Curvature of the universe and the dark energy potential

Sergio del Campo
Instituto de Física, Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile
(Received 3 July 2002; published 27 December 2002)

The flatness of an accelerating universe model (characterized by a dark energy scalar field χ) is mimicked from a curved model that is filled with, apart from the cold dark matter component, a quintessencelike scalar field Q. In this process, we characterize the original scalar potential $V(Q)$ and the mimicked scalar potential $V(\chi)$ associated with the scalar fields Q and χ, respectively. The parameters of the original model are fixed through the mimicked quantities that we relate to the present astronomical data, such as the equation of state parameter w_χ and the dark energy density parameter Ω_χ.

DOI: 10.1103/PhysRevD.66.123513 PACS number(s): 98.80.Hw, 98.80.Bp

I. INTRODUCTION

At the moment, we do not precisely know the amount of matter present in the universe, and we do not as yet know what the geometry of the universe is. Astronomical observations conclude that the matter density related to baryonic and nonbaryonic cold dark matter is much less than the critical density value [1]. However, measurements of anisotropy in the cosmic microwave background radiation indicate that the total matter present in the universe is very much the same as its critical value [2–5]. These measurements agree with the theoretical prediction of inflationary universe scenarios [6], which predict that our universe should become flat after leaving the inflationary era. In light of these results, it seems that there exists an important amount of matter that we are not considering.

On the other hand, recent measurements of type Ia supernova at high redshift indicate that our universe is accelerating [7,8]. The simplest description of this acceleration is that it can be characterized as a cosmological constant, which contributes to a negative pressure. Other approaches have been employed to explain this acceleration. We distinguish the one related to the quintessence model. This model is characterized by an evolving scalar field χ and its scalar-field potential $V(\chi)$ [9].

Any cosmological model becomes characterized by the total matter density parameter Ω_T. This parameter is defined by the ratio between the total matter (ρ_T) and the critical energy (ρ_C) densities. For instance, in the case of the cosmological constant (Λ) cold dark matter (CDM) model, this parameter is given by $\Omega_T=\Omega_M+\Omega_\Lambda$, in which $\Omega_M=\rho_M^0/\rho_C=(8\pi G/3H_0^2)\rho_M^0$ and $\Omega_\Lambda=(\Lambda/8\pi G)1/\rho_C=\Lambda/3H_0^2$. Here, ρ_M^0 is the actual value of the nonrelativistic matter density and H_0 is the actual value of the Hubble parameter. Ω_Λ is the fraction of the critical energy density contained in a smoothly distributed vacuum energy referred to as Λ, and Ω_M represents the matter density related to the baryonic and nonbaryonic CDM densities. The constant G is the Newton constant, and we use $c=1$ for the speed of light. From now on, all quantities with upper or lower zero indexes specify current values.

Because of measurements and theoretical arguments, it seems natural to consider flat universe models, but one interesting question to ask is whether this flatness is due to a sort of compensation among different components that enter into the dynamical equations. In this respect, our main goal in this paper is to address this sort of question. In the literature, we have found some descriptions along these lines. For instance, closed models with an important matter component with equation of state given by $P=-\rho/3$ have been studied. Here, the universe expands at constant speed [10]. Other authors, by using the same properties for the universe, have added a nonrelativistic matter density in which Ω_T is less than 1, thus describing an open universe [11]. Also, flat decelerating models have been simulated [12]. The common fact in all of these models is that, even if the starting geometry presents curvature, all models are indistinguishable from flat geometries at low redshift.

In this paper, we wish to consider universe models that have curvature and are composed of two matter components. One of these components is the usual nonrelativistic dust matter; the other corresponds to a sort of quintessence-type matter, described by a scalar field that we designate by Q. This field is fundamental in the sense that it is introduced from a Lagrangian. In order to mimic a flat quintessence CDM, we introduce a new scalar field χ, such that this field is constrained by recent astronomical data. Therefore, in a flat background the scalar field χ together with the CDM component form the basis for the χCDM model, a model that is restricted by the present observations. In this way, it is the scalar field χ, not the Q field, that is related to the observable quantities [13]. Thus, we assume that the effective equation of state for χ is given by $P_\chi=w_\chi\rho_\chi$, where w_χ is the observable effective equation of state parameter [9]. We note that the astronomical observations (related to the type Ia supernovae measurements) put an upper limit on the present value of this parameter, $w_\chi<-\frac{1}{3}$ [8].

