Cosmological limit on the neutrino mass

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We perform a careful analysis of constraints on the neutrino mass from current cosmological data. Combining data from the cosmic microwave background and the 2dF galaxy survey yields an upper limit on the sum of the three neutrino mass eigenstates of $\Sigma m_\nu \leqslant 3$ eV (95% C.L.), without including additional priors. Including data from SNIa observations, big bang nucleosynthesis, and HST Hubble key project data on H_0 tightens the limit to $\Sigma m_\nu \leqslant 2.5$ eV (95% C.L.). We also perform a Fisher matrix analysis which illustrates the cosmological parameter degeneracies affecting the determination of Σm_ν.

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I. INTRODUCTION

The absolute value of the neutrino masses is very difficult to measure experimentally. On the other hand, mass differences between neutrino mass eigenstates (m_1, m_2, m_3) can be measured in neutrino oscillation experiments. Observations of atmospheric neutrinos suggest a squared mass difference of $\delta m^2 \simeq 3\times 10^{-3}$ eV^2 [2,1]. While there are still several viable solutions to the solar neutrino problem, the so-called large-mixing-angle solution gives by far the best fit with $\delta m^2 \simeq 5\times 10^{-5}$ eV^2 [3,4].

In the simplest case where neutrino masses are hierarchical, these results suggest that $m_1 \sim 0$, $m_2 \sim \delta m_{\rm solar}$, and $m_3 \sim \delta m_{\rm atmospheric}$. If the hierarchy is inverted [5–10], one finds instead $m_3 \sim 0$, $m_2 \sim \delta m_{\rm atmospheric}$, and $m_1 \sim \delta m_{\rm atmospheric}$. However, it is also possible that neutrino masses are degenerate [11–21], $m_1 \sim m_2 \sim m_3 \gg \delta m_{\rm atmospheric}$, in which case oscillation experiments are not useful for determining the absolute mass scale.

Experiments which rely on kinematical effects of the neutrino mass offer the strongest probe of this overall mass scale. Tritium decay measurements have been able to put an upper limit on the electron neutrino mass of 2.2 eV (95% C.L.) [22]. However, cosmology at present yields an even stronger limit which is also based on the kinematics of the neutrino mass. Neutrinos decouple at a temperature of 1–2 MeV in the early universe, shortly before electron-positron annihilation. Therefore, their temperature is lower than the photon temperature by a factor $(4/11)^{1/3}$. This again means that the total neutrino number density is related to the photon number density by

$$n_\nu = \frac{9}{11} n_\gamma. \tag{1}$$

Massive neutrinos with masses $m \gg T_0 \sim 2.4\times 10^{-4}$ eV are nonrelativistic at present and therefore contribute to the cosmological matter density [23–25]

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{92.5 \text{ eV}}, \tag{2}$$

calculated for a present-day photon temperature $T_0 = 2.728$ K. Here, $\Sigma m_\nu = m_1 + m_2 + m_3$. However, because they are so light, these neutrinos free stream on a scale of roughly $k \simeq 0.03 m_{\rm eV} \Omega_m^{1/2} h$ Mpc^{-1} [26–28]. Below this scale, neutrino perturbations are completely erased and therefore the matter power spectrum is suppressed, roughly by $\Delta P/P \sim -8\Omega_\nu/\Omega_m$ [26].

This power spectrum suppression allows for a determination of the neutrino mass from measurements of the matter power spectrum on large scales. This matter spectrum is related to the galaxy correlation spectrum measured in large-scale-structure (LSS) surveys via the bias parameter, $b^2 \equiv P_g(k)/P_m(k)$. Such analyses have been performed several times before [29,30], most recently using data from the 2dF galaxy survey [31]. This investigation finds an upper limit of 1.8–2.2 eV for the sum of neutrino masses. However, this result is based on a relatively limited cosmological parameter space.

It should also be noted that, although massive neutrinos have little impact on the cosmic microwave background (CMB) power spectrum, it is still necessary to include CMB data in any analysis in order to determine other cosmological parameters.

In the present paper, we perform an extensive analysis, carefully discussing the issue of parameter degeneracies. The next section is devoted to a Fisher matrix analysis of the problem which establishes possible parameter degeneracies and yields a general idea of the precision with which the neutrino mass can be measured. Section III describes a full numerical likelihood analysis of data from CMB and LSS, which yields a robust limit on the neutrino mass. Finally, Sec. IV contains a discussion and conclusion.

