Degeneracy inherent in the observational determination of the dark energy equation of state

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Using a specific model for the expansion rate of the Universe as a function of scale factor, it is demonstrated that the equation of state of the dark energy cannot be determined uniquely from observations at redshifts $z \lesssim$ a few unless the fraction of the mass density of the Universe in nonrelativistic particles, Ω_M, somehow can be found independently. A phenomenological model is employed to discuss the utility of additional constraints from the formation of large scale structure and the positions of cosmic microwave background peaks in breaking the degeneracy among models for the dark energy.

I. INTRODUCTION

Although observations of acoustic peaks in the cosmic microwave background (CMB) fluctuation spectra are sensitive to the curvature of spacetime, and appear to require some form of cosmological dark energy [1–6], they may be less useful for discriminating among different equations of state for the dark energy. Various studies (e.g. [7,8]) have concluded that the signatures of different types of spatially smooth, evolving dark energy on the CMB spectra are relatively undiscriminating, in part because CMB fluctuations were formed at high redshift, when the dark energy was not a prominent constituent, although spatial fluctuations in the dark energy could lift the degeneracy considerably [9]. By contrast, observations of sources at small to moderate redshifts ($z \lesssim$ a few) would probe epochs where the dark energy is prominent, leading to suggestions that observations of type Ia supernovae [10–16] or of galaxy counts [17–19] could be used to determine the nature of the dark energy most effectively. However, the efficacy of these observational programs is controversial, as there is considerable degeneracy among the predictions of different dark energy models for, for example, the luminosity distance as a function of redshift [20].

One approach to the analysis of low redshift data would be to presume particular classes of models, perhaps parametrized by a series expansion of $w(z) = p(z)/\rho(z)$, and then attempt to constrain the model parameters by a likelihood or Bayesian method (e.g. [10,11,20,21]). A second approach is to fit the data by a parametric representation of, for example, the luminosity distance as a function of redshift, and then analyze the results to constrain the properties of the dark energy indirectly (e.g. [12–16]). Although the first approach is preferable (because it generally avoids difficulties associated with differentiating data, and allows a simpler assessment of uncertainties in derived parameters, in addition to outlining its implicit assumptions more clearly), the second approach is more useful for understanding whether or not such observations can ever yield a unique solution for the dark energy equation of state.

In this paper note, the following question is considered: Suppose one could analyze data from $z \lesssim$ a few to determine the expansion rate of the Universe as a function of scale factor, $H(a)$, *exactly*. Could one then determine the equation of state of the dark energy exactly from these measurements alone? The answer, as we shall see, is *no*: a *separate* determination of the fraction of the Universe in the form of nonrelativistic particles is needed [22].

II. A SIMPLE ILLUSTRATIVE MODEL

The necessity of separately determining the density of nonrelativistic particles in order to deduce the equation of state of the dark energy can be demonstrated most easily by constructing a specific example. Suppose the data show that the Universe is expanding according to the simple relationship

$$H^2 = H_0^2 \left(\Omega_1 + \frac{\Omega_2}{a^3} \right), \tag{1}$$

where $a \equiv (1+z)^{-1}$. One (tempting) interpretation of this result would be that the Universe consists of two components, nonrelativistic matter contributing a fraction Ω_2 of the closure density, and a cosmological constant contributing a fraction $\Omega_1 = 1 - \Omega_2$. However, this interpretation is not unique: there are models involving a scalar field ϕ with nonconstant effective potentials $V(\phi)$ that can lead to Eq. (1).

To see this, we can construct an explicit model with both a scalar field and nonrelativistic matter, so that

$$H^2 = \frac{8\pi G \rho_\phi}{3} + \frac{H_0^2 \Omega_M}{a^3} \tag{2}$$

with $\Omega_M \neq \Omega_2$ in general. The scalar field energy density is therefore

$$\rho_\phi = \rho_0 \left(\Omega_1 + \frac{\Omega_2 - \Omega_M}{a^3} \right), \tag{3}$$

where $\rho_0 = 3H_0^2/8\pi G$. Differentiate ρ_ϕ with respect to a to find

$$\frac{d\rho_\phi}{da} = -\frac{3\rho_0(\Omega_2 - \Omega_M)}{a^4} = -\frac{3(\rho_\phi + P_\phi)}{a} \tag{4}$$