We also assume that the scalar field Q is characterized by a similar equation of state,

$$P_Q=w_Q\rho_Q, \tag{1}$$

where the parameter w_Q is, in general, a variable quantity. Therefore, our goal in this paper is to investigate the conditions under which a scenario with positive (or negative) curvature may mimic a flat universe at low redshifts. This approach forces us to determine the exact contribution of the scalar field Q, together with the curvature term, that gives rise to the flat quintessence accelerating χCDM universe. However, we should comment that absorbing the curvature term in a redefinition of the Q field is certainly not equivalent to getting a really flat universe, since the curvature is a geometrical property, which follows directly from the metric tensor and which enters into the Friedmann-Robertson-Walker (FRW) line element. As we mentioned above, these two models become indistinguishable only for low redshifts and are similar to the cases studied in Refs. [11,12]. We will return to this point later on. Here, we emphasize that our approach rests on the fact that there is a clear similitude between the curvature and the flat universe models at low redshift.

II. THE DYNAMIC FIELD EQUATIONS

We start with the following effective action:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R+\frac{1}{2}(\partial_\mu Q)^2-V(Q)+L_M\right], \quad (2)$$

where R is the scalar curvature, $V(Q)$ is the scalar potential associated to the field Q, and L_M is related to any ordinary matter component.

We shall assume that the Q field is homogeneous, i.e., it is a time-depending quantity only, $Q=Q(t)$, and the spacetime is isotropic and homogeneous, with the metric corresponding to the Friedmann-Robertson-Walker metric

$$ds^2=dt^2-a(t)^2\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)\right], \quad (3)$$

where $a(t)$ represents the scale factor and the parameter k takes the values $k=-1,0,1$ corresponding to an open, flat, closed three-geometry, respectively. With these assumptions, the action (2) yields the following field equations: the time-time component of the Einstein equation,

$$H^2=\frac{8\pi G}{3}(\rho_M+\rho_Q)-\frac{k}{a^2}; \quad (4)$$

the evolution equation for the scalar field Q,

$$\ddot{Q}+3H\dot{Q}=-\frac{\partial V(Q)}{\partial Q}; \quad (5)$$

and the energy conservation law for the ordinary matter,

$$\dot{\rho}_M+3H(\rho_M+P_M)=0. \quad (6)$$

In these equations, the overdots denote derivatives with respect to t, $H=\dot{a}/a$ defines the Hubble expansion rate, and ρ_M and ρ_Q are the effective matter energy density and the average energy densities, respectively. The Q-energy density is defined by

$$\rho_Q=\frac{1}{2}\dot{Q}^2+V(Q). \quad (7)$$

We introduce its average pressure P_Q by means of

$$P_Q=\frac{1}{2}\dot{Q}^2-V(Q). \quad (8)$$

These two latter quantities are related by the equation of state, Eq. (1). Thus, expression (5) becomes

$$\dot{\rho}_Q+3H(\rho_Q+P_Q)=0, \quad (9)$$

which, similar to the ordinary matter, represents an energy balance for the scalar field Q. From now on, we consider the ordinary matter to correspond to dust, which becomes characterized by the equation of state $P_M=0$. For this case, we could solve the energy Eq. (6) analytically, in which case we get $\rho_M\propto a^{-3}$.

Summarizing, we have a combination of two noninteracting perfect fluids: one, a dust matter component (ρ_M); the other, the scalar field (ρ_Q) component.

Equation (4) may be written as

$$H^2=H_0^2\left[\Omega_M\left(\frac{\rho_M}{\rho_M^0}\right)+\Omega_Q\left(\frac{\rho_Q}{\rho_Q^0}\right)+\Omega_k\left(\frac{a_0}{a}\right)^2\right]. \quad (10)$$

Here, the actual curvature density Ω_k and the quintessence density Ω_Q parameters are defined by $\Omega_k=-k(1/a_0 H_0)^2$ and $\Omega_Q=(8\pi G/3H_0^2)\rho_Q^0$, respectively.

In the next section, we study the model that arises when the matter component ρ_M together with Eqs. (4), (5), and the equation of state for the field Q complement each other in such a way that a flat CDM accelerated universe originates, specifically the χCDM model.

