Astrophysical detection of heavy-particle-induced spectral shifts in muonic iron

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By significantly increasing the nuclear mass, a strongly interacting massive particle (SIMP) bound to an iron nucleus would cause a characteristic change in the spectrum of muonic iron. At temperatures high enough that such atoms are completely stripped of electrons, the effect is directly observable as a 0.2% shift in the energies of high angular momentum states. This phenomenon provides a new test for the existence of SIMPs, which have been proposed as dark matter candidates, and as candidates for the lightest supersymmetric particle.

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I. INTRODUCTION

Various theoretical arguments have raised the possibility that strongly interacting massive particles (SIMPs) exist, and might thus constitute a heretofore unknown class of elementary particles [1–14]. SIMPs would be electrically neutral and could be detected in the laboratory through their interactions with ordinary matter. Specifically, SIMPs could bind to the nuclei of atoms, and would manifest themselves as anomalously heavy isotopes of known elements. Searches for SIMPs bound to light nuclei have been reported in Refs. [15,16], and in Refs. [17,18] limits were set on the abundances of anomalous isotopes in gold with masses up to 1.67 TeV/c^2, and for iron isotopes with masses up to 0.65 TeV/c^2. These experimental results were then used in Ref. [19] to infer limits on the SIMP contribution Ω_S to the cosmological density parameter Ω.

Although it would be clearly desirable to set experimental limits on SIMPs with masses substantially greater than ≈ 2 TeV/c^2, this may be difficult to accomplish in laboratory experiments due to the limited energies that are available. In the present paper we propose an astrophysical search for anomalously heavy isotopes, which is sensitive to a complementary range of SIMP masses ($M_S \gtrsim 100$ GeV/c^2). This proposal exploits the effect of increased nuclear mass, arising from the binding of a SIMP to an iron nucleus, on the binding energy of muonic-iron (μ-Fe) atoms. By greatly increasing the nuclear mass, the presence of a SIMP in the nucleus effectively eliminates the well-known reduced mass correction in a hydrogenic atom. We choose to study muonic atoms instead of electronic atoms because the muon's larger mass ($m_\mu = 207\ m_e$) amplifies the reduced mass correction. The two essential conditions for measuring this effect are (i) that the muonic atom is completely stripped of electrons and (ii) that we measure states having high angular momentum ($\ell \geqslant 2$). Under these conditions, the presence of a SIMP in the nucleus of muonic iron causes a 0.2% shift in its binding energies, which may be experimentally detectable in astrophysical systems. Iron was chosen as a candidate for observing this effect since ionized iron is present in the interstellar medium (ISM) of many galaxies, and also in the intracluster medium (ICM) of rich galaxy clusters. The high plasma temperatures of the ICM, $T_e \sim 10^7 - 10^8$ K, and its particle density, $n_e \sim 10^{-3}$ cm^{-3}, ensures that iron ions are fully ionized under such conditions (see, e.g., [20] for a review). Moreover, supernova remnants heat the ISM producing iron ions with high ionization states.

The ICM is particularly useful for our purposes since its physical state is quite simple: this plasma is in hydrostatic equilibrium with the overall gravitational potential, and it is at an almost constant temperature. Iron L and K_α-K_β lines are widely observed with high precision in the spectra of galaxy clusters using the available capabilities on board the XMM and Chandra x-ray satellites. The iron abundance in the ICM is in the range $(1/3-1)$ of the solar value.