II. FISHER MATRIX ANALYSIS

Measuring neutrino masses from cosmological data is quite involved since for both CMB and LSS the power spectra depend on a plethora of different parameters in addition to the neutrino mass. Furthermore, since the CMB and matter

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power spectra depend on many different parameters, one might worry that an analysis which is too restricted in parameter space could give spuriously strong limits on a given parameter. Therefore, it is desirable to study possible parameter degeneracies in a simple way before embarking on a full numerical likelihood analysis.

It is possible to estimate the precision with which the cosmological model parameters can be extracted from a given hypothetical data set. The starting point for any parameter extraction is the vector of data points, x. This can be in the form of the raw data, or in compressed form, typically the power spectrum [C_l for CMB and $P(k)$ for LSS].

Each data point has contributions from both signal and noise, $x = x_{\rm signal} + x_{\rm noise}$. If both signal and noise are Gaussian-distributed, it is possible to build a likelihood function from the measured data which has the following form [32]:

$$\mathcal{L}(\Theta) \propto \exp\left(-\frac{1}{2} x^\dagger [C(\Theta)^{-1}] x \right), \tag{3}$$

where $\Theta = (\Omega, \Omega_b, H_0, n_s, \tau, \dots)$ is a vector describing the given point in model parameter space and $C(\Theta) = \langle x x^T \rangle$ is the data covariance matrix. In the following, we shall always work with data in the form of a set of power spectrum coefficients, x_i, which can be either C_l or $P(k)$.

If the data points are uncorrelated so that the data covariance matrix is diagonal, the likelihood function can be reduced to $\mathcal{L} \propto e^{-\chi^2/2}$, where

$$\chi^2 = \sum_{i=1}^{N_{\rm max}} \frac{(x_{i,\rm obs} - x_{i,\rm theory})^2}{\sigma(x_i)^2} \tag{4}$$

is a χ^2 statistics and $N_{\rm max}$ is the number of power spectrum data points [32].

The maximum likelihood is an unbiased estimator, which means that

$$\langle \Theta \rangle = \Theta_0 . \tag{5}$$

Here Θ_0 indicates the true parameter vector of the underlying cosmological model and $\langle \Theta \rangle$ is the average estimate of parameters from maximizing the likelihood function.

The likelihood function should thus peak at $\Theta \simeq \Theta_0$, and we can expand it to second order around this value. The first-order derivatives are zero, and the expression is thus

$$\chi^2 = \chi^2_{\rm min} + \sum_{i,j} (\theta_i - \theta) \left(\sum_{k=1}^{N_{\rm max}} \frac{1}{\sigma(x_k)^2} \left[\frac{\partial x_k}{\partial \theta_i} \frac{\partial x_k}{\partial \theta_j} - (x_{k,\rm obs} - x_k) \frac{\partial^2 x_k}{\partial \theta_i \partial \theta_j} \right] \right) (\theta_j - \theta), \tag{6}$$

where i,j indicate elements in the parameter vector Θ. The second term in the second derivative can be expected to be very small because $(x_{k,\rm obs} - x_k)$ is in essence just a random measurement error which should average out. The remaining term is usually referred to as the Fisher information matrix,

$$F_{ij} = \frac{\partial^2 \chi^2}{\partial \theta_i \partial \theta_j} = \sum_{k=1}^{N_{\rm max}} \frac{1}{\sigma(x_k)^2} \frac{\partial x_k}{\partial \theta_i} \frac{\partial x_k}{\partial \theta_j}. \tag{7}$$

The Fisher matrix is closely related to the precision with which the parameters, θ_i, can be determined. If all free parameters are to be determined from the data alone without any priors, then it follows from the Cramer-Rao inequality [33] that

$$\sigma(\theta_i) = \sqrt{(F^{-1})_{ii}} \tag{8}$$

for an optimal unbiased estimator, such as the maximum likelihood [34].

In order to estimate how degenerate parameter i is with another parameter, j, one can calculate how $\sigma(\theta_i)$ changes if parameter j is kept fixed instead of free in the analysis. Starting from the 2×2 submatrix

$$S_{ij} = (F^{-1})_{ij}, \tag{9}$$

one then finds

$$\sigma_{j\ \rm fixed}(\theta_i) = \sqrt{\frac{1}{(S^{-1})_{ii}}}. \tag{10}$$

We therefore define the quantity

$$r_{ij} = \frac{\sigma_{j\ \rm fixed}(\theta_i)}{\sigma(\theta_i)} \leqslant 1 \tag{11}$$

as a measure of the degeneracy between parameters i and j.

