Weak production of Λ particles near threshold in electron-proton scattering

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We obtain differential cross sections for the reaction $e^- + p \to \Lambda + \nu_e$, for several incident electron energies near threshold. We are motivated to examine this reaction because it has been proposed as a possible experiment at the Thomas Jefferson National Accelerator Facility to measure the electron neutrino mass. We choose electron energies of 194.25 MeV, 195 MeV, 200 MeV, and 205 MeV so that trends in the cross section may be examined. This calculation is phenomenologically based and makes use of $SU(3)$ relations, and the form factors so obtained accurately describe Λ beta decay and so should be correct in this region. We obtain contributions of the individual form factors to the differential cross section and show that the axial vector current form factor increasingly dominates as the electron energy nears threshold. Finally we discuss how the behavior of the cross section in the near threshold region affects its use as a tool to study the electron neutrino mass as has recently been proposed.

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I. INTRODUCTION

It has recently been proposed [1] that a measurement be undertaken at the Thomas Jefferson National Accelerator Facility (TJNAF) to measure the electron neutrino mass via the reaction $e^- + p \to \nu_e + \Lambda$ run at near threshold energies. The proposed experiment would make use of the polarized electron beam capabilities at TJNAF rather than the exact form of the differential cross section. Because the quantity $(1 - \gamma_5)$, is a projector only for massless particles, the group is interested in considering the possibility of nonrelativistic neutrinos such that the ratio m_ν / E_ν approaches 1. In this regime the $(1-\gamma_5)$ factor in the weak lepton current allows contributions from right handed neutrinos and these contributions become large as the neutrino momentum approaches zero. The group would use both left handed and right handed polarized electrons and look for deviations from expected results. They would also use right handed electrons somewhat above the nonrelativistic neutrino region to provide a handle on their background.

The experiment would make use of a missing mass determination at a resolution of less than 100 keV to separate the neutrino events from scattered electron events. The actual experiment [2] would in effect be an asymmetry measurement to measure the difference between the number of events initiated by right helicity electrons and those initiated by left helicity electrons, $(N_R - N_L)/(N_R + N_L)$. The group expects about one in two to three hundred events to be right handed and needs from 5000 to 50 000 events to obtain adequate statistics. They expect to be able to measure or at least set an upper limit on the electron neutrino mass of 1 eV. This is better than tritium beta decay experiments and complements the various oscillation experiments which measure mass squared differences between different flavors of neutrinos rather than absolute masses. This is discussed at length in their proposal [2].

Nonetheless it is still necessary to obtain cross sections in this region to determine the feasibility of such an experiment and, as we shall see, the exact behavior of the cross section in the near threshold region provides serious limits for such an experiment. Weak Λ production in electron proton scattering has been studied at higher energies [3,4]. However all of this work has been for electrons in the GeV region and there are some interesting considerations in the near threshold region to which this Brief Report is addressed.

In this paper we look at the process $e^- + p \to \nu_e + \Lambda$ near threshold which occurs at $E = 194.1753$ MeV. We choose energies 205, 200, 195, and 194.25 MeV to provide a representative set of energies from which trends may be determined. We obtain the differential cross sections for the outgoing Λ in the laboratory frame. We also obtain the contributions to the differential cross sections from the individual form factor for $E = 195$ MeV and discuss what might be learned in this energy range. Finally we discuss the possibilities for the proposed experiment.

We undertake a calculation as model independent as possible and make use, where available, of experimental data along with $SU(3)$ relations. We note that in general $SU(3)$ relations are not as accurate as $SU(2)$ relations for predicting baryon form factors. However they work well for hyperon decays [5–8] which are low q^2 processes and give results which are consistent with a detailed model calculation [4,3] for the Λ reaction at intermediate energy. We thus make use of them for the low q^2 process of interest here.