Interestingly, the ICM also possesses a relatively high abundance of muons. Two main channels of muon production can be envisaged. (i) Cosmic ray protons can be accelerated at interstellar/intergalactic shocks and/or injected into the ISM/ICM by relativistic jets of micro-quasars and radio galaxies. These energetic protons can then collide with the thermal protons in the ISM/ICM producing muons through $pp \to \pi^{0,\pm} + X$, followed by $\pi^\pm \to \mu^\pm \nu_\mu(\overline{\nu_\mu})$ (see, e.g., [21]). (ii) Dark matter (assumed to be neutralino χ) annihilation in the central regions of galaxies and galaxy clusters would create muons through $\chi\chi \to \pi^{0,\pm} + X$, followed again by $\pi^\pm \to \mu^\pm \nu_\mu(\overline{\nu_\mu})$ (see, e.g., [22]). Muon production in the ICM via the previous mechanisms is a stationary process since (i) cosmic ray protons are stored in the ICM for durations comparable to the Hubble time [21], and thermal protons are in hydro- and virial-equilibrium with the galaxy cluster potential, and (ii) dark matter, which provides more than 80% of the total galaxy cluster mass, can annihilate efficiently in their cores yielding a continuous reservoir of muons. We can be certain that muon production in the

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ISM/ICM is occurring, since it is a necessary intermediate step in the production of relativistic electrons [$\mu^{\pm} \rightarrow e^{\pm} + (\overline{\nu_{\mu}})\nu_{\mu} + \nu_e(\overline{\nu_e})$], which are required to explain the diffuse radio emission observed in many galaxy clusters (see, e.g., [22] and references therein).

We conclude that the two essential conditions (highly ionized iron and a muon-rich medium) needed to measure the SIMP-induced spectral shift are fulfilled in the astrophysical environments of galaxies and galaxy clusters. In what follows we first discuss in detail the effect of increased nuclear mass on the binding energy of muonic-iron. We then explain the conditions under which this mass shift is the dominant effect in an anomalous isotope. Finally we consider the possibility of observing the SIMP-induced shift in an astrophysical environment.

II. THE REDUCED-MASS SHIFT

The bound state energies of a hydrogenic atom (with point nucleus) are proportional to the reduced mass, given by

$$\frac{m_e M}{M+m_e} \cong m_e\left(1-\frac{m_e}{M}\right) \tag{1}$$

where m_e is the electron mass and M is the nuclear mass. Muonic atoms are formed when a negative muon, having charge $-|e|$ and mass $m_{\mu}=206.7\ m_e$ is captured in the Coulomb field of a nucleus. Muonic atoms can be treated as if they were quasi-stable because the muon cascades from its initial state to the atomic ground state over a time scale much shorter than the characteristic time for muon capture by the nucleus, $\mu^- + p \rightarrow n + \nu_{\mu}$ [23]. Since the muon is much more massive than the electron, the reduced mass correction for muonic atoms is ≈ 207 times larger than for normal atoms.

If a SIMP is captured by the nucleus of a muonic atom, the fractional change in the energy of a level n due to the change in nuclear mass is given by

$$\frac{\Delta E_n(M_S)}{E_n} = \frac{1+m_{\mu}/M}{1+m_{\mu}/(M+M_S)} - 1 \tag{2}$$

where M is the nuclear mass, M_S is the SIMP mass, and where we have assumed that M_S is much larger than the SIMP-nucleus binding energy. For small M_S, the change in energy is approximately

$$\frac{\Delta E_n(M_S)}{E_n} \cong \frac{m_{\mu}}{M^2} M_S. \tag{3}$$

For large M_S, the nucleus effectively acquires an infinite mass, eliminating the original reduced mass correction:

$$\frac{\Delta E_n(M_S)}{E_n} \cong \frac{m_{\mu}}{M}. \tag{4}$$

In muonic-iron (μ-Fe), the presence of a large-mass SIMP in the nucleus causes a fractional change of 0.002 in the binding energies (see Fig. 1). This mass shift is the effect we hope to measure.