In order to simulate a flat universe, we assume that the energy density ρ_Q and the curvature term combine so that

$$\frac{8\pi G}{3}\rho_Q(t)-\frac{k}{a^2(t)}=\frac{8\pi G}{3}\rho_\chi(t), \quad (11)$$

or equivalently,

$$\Omega_Q\left(\frac{\rho_Q}{\rho_Q^0}\right)+\Omega_k\left(\frac{a_0}{a}\right)^2=\Omega_\chi\left(\frac{\rho_\chi}{\rho_\chi^0}\right), \quad (12)$$

where, similar to the definitions of Ω_M and Ω_Q, we have defined $\Omega_\chi=(8\pi G/3H_0^2)\rho_\chi^0$. This latter quantity is related to the recent astronomical measurement of distant supernova of type Ia.

Note that we can get an explicit expression (as a function of time) for the unknown energy density, ρ_χ, if we know both the scale factor $a(t)$ and the time dependence of the Q density, ρ_Q. Note also that for $\Omega_k>0$ (open universes) we must have $\rho_\chi>(|\Omega_k|/\Omega_\chi)\rho_\chi^0(a_0/a)^2$, since $\rho_Q>0$. At this point we should comment on the difference between the two scalar fields Q and χ. As we saw, the scalar field Q is defined from the fundamental Lagrangian, from which we could define the stress-energy density tensor with the property of a perfect fluid behavior, which allows us to introduce the pressure P_Q. However, we could not say the same for χ. The definition of the stress-energy density tensor associated with the χ field is more subtle, as we can see in Eq. (11) [or Eq. (12)]. We hope to address this problem in subsequent work. Here, we just take P_χ as an effective pressure that follows the equation of state $P_\chi = w_\chi \rho_\chi$, where w_χ is determined from the observational data.

III. CHARACTERISTICS OF THE MODEL

In this section, we will impose explicit conditions under which a curved universe ($k=\pm 1$) may look similar to a flat universe ($k=0$) at low redshift. This flat model is defined by expression (11) [or (12)], which reduces the time-time Einstein Eq. (10) to

$$H^2 = H_0^2\left[\Omega_M\left(\frac{\rho_M}{\rho_M^0}\right) + \Omega_\chi\left(\frac{\rho_\chi}{\rho_\chi^0}\right)\right]. \tag{13}$$

Equations (12) and (13), along with the evolution equations for the scalar fields χ and Q, form the basic set of equations that describes our model.

Note that, when Eq. (12) is evaluated at the present time, we obtain the following relation among the omega parameters:

$$\Omega_Q = \Omega_\chi - \Omega_k. \tag{14}$$

From this relation we observe that, for $k=1$, Ω_Q must be greater than Ω_χ, since $\Omega_{k=1}<0$. For $k=-1$, $\Omega_k>0$, thus $\Omega_\chi>\Omega_Q$. Notice also that Eq. (13) gives the additional expression $\Omega_M+\Omega_\chi \equiv \Omega_T = 1$ when evaluated at the present time. From the present observational values of Ω_χ and Ω_k, we could get the actual value of the parameter Ω_Q. Note also that when Eqs. (10) and (13) are compared, we observe that the variables ρ_M and H [and also the scale factor a, as we can see from Eq. (11) or equivalently Eq. (12)] appear *identical* in the two scenarios.

In order to see that a curvature model at low redshift is indistinguishable from a flat one, we could consider the luminosity distance D_L as a function of the redshift z or the angular distance [12]. Let us take the first one. The luminosity distance between a source at a redshift $z>0$ and $z=0$ related to a curvature model is obtained from the expression

$$D_L^{k\neq 0}(z) \propto \frac{(1+z)}{\sqrt{|\Omega_k|}}\sin[\sqrt{|\Omega_k|}\,\xi(z)]. \tag{15}$$

For a flat universe model, we obtain

$$D_L^{k=0}(z) \propto (1+z)\xi(z). \tag{16}$$

In these expressions, we have defined $\xi(z)$ by means of $\xi(z) = 1/a_0\int_0^z dz/H(z)$, which represents the polar-coordinated distance between a source at z and another at $z=0$ in the same line of sight. For $z\ll 1$, we could show that $\xi(z)\propto z$. Thus, since $|\Omega_k|\ll 1$ we observe that the luminosity distance in both cases coincides.

Therefore, we expect that the differences between the curvature and the flat models happen to high enough redshifts.