II. MATRIX ELEMENTS

The Λ process from threshold to intermediate energies is well described by the matrix element

$$\langle \nu_e \Lambda | H_w | e^- p \rangle = \frac{G}{\sqrt{2}} \sin\theta_C \overline{u}_\nu \gamma^\lambda (1-\gamma_5) u_e \langle \Lambda | J_\lambda^\dagger(0) | p \rangle \tag{1}$$

where the weak hadronic current is given by

$$J_\mu(0) = V_\mu(0) - A_\mu(0). \tag{2}$$

Here V_μ and A_μ are the vector and axial vector parts of the weak, strangeness changing, hadronic current. In Eq. (1) the hadronic current matrix element is not known and so we write it in terms of form factors using the notation of Ref. [3],

$$\langle\Lambda|V_\mu^\dagger(0)|p\rangle = \overline{u}_f\left(\gamma_\mu F_V(q^2)+i\frac{F_M(q^2)\sigma_{\mu\nu}q^\nu}{2m_p}-F_S(q^2)\frac{q_\mu}{2m_p}\right)u_i \tag{3}$$

and

$$\langle\Lambda|A_\mu^\dagger(0)|p\rangle=\overline{u}_f\left(\gamma_\mu\gamma_5 F_A(q^2)+\frac{q_\mu\gamma_5 F_P(q^2)}{m_\pi}+\frac{iF_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5}{2m_p}\right)u_i \tag{4}$$

where i and f refer to the proton and Λ hyperon respectively. The six form factors describing the matrix elements given by Eqs. (3) and (4) contain the structure of the particles. A determination of these form factors enables a calculation of the differential cross section to be undertaken.

For the reaction described here with any initial charged lepton, all terms in the transition matrix element squared containing either F_P or F_S are also proportional to the lepton mass squared and are thus highly suppressed for the case of the electron. We therefore need only determine the form factors, F_V, F_M, F_A and F_E. These would, in the case of the nonstrangeness changing current, be determined by $SU(2)$ relations. Here however we must use $SU(3)$ relations. These are well known and the form factors may be written [3] as

$$F_r^{ijk}=-if^{ijk}\tilde{F}_r+d^{ijk}\tilde{D}_r \tag{5}$$

where we use a tilde to distinguish the $SU(3)$ functions from the form factors and where i, j and k refer to the current octet number, the initial baryon octet number and the final baryon octet number respectively. Here r stands for V,M,A,E or 1,2, and 3 if an electromagnetic current is described. For the process under consideration here Eq. (5) becomes

$$F_r=\frac{-1}{\sqrt{6}}(3\tilde{F}_r+\tilde{D}_r). \tag{6}$$

Applying Eq. (5) to the electromagnetic current, $V_\mu^3+(1/\sqrt{3})V_\mu^8$, first to the proton and then to the neutron cases one obtains [3] $\tilde{D}_V=0$, $\tilde{F}_V=F_1^p$, $\tilde{D}_M=\frac{3}{2}F_2^n$, and $\tilde{F}_M=-F_2^p-\frac{1}{2}F_2^n$ where we have suppressed the q^2 dependence of the form factors. Using these relations and Eq. (6) we can immediately find F_V and F_M for our process of interest. We have previously determined these form factors [3] which are consistent with hyperon decay data and are given by

$$F_r(q^2)=F_r(0)/(1-q^2/M_r^2)^2 \tag{7}$$

where $r=V,M,A,E$ again. For the vector current form factor, $F_V(0)=1.2247$, $M_V=0.98$ GeV/c^2 and for the weak magnetism form factor, $F_M(0)=1.793/2m_p$, $M_M=0.71$ GeV/c^2.

The axial vector current form factor may be determined from Λ beta decay data via the decay $\Lambda\to p+e^-+\overline{\nu}_e$. This reaction has been well studied [9] and from existing data one finds

$$\frac{F_A(0)}{F_V(0)}=0.718\pm0.015. \tag{8}$$

From this result we see that the parameters for Eq. (7) with $r=A$ are $F_A(0)=.8793$. A dipole mass of $M_A=1.25$ GeV/c^2 is consistent with the decay data.

We now need only F_E. From a theoretical model dependent calculation [4] we obtain an estimate for this form factor given by $F_E(0)=0.705/2m_p$ and $M_E\sim M_M$. It will turn out that the contributions from F_E are highly suppressed and in fact play no real role in this calculation. We are thus in a position to calculate the differential cross sections.