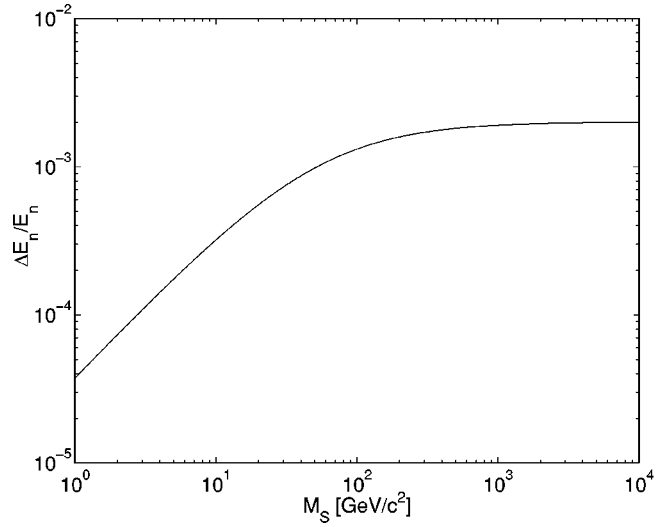

FIG. 1. Fractional change in the bound state energies of muonic iron due to the change in reduced mass arising from the addition of a SIMP of mass M_S to the nucleus. For large M_S, the result is to shift all energies by 0.2%, since the reduced mass correction to the Bohr formula vanishes as $M_S \rightarrow \infty$. Other effects, discussed in the text, may obscure the mass shift, but for $\ell \geqslant 2$ states it remains the dominant effect.

III. CORRECTIONS TO THE REDUCED MASS SHIFT

The presence of a SIMP in the nucleus leads to other corrections in addition to that arising from the reduced mass. In this section we consider three such effects: changes in the electron cloud due to increased nuclear mass, changes in the nuclear charge distribution, and quantum-electrodynamic (QED) contributions. The effect of the electron cloud is complicated, and of comparable size to the mass shift [24]. Hence we will assume that the muonic iron atom is completely stripped of electrons, as is expected to be the case in the astrophysical environments of interest to us. We can then show that for muonic states having angular momentum quantum number $\ell \geqslant 2$, the remaining corrections arising from changes in the nuclear charge distribution and from QED corrections are negligible compared to the reduced mass shift. Hence a shift at the 10^{-3} level in the spectrum arising from transitions among states with $\ell \geqslant 2$ could be interpreted as possible evidence for a SIMP bound to an $^{56}_{26}$Fe nucleus.

A. Finite size shift

In muonic iron, the energies of all nonzero angular momentum states can be well approximated by the Bohr formula

$$E_n = Z^2\left(\frac{\mu_{\mu}}{\mu_e}\right)\frac{E_{0H}}{n^2}, \tag{5}$$

where $E_{0H} = -13.6$ eV is the ground state energy of hydrogen, and μ_{μ} (μ_e) is the muonic (electronic) reduced mass. We can thus estimate the effect of the finite nuclear size by using first-order perturbation theory with hydrogenic wave functions describing the unperturbed state. In fact, Rosenthal

TABLE I. Fractional finite size shift due to an assumed 10% increase in nuclear charge radius for several low-lying states.

n	ℓ	$\delta R/R_0$	$\delta\Delta E_{n\ell}(\delta R)/E_n$
1	0	0.1	2×10^{-2}
2	0	0.1	1×10^{-2}
2	1	0.1	2×10^{-4}
3	0	0.1	1×10^{-2}
3	1	0.1	2×10^{-4}
3	2	0.1	2×10^{-7}

and Breit found that this procedure overestimates the actual shift [24], so it will serve as an upper bound. Assuming a uniform nuclear charge distribution for simplicity, the finite size energy shift found from first-order perturbation theory is given by

$$\Delta E_{n\ell}(R)=\langle n\ell m|\Delta V(R,r)|n\ell m\rangle$$
$$=4\pi\int_0^R dr r^2\Delta V(R,r)|\Psi_{n\ell}|^2. \quad (6)$$

Here

$$\Delta V(R,r)=V_{finite}-V_{Coulomb}=\frac{3}{2}\frac{Z\alpha}{R}\left[1-\frac{1}{3}\left(\frac{r}{R}\right)^2-\frac{2}{3}\left(\frac{R}{r}\right)\right] \quad (7)$$

is the difference between the Coulomb potential arising from a point charge and that due to a uniform charge distribution of radius R, and

$$|\Psi_{n\ell}|^2=\frac{1}{4\pi}\frac{2Z^3}{na_0}\left\{\frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}\right\}e^{-\rho}\rho^{2\ell}L_{n+\ell}^{2\ell+1}(\rho)^2 \quad (8)$$