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where the second equality follows from conservation of energy for the scalar field. Thus, we find simply that $P_\phi = -\rho_0 \Omega_1$, and

$$\dot\phi^2 = \rho_\phi + P_\phi = \frac{\rho_0(\Omega_2 - \Omega_M)}{a^3}$$

$$V(\phi) = \frac{\rho_\phi - P_\phi}{2} = \rho_0\left(\Omega_1 + \frac{(\Omega_2 - \Omega_M)}{2a^3}\right). \quad (5)$$

Note that the first of Eqs. (5) requires that $\Omega_2 \geq \Omega_M$. We can determine $V(\phi)$ by solving the first of Eqs. (5) for $\phi(a)$, inverting to find $a(\phi)$, and substituting into the second of Eqs. (5). Combining Eqs. (1) and the first of Eqs. (5) implies

$$a^2\left(\frac{d\phi}{da}\right)^2 = \frac{\dot\phi^2}{H^2} = \frac{3(\Omega_2 - \Omega_M)}{8\pi G \Omega_2(1 + \Omega_1 a^3/\Omega_2)}. \quad (6)$$

Define

$$x = \frac{\Omega_1 a^3}{\Omega_2} \quad \text{and} \quad \phi_g^2 = \frac{\Omega_2 - \Omega_M}{24\pi G \Omega_2}; \quad (7)$$

then if $\psi \equiv \phi/\phi_g$,

$$x^2\left(\frac{d\psi}{dx}\right)^2 = \frac{1}{1+x}, \quad (8)$$

which has the general solution

$$\psi = \psi_\infty \pm 2\sinh^{-1}\left(\frac{1}{\sqrt{x}}\right), \quad (9)$$

where ψ_∞ is a constant of integration. The sign in this solution can be chosen arbitrarily; choosing the $-$ sign gives a solution in which ψ increases with increasing a. If we let $\psi = \psi_0$ today, then we find

$$\psi = \psi_0 + 2\sinh^{-1}\sqrt{\frac{\Omega_2}{\Omega_1}} - 2\sinh^{-1}\sqrt{\frac{\Omega_2}{\Omega_1 a^3}}, \quad (10)$$

which we may invert to find

$$\frac{1}{a^3} = \left[\cosh\left(\frac{\hat\psi}{2}\right) - \sqrt{\frac{1}{\Omega_2}}\sinh\left(\frac{\hat\psi}{2}\right)\right]^2, \quad (11)$$

where $\hat\psi \equiv \psi - \psi_0$. The second of Eqs. (5) then determines the potential to be [23]

$$V(\phi) = \rho_0\left\{\Omega_1 + \frac{1}{2}(\Omega_2 - \Omega_M)\right.$$

$$\left. \times\left[\cosh\left(\frac{\hat\psi}{2}\right) - \sqrt{\frac{1}{\Omega_2}}\sinh\left(\frac{\hat\psi}{2}\right)\right]^2\right\}. \quad (12)$$

Note that $\hat\psi \to 2\coth^{-1}\sqrt{1/\Omega_2}$ and $V(\phi) \to \rho_0\Omega_1$ asymptotically. Also, when $\Omega_2 = 1 - \Omega_1 \to 1$, the potential becomes exponential: $V(\phi) \to \frac{1}{2}\rho_0(1 - \Omega_M)e^{-\hat\psi}$.

The key question here revolves around prior assumptions. The "simplest" interpretation of Eq. (1) is that the energy density of the Universe is the sum of contributions due to nonrelativistic matter and a cosmological constant. However natural this interpetation may seem to be, it only follows if one assumes *either* that the dark energy is in the form of a cosmological constant, or that the mass density in nonrelativistic form has closure parameter Ω_2. If one is somewhat more agnostic on either of these points, then a range of possible interpretations accounts for the data equally well. The degeneracy is not just a property of models for which $H^2(a)$ is given by Eq. (1). If the data were to favor a model of the form

$$H^2(a) = H_0^2\left[\Omega_1 g(a) + \frac{\Omega_2}{a^3}\right], \quad (13)$$

with $g(a)$ some function, and $\Omega_1 g(1) + \Omega_2 = 1$, then without additional prior assumptions the most one can say for sure is that $\rho_\phi = \rho_0[\Omega_1 g(a) + (\Omega_2 - \Omega_M)/a^3]$, which leaves open the possibility of a range of different solutions for $V(\phi)$.

