_b+L_f, \tag{4.1}$$

where

$$L_b=\partial_{+}u\partial_{-}v-m^2x_i^2+\partial_{+}x_i\partial_{-}x_i+\partial_{+}x_{\alpha}\partial_{-}x_{\alpha}+\mu\sum_{(i,j)=(1,2),(3,4)}x^i(\partial_{+}u\partial_{-}x^j-\partial_{-}u\partial_{+}x^j), \tag{4.2}$$

$$L_f=iS_R(\partial_{+}-mM)S_R+iS_L(\partial_{-}+mM)S_L, \tag{4.3}$$

with

$$m \equiv \alpha' p^u \mu = 2\alpha' p_v \mu, \tag{4.4}$$

$$M = -\frac{1}{2}(\gamma^{12} + \gamma^{34}). \tag{4.5}$$

Eight-component real spinors (S_L, S_R) have been obtained from 16-component Majorana-Weyl spinors in the left and the right sectors after solving the light-cone gauge conditions.

The equations of motion and boundary conditions for bosons x^i, x^α in our case are as follows:

$$\partial_+\partial_- x_{i_1} + m^2 x_{i_1} - m\epsilon^{i_1 j_1}(\partial_- x^{j_1} - \partial_+ x^{j_1}) = 0, \quad \partial_+\partial_- x_\alpha = 0, \tag{4.6}$$

$$\partial_\sigma x^\alpha|_{\sigma=0} = \partial_\tau x^\alpha|_{\sigma=\pi} = 0, \quad x^i|_{\sigma=0,\pi} = \text{const.} \tag{4.7}$$

The solution to the bosonic equations of motion, with the boundary conditions specified above, is given by [defining $X^{\hat{1}} = (1/\sqrt{2})(x^1 + ix^2)$ and $X^{\hat{2}} = (1/\sqrt{2})(x^3 + ix^4)$] [35]

$$X^\alpha(\sigma,\tau) = i\sum_{r\in(z+1/2)} \frac{1}{r}\alpha_r^\alpha e^{-ir\tau}\cos r\sigma \ (\alpha = 5,\ldots,8), \tag{4.8}$$

$$X^{\hat{i}}(\sigma,\tau) = e^{-2im\sigma}\left[x_0 + (x_1 e^{2im\sigma} - x_0)\frac{\sigma}{\pi} + i\sum_{n\neq 0}\frac{1}{n}\alpha_n^i e^{-in\tau}\sin n\sigma\right]. \tag{4.9}$$

To consider the equations of motion of fermions, we note that the matrix M evidently breaks the SO(8) symmetry further, and thereby splits the fermions in the $8 \to 4 + 4$ way $S_L \to (\tilde{S}_L, \hat{S}_L), S_R \to (\tilde{S}_R, \hat{S}_R)$:

$$\gamma^{1234}\begin{pmatrix}\tilde{S}_{L,R}\\ \hat{S}_{L,R}\end{pmatrix} = \begin{pmatrix}-\tilde{S}_{L,R}\\ \hat{S}_{L,R}\end{pmatrix}. \tag{4.10}$$

In this connection, one also introduces 4×4 matrices Λ and Σ:

$$\gamma^{12}\begin{pmatrix}\tilde{S}_{L,R}\\ \hat{S}_{L,R}\end{pmatrix} = -\begin{pmatrix}\Lambda\tilde{S}_{L,R}\\ \Sigma\hat{S}_{L,R}\end{pmatrix}, \tag{4.11}$$

with $\Lambda^2 = \Sigma^2 = -1$. Λ and Σ in the above equation are 4×4 antisymmetric matrices with eigenvalues $\pm i$. Using this notation, one has

$$M\begin{pmatrix}\tilde{S}_{L,R}\\ \hat{S}_{L,R}\end{pmatrix} = \begin{pmatrix}\Lambda\tilde{S}_{L,R}\\ 0\end{pmatrix}. \tag{4.12}$$

The equations of motion written in terms of $(\tilde{S}_L, \hat{S}_L)$ and $(\tilde{S}_R, \hat{S}_R)$ are then of the form

$$\partial_+(e^{2m\tau}\tilde{S}_R) = 0, \quad \partial_-(e^{-2m\tau}\tilde{S}_L) = 0, \tag{4.13}$$

$$\partial_+\hat{S}_R = 0, \quad \partial_-\hat{S}_L = 0. \tag{4.14}$$

Now we will write down the boundary conditions for the fermions in the mixed sector. As the equations of motion and the boundary condition for the components $\hat{S}_{L,R}$ are identical to the ones in flat space, we only concentrate on finding an explicit solution for $\tilde{S}_{L,R}$ below. Following [10,21,35], one can write down the boundary conditions for the fermions as