When the quintessence component has a constant equation of state parameter, $w_\chi^0 \equiv w_\chi$, which is a negative number that lies in the range $-1<w_\chi<-0.3$, we could immediately solve for the density ρ_χ as a function of the scale factor,

$$\rho_\chi(a) = \rho_C\Omega_\chi\left(\frac{a_0}{a}\right)^{3(1+w_\chi)}. \tag{17}$$

Thus, Eq. (13) becomes

$$H(a) = H_0\left(\frac{a_0}{a}\right)^{3/2}\sqrt{\Omega_M + \Omega_\chi\left(\frac{a_0}{a}\right)^{3w_\chi}}. \tag{18}$$

The solution of Eq. (18) is given by

$$t = \frac{2}{3H_0\sqrt{\Omega_M}}\left(\frac{a}{a_0}\right)^{3/2} \times {}_2F_1\left(\frac{1}{2}, -\frac{1}{2w_\chi}; 1-\frac{1}{2w_\chi}; -\left(\frac{\Omega_k+\Omega_Q}{\Omega_M}\right)\left(\frac{a}{a_0}\right)^{-3w_\chi}\right), \tag{19}$$

where ${}_2F_1$ represents the generalized hypergeometric function. The initial condition that we have used in solving Eq. (18) is $a=0$ at $t=0$.

We use the definition of P_χ and ρ_χ in terms of the scalar field χ, together with the equation of state that relates these quantities, to obtain χ as a function of the scale factor. The result is

$$\chi(a) = \chi_0\left(\frac{a}{a_0}\right)^{-3w_\chi/2} \times \left[\frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; -\left(\frac{\Omega_k+\Omega_Q}{\Omega_M}\right)\left(\frac{a}{a_0}\right)^{-3w_\chi}\right)}{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; -\left(\frac{\Omega_k+\Omega_Q}{\Omega_M}\right)\right)}\right], \tag{20}$$

where χ_0 is given by

$$\chi_0 = \tilde{\chi}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; -\left(\frac{\Omega_k+\Omega_Q}{\Omega_M}\right)\right),$$

with $\tilde{\chi} = \sqrt{4\rho_c/9H_0^2}\sqrt{(\Omega_k+\Omega_Q)(1+w_\chi)/\Omega_M w_\chi^2}$.

In a similar way, we get for the scalar potential V_χ

$$V_\chi(a) = V_\chi^0\left(\frac{a_0}{a}\right)^{3(1+w_\chi)}, \tag{21}$$

where $V_\chi^0 = \frac{1}{2}(1-w_\chi)\rho_C(\Omega_k+\omega_Q)$.

Figure 1 shows the plot of the scalar potential V_χ as a function of χ, for three different values of the equation of state parameter w_χ. The parameters $\Omega_\chi(=\Omega_k+\Omega_Q)$ and Ω_M have been fixed at values 0.65 and 0.35, respectively. This form of potential has been described in the literature [14]. Note that, in the limit $w_\chi\rightarrow -1$ the potential V_χ

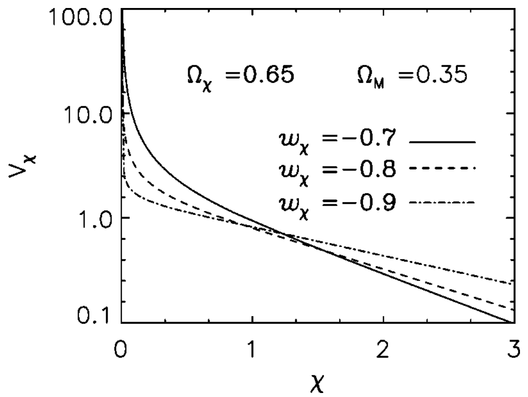

FIG. 1. This graph shows the scalar potential V_χ as a function of the scalar field χ for $w_\chi=-0.7$, -0.8, and -0.9. We use $\Omega_M=0.35$ and $\Omega_\chi=0.65$.

$\to$const$\equiv\rho_C\Omega_\chi$, i.e., the model becomes equivalent to the Λ cold dark matter scenario [9].