In order to perform an actual calculation, we use the most present data from CMB and LSS.

CMB data set. Several data sets of high precision are now publicly available. In addition to the Cosmic Background Explorer (COBE) [35] data for small l, there are data from BOOMERANG [36], MAXIMA [37], DASI [38] and several other experiments [39,40]. Wang, Tegmark, and Zaldarriaga [39] have compiled a combined data set from all these available data, including calibration errors. In the present work, we use this compiled data set, which is both easy to use and includes all relevant present information. Altogether there are 24 CMB data points in this compilation.

LSS data set. At present, by far the largest survey available is the 2dF [41], of which about 147 000 galaxies have so far been analyzed. Tegmark, Hamilton, and Xu [42] have calculated a power spectrum, $P(k)$, from these data, which we use in the present work. The 2dF data extend to very small scales where there are large effects of nonlinearity.

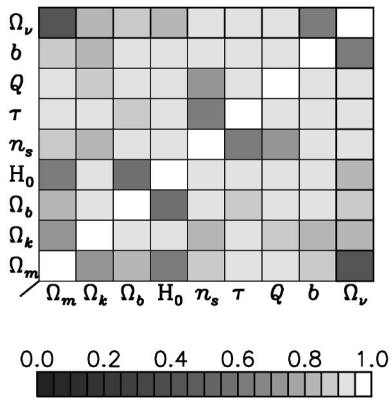

FIG. 1. Values of the parameter r_{ij}, defined in Eq. (11).

Since we only calculate linear power spectra, we use (in accordance with standard procedure) only data on scales larger than $k=0.2h$ Mpc^{-1}, where effects of nonlinearity should be minimal. Making this cut reduces the number of power spectrum data points to 18.

For calculating the theoretical CMB and matter power spectra, we use the publicly available CMBFAST package [43]. As the set of cosmological parameters, we choose Ω_m, the matter density; $\Omega_k=1-\Omega_m-\Omega_\Lambda-\Omega_\nu$, the curvature parameter; Ω_b, the baryon density; H_0, the Hubble parameter; n_s, the scalar spectral index of the primordial fluctuation spectrum; τ, the optical depth to reionization; Q, the normalization of the CMB power spectrum; b, the bias parameter; and Ω_ν, the neutrino density. In all cases we take the number of massive neutrinos to be 3. The reason is that if neutrinos are to have an impact on CMB and matter spectra, their masses must be much larger than the mass splitting inferred from atmospheric neutrino observations ($\delta m \sim 0.05-0.1$ eV), and therefore neutrino masses will be degenerate with $m_1 \sim m_2 \sim m_3 \gg \delta m_{\rm atmospheric}$.

In principle, one might include even more parameters in the analysis, such as r, the tensor to scalar ratio of primordial fluctuations. However, r is most likely so close to zero that only future high-precision experiments may be able to measure it. The same is true for other additional parameters. Deviations from the slow-roll prediction of a simple power-law initial spectrum [44–47] or additional relativistic energy density [48–56] could be present. However, such effects only appear in cosmological models which are more complicated than the "standard" cold dark matter model with a cosmological constant (ΛCDM). The parameters we use fully describe the features of the simplest working model.

In the end, one can check the consistency of the numerical parameter extraction by calculating the χ^2 per degree of freedom. In Sec. III, we find that the best fit is consistent with expectations and therefore it is unlikely that there are other parameters significantly affecting the power spectra.

Figure 1 shows the matrix r_{ij}, calculated for the Wang-Tegmark-Zaldarriaga (WTZ) + 2dF data set errors, around a reference cosmological model with parameters $\Omega_m=0.3$, $\Omega_\Lambda=0.7$, $\Omega_b h^2=0.020$, $H_0=70$ km s^{-1} Mpc^{-1}, $n_s=1.0$, and $\tau=0$, i.e., the ΛCDM concordance model. Note that r_{ii} is an ill-defined quantity. For plotting purposes, we simply set $r_{ii}=1$, but this has no physical significance.

From this general matrix, one can study the m_ν-related degeneracies more closely. Clearly the two parameters most degenerate with m_ν are Ω_m, the matter density, and b, the bias. This is not surprising because a nonzero neutrino mass has little effect on the CMB acoustic peaks and a large effect on the matter power spectrum on scales below the free-streaming scale.