$$\tilde{S}_L|_{\sigma=0} = -\tilde{S}_R|_{\sigma=0}, \tag{4.15}$$

$$\tilde{S}_L|_{\sigma=\pi} = \tilde{S}_R|_{\sigma=\pi}, \tag{4.16}$$

$$\hat{S}_L|_{\sigma=0,\pi} = \hat{S}_R|_{\sigma=0,\pi}. \tag{4.17}$$

The solution for the $\tilde{S}_{L,R}$ equations of motion (4.13), with the above boundary condition, can be read from [35], and has the following form:

$$\tilde{S}_L = -e^{-2m\sigma\Lambda}\sum_{r\in(z+1/2)} s_r e^{-ir(\tau+\sigma)}, \tag{4.18}$$

$$\tilde{S}_R = e^{-2m\sigma\Lambda}\sum_{r\in(z+1/2)} s_r e^{-ir(\tau-\sigma)}. \tag{4.19}$$

The canonical quantization conditions as well as the worldsheet Hamiltonian for the D1-D5 system discussed above can also be written in a straightforward manner following the procedure in [35]. We skip these details.

B. RR *pp* wave

Now we present the worldsheet analysis of the (D1-D5) system discussed earlier in Sec. II B. This can be done by realizing that the pp-wave background for these solutions is given by a T-dual configuration of the ones presented in [10,21]. More explicitly, in the worldsheet action in the present case,

$$L = L_B + L_F, \tag{4.20}$$

where

$$L_B = \partial_+ u\partial_- v - m^2 x_i^2 + \partial_+ x_i \partial_- x_i + \partial_+ x_\alpha \partial_- x_\alpha, \tag{4.21}$$

$$L_F = iS_R\partial_+ S_R + iS_L\partial_+ S_L - 2imS_L M S_R, \tag{4.22}$$

with

$$m \equiv \alpha' p^u \mu = 2\alpha' p_v \mu, \tag{4.23}$$

$$M = -\tfrac{1}{2}(\gamma^{12} + \gamma^{34})\gamma^{56}, \tag{4.24}$$

the terms involving fermions are easily seen to be related to the ones in [10,21] through T dualities along x^5 and x^6. This in fact leads to the relation

$$S'_R = \gamma^{56} S_R, \tag{4.25}$$

and reproduces the original action in [10]. The mode expansion for fermions as well as canonical quantization conditions can therefore also be written down in a straightforward manner. We end this section by pointing out that, since the D-brane solutions found in this paper preserve less than 1/2 supersymmetry, some of the restrictions on the brane directions imposed using zero mode considerations [20] do not directly apply above.

V. BRANES IN "LITTLE STRING THEORY"

Supergravity backgrounds

In this section, we discuss the branes in the Penrose limit of "little string theory" (LST). pp waves of nonlocal theories have been discussed recently in the literature [36–38]. Among them "little string theory" arises on the world volume of NS5-brane, when a decoupling limit $g_s\to 0$ with fixed α' is taken [39,40]. To construct our solution, we start with the NS5-brane solution given by the metric and dilaton:

$$ds^2=-dt^2+dy_5^2+H(r)(dr^2+r^2d\Omega_3^2),$$
$$e^{2\Phi}=g_s^2H(r), \tag{5.1}$$

with $H(r)=1+Nl_s^2/r^2$. The near horizon limit of the above solution is the linear dilaton geometry, which in the string frame is given by

$$ds^2=Nl_s^2\left(-d\tilde{t}^{\,2}+\cos^2\theta d\psi^2+d\theta^2+\sin^2\theta d\phi^2+\frac{dr^2}{r^2}\right)+dy_5^2, \tag{5.2}$$

with $t=\sqrt{N}l_s\tilde{t}$. The pp-wave background of LST is then found by applying the Penrose limit to Eq. (5.2). We, however, consider the case after applying S duality on Eq. (5.2). Applying the S duality transformation and then taking the Penrose limit (as described in [36]), the background solution is given by

$$ds^2=-4dx^+dx^--\mu^2\vec{z}^2dx^{+2}+(d\vec{z})^2+dx^2+dy_5^2,$$
$$e^{2\phi}=\text{const},$$
$$F_{+12}=C\mu, \tag{5.3}$$

where F_{+12} is the three-form field strength in the $\vec{z}$ plane. Now we proceed to analyze the existence and stability of branes in this background. The supergravity solution for a system of D5-branes in this background is given by

$$ds^2=f^{-1/2}\left(-4dx^+dx^--\mu^2\vec{z}^2(dx^+)^2+\sum_{i=1}^{2}(dz_i)^2+\sum_{p=1}^{2}(dy_p)^2\right)+f^{1/2}\sum_{a=1}^{4}(dx_a)^2,$$
$$e^{2\Phi}=f^{-1},\quad F_{+12}=2\mu,$$

$$F_{mnp}=\epsilon_{mnpr}\partial_r f,\quad f=1+\frac{Ng_sl_s^2}{r^2}. \tag{5.4}$$

One notices that the background has only one constant three-form field strength (F_{+12}). We have once again verified that the solution presented above satisfies type IIB field equations.