During the evolution of this model, we distinguish two periods: one, where the regular matter dominates ($\rho_M \gg \rho_\chi$); the other, where $\rho_\chi \gg \rho_M$. It is not hard to see that the time at which ρ_M becomes equal to ρ_χ is given by

$$t_{\rm eq}=\frac{2}{3H_0\sqrt{\Omega_M}}\left(\frac{\Omega_M}{\Omega_\chi}\right)^{-1/2w_\chi} \times {}_2F_1\left(\frac{1}{2},-\frac{1}{2w_\chi};1-\frac{1}{2w_\chi};-1\right). \tag{22}$$

Notice that the age of the universe in this model is given by

$$t_0=\frac{2}{3H_0\sqrt{\Omega_M}}\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2w_\chi};1-\frac{1}{2w_\chi};-\frac{\Omega_\chi}{\Omega_M}\right), \tag{23}$$

where, similar to $t_{\rm eq}$, it depends on the observable parameters Ω_χ, Ω_M, and w_χ. For $\Omega_M=0.3$, $\Omega_\chi=0.7$, and $w_\chi=-0.8$, this gives $t_0H_0\sim0.93$, which lies in the observational range $t_0H_0=0.94\pm0.14$ of the measurement of the age of the universe [15]. Figure 2 shows Ω_χ as a function of Ω_M for the range of t_0H_0 specified above. The dark region represents the value $w_\chi=-0.9$.

We are now going to describe the properties of the scalar field Q. Following an approach analogous to that used for the scalar field χ, we find that the parameter w_Q is given by

$$w_Q(a)=-|w_Q^0|\left(\frac{1+\beta}{1-3\beta w_\chi}\right)\left[\frac{1-3\beta w_\chi\left(\frac{a}{a_0}\right)^{-3w_\chi-1}}{1+\beta\left(\frac{a}{a_0}\right)^{-3w_\chi-1}}\right], \tag{24}$$

where $\beta=\Omega_\chi/|\Omega_k|$ and w_Q^0 is the actual value of w_Q defined by $w_Q^0=P_Q^0/\rho_Q^0$. Figure 3 shows its dependence with the redshift z defined as $z\equiv a_0/a-1$, for three different values

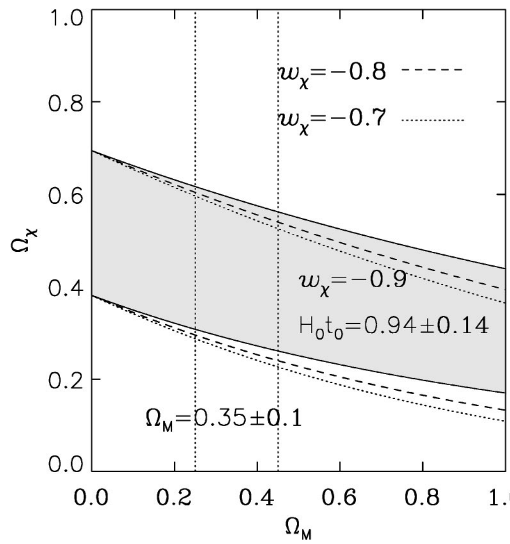

FIG. 2. This graph represents Ω_χ as a function of Ω_M for the range of $t_0H_0=0.94\pm0.14$ for three different values of the equation state parameter, $w_\chi=-0.7,-0.8,-0.9$. The dark region corresponds to $w_\chi=-0.9$. The vertical dotted lines show the observational range for the Ω_M parameter.

of the w_χ parameter. For completeness, we have chosen $\Omega_Q=0.85$ and $\Omega_\chi=0.65$ in this plot, i.e., we have considered a closed model.

The scalar field Q results in

$$Q(a)=\overline{Q}\int_0^{a/a_0}\sqrt{\frac{1+\frac{3}{2}\beta(1+w_\chi)x^{-(1+3w_\chi)}}{x+\frac{\Omega_\chi}{\Omega_M}x^{-(1+3w_\chi)}}}dx, \tag{25}$$

where $\overline{Q}=\sqrt{1/4\pi G(|\Omega_k|/\Omega_M)}$.

The scalar potential, $V_Q(a)$, is found to be given by

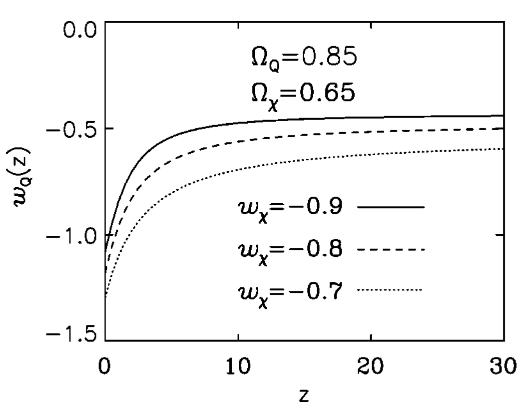

FIG. 3. The graph shows w_Q (in unit of $|w_Q^0|$) as a function of the redshift z for three different values of the equation state parameter, $w_\chi=-0.7,-0.8,-0.9$. We have used the values $\Omega_Q=0.85$ and $\Omega_\chi=0.65$, corresponding to a closed model.