Therefore, any parameter which behaves in a similar way will cause degeneracy. This is indeed the case for Ω_m; if this parameter is changed while keeping Ω_k fixed, there is little effect on CMB. On the other hand, changing Ω_m changes the normalization of the matter power spectrum at small scales relative to large scales. Changing b also mimics a nonzero neutrino mass. The reason is that present-day LSS data have large error bars around the free-streaming scale, for light neutrinos in the eV range. On low scales the effect of massive neutrinos is simply to lower the fluctuation level roughly as [26]

$$\frac{\Delta P}{P} \simeq -8\frac{\Omega_\nu}{\Omega_m}, \tag{12}$$

i.e., it is scale-independent and therefore indistinguishable from changing the bias, b. This degeneracy can be broken by precision measurements around the free-streaming scale where the break in the power spectrum occurs. A good example of how the mass limit on neutrinos can be tightened if bias is fixed comes from Ref. [30]. Here the mass limit comes from comparing the overall normalization of the spectra at COBE scales [35] with those on cluster scales [57]. However, we believe that, at present, keeping bias as a free parameter yields a much more robust constraint. To a much lesser extent, the neutrino mass is also degenerate with the Hubble parameter.

It should be noted that there is little degeneracy with n_s, the spectral index. In Refs. [26,31] a significant degeneracy between Ω_ν and n_s was found when only LSS data are considered. However, this is broken when CMB data are included (as is also noted in Ref. [26]), the reason being that changing n_s affects both CMB and matter power spectra, not just the matter spectrum.

TABLE I. The different priors on parameters other than $\Omega_\nu h^2$ used in the analysis.

Prior type	Ω_m	$\Omega_b h^2$	h	n	τ	Q	b
CMB+LSS	0.1–1	0.008–0.040	0.4–1.0	0.66–1.34	0–1	free	free
CMB+LSS+BBN+H_0	0.1–1	0.020 ± 0.002	0.70 ± 0.07	0.66–1.34	0–1	free	free
CMB+LSS+BBN+H_0+SNIa	0.28 ± 0.14	0.020 ± 0.002	0.70 ± 0.07	0.66–1.34	0–1	free	free

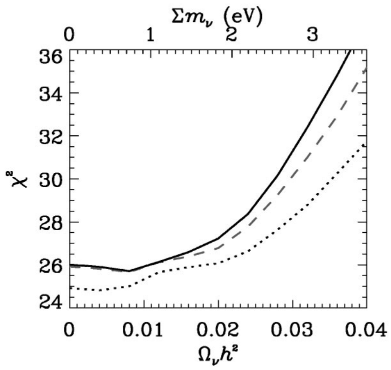

FIG. 2. χ^2 as a function of $\Omega_\nu h^2$, plotted for the three different priors. The dotted curve is for CMB+LSS, the dashed for CMB+LSS+BBN+H_0, and the full curve for CMB+LSS+BBN +H_0+SNIa.

Clearly, it would be desirable to fix the parameters with which the neutrino mass is most degenerate; Ω_m, b, and H_0. As for Ω_m, one can use the Supernova Type Ia (SNIa) result $\Omega_m = 0.28 \pm 0.14$ which applies to a flat universe [58]. However, this value is not much more restrictive than what is found from the CMB+LSS data alone. Fixing the bias is much more difficult since it is not a physically well-understood parameter. In Elgaroy *et al.* [31] bias was kept as a free parameter, and we follow this line. H_0 has been determined precisely by the Hubble Space Telescope (HST) key project to be $H_0 = 70 \pm 7$ km s^{-1} Mpc^{-1} [59].

If all the above parameters are included in the Fisher matrix analysis, the estimated 1σ precision on m_ν is 1.8 eV, equivalent to a 95% confidence limit of 3.6 eV.

III. NUMERICAL RESULTS

The Fisher matrix analysis can only give a general idea of the constraints which can be found from a given data set. In reality, the likelihood is non-Gaussian and away from the best-fit point the formalism breaks down. In order to get reliable estimates, it is necessary to perform a full numerical likelihood analysis over the space of cosmological parameters.