One can then write down [by applying S duality on Eq. (5.4)] the NS5-brane in a pp-wave background of the little string theory [36] as

$$ds^2=-4dx^+dx^--\mu^2\vec{z}^2(dx^+)^2+\sum_{i=1}^{2}(dz_i)^2+\sum_{p=1}^{2}(dy_p)^2+f\sum_{a=1}^{4}(dx_a)^2,$$
$$e^{2\Phi}=f,\quad H_{+12}=2\mu,$$
$$H_{mnp}=\epsilon_{mnpr}\partial_r f, \tag{5.5}$$

with the H's being the NS sector three-form field strengths.

We will now consider the dilatino and gravitino variation of the solution presented in Eq. (5.4) to study the supersymmetry properties. The dilatino variation gives the equations

$$\Gamma^{\hat{a}}\epsilon_\pm+\frac{1}{3!}\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}\Gamma^{\hat{b}\hat{c}\hat{d}}\epsilon_\mp=0, \tag{5.6}$$

$$\Gamma^{\hat{+}\hat{1}\hat{2}}\epsilon_\mp=0. \tag{5.7}$$

The gravitino variation leads to the equations

$$\delta\Psi_+^\pm\equiv\partial_+\epsilon_\pm+\frac{\mu^2z_{\hat{i}}}{2}\Gamma^{\hat{+}\hat{i}}\epsilon_\pm+\frac{1}{16}\mu^2\vec{z}^2\frac{f,\hat{a}}{f^{3/2}}\Gamma^{\hat{+}\hat{a}}\epsilon_\pm-\frac{\mu}{4}\Gamma^{\hat{+}\hat{1}\hat{2}}\Gamma^-\epsilon_\mp=0, \tag{5.8}$$

$$\delta\Psi_-^\pm\equiv\partial_-\epsilon_\pm=0, \tag{5.9}$$

$$\delta\Psi_i^\pm\equiv\partial_i\epsilon_\pm-\frac{\mu}{4}\Gamma^{\hat{+}\hat{1}\hat{2}}\delta_{i\hat{i}}\Gamma^{\hat{i}}\epsilon_\mp=0, \tag{5.10}$$

$$\delta\Psi_p^\pm\equiv\partial_p\epsilon_\pm-\frac{\mu}{4}\Gamma^{\hat{+}\hat{1}\hat{2}}\delta_{p\hat{p}}\Gamma^{\hat{p}}\epsilon_\mp=0, \tag{5.11}$$

$$\delta\Psi_a^\pm\equiv\partial_a\epsilon_\pm+\frac{1}{8}\frac{f,a}{f}\epsilon_\pm-\frac{\mu}{4}f^{1/2}\Gamma^{\hat{+}\hat{1}\hat{2}}\delta_{a\hat{a}}\Gamma^{\hat{a}}\epsilon_\mp=0. \tag{5.12}$$

In writing this set of equations we have used the condition (5.6). Imposing the condition $\Gamma^{\hat{+}}\epsilon=0$, we further reduce them to

$$\partial_+\epsilon_\pm-\frac{\mu}{4}\Gamma^{\hat{+}\hat{1}\hat{2}}\Gamma^{\hat{-}}\epsilon_\mp=0, \tag{5.13}$$

$$\partial_- \epsilon_\pm = 0, \quad \partial_i \epsilon_\pm = 0, \quad \partial_p \epsilon_\pm = 0, \quad \partial_a \epsilon_\pm = -\frac{1}{8}\frac{f_{,a}}{f}\epsilon_\pm . \tag{5.14}$$

Since Eqs. (5.13) and (5.14) are integrable ones, in this case we get 1/4 supersymmetry. It will also be nice to give a worldsheet construction for such D-branes.

VI. CONCLUSION

In this paper we have presented several supersymmetric Dp- and (Dp-Dp')-brane configurations in the pp-wave background and analyzed their supersymmetry properties. We have also presented the (D1-D5)-brane construction from the point of view of the massive Green-Schwarz formalism in the light-cone gauge in NS-NS and RR pp-wave of $AdS_3 \times S^3 \times R^4$. It will be interesting to study the gauge theory duals of the branes presented in this paper by using operators such as "defects." One could possibly also look at the black hole physics using the (D1-D5) system presented here in an attempt to understand their properties.

ACKNOWLEDGMENT

We thank Rashmi R. Nayak, Koushik Ray, and Sanjay for useful discussions.

