# Toward a possible solution to the cosmic coincidence problem

Kim Griest
*Physics Department 0319, University of California, San Diego, La Jolla, California 92093*
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We suggest a paradigm that might allow for a nonanthropic solution to the cosmic coincidence problem of why the density of vacuum energy and matter are nearly equal today. The fact that the half life of uranium 238 is near to the age of the solar system is not considered a coincidence since there are many nuclides with a wide range of half lives implying that there is likely to be some nuclide with a half life near to any given time scale. Likewise it may be that the vacuum field energy causing the universal acceleration today is just one of a large ensemble of scalar field energies, which have dominated the Universe in the past and then faded away. Predictions of the idea include the following: the current density of vacuum energy is decreasing, the ratio of vacuum pressure to vacuum density, $w$, is changing and not equal to $-1$, there were likely periods of vacuum domination and acceleration in the past and may be additional periods in the future, and the eventual sum of all scalar field vacuum densities may be zero.

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## INTRODUCTION

The discovery that the Universe is accelerating [1,2], probably due to dominance of vacuum energy, has pushed two uncomfortable fine-tuning problems to the forefront of the physics community. The first problem [3] is that the scale of this vacuum energy density, $(.002\ \mathrm{eV})^4$, is vastly different from the MeV to Planck scale particle physics vacuum energies that must sum to give this number. In the past, one hoped that some unknown symmetry principle would require these energies to ultimately sum to zero, but the existence of a nonzero sum naively dashes these hopes. The second fine-tuning problem occurs because traditional vacuum energy density does not change with time while matter and radiation density change rapidly as the Universe expands. At early times the Universe was radiation and then matter dominated, and only recently, at $z \sim 1.7$, did it become vacuum dominated. The importance of this "cosmic coincidence" is that in the normal scenario, once vacuum energy becomes dominate, it stays dominate, and if this had occurred at any earlier epoch, the evolution of the Universe would have been completely different and most likely we would not be here to discuss it. Currently, there is no convincing fundamental physics idea for why vacuum dominance happened only recently, and this has led many workers [4–6] to conclude that some sort of anthropic principle must be at work. The anthropic idea [3] is that there is an ensemble of universes with different values of the vacuum energy, most of which do not allow life to develop. Therefore the cosmic coincidence is "explained" by saying the existence of intelligent life selects only those values of vacuum energy density near that which has been measured in the supernova type Ia (SN Ia) observations. Many nonanthropic ideas have been proposed, which reduce the coincidence, usually by having the vacuum energy "track" the matter density in some way so that the ratio is not so large [7], but all of these have been criticized as in fact involving fine-tuning in some way [4–6]. At this point, some workers have concluded that only anthropic ideas have a chance of explaining the cosmic coincidence [5].

The goal of this paper is to propose a new class of solutions to the cosmic coincidence problem which is not anthropic and which may allow removal of fine-tuning. The idea presented here is similar to, but somewhat more general than, that of the tracking oscillating potential model of Dodelson, Kaplinghat, and Stewart [8]. This class of models also makes it possible to change the current cosmological constant problem from why the vacuum energies sum to a small number, back to the older problem of why the vacuum energies sum to zero. We do not have anything to say about the basic cosmological constant problem of why this sum should be zero in the first place. To understand our class of potential solutions, an analogy may be helpful.

## AN ANALOGY

Suppose we demand a nonanthropic solution to the cosmic coincidence problem. What would such a solution look like? Physics offers several examples of potential cosmic coincidences that we do not consider as such. For example, U238 is a common radioactive substance with a half life of 4.5 Gyr, almost exactly the age of the solar system. The reason that this is not a cosmic coincidence is clear in this case. There are thousands of nuclides with half lives ranging over an enormous range of time scales, from microseconds (e.g. U222, $\tau = 1\ \mu\mathrm{s}$), to seconds (e.g. U226, $\tau = 0.5$ s), to days (e.g. U231, $\tau = 4.2$ d), to millennia (e.g. U233, $\tau = 1.6 \times 10^5$ yr) to the age of the Universe (e.g. U238), to $10^{20}$ years (e.g. Se82, $\tau = 1.4 \times 10^{20}$ yr), and up.