is the radial probability distribution for the muonic heavy-hydrogen wave functions with $\rho=2Zr/na_0$, and $a_0=1/\alpha m_\mu$ [25]. We now define the fractional finite-size energy shift due to an increase in nuclear charge radius for comparison to the mass shift:

$$\frac{\delta\Delta E_{n\ell}(\delta R)}{E_n}=\frac{\Delta E_{n\ell}(R_0+\delta R)-\Delta E_{n\ell}(R_0)}{E_n} \quad (9)$$

where $R_0=5.4$ fm is the radius of the $^{56}_{26}$Fe nucleus and δR is the increase in the nuclear charge radius caused by the presence of a SIMP.

Making the standard approximation that the nuclear volume is proportional to the number of nucleons gives $\delta R/R_0=0.006$ for the addition of a *neutron* to an $^{56}_{26}$Fe nucleus. However, that approximation may fail for the addition of a SIMP to the nucleus, hence we invoke the weaker assumption $\delta R/R_0<0.1$. Under this assumption, the fractional change $\delta\Delta E_{n\ell}(\delta R)/E_n$ in the finite size effect for states having $\ell\neq0$ is at least an order of magnitude smaller than the mass shift (see Table I and Fig. 2). If $\ell\geq2$, the finite size effect itself, $\Delta E_{n\ell}(R_0)$, is completely negligible.

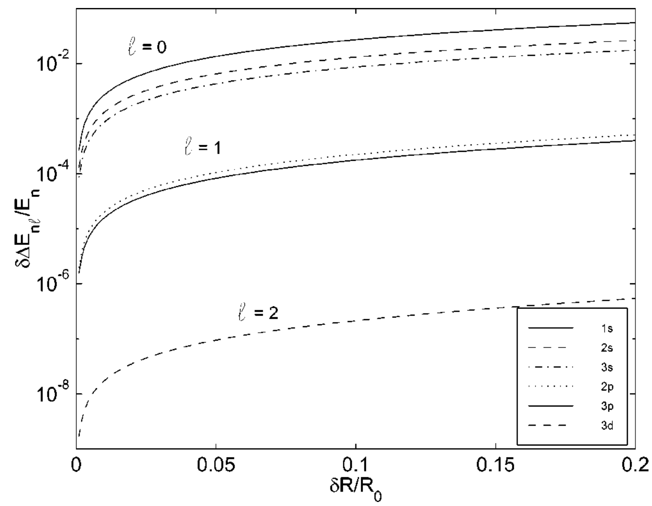

FIG. 2. Fractional change in the bound state energies of muonic iron due to a change in the radius of a uniform nuclear charge distribution (finite size shift). As noted in the text, these plots are upper limits for the effect. A SIMP bound to the nucleus would give rise to such a shift, which might obscure the mass shift in low-lying states. Although the degree to which a SIMP would affect the nuclear charge distribution is unknown, in states with angular momentum quantum number $\ell\geq2$ this effect is negligible. (See also Table I.)

B. Nuclear polarization

Another correction related to the finite size effect is nuclear polarization, which refers to effects of internal nuclear degrees of freedom expressed in first- or second-order perturbation theory [26]. Ericson and Hüfner [27], show that the energy shift in high orbital angular momentum leptonic energy levels due to nuclear effects is dominated by the polarizability shift. Thus other effects, such as strong interactions and form factors of the nuclear charges, may be neglected when analyzing transitions at high ℓ. Using the Fermi-function

$$\rho(r)=\frac{\rho_o}{1+exp[(r-c)/a]} \quad (10)$$

with $a=0.52$ and $c=(1.183A^{-1/3}-0.353)$ to describe the nuclear charge distribution, Rinker and Speth [28] calculated the nuclear polarization contributions to shifts in the muon binding energy for a number of elements in the Periodic Table. They show that for $^{56}_{26}$Fe, this correction is effectively absent for muons in the $3d_{3/2}$ (or any higher) excited states. Since this is also the case for $^{79}_{34}$Se, we can presume that any shift in the muon binding energy due to the addition of a neutral SIMP to $^{56}_{26}$Fe is negligible.