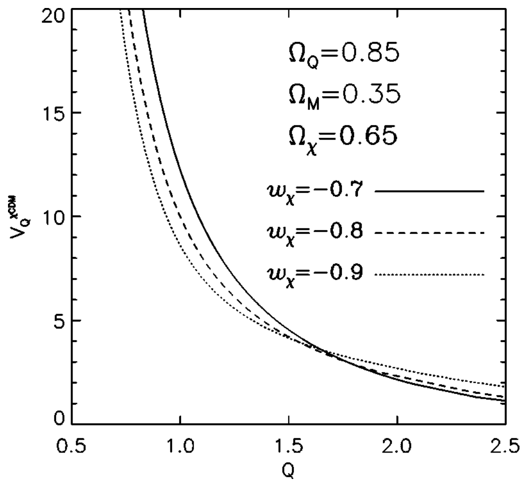

FIG. 4. The graph shows V_Q (in unit of $\rho_C/2$) as a function of Q (in unit of $1/\sqrt{4\pi G}$) for three different values $w_\chi=-0.7,-0.8$, and -0.9. We use the values $\Omega_Q=0.85$, $\Omega_\chi=0.65$, and $\Omega_M=0.35$ associa!ted with a closed model.

$$V_Q(a)=V_Q^0\left(\frac{a_0}{a}\right)^2\left[\frac{4+3\beta(1-w_\chi)\left(\frac{a}{a_0}\right)^{-(1+3w_\chi)}}{4+3\beta(1-w_\chi)}\right], \tag{26}$$

where $V_Q^0=[\rho_C\Omega_Q/2(1-w_\chi)][4/3+\beta(1-w_\chi)]$. Figure 4 shows the potential V_Q as a function of the scalar field Q for three different values of w_χ. Again, we have considered a closed model with $\Omega_Q=0.85$, $\Omega_M=0.35$, and $\Omega_\chi=0.65$. For open models, i.e., for $\Omega_\chi>\Omega_Q$, these curves are very similar.

Note that the potential decreases when Q increases. We observe that this potential asymptotically tends to vanish for $a\to\infty$. This implies that, asymptotically, the effective equation of state becomes $P_Q=\rho_Q$ (for $\dot{Q}\neq 0$), corresponding to a stiff fluid.

IV. THE QUINTESSENCE SCALAR POTENTIAL

One of the possible modifications to the basic idea of quintessence includes the use of other potentials [16–18]. It may be interesting to investigate the situation in which the scalar potential $V(\chi)$ is of the form [19]

$$V(\chi)=\alpha\dot{\chi}^2+\gamma, \tag{27}$$

where α and γ are two constants. We may write γ as $\gamma=(\alpha_--\alpha_+w_\chi)\rho_\chi^0$, where $\alpha_\pm=\frac{1}{2}\pm\alpha$.

Combining Eq. (27) with the evolution equation for the field χ, we get $\dot{\chi}$ as a function of the scale factor a, and thus we get

$$\rho_\chi(a)=\rho_\chi^0\left[\alpha_+\left(\frac{a_0}{a}\right)^{3/\alpha_+}+\alpha_-\right]+\alpha_+P_\chi^0\left[\left(\frac{a_0}{a}\right)^{3/\alpha_+}-1\right] \tag{28}$$

and

$$P_\chi(a)=P_\chi^0\left[\alpha_-\left(\frac{a_0}{a}\right)^{3/\alpha_+}+\alpha_+\right]+\alpha_-\rho_\chi^0\left[\left(\frac{a_0}{a}\right)^{3/\alpha_+}-1\right], \tag{29}$$

where the constants ρ_χ^0 and P_χ^0 represent the actual energy density and pressure, respectively, and are related to an arbitrary integration constant. From these two latter relations, we get

$$w_\chi(a)=\frac{w_\chi\left[\alpha_-\left(\frac{a_0}{a}\right)^{3/\alpha_+}+\alpha_+\right]+\alpha_-\left[\left(\frac{a_0}{a}\right)^{3/\alpha_+}-1\right]}{\left[\alpha_+\left(\frac{a_0}{a}\right)^{3/\alpha_+}+\alpha_-\right]+w_\chi\alpha_+\left[\left(\frac{a_0}{a}\right)^{3/\alpha_+}-1\right]}, \tag{30}$$

where, just as before, we take $w_\chi=P_\chi^0/\rho_\chi^0$. Note that for $w_\chi=\alpha_-/\alpha_+$ we get $w_\chi(a)=w_\chi=\text{const}$ This case corresponds to using $\gamma=0$ in Eq. (27). As we will soon see, this case allows us to write down an explicit expression for the quintessence scalar potential, $V(\chi)$. Throughout this paper, we will address this case only.