Thus there is no surprise that for any given time scale, such as the current age of the Solar System, there is some nuclide which is decaying on just this time scale. Note that if U238 were the only nuclide in existence and everything were made of it, then there would be a cosmic coincidence problem quite similar to the vacuum energy cosmic coincidence problem. Also, if the range of radioactive decay constants were not exponentially distributed over such a wide range of time scales, then again even with hundreds of nuclides, finding one with $\tau = 4.6$ Gyr would be unlikely. In this example, it is the exponential sensitivity of radioactive decay to the nuclear wave function that allows a small change, such as the addition of one neutron, to make a large change in the half life.

Thus we are inspired to seek a possible class of models to solve the vacuum energy cosmic coincidence by positing not just one scalar field whose vacuum energy is making its appearance today, but an ensemble of fields, whose vacuum energies span an exponentially large range of energy densities, some of which have dominated the Universe for short periods of times at many time scales in the past. We note that we have no *a priori* reason for the existence of such an ensemble of vacuum fields, but we are using the cosmic coincidence problem as a clue that such an ensemble might exist.

In this class of models, the specific field (or fields) responsible for the current acceleration is not special, but just happens to be the one dominant at this time. We note that in this context, the purported inflaton field responsible for cosmic inflation in the very early Universe could be just another one (or more than one) of these fields.

This type of solution to the cosmic coincidence problem makes some interesting predictions:

(i) The vacuum energy is not a cosmological "constant." The current phase of acceleration is temporary and will eventually finish; $w=p/\rho$ is not $w=-1$ exactly.

(ii) There probably were several periods of acceleration and vacuum dominance in the past, followed by radiation and/or matter domination, and finally vacuum dominance again today.

(iii) There will likely be additional periods of acceleration in the future. Thus predicting the ultimate fate of the Universe will not be possible without an understanding of the origin of all these fields and their vacuum energies.

(iv) The sum of all these changing vacuum energies may well eventually be zero; that is, the minimum of the potential of all these fields may be a zero that we are evolving toward. Just as even very long lived nuclides will eventually decay, it is thus possible that the actual cosmological "constant" is zero, and we are just part of the way there. We offer no suggestion here as to why the sum should be exactly zero, we just note that in this scheme it is possible. However, just as some nuclides are stable, it is also possible that the final cosmological constant is not zero.

We note that the tracking oscillating model of Dodelson, Kaplinghat, and Stewart [8] is similar to our class of models. It uses a single scalar field with a potential with many wiggles and also predicts many periods of acceleration in the past. It addresses the cosmic coincidence problem in a way similar to ours. One difference is that the single potential must be quite complicated to allow such behavior, and it seems interesting and somewhat more generic and flexible to consider an ensemble of scalar fields with simple potentials.

## CONSTRAINTS

There are several observations that constrain vacuum energy density in the Universe and which any model of an accelerating Universe must satisfy. Our model contains several fields and so differs somewhat from single field quintessence/dark energy models [7]. However, given that we do not have a specific model, we will not attempt an accurate tallying of the constraints, but only mention a few constraints that should be considered.

First, if the density of scalar field energy, $\rho_{vac}$, is more than a few percent of the radiation density, $\rho_{rad}$, at the time of big bang nucleosynthesis (BBN), the universal expansion rate will increase enough to make a significant difference in the predicted abundances of helium and deuterium. Since these abundances are fairly well measured, we demand $\rho_{vac}/\rho_{rad}<.02$ when $10^8<z<10^{10}$ [9].

Second, the equation of state $w$ must be sufficiently negative for the scalar field energy solutions to match the current type Ia supernova measurements and be consistent with the cosmic microwave background (CMB) and large scale structure measurements. We require $w<-0.79$ (95% C.L.) [10]. In addition we can directly calculate the absolute magnitude-redshift relation and compare to the supernovae reported in Perlmutter *et al.* [1], demanding that, within errors, it be as good a fit as a cosmological constant model. Also, since the growth of large scale structure can be reduced by periods of vacuum dominance, or near vacuum dominance, there are several observational constraints on the power spectrum that are possible. For simplicity we will merely calculate the linear growth factor from the time of matter-radiation equality until the present, and compare it to that expected from a cosmological constant model.