This model illustrates that if one makes no prior assumptions about the form of the quintessence potential $V(\phi)$, then it is impossible to fit low redshift data alone to find $V(\phi)$ uniquely unless Ω_M is known independently—in this example, a range of models, parametrized by the value of $f \equiv \Omega_M/\Omega_2$, will fit equally well. Constraints on the value of f limit the range of acceptable models. Although minimally $f \leq 1$, we know that this model would be unable to account for the formation of large scale structure if f is too small. If we assume, to get a rough estimate of the allowed values of f, that only the nonrelativistic matter can clump (i.e. that the dark energy and relativistic matter remain smooth), then, in linear theory, the growing mode of dark matter energy perturbations, $D_+(a)$, mainly amplifies perturbations between $a_{eq} = \Omega_{rad}/\Omega_2$ and $a_1 = (\Omega_2/\Omega_1)^{1/3}$. During this phase, $H^2 \propto a^{-3}$, and $D_+(a) \propto a^{\sigma_+}$, where (e.g. [24]) $\sigma_+(f) = \frac{1}{4}(\sqrt{1+24f} - 1)$. The overall linear theory growth factor is $\simeq (a_1/a_{eq})^{\sigma_+(f)}$, so the ratio of the growth factor for $f \neq 1$ to its value for $f = 1$ is approximately $(a_1/a_{eq})^{1/4(\sqrt{1+24f}-5)} \approx (a_1/a_{eq})^{-3(1-f)/5}$, where the approximation assumes $1 - f \ll 1$. Thus, we estimate that for the growth factor to be within a factor of 2 of its value for $f = 1$, we must have $1 - f \lesssim 0.12$. More detailed calculations that include the gravitational effects on the quintessence field and radiation (e.g. [25,26]) show rigorously that the growth of large scale structure from small perturbations in the early Universe depends on $1 - f$ sensitively, as we have estimated more crudely here.

Thus, primarily from the requirement that linear density perturbations can grow substantially from small values in the early Universe, we know that if Eq. (1) were truly exact, f must be close to 1. Imposing limits on the range of plausible Ω_M from independent, physical and phenomenological arguments amounts to using prior information to restrict the pos-

sibilities to be compared with the data from observations at $z \lesssim$ a few. (The use of prior information from large scale structure and CMB anisotropies has been discussed in e.g. [14,27–29]; see also [30] for a discussion of the prospects for measuring the quintessence potential using several different approaches.) This does not eliminate the degeneracy implicit in Eq. (1) altogether, but it does diminish its significance by constraining the range of acceptable values of f severely. However, the fact remains that if one were to try to determine the equation of state of the dark energy from observations that, hypothetically, yield Eq. (1), then one could never hope to find a unique result from an analysis that ignores constraints from the development of large scale structure (or any other considerations that yield independent information about Ω_M, such as peculiar velocities, CMB fluctuations and weak lensing). Moreover, even if such restrictions are imposed, unless Ω_M is determined *precisely* by them, some degeneracy remains in the construction of $V(\phi)$ from observations at $z \lesssim$ a few.

III. CAN Ω_M AND Ω_2 DIFFER SUBSTANTIALLY?

If Eq. (1) with Ω_2 substantially different from Ω_M is an excellent approximation at low to moderate redshifts, it can *never* be exact. In this case, there must be a transition in $H^2(a)$, so that the component of the density that scales like a^{-3} is predominantly due to nonrelativistic particles for a period sufficient to grow large scale structure. The implicit deviations from Eq. (1) would contain information on Ω_M as well as Ω_2, and probably additional parameters as well. In this situation, it might be possible to ascertain the equation of state of the dark energy from measurements at low to moderate redshifts alone, provided that the imprint of Ω_M and the additional parameters in $H^2(a)$ can be discerned from these

observations. The question is, if Eq. (1) is an accurate (but inexact) fit at $z \lesssim$ a few, and Ω_2 differs substantially from Ω_M, how well could we determine Ω_M, and hence break the degeneracy, observationally?