We could solve the time-time Einstein equation (13) to obtain in this case

$$t=\frac{1}{H_0}\frac{2}{\sqrt{\Omega_M}}\left(\frac{a}{a_0}\right)^{1/2}{}_2F_1\left(\frac{1}{2},\frac{\alpha_+}{\alpha_-};\frac{5}{6};-\frac{\Omega_\chi}{\Omega_M}\left(\frac{a}{a_0}\right)^{3\alpha_-/\alpha_+}\right). \tag{31}$$

The quintessence scalar field χ in terms of the scale factor is given by

$$\chi(a)=\chi_0+\varepsilon\,\text{arcsinh}\left[\delta\left(\frac{a}{a_0}\sqrt{1+\delta^2}-\sqrt{1+\delta^2\left(\frac{a}{a_0}\right)^2}\right)\right], \tag{32}$$

where

$$\varepsilon=\frac{2}{3H_0|w_\chi|}\sqrt{\frac{2V_\chi^0}{\Omega_\chi}\frac{1}{\alpha_-+\alpha_+}}\quad\text{and}\quad\delta=\left(\frac{\Omega_\chi}{\Omega_M}\right)^{1/3|w_\chi|}.$$

Equation (32) allows us to write down an explicit expression for the dark energy scalar potential $V(\chi)$. We get

$$V(\chi)=V_\chi^0\left\{\frac{\delta}{\sinh[\varepsilon^{-1}(\chi-\chi_0)+\text{arcsinh}(\delta)]}\right\}^{3/\alpha_+}, \tag{33}$$

where V_χ^0 represents the actual value of this potential. The hyperbolic potential (33) has been used in the literature [19–21]. This potential was studied for getting a tracker solution from the corresponding field equations.

One of the characteristics of the scalar field Q is given by the equation of state parameter $w_Q(a)$, which is given by

$$w_Q(a) = \frac{(3w_Q^0+1)(w_\chi+1)\alpha_-\left(\frac{a_0}{a}\right)^{3/\alpha_+}+(w_Q^0-w_\chi)\left(\frac{a_0}{a}\right)^2}{(3w_Q^0+1)(w_\chi+1)\alpha_-\left(\frac{a_0}{a}\right)^{3/\alpha_+}-(w_Q^0+w_\chi)\left(\frac{a_0}{a}\right)^2}. \tag{34}$$

Here, the quantities w_Q^0 are defined by

$$w_Q^0=\frac{w_\chi\Omega_\chi+\Omega_k/3}{\Omega_\chi-\Omega_k}=\left(w_\chi+\frac{1}{3}\right)\frac{\Omega_\chi}{\Omega_Q}-\frac{1}{3}, \tag{35}$$

where we have used the relation $\Omega_k+\Omega_Q=\Omega_\chi$ in the latter expression.

With $x\equiv a/a_0$, the scalar potential associated to the Q field is given by

$$V_Q(x)=\rho_Q^0\left[\left(\frac{\alpha_++\alpha_-}{2}\right)(1+w_\chi)x^{-3/\alpha_+}-\frac{2}{3}\frac{\Omega_k}{\Omega_M}x^{-2}\right], \tag{36}$$

and the corresponding scalar field Q is expressed by means of the following integral:

$$Q(x)=\frac{\sqrt{\rho_Q^0}}{H_0}\int_x^1\frac{dz}{z}\times\sqrt{\frac{(1+w_\chi)\frac{\Omega_\chi}{\Omega_Q}z^{-3\alpha_-/\alpha_+}-\frac{2}{3}\frac{\Omega_k}{\Omega_Q}z}{1+(1+w_\chi)\frac{\Omega_\chi}{\Omega_M}\alpha_+z^{-3\alpha_-/\alpha_+}}}. \tag{37}$$

A numerical integration shows that this potential presents the same characteristic as that described in the previous case, i.e., $V(Q)$ decreases when Q increases, reaching the limit $V(Q)\to 0$ for $Q\to\infty$.