C. QED corrections

QED corrections arise from the interaction of the lepton field with the quantized electromagnetic field. Two such effects are important: modifications of muon energy levels by the emission and absorption of photons, and vacuum polarization effects arising from the modification of the photon propagator by virtual pair creation and annihilation [26]. Vacuum polarization corrections dominate over other QED corrections for muonic atoms, because of the large overlap between the nucleus and the muon wave function [29]. For a hydrogen-like atom with a point nucleus the vacuum polarization correction $\delta E_{n\ell}$ for an energy level $|n,\ell\rangle$ is given by [30]

$$\delta E_{n\ell}=\frac{-Z\alpha e^2}{15\pi m_{\mu,e}^2}\delta_{\ell,0}|\psi_{n\ell}(0)|^2=-\frac{4}{15\pi}\frac{Z^2\alpha^5}{n^3}m_{\mu,e}\delta_{\ell,0} \tag{11}$$

where $\psi_{n\ell}(0)$ is the wave function of e or μ at the origin, and $m_{\mu,e}$ denotes m_μ or m_e. It follows that for a point-like nucleus $\delta E_{n\ell}=0$ for states with $\ell\neq 0$. However, we are interested in the changes in $\delta E_{n\ell}$ when a SIMP is captured by a finite nucleus. In this case the contribution to $\delta E_{n\ell}$ no longer vanishes for $\ell\neq 0$, but it is greatly reduced. A calculation of the finite size effect on vacuum polarization was carried out by Arafune [31], and by Brown *et al.* [32]. Brown calculates a shift of -0.11 eV for the $3d_{5/2}$ state of $^{40}_{20}$Ca which corresponds to a fractional shift of

$$\frac{\Delta E_{QED}}{E_n}=\frac{0.11\ \text{eV}}{0.125\ \text{MeV}}\approx 8.8\times 10^{-7}, \tag{12}$$

where E_n is the nonrelativistic binding energy. Although the fractional shift increases with Z, it is still less than 10 eV for $n\geqslant 3$ in $^{137}_{56}$Ba. This leads to a fractional shift of

$$\frac{\Delta E_{QED}}{E_n}\approx 1.6\times 10^{-5}, \tag{13}$$

and hence any modification of the finite-size correction to the vacuum polarization is negligible in iron.

IV. CONCLUSIONS

We have shown that in muonic iron atoms, a SIMP of mass greater than ≈ 100 GeV/c^2 bound to the nucleus causes a 0.2% shift in the muon binding energy which arises from a change in its reduced mass. For states having angular momentum $\ell\geqslant 2$, interactions with the nuclear charge distribution may be neglected, and if the atom is completely stripped of electrons, the reduced mass shift is the dominant effect. This phenomenon provides a new test for the existence of SIMPs manifested as anomalous atomic isotopes. We see that the proposed astrophysical test complements laboratory searches for SIMPs by allowing limits to be inferred on SIMPs of arbitrarily large mass M_S. Whereas laboratory tests are more sensitive for SIMPs of relatively low mass, the shift of spectral lines in (μ-Fe) becomes more sensitive the larger the SIMP mass is, since the reduced mass correction is more completely eliminated as $M_S\to\infty$.

Muonic iron ions could be observed in the spectra of nearby galaxy clusters and galaxies with the high resolution spectrometers on board the next generation γ-ray observatories such as INTEGRAL. It is also likely that other muonic atoms could be observed with the high-resolution spectrometers planned for the next generation x-ray observatories such as ASTROE-2, XEUS and Constellation-X, and with the instruments already operating on Chandra and XMM. Such a possibility is of crucial relevance for testing the existence of SIMPs or even for inferring constraints on their presence in astrophysical environments. Moreover, it opens a new window on the possibility of studying dark matter composition through direct astrophysical observations. It is appealing to anticipate that some astrophysical feature of galaxy clusters might yield constraints on the fundamental properties of a possible dark matter particle.

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