To make these issues more concrete, suppose in reality Eq. (1) is actually the large a limit of a more general, exact relationship, say

$$H^2(a) = H_>^2(a) F\left(\frac{a}{a_t}\right) + H_<^2(a)\left[1 - F\left(\frac{a}{a_t}\right)\right], \quad (14)$$

where $F(q) \to 0$ as $q \to 0$ and $F(q) \to 1$ as $q \to \infty$, with $a_t < 1$ a transition value of the scale factor, $H_>^2(a)$ given by Eq. (1), and, for example,

$$H_<^2(a) = H_0^2\left(\Omega_< + \frac{\Omega_M}{a^3}\right); \quad (15)$$

since the model is flat,

$$1 = (\Omega_1 + \Omega_2) F_0 + (\Omega_< + \Omega_M)(1 - F_0), \quad (16)$$

where $F_0 \equiv F(1/a_t)$. Such a model could be consistent with the growth of large scale structure, but might still be hard to distinguish from Eq. (1) based on observations at low to moderate redshift, depending on the value of a_t, and the form of $F(a/a_t)$. Equation (14), although admittedly ad hoc, is useful for studying the extent to which observations of various sorts could discern that $\Omega_2 \neq \Omega_M$, and pin down the difference sufficiently to break the degeneracy discussed above.

For a flat cosmology, Eq. (14) implies that the radial coordinate for a source at $a = (1+z)^{-1}$ is

$$r_S(a; \Omega_M, \Omega_2, F) = \eta(1) - \eta(a) = \int_a^1 \frac{db}{b^2 H(b)}$$

$$= H_0^{-1} \int_a^1 \frac{db}{\sqrt{b\{\Omega_e b^3 + 1 - \Omega_e + [(\Omega_< - \Omega_1) b^3 + \Omega_M - \Omega_2][F_0 - F(b/a_t)]\}}} \quad (17)$$

where $\Omega_e \equiv \Omega_1 F_0 + \Omega_<(1 - F_0)$, and $d\eta \equiv dt/a = da/a^2 H(a)$. [Here, the notation $r_S(a; \Omega_M, \Omega_2, F)$ means the radial coordinate as a function of a given Ω_M and $\Omega_2 \geqslant \Omega_M$, as well as a set of parameters required to define $F(a/a_t)$.] If $F(a/a_t)$ does not vary much over the range of a covered by the observations, then the data would only determine Ω_e accurately, and would yield little useful information on Ω_2 or Ω_M: only the combination $\Omega_2 F_0 + \Omega_M(1 - F_0) = 1 - \Omega_e$ could be deduced. This situation is possible provided that a_t is fairly small, below the range covered by observations, and $F(a/a_t)$ varies relatively slowly as a function of a/a_t for large values of a/a_t. In this case, we cannot expect to eliminate the degeneracy among models directly from observations of low to moderate redshift sources alone. Constraints from other sorts of observations that probe Ω_M directly would be needed to deduce the quintessence model as accurately as possible.

More quantitative statements only can be made in terms of specific $F(a/a_t)$, and require some exploration of parameters. Let us examine one choice: $\Omega_1 + \Omega_2 = 1 = \Omega_< + \Omega_M$, in which case Eq. (16) is satisfied exactly, and Eq. (17) becomes

$$r_S(a; \Omega_2, \Omega_M, F) = H_0^{-1} \int_a^1 \frac{db}{\sqrt{b\{\Omega_2 + (1 - \Omega_2) b^3 - (\Omega_2 - \Omega_M)(1 - b^3)[1 - F(b/a_t)]\}}}. \quad (18)$$

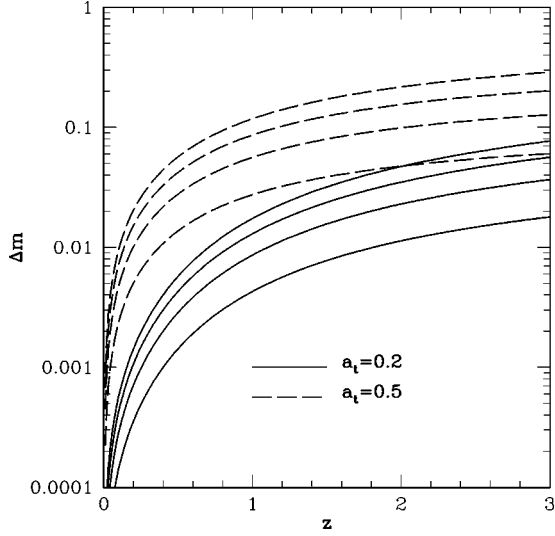FIG. 1. Magnitude differences among models with $\Omega_2=0.5$ for $a_t=0.2$ (solid) and $a_t=0.5$ (dashed) as functions of z for (from the top for each a_t) $\Omega_M=0.1,0.2,0.3$ and 0.4.