Finally, one interesting parameter to determine in this kind of model is the deceleration parameter, which is defined by $q=-\ddot{a}/\dot{a}H^2$, and when evaluated to present time it becomes

$$q_0=\frac{1}{2}\Omega_M-\frac{1}{2}(3|w_\chi|-1)(\Omega_k+\Omega_Q). \tag{38}$$

For $w_\chi<-1/3(1+\Omega_M/\Omega_\chi)$, the parameter q_0 becomes negative, in agreement with the observed acceleration of the universe.

V. CONCLUSIONS

In this paper we have described a curvature universe model in which, apart from the usual CDM component, we have included a quintessencelike scalar field Q. We have fine-tuned the energy density, associated with this field, together with the curvature, for mimicking a flat model which resembles the quintessence (or dark energy) χCDM model, which is characterized by $\Omega_T=\Omega_M+\Omega_\chi=1$. We have assumed for Q an effective equation of state $P_Q=w_Q\rho_Q$, with $w_Q<0$. We have determined the form of the potential associated with this field, which effectively has the property of a quintessence potential, i.e., $V(Q)$ decreases when Q increases, approaching zero asymptotically. Under the assumption that the equation of state parameter was constant, we could describe explicitly the characteristics of the dark energy scalar field, χ. Certainly, the basic idea of quintessence rests on the determination of the scalar potential as a function of this field.

Under an appropriate choice for the dark energy scalar potential, $V(\chi)$ (in terms of $\dot{\chi}$), we determine an explicit expression for this potential as a function of χ. In this case, we also gave an expression for the scalar field Q and its potential. Here, we determine the deceleration parameter which, under an appropriate choice of the parameters that characterize the model, became positive, in agreement with the acceleration detected by astronomical observations.

ACKNOWLEDGMENTS

I am grateful to Francisco Vera for plotting assistance. Discussions with M. Cataldo, N. Cruz, S. Lepe, F. Peña, and P. Salgado are acknowledged. I also acknowledge the Universidad de Concepción and Universidad del Bio-Bio for partial support of the Dichato Cosmological Meeting, where part of this work was done. This work was supported by Comision Nacional de Ciencias y Technologia through Fondecyt Projects No. 1000305 and 1010485 grants, and from UCV-DGIP No. 123.752.

[1] S. White *et al.*, Nature (London) **366**, 429 (1993).
[2] J. Mather *et al.*, Astrophys. J. Lett. **420**, L439 (1994).
[3] M. Roos and S. Harun-or-Rashid, "The Flatness of the Universe is Robust," astro-ph/0005541.
[4] P. De Bernardis *et al.*, Nature (London) **404**, 955 (2000).
[5] S. Hanany *et al.*, Astrophys. J. Lett. **545**, L5 (2000).
[6] A. Guth, Phys. Rev. D **23**, 347 (1981).
[7] S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998).
[8] P.M. Garnavich *et al.*, Astrophys. J. **509**, 74 (1998).
[9] R.R. Caldwell, R. Dave, and P. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[10] E.W. Kolb, Astrophys. J. **344**, 543 (1989).
[11] M. Kamionkowski and N. Toumbas, Phys. Rev. Lett. **77**, 587 (1996).
[12] N. Cruz, S. del Campo, and R. Herrera, Phys. Rev. D **58**, 123504 (1998); M. Cataldo and S. del Campo, *ibid.* **62**, 023501 (2000); S. del Campo and N. Cruz, Mon. Not. R. Astron. Soc. **317**, 825 (2000).

[13] L. Wang, R.R. Caldwell, J.P. Ostriker, and P.J. Steinhardt, Astrophys. J. **530**, 17 (2000).

[14] T.D. Saini, S. Raychaudhury, V. Sahni, and A. Starobinsky, Phys. Rev. Lett. **85**, 1162 (2000).

[15] M.S. Turner, ‘‘Time at the Beginning,’’ astro-ph/0106262.

[16] A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000).

[17] P. Brax and J. Martin, Phys. Lett. B **468**, 40 (1999).

[18] I. Zlatev and P.J. Steinhardt, Phys. Lett. B **459**, 570 (1999).

[19] Y. Gong, Class. Quantum Grav. **19**, 4537 (2002).

[20] T. Matos and L. Ureña-López, Phys. Rev. D **63**, 063506 (2001).

[21] V.B. Johri, ‘‘Search for Tracker Potential in Quintessence Theory,’’ astro-ph/0108247.