[1] R. Penrose, in *Differential Geometry and Relativity* (Reidel, Dordrecht, 1976), pp. 271–275.
[2] G. T. Horowitz and A. A. Tseytlin, Phys. Rev. D **51**, 2896 (1995).
[3] R. Gueven, Phys. Lett. B **482**, 255 (2000).
[4] R. R. Metsaev, Phys. Lett. B **468**, 65 (1999); Nucl. Phys. **B625**, 70 (2002); R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D **65**, 126004 (2002).
[5] M. Blau, J. Figuero-O'Farrill, C. Hull, and G. Papadopoulos, J. High Energy Phys. **01**, 047 (2000); M. Blau, J. Figuero-O'Farrill, and G. Papadopoulos, Class. Quantum Grav. **19**, L87 (2000); **19**, 4753 (2002).
[6] A. A. Tseytlin, "On Limits of Superstring in $AdS(5) \times S^5$," hep-th/0201112.
[7] D. Berenstein, J. Maldacena, and H. Nastase, J. High Energy Phys. **04**, 013 (2002).
[8] N. Itzhaki, I. R. Klebanov, and S. Mukhi, J. High Energy Phys. **03**, 048 (2002).
[9] J. Gomis and H. Ooguri, Nucl. Phys. **B635**, 106 (2002).
[10] J. G. Russo and A. A. Tseytlin, J. High Energy Phys. **04**, 021 (2002).
[11] L. A. Pando Zayas and J. Sonnenschein, J. High Energy Phys. **05**, 010 (2002).
[12] M. Alishahiha and M. M. Sheikh-Jabbari, Phys. Lett. B **535**, 328 (2002).
[13] C. S. Chu and P. M. Ho, Nucl. Phys. **B636**, 141 (2002).
[14] P. Lee and J. W. Park, "Open Strings in pp-Wave Background from Defect Conformal Field Theory," hep-th/0203257.
[15] V. Balasubramanian, M. X. Huang, T. S. Levi, and A. Naqvi, J. High Energy Phys. **08**, 037 (2002).
[16] H. Takayanagi and T. Takayanagi, J. High Energy Phys. **05**, 012 (2002).
[17] P. Meessen, Phys. Rev. D **65**, 087501 (2002).
[18] M. Cvetic, H. Lu, and C. N. Pope, "Penrose Limits, pp-Waves and Deformed M2-branes," hep-th/0203082.
[19] J. P. Gauntlett and C. M. Hull, J. High Energy Phys. **06**, 013 (2002).
[20] A. Dabholkar and S. Parvizi, Nucl. Phys. **B641**, 223 (2002).
[21] A. Kumar, R. R. Nayak, and Sanjay, Phys. Lett. B **541**, 183 (2002).
[22] K. Skenderis and M. Taylor, J. High Energy Phys. **06**, 025 (2002).
[23] M. Alishahiha and A. Kumar, Phys. Lett. B **542**, 130 (2002).
[24] P. Bain, P. Meessen, and M. Zamaklar, "Supergravity Solutions for D-Branes in Hpp-Wave Backgrounds," hep-th/0205106.
[25] H. Singh, "M5-branes with 3/8 Supersymmetry in pp-Wave Background," hep-th/0205020.
[26] S. S. Pal, Mod. Phys. Lett. A **17**, 1735 (2002).
[27] M. J. Duff, Ramzi R. Khuri, and J. X. Lu, Phys. Rep. **259**, 213 (1995).
[28] M. Alishahiha, H. Ita, and Y. Oz, J. High Energy Phys. **06**, 002 (2000).
[29] G. Papadopoulos, J. Russo, and A. A. Tseytlin, Class. Quantum Grav. **17**, 1713 (2000).
[30] J. C. Breckenridge, G. Michaud, and R. C. Myers, Phys. Rev. D **55**, 6438 (1997).
[31] A. Kumar, S. Mukherji, and K. L. Panigrahi, J. High Energy Phys. **05**, 050 (2002).
[32] A. A. Tseytlin, Nucl. Phys. **B475**, 149 (1996).
[33] J. H. Schwarz, Nucl. Phys. **B226**, 269 (1983).
[34] S. F. Hassan, Nucl. Phys. **B568**, 145 (2000).
[35] Y. Michishita, "D-branes in NSNS and RR pp-Wave Backgrounds and S-Duality," hep-th/0206131.
[36] V. E. Hubeny, M. Rangamani, and E. Verlinde, J. High Energy Phys. **10**, 020 (2002).
[37] Y. Oz and T. Sakai, Phys. Lett. B **544**, 321 (2002).
[38] M. Alishahiha and A. Kumar, J. High Energy Phys. **09**, 031 (2002).
[39] M. Berkooz, M. Rozali, and N. Seiberg, Phys. Lett. B **408**, 105 (1997).
[40] N. Seiberg, Phys. Lett. B **408**, 98 (1997).