Equation (18) may be used to compute magnitude differences among models with given Ω_2, Ω_M and $F(a/a_t)$; for illustrative purposes, let us choose $1-F(a/a_t)=e^{-a/a_t}$. Figure 1 depicts the magnitude difference

$$\Delta m(z;\Omega_2,\Omega_M,a_t)$$

$$=5.0\log_{10}\left[\frac{r_S((1+z)^{-1};\Omega_M,\Omega_2,a_t)}{r_S((1+z)^{-1};\Omega_2,\Omega_2,a_t)}\right] \quad (19)$$

relative to the model with $\Omega_M=\Omega_2=0.5$ as a function of z assuming $a_t=0.2$ (solid) and $a_t=0.5$ (dashed) for (from top to bottom in each case) $\Omega_M=0.1,0.2,0.3$ and 0.4. Not surprisingly, the magnitude differences are smaller for the larger value of a_t, and decrease as $\Omega_M\to\Omega_2$. Nevertheless, the *largest* magnitude difference shown for $z\lesssim3$ is 0.29 mag, for $(z,a_t,\Omega_M)=(3,0.5,0.1)$; for $(z,a_t,\Omega_M)=(3,0.2,0.1)$, the difference is 0.077 mag.

For comparison, the magnitude differences between flat cosmologies with a cosmological constant and $\Omega_M=0.2$ and $\Omega_M=0.3$ are $\Delta m=0.14$, 0.21 and 0.25 at $z=1$, 2 and 3, respectively; the corresponding magnitude differences between $\Omega_M=0.4$ and $\Omega_M=0.3$ are $\Delta m=-0.12$, -0.17 and -0.19. These values of Ω_M span the $\approx1\sigma$ ranges reported in Ref. [31] for data on type Ia supernovae, which are generally at $z\lesssim1$, presuming flat cosmology. Thus, it would be hard to tell if the Universe is actually a "kinky" cosmology like Eq. (14) from existing type Ia samples.

Figure 2 shows results for

$$\frac{\Delta\theta}{\theta}(\Omega_M;\Omega_2,a_t)$$

$$=\left|\frac{\eta(10^{-3};\Omega_M,\Omega_2,a_t)r_S(10^{-3};\Omega_2,\Omega_2,a_t)}{r_S(10^{-3};\Omega_M,\Omega_2,a_t)\eta(10^{-3};\Omega_2,\Omega_2,a_t)}-1\right|,$$
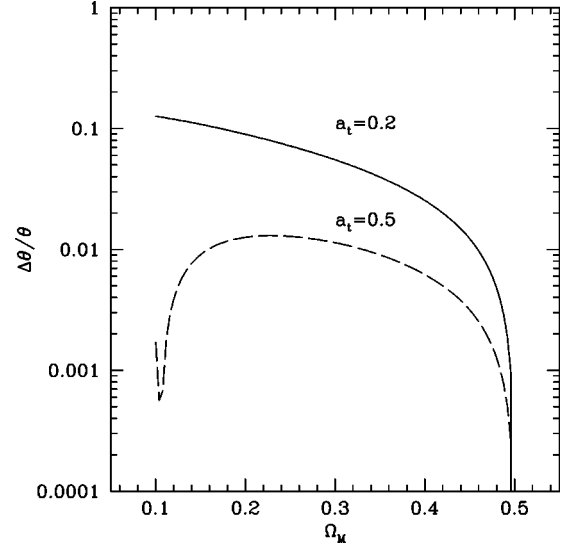
$$(20)$$

FIG. 2. Fractional deviations of the angular size of the (sonic) horizon at $a=10^{-3}$ from the $\Omega_M=\Omega_2=0.5$ model as functions of $\Omega_M\lesssim\Omega_2$. The solid line is for $a_t=0.2$, and the dashed line for $a_t=0.5$. For computing the angular sizes, $H_0=70$ km s^{-1} Mpc^{-1} was adopted.

the fractional difference between the angular size of the (sonic) horizon at $a=10^{-3}$ as a function of $\Omega_M\lesssim\Omega_2$ and its value for $\Omega_M=\Omega_2$ for the two different values of a_t. (Deviations of the sound speed from $1/\sqrt{3}$ have not been included.) Larger fractional deviations arise for smaller Ω_M and a_t. The largest deviation shown in the two figures is 12.7% for $(\Omega_M,a_t)=(0.1,0.2)$; the largest fractional deviation for $a_t=0.5$ is about 1.3%, and occurs near $\Omega_M=0.225$.

