Universal description of *S*-wave meson spectra in a renormalized light-cone QCD-inspired model

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A light-cone QCD-inspired model, with the mass squared operator consisting of a harmonic oscillator potential as confinement and a Dirac delta interaction, is used to study *S*-wave meson spectra. The two parameters of the harmonic potential and quark masses are fixed by the masses of $\rho(770)$, $\rho(1450)$, J/ψ, $\psi(2S)$, $K^*(892)$, and B^*. We apply a renormalization method to define the model, in which the pseudoscalar ground state mass fixes the renormalized strength of the Dirac delta interaction. The model presents a universal and satisfactory description of both singlet and triplet states of *S*-wave mesons and the corresponding radial excitations.

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I. INTRODUCTION

In the effective light-cone QCD theory [1,2] the lowest Fock component of the hadron wave function is an eigenfunction of an effective mass squared operator with constituent quark degrees of freedom and parametrized in terms of an interaction which contains a Coulomb-like potential and a Dirac delta term. The Fock-state components of the hadron light-front wave function can be constructed recursively from the lowest Fock-state component. The interaction in the mass operator comes from an effective one-gluon exchange where the Dirac delta term corresponds to the hyperfine interaction. The masses of the ground state of the pseudoscalar mesons and in particular the pion structure [3] were described reasonably, with a small number of free parameters, which is only the canonical number plus one—the renormalized strength of the Dirac delta interaction.

The model was extended to include the confining interaction and used to study the splitting of the excited pseudoscalar states from the excited ${}^3S_1$ vector meson states as a function of the ground state pseudoscalar mass [4]. In Ref. [4], the Coulomb-like and the confining interactions were substituted by a harmonic oscillator potential, which allowed an analytic formulation. The parameters of the confining interaction in the mass squared operator were fitted to the ${}^3S_1$ meson ground state mass and to the slope of the trajectory of excited states with the radial quantum number [5]. With the renormalized strength of the Dirac delta interaction fixed by pseudoscalar masses, it was shown that the π-ρ mass splitting, due to the attractive Dirac delta interaction, is the source of the splitting between the masses of the excited states. A reasonable agreement with the data [6] was obtained. In addition, in the light-cone framework, the mass squared operator which contains the Dirac delta plus confining harmonic oscillator potential gave a natural explanation of the observation of the almost linear relationship between the mass squared of excited states with radial quantum number n [5]. This reveals some of the physics that is included in the work of Ref. [5] and shows the relation between the π and ρ spectrum, through the pion mass scale which defines the renormalization condition of the model.

In the present paper, this simple model, with the Dirac delta interaction acting in the ${}^1S_0$ channel only and the harmonic oscillator potential as confinement, is used to investigate the *S*-wave meson spectra from π-ρ to η_b-Υ and make predictions for η_t-θ (we do not study η-ω and η'-ϕ). Instead of using flavor-dependent parameters, parameters are found for the harmonic oscillator potential which is valid for not only light mesons but also heavy mesons. In other words, the parameters are universal. It is shown that the linear relationship between the mass squared of excited states and the radial quantum number is still qualitatively valid even for heavy mesons like Υ. The simple model presents reasonable agreement with available data and/or with the meson mass spectra given by Godfrey and Isgur [7].

This paper is organized as follows. In Sec. II, we give very briefly the extension of the light-cone QCD-inspired theory for which the mass squared operator of a constituent quark-antiquark system includes a confining interaction [4]. The renormalization of the theory using the subtracted equations for the transition matrix [8] of the model can be found in Refs. [3,4] and thus is omitted here. In the same section, we present the Dirac delta term plus harmonic oscillator potential approach and solve it with the T matrix method developed in Refs. [3,4]. The results and discussion are presented in Sec. III. A brief summary is given in the last section.

II. EXTENDED LIGHT-CONE QCD-INSPIRED THEORY WITH DIRAC DELTA AND HARMONIC OSCILLATOR CONFINING POTENTIAL

In this section we review our previous work [4], in which we have extended the renormalized effective QCD theory of

TABLE I. Parameters used in the present paper. c_0 and c_2 of harmonic oscillator potential and masses of up, down, and charm quarks are fixed from masses of $\rho(770)$, $\rho(1450)$, $J/\psi(1S)$, and $\psi(2S)$, with the assumption of $m_u = m_d$. Strange and bottom quark masses are determined by masses of K^* and B^*. The top quark masses $m_t = 35$ GeV (as in Godfrey and Isgur [7]), and $m_t = 175$ GeV from [6] are used to predict spectra for t-quark mesons. The data for meson masses are taken from Hagiwara *et al.* [6].