III. RESULTS AND DISCUSSION

The differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\frac{m_e m_\nu G^2 m_f p_f|M|^2}{(2\pi)^2 E8\left|m_i+E-\frac{EE_f\cos\theta}{p_f}\right|} \tag{9}$$

where p_f and E_f are here the magnitude of the three momentum and the energy of the final state Λ respectively. The quantity $|M|^2$ is easily found from Eq. (1) and is given in Ref. [3]. We note that there is a maximal angle for the outgoing Λ which increases as the electron energy increases [3] and that there is a mild singularity in the differential cross section [3] at the maximal angle. This is easily removed by the use of a wave packet for the outgoing hyperon as is discussed in Ref. [10].

Because this mild singularity which is entirely kinematical in origin plays an important role in the behavior of the differential cross sections we give an exact result for the maximal angle of the outgoing Λ:

$$\sin^2(\theta_{max})=1-\frac{2\delta}{m_f}-\frac{\delta m_e^2 E}{m_f^2p^2}+\frac{m_e^2\delta}{m_f p^2}-\frac{2\delta E}{p^2}+\frac{\delta^2}{m_f^2}+\frac{\delta^2}{p^2}-\frac{m_e^2\delta^2}{2p^2m_f^2}+\frac{3E\delta^2}{m_f p^2}-\frac{\delta^3 E}{m_f^2p^2}-\frac{\delta^3}{m_f p^2}+\frac{\delta^4}{m_f^2p^2}+\frac{m_e^4}{4m_f^2p^2}+\frac{Em_e^2}{m_f p^2}. \tag{10}$$

We note that if E becomes large, most of the terms in Eq. (10) disappear leading to a limiting value of 57.24 degrees for the maximal angle. As the maximal angle is approached, the denominator in Eq. (9) develops a mild singularity as θ approaches θ_{max}. This singularity is easily removed [3] by

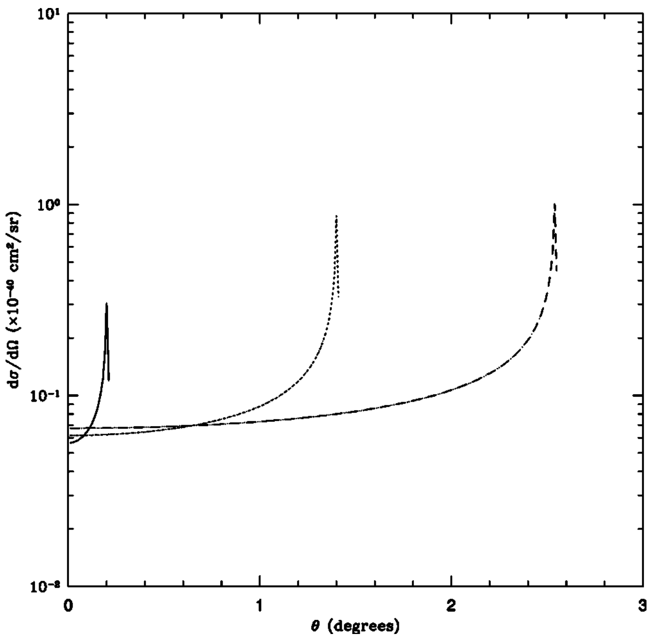

FIG. 1. Differential cross section for reaction $e^- + p \rightarrow \Lambda + \nu_e$ as a function of outgoing Λ laboratory angle. The solid, small dashed, and dot dashed curves are for incoming electron energies of 195, 200, and 205 MeV respectively.