In this full numerical likelihood analysis, we use a slightly restricted parameter space with the following free parameters: Ω_m, Ω_b, H_0, n_s, Q, b, and τ. We restrict the analysis to flat models, $\Omega_k = 0$. This has very little effect on the analysis because there is little degeneracy between m_ν and Ω_k. In order to study the effect of the different priors we calculate three different cases, the priors for which can be seen in Table I. The big bang nucleosynthesis (BBN) prior on $\Omega_b h^2$ comes from Ref. [60]. The actual marginalization over parameters other than $\Omega_\nu h^2$ was performed using a simulated annealing procedure [61].

Figure 2 shows χ^2 for the three different cases as a function of the m_ν. The best-fit χ^2 values are 24.81, 25.66, and 25.71 for the three different priors, respectively. In comparison, the numbers of degrees of freedom are 34, 35, and 36, meaning that the fits are compatible with expectations, roughly within the 68% confidence interval.

We identify the 95% confidence limit on m_ν with the point where $\Delta\chi^2 = 4$. These limits are shown in Table II. For the most restrictive prior we find a 95% confidence upper limit of $\Sigma m_\nu \leq 2.47$ eV. This is compatible with the findings of Ref. [31], which finds that $\Sigma m_\nu \lesssim 1.8 - 2.2$ eV for a slightly more restrictive parameter space.

Based on the present analysis, we consider $\Sigma m_\nu \leq 3$ eV (95% C.L.) a robust upper limit on the sum of the neutrino masses. This corresponds roughly to the value found for the CMB+LSS data alone without any additional priors. Even though this value is significantly higher than what is quoted in Ref. [31], it is still much more restrictive than the value $\Sigma m_\nu \leq 4.4$ eV [39] found from CMB and PSCz [62] data. As is also discussed in Ref. [31], the main reason for the improvement is the much greater precision of the 2dF survey, compared to the PSCz data [62].

IV. DISCUSSION

We have studied cosmological constraints on the neutrino masses from present CMB and LSS data. Initially a Fisher matrix analysis was performed which illustrates the main degeneracies of m_ν with other cosmological parameters used in the analysis. From this simplified analysis it was estimated that the precision on Σm_ν should be roughly 3.6 eV at 95% confidence from CMB+LSS data alone.

However, in order to obtain reliable estimates, we performed a full numerical likelihood analysis. Using reasonable priors on Ω_m, $\Omega_b h^2$, and H_0 we obtained a limit of $\Sigma m_\nu \leq 2.47$ eV at 95% confidence, while using no priors on the CMB+LSS data yielded $\Sigma m_\nu \leq 3$ eV, again at 95% confidence. We believe this to be a robust upper limit.

Our analysis shows, not surprisingly, that priors are extremely important for parameter estimation of m_ν. Our most restrictive prior yields a result similar to that found by Ref. [31], while our no-prior case yields a significantly looser constraint.

The Fisher matrix analysis showed that the parameter most degenerate with m_ν is the bias parameter, b. In order to obtain much stronger limits, one must either determine b

TABLE II. Best fit χ^2 and upper limits on $\Sigma m_{\nu,\max}$ for the three different priors.

Prior type	Best fit χ^2	$\Sigma m_{\nu,\max}$ (eV) (95% C.L.)
CMB+LSS	24.81	2.96
CMB+LSS+BBN+H_0	25.66	2.65
CMB+LSS+BBN+H_0+SNIa	25.71	2.47

precisely in an independent way or obtain better LSS power spectrum statistics around the scale corresponding to the free-streaming scale for neutrinos, $k \simeq 0.02-0.03h$ Mpc^{-1}. The Sloan Digital Sky Survey [63] will measure the power spectrum shape with higher precision on the relevant scale, and these data, combined with CMB data from the Microwave Anisotropy Probe (MAP) experiment [64], will either push the limit by a factor of at least a few or indeed detect a nonzero neutrino mass directly. It was estimated in Ref. [26] that $\Sigma m_\nu \lesssim 0.65$ eV can be reached.

Finally, we note that the present cosmological limit is significantly stronger than current laboratory limits. The Mainz tritium experiment [22] currently quotes a 95% upper limit to the ν_e mass of 2.2 eV, which translates to a sum of roughly 6.5–7 eV for the three mass eigenstates. As is also noted in Elgaroy *et al.* [31], the cosmological limit is compatible with the very controversial detection of neutrinoless double beta decay by the Heidelberg-Moscow experiment [65]. If this finding is confirmed, it would imply a sum of masses of order 1 eV, within range of the MAP+SDSS data.

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