Next, additional vacuum energy at the time of photon decoupling can shift the well measured acoustic peaks in the CMB, providing the constraint $\rho_{vac}/\rho_{other}<0.64$, at $z\approx 1100$ [11,12] where $\rho_{other}$ includes dark matter, radiation, and baryons. To preserve the peak positions, we can also demand that the angular diameter distance to the surface of last scattering at $z\approx 1100$ not be too different from that implied by a cosmological constant model.

We easily find models in which these constraints are satisfied, basically by choosing the field content and parameters such that none of the additional scalar fields is very important during either the decoupling or nucleosynthesis epochs, and such that the field that dominates today has $w$ in an appropriate range. From one point of view it thus requires fine tuning of many parameters to satisfy these constraints and so these restrictions are a weakness of our idea. However, from another point of view, these constraints and the data from which they derive are actually just measurements of the initial scalar field content and parameters. That is, had these values been different, then the present values of our current cosmological parameters and the positions of the CMB peaks would now be different.

## EXAMPLE MODEL

Since this class of solutions is motivated purely by solving the cosmic coincidence problem, there are no restrictions on the types of fields or forms of potentials that may be used. As a simple example consider an ensemble of $N$ scalar fields $\phi_i$, $i=1,\ldots,N$ that do not interact with ordinary matter or each other, and which have potentials of the form

$$V_i(\phi_i)=\lambda_i\phi_i^{\alpha_i}. \tag{1}$$

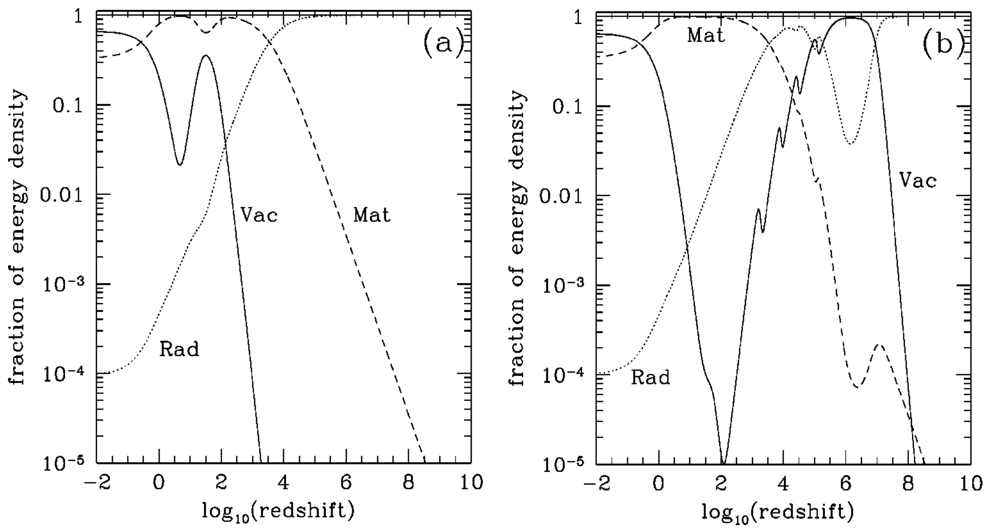


FIG. 1. The fraction of energy density in various components vs redshift for simple two-field models. The thick dark line represents (false) vacuum energy, the dashed line represents matter, and the thin dotted line represents radiation. See text for model descriptions.