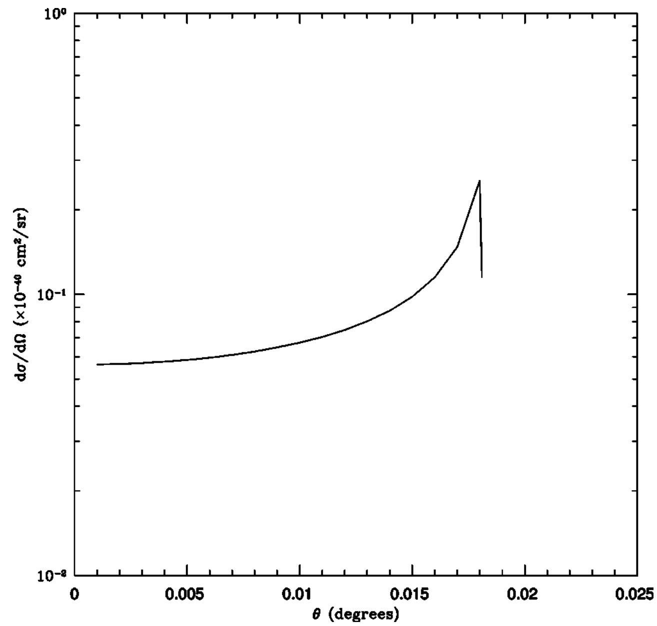

FIG. 2. Differential cross sections for the reaction $e^- + p \rightarrow \Lambda + \nu_e$ as a function of outgoing Λ laboratory angle. The incident electron energy is 194.25 MeV.

using a wave packet for the outgoing Λ. However the denominator does drive up the differential cross section before this condition is reached as can be seen in Fig. 1 and as the maximal angle is approached this rise is rapid and substantial.

There are two other factors which affect the size of the cross section which must be considered. First, every term in the transition matrix element squared, $|M|^2$ is proportional to E, the electron energy and ν (which may stand for either the neutrino energy or momentum) to at least the first power in each of them. There is a factor of E in the denominator of the differential cross section, Eq. (9), which cancels at least one power of E in $|M|^2$ but ν is not cancelled. Because E is always at least over 194 MeV, the electron is highly relativistic and no distinction need be made between the electron energy and momentum. As the neutrino momentum becomes small all terms in $|M|^2$ will either approach zero or be proportional to the neutrino mass and thus $|M|^2$ will become negligible because the neutrino mass is very small.

Secondly, the form factors in $|M|^2$ are all dipole and fall rapidly with increasing q^2. Thus low values of q^2 yield the largest form factors but low q^2 implies a large neutrino momentum which also drives up $|M|^2$. Thus in general small values of q^2 imply large values for $|M|^2$. Moreover as the maximal angle for the outgoing Λ is approached, $|q^2|$ falls to its minimum value.

The final result is that all factors contributing to the differential cross section, i.e., the kinematical rise near the maximal angle, the increase in size of the matrix element as E_ν increases, and the rise in the form factors, occur when $|q^2|$ is small. But this is precisely where E_ν is large and therefore highly relativistic. These are exactly the wrong conditions for looking for neutrino helicity effects.

In Fig. 1 we plot the differential cross section for the outgoing Λ particle in the laboratory frame for incident electron energies of 195, 200, and 205 MeV. As can be seen, the peaks increase as the electron energy increases. The maximal angle also increases. This is well known [3]. As the electron energy approaches infinity a limiting maximal angle of 57.24 degrees is obtained.

In Fig. 2 we show the same differential cross section for an incoming electron energy of 194.25 MeV. Because threshold is 194.1753 MeV, this case represents an energy of about 75 keV above threshold. This would still lead to a fairly relativistic neutrino energy but some features become clear. At this point the maximal angle is only .02 degrees and is approaching zero. This has one beneficial effect in that the range of q^2 is not very great and so the difference between the differential cross section for the highest and lowest q^2 is less than an order of magnitude (about a factor of 5) and the difference between the high and low values of the differential cross section shrinks as threshold is approached and the maximal angle approaches zero. Thus if a differential cross section in the range of 10^{-42} to 10^{-43} cm^2/sr can be measured, a meaningful experiment might be possible. We note that the small maximal angle is not in itself a bar to this experiment because the Λ decays to a nucleon and pion which would be actually observed and for these the angle need not be small.

Finally in Fig. 3 we plot the contributions of the form factors to the differential cross section. If these contributions are compared to those of Ref. [3], it is clear that the axial vector current contributions have increasing importance at lower energies. This is very interesting and it would be useful to observe the differential cross sections directly in these lower energy ranges. The experimental evidence which we presently have for the magnitude of these form factors is