Although this is only one example, based on a particular (ad hoc) choice of $F(a/a_t)$, it illustrates that while data at both low to moderate z as well as CMB observations can yield information that lifts the degeneracy between Ω_2 and Ω_M, considerable precision may be needed to determine the equation of state of the dark energy accurately. Note that no attempt has been made to fit simulated data based on Eq. (20) to figure out how well $H(a)$ could be determined in practice; the comparisons made in Figs. 1 and 2 are based on exact functions. The observational challenge of distinguishing among models at these levels could be even greater than these relatively small differences would indicate. On the other hand, Figs. 1 and 2 were constructed based on a particular (ad hoc) model for $H^2(a)$ designed for a phenomenological study of how easily information on Ω_M could be gleaned from luminosity distances and CMB acoustic peaks. The task could be easier or harder in cosmologies based on particular physical theories for the dark energy, depending on the details of the model.

IV. DISCUSSION

To conclude, the point of this paper is *not* to suggest that it is absolutely inconceivable that the equation of state of the

dark energy could be measured, but rather to show, via a specific example, how measurements at $z \lesssim 3$ admit a continuum of interpretations in terms of evolving dark energy fields *unless* Ω_M can be determined separately somehow. The specific example, Eq. (1), can be intepreted ''naturally'' in terms of a Universe containing a mixture of nonrelativistic particle dark matter plus a cosmological constant, but can be interpreted equally well in terms of a quintessence model with a range of potentials, Eq. (12). This specific example sheds some light on why simulated analyses of low to medium redshift data appear to allow degenerate interpretations: it is always possible that the component of the total energy density of the Universe that evolves like a^{-3} is only partially due to nonrelativistic particles, with the rest arising from quintessence. Unique interpretations can only be obtained in terms of specific models for the quintessence that forbid such a conspiracy *a priori*, by either specifying the form of $V(\phi)$ or fixing the value of Ω_M.

Realistically, the extent to which the $\rho \propto a^{-3}$ constituent must be due to nonrelativistic particles is constrained by the requirement that large scale structure formation evolve ''normally,'' that is, unimpeded by the existence of a component with density proportional to a^{-3} that is incapable of clustering. If Eq. (1) were truly exact, then Ω_2 would have to be close to Ω_M for large scale structure to grow, but even in this case, a limited degeneracy remains in the determination of $V(\phi)$. If Ω_2 and Ω_M differ substantially, then Eq. (1) cannot be exact, raising the question of how well one could discern the two parameters separately from observations. To examine this issue, a modified (phenomenological) expansion law, Eq. (14), that encodes information on both Ω_2 and Ω_M was introduced. In the context of a particular model that is consistent with large scale structure formation, but reduces to Eq. (1) with $\Omega_2 \gtrsim \Omega_M$ at large enough a, we have seen that information about Ω_M could be gleaned both from measurements of the positions of CMB peaks and from luminosity distance determinations. However, the deviations among models with various $\Omega_M \lesssim \Omega_2$ can be quite small, which would still pose a substantial challenge for programs that aim to determine the equation of state of the dark energy.

Of course, in the context of *specific* models for the quintessence field, embodied in particular forms for $V(\phi)$, the analyses may not encounter pronounced degeneracies. At present, though, there is little compelling reason to assume any particular $V(\phi)$ *a priori*. When analyzing data for a particular form of $V(\phi)$, one is primarily engaged in estimating the parameters of the model, as well as Ω_M. Thus, one might be able to find the best fit model of a particular type [e.g. $V(\phi) = \text{const}$], without being able to tell if the underlying model would be favored by the data if other possibilities were admitted. If the allowed ranges of parameters for a particular quintessence model do not shrink with the accumulation of data, one can conclude that the model is inadequate with confidence. However, there is no guarantee that a given model is correct even if the data seem to converge on a unique set of parameters, as Eqs. (1) and (12) illustrate.

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