The total scalar field potential is then just $V=\Sigma_i V_i(\phi_i)$. To first approximation (ignoring gravity, finite temperature effects, parametric resonance, etc.) each field is governed by the standard equations $\ddot{\phi}_i+3H\dot{\phi}_i+V_i'(\phi_i)=0$, $H^2=1/3M_{\rm Pl}^2[\rho_{\rm other}+\frac{1}{2}\Sigma_i\dot{\phi}_i^2+\Sigma_i V_i(\phi_i)]$, where $H(z)\equiv\dot{a}/a$, and the Planck mass is $M_{\rm Pl}=(8\pi G_N)^{-1/2}=2.44\times10^{18}$ GeV, and where $a$ is the scale factor of the Universe, $\rho_{\rm other}$ is the total energy density of the other contributing fields, the dot represents differentiation with respect to time, and the prime indicates differentiation with respect to $\phi_i$. Note that with $\alpha=4$, the $\lambda_i\phi_i^{\alpha_i}$ form for the potential is that used in chaotic inflation [13], and if $\phi_i$ starts at a nonzero value, it will eventually approach zero under the influence of its potential as long as $\alpha_i$ is an even integer. The $\phi_i$ field may come to dominate the energy density of the Universe depending upon the magnitude of its initial value and the speed at which it goes toward zero, which is determined by $\lambda_i$ and $\alpha_i$. Thus in this model, the period of vacuum dominance will not last forever, since eventually $\phi_i$ reaches zero and then oscillates around zero. During the oscillation phase, $\phi_i$ effectively behaves as material with $\overline{w}_i=(\alpha_i-2)/(\alpha_i+2)$, and density scaling as $\rho_i\propto a^{-3(1+\overline{w}_i)}$ [14], unless it couples and decays into ordinary particles. Thus any potential with $\alpha_i>4$ will eventually fade away faster than radiation or matter unless particles are produced.

Naively, with more than one scalar field this dominance of the "false vacuum" can happen several times, with first $\phi_1$ coming to dominate and then fade away, and then $\phi_2$ coming to dominance and so on. Since the values of $\phi_i(init)$, $\lambda_i$, and $\alpha_i$ determine when and how long each $\phi_i$ dominates, an appropriate ensemble of such values could give a series of periods of vacuum domination and universal acceleration, followed by periods of radiation and/or matter domination, depending upon how each $\phi_i$ decays and when the next field rises to dominance.

Very roughly the time of vacuum dominance for any $\phi_i$ occurs when its energy density $\rho_i=V_i(\phi_i)+\frac{1}{2}\dot{\phi}_i^2$ equals the radiation (or matter) energy density $\rho_{rad}\approx8.6\times10^{-5}a^{-4}\rho_{crit0}$. That is, supposing for simplicity that the scalar field kinetic energy is not large compared with the potential energy, we have $a_i(vacuum\ dom)\sim[\lambda_i\phi_i^{\alpha_i}(init)/8.6\times10^{-5}\rho_{crit0}]^{-1/4}$. Thus if the values of $V_i(init)$ are distributed over a wide range of values so that the $a_i(vacuum\ dom)$ are distributed over epochs from the Planck time at $a=10^{-27}$ to today at $a=1$, then our suggested scenario might take place. Of course there are many ways that the $\lambda_i$ and the $\phi_i(init)$ might be distributed to make this work, and we will not speculate on the precise form of the distribution since we have no understanding of it, except for possibly at two points: the inflaton in the very early Universe and the current vacuum dominated epoch.

For illustration purposes we consider two simple two-field cases, both of which satisfy the above constraints on $w$, fit the SN1A data as well as a cosmological constant model with $\Omega_V=\Omega_\Lambda=0.7$, have nearly the same distance to CMB last scattering, and satisfy the constraints on the fraction of vacuum energy at decoupling and BBN. The first example, shown in Fig. 1(a), has $\alpha_1=6$, $\lambda_1=10^{-175}$, $V_1(init)=5\times10^{-4}$, $\alpha_2=4$, $\lambda_2=10^{-125}$, $V_2(init)=10^{-8}$, where $V_i(init)$ is given in units of $m_{Pl}^2\mathrm{Mpc}^{-2}$ to help with numerics and $\phi_i$ is found from Eq. (1). In these units $\rho_{crit0}=1.6\times10^{-8}$, $M_{\rm Pl}=1.94\times10^{28}$, $\phi$ has units of $(m_{Pl}^{1/2}\mathrm{Mpc}^{-1/2})$, and $\lambda$ units $(m_{Pl}^{1/2}\mathrm{Mpc}^{-1/2})^{4-\alpha}$. This example has the current accelerating expansion coming from $\phi_2$, but earlier, at $z\approx1000$, $\phi_1$ started to become important, reaching more than 30% of the matter density at $z\sim300$, and then fading away. Today this model gives $\Omega_V=.67$ and $w=-.97$. This example might be ruled out since the linear growth factor calculated from matter-radiation equality is about 30% smaller than for the $\Omega_\Lambda=0.7$ model (about the same as for an $\Omega_\Lambda=0.78$ model). Of course the value of $V_1(init)$ could be reduced to lessen the size of these effects.