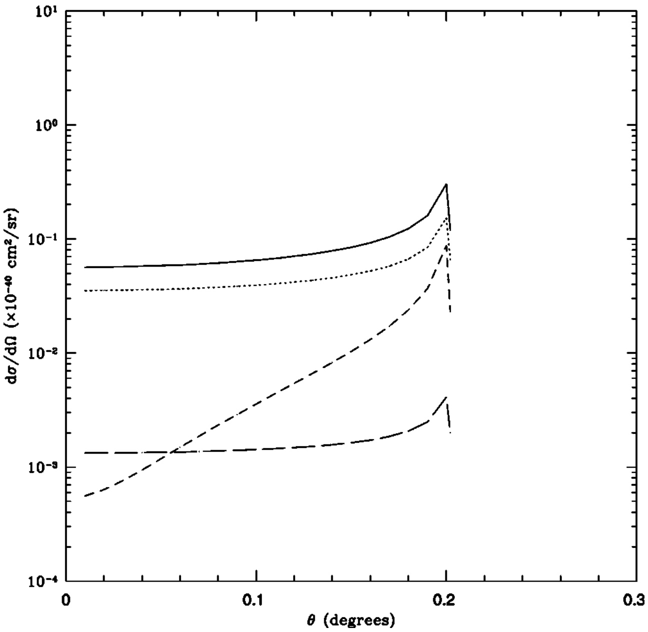

FIG. 3. Contributions of the various form factors to the differential cross section as a function of outgoing Λ laboratory angle for an incident electron energy of 195 MeV. The solid, small dashed, dashed, and double dashed curves are the contributions of all form factors, F_A, F_V, and F_M, respectively. The curves, except for the solid one, are obtained by setting all form factors but one at zero.

from beta decay, and this produces a decay spectrum. An experiment at TJNAF or a similar facility would allow these form factors to be observed by the use of monoenergetic electrons and would allow a much cleaner determination of $F_A(0)$ and $F_V(0)$. This would provide more stringent tests for the SU(3) model used here and for microscopic models.

We note here that the SU(3) model used here is primarily based on the valence quark arrangement of the proton and the Λ particles. However because the form factors it relates are phenomenological, it certainly includes some sea quark contributions as well. A recent experiment [11] measured the strange quark contributions (necessarily sea quarks) to the proton weak magnetism form factor and found them to be very small. Because contributions from the transition weak magnetism from factor are highly suppressed in our process (see Fig. 3) it is unlikely that related sea quark contributions can be seen in it. We hope to consider such contributions in an asymmetry calculation which we plan for a later paper.

To conclude, the reaction $e^- + p \rightarrow \Lambda + \nu_e$ might be suitable for a helicity experiment to determine the neutrino mass if cross sections of the order of 10^{-42} cm^2/sr can be observed. Magnitudes of this size are not unusual in neutrino experiments but might prove difficult for a facility such as TJNAF. It is also substantially lower than has been estimated in the proposal for the neutrino experiment [1] discussed here where 10^{-39} to 10^{-40} cm^2/sr has been assumed. Thus it is likely that between 50 000 and 500 000 events might need to be observed rather than the order of 5000 as originally proposed. However at slightly greater than threshold energies experiments might be performed which would allow the separation of the vector and axial vector contributions to the differential cross sections. Because there would be no particular need to examine the case for low neutrino energy it would be possible to work in the region where the cross section is large, which here would be in the range of 10^{-40} cm^2/sr or two to three orders of magnitude greater than would be the case for observing the neutrino mass. As noted above, this could provide useful tests of models. Thus the process at low energy does have some intrinsic interest.

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[1] O.K. Baker, Proposal to the Jefferson Lab PAC, CEBAF (2001).
[2] O.K. Baker (private communication).
[3] S.L. Mintz, Nucl. Phys. **A657**, 303 (1999).
[4] W-Y.P. Hwang and E.M. Henley, Phys. Rev. D **38**, 798 (1988).
[5] J. Dworkin *et al.*, Phys. Rev. D **41**, 780 (1990).
[6] J. Wise *et al.*, Phys. Lett. **98B**, 123 (1981).
[7] M. Bourquin *et al.*, Z. Phys. C **12**, 307 (1982).
[8] L.J. Carson, R.J. Oakes, and C.R. Wilcox, Phys. Rev. D **37**, 3197 (1988).
[9] Particle Data Group, D. Groom *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[10] S.L. Mintz and M. Pourkiavani, Phys. Rev. C **37**, 2249 (1988).
[11] R. Hasty *et al.*, Science **290**, 2117 (2000).