The second example shows a typical period of complete vacuum dominance in the early Universe [Fig. 1(b)]. Here the parameters used are $\alpha_1=10$, $\lambda_1=10^{-275}$, $V_1(init)$

$=10^{16}$, $\alpha_2=4$, $\lambda_2=10^{-124}$, $V_2(init)=10^{-8}$. Here the false vacuum dominates between redshifts of $10^5$ and $10^7$ and also near $z=0$, and is again a small contributor at the time of BBN and photon decoupling. Today this model gives $\Omega_V=.65$ and $w=-.91$. The linear growth factor is nearly the same as for a cosmological constant model. Note in all these plots we took the initial values of $\dot{\phi}_i(init)$ to be zero at $a_{init}=10^{-10}$, but very similar results are obtained when we use equipartition of kinetic and potential energy by setting $\dot{\phi} i(init)=\pm[2V_i(init)]^{1/2}$.

## DISCUSSION

This paper suggests a class of models that might explain the cosmic coincidence problem without invoking the anthropic principle. The basic idea is that there is an ensemble of scalar (or pseudoscalar) fields with exponentially distributed parameters that cause them to dominate the universal expansion at random times throughout the history of the Universe. This paper does not attempt to solve the basic cosmological constant problem of why the energies of all the scalar fields sum to near zero today, but does allow for consistency between the present accelerating expansion and a zero sum. The simple polynomial models used here do require fine-tuning of their initial values and coupling constants. The initial values determine the time of domination and the values of the λ's determine whether or not vacuum domination takes place and for how long. Note that in models with attractor potentials there is no need to fine-tune the initial values, and so these models have received the bulk of the attention of the community. In our scenario, we could probably remove this fine-tuning by considering an ensemble of hybrid potentials which have tracking behavior at early times and then asymptote to polynomials with $\alpha>4$ after dominance, but for simplicity's sake we did not pursue this option in this paper. The basic point is that if one wants a nonanthropic solution to the cosmic coincidence problem, one probably wants an ensemble of fields with properties such as we discussed. This clue may motivate field or string theorists to find a way of naturally generating such an unusual set of initial conditions and coupling constants, or to find an ensemble of hybrid potentials that switch from tracking to decay in the right way.

In summary, we have not investigated any models in detail and have no first principle reason for why such an ensemble of fields should exist or why their parameters should be properly distributed, but we are using the idea of a nonanthropic solution to the cosmic coincidence problem as the main motivation. However, we do note that the Higgs field in the standard model of particle physics is a scalar field that contributes to the universal vacuum energy and whose contribution must be cancelled by a large negative contribution from either a cosmological constant or from the vacuum energy of another field (or fields). Including the inflaton and the field giving rise to the current universal acceleration we thus probably have at least three important scalar fields contributing to standard cosmology, making our suggestion of a large ensemble of such fields more palatable.

This idea has several advantages, including a ''unification'' of the inflaton field and the field that currently causes universal acceleration, as well as perhaps detectable periods of vacuum dominance in the past, and predictions that the current era of vacuum dominance will end and that $w$ is not precisely unity. This last prediction might be testable in proposed experiments to measure the value and time derivative of $w$ [15].

There are many possibilities and open questions that should be addressed. A general question is what kind of potentials and initial values can give rise to realistic implementations of this idea, and what kind of theories could give rise to such an ensemble of fields and initial data? Other important questions are probably difficult to address without a more specific field theoretic framework; for example, one should consider how each of these small periods of vacuum dominance end: Are particles created? Do substantial adiabatic or iso-curvature fluctuations result? Is the Universe reheated? Is the power spectrum affected? Even in a model dependent way it would be interesting to explore how much and what kind of late time vacuum domination, or near domination, is allowed by current observations. It may even be that periods of vacuum dominance, or near dominance, could help the fit between theory and observation, or that the leftover oscillating fields could make up some of the dark matter. In general one could ask what sets of fields and initial data could give rise to our current Universe and what observable effects would remain today?

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