Parameter	c_0 (MeV)	c_2 (GeV^3)	$m_u = m_d$ (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)	m_t (GeV)
Value	807	7.13×10^{-2}	265	478	1749	5068	35/175

Ref. [3] to include confinement. In the effective theory the bare mass operator equation for the lowest light-front Fock-state component of a bound system of a constituent quark and antiquark of masses m_1 and m_2 is described as [1,2]

$$\begin{aligned} M^2\psi(x,\vec{k}_\perp) &= \left[\frac{\vec{k}_\perp^2+m_1^2}{x}+\frac{\vec{k}_\perp^2+m_2^2}{1-x}\right]\psi(x,\vec{k}_\perp) \\ &\quad -\int \frac{dx'\,d\vec{k}'_\perp\,\theta(x')\,\theta(1-x')}{\sqrt{x(1-x)x'(1-x')}} \\ &\quad \times\left(\frac{4m_1m_2}{3\pi^2}\,\frac{\alpha}{Q^2}-\lambda-W_{\text{conf}}(Q^2)\right) \\ &\quad \times\psi(x',\vec{k}'_\perp), \end{aligned} \tag{1}$$

where M is the mass of the bound state and ψ is the projection of the light-front wave function in the quark-antiquark Fock state. The confining interaction is included in the model by $W_{\text{conf}}(Q^2)$. The momentum transfer Q is the space part of the four-momentum transfer and the strength of the Coulomb-like potential is α. The singular interaction is active only in the pseudoscalar meson channel with λ as the bare coupling constant.

For convenience the mass operator equation is transformed to the instant form representation [9], which in operator form is written as [4]

$$(M_0^2+V+V^\delta+V_{\text{conf}})|\varphi\rangle = M^2|\varphi\rangle, \tag{2}$$

where the free mass operator $M_0(=E_1+E_2)$ is the sum of the energies of quarks 1 and 2 ($E_i=\sqrt{m_i^2+k^2}$, $i=1,2$, and $k\equiv|\vec{k}|$), V is the Coulomb-like potential, V^δ is the short-range singular interaction, and V_{conf} gives the quark confinement.

We simplify Eq. (2) by omitting the Coulomb term to the form [4]

$$[M_{\text{ho}}^2+g\,\delta(\vec{r})]\varphi(\vec{r}) = M^2\varphi(\vec{r}), \tag{3}$$

where the bare strength of the Dirac delta interaction is g, and the mass squared operator is [9]

$$M_{\text{ho}}^2 = [C(k)k^2+m_s^2]+2m_s v(r), \tag{4}$$

in units of $\hbar=c=1$, $m_s=m_1+m_2$. The dimensionless factor of k is

$$C(k) = 2+\frac{E_1+m_1}{E_2+m_2}+\frac{E_2+m_2}{E_1+m_1}. \tag{5}$$

In the following, we approximate $C(k)$ as m_s/m_r [9], with $m_r = m_1m_2/(m_1+m_2)$.

The harmonic oscillator potential is introduced as a confinement

$$v(r) = -c_0+\frac{1}{2}c_2r^2, \tag{6}$$

where c_0 and c_2 are two universal parameters valid for all of the mesons. The eigenvalue Eq. (4) is given now by

$$2m_s\left(-\frac{1}{2m_r}\nabla^2+\frac{1}{2}c_2r^2+\frac{1}{2}m_s-c_0\right)\Psi_n(\vec{r}) = M_n^2\Psi_n(\vec{r}), \tag{7}$$

with $\Psi_n(\vec{r})$ the eigenstate of the harmonic oscillator potential and the corresponding eigenvalue

TABLE II. The physical nomenclature of the mesons as a reminder. Pseudoscalar mesons are given on the left, vector mesons on the right of each sector. Diagonal sectors are marked in bold to guide the eye.

	$\bar{d}$	$\bar{u}$	$\bar{s}$	$\bar{c}$	$\bar{b}$	$\bar{t}$
d	$\boldsymbol{\pi^0,\eta,\eta'\|\|\rho^0,\omega,\phi}$	$\pi^-\|\|\rho^-$	$K^0\|\|K^{*0}$	$D^-\|\|D^{*-}$	$B^0\|\|B^{*0}$	$T^-\|\|T^{*-}$
u	$\pi^+\|\|\rho^+$	$\boldsymbol{\pi^0\,\eta,\eta'\|\|\rho^0,\omega,\phi}$	$K^+\|\|K^{*+}$	$\bar{D}^0\|\|\bar{D}^{*0}$	$B^+\|\|B^{*+}$	$\bar{T}^0\|\|\bar{T}^{*0}$
s	$\bar{K}^0\|\|\bar{K}^{*0}$	$K^-\|\|K^{*-}$	$\boldsymbol{\eta,\eta'\|\|\omega,\phi}$	$D_s^-\|\|D_s^{*-}$	$B_s^0\|\|B_s^{*0}$	$T_s^-\|\|T_s^{*-}$
c	$D^+\|\|D^{*+}$	$D^0\|\|D^{*0}$	$D_s^+\|\|D_s^{*+}$	$\boldsymbol{\eta_c\|\|\psi}$	$B_c^+\|\|B_c^{*+}$	$\bar{T}_c^0\|\|\bar{T}_c^{*0}$
b	$\bar{B}^0\|\|\bar{B}^{*0}$	$B^-\|\|B^{*-}$	$\bar{B}_s^0\|\|\bar{B}_s^{*0}$	$B_c^-\|\|B_c^{*-}$	$\boldsymbol{\eta_b\|\|\Upsilon}$	$T_b^-\|\|T_b^{*-}$
t	$T^+\|\|T^{*+}$	$T^0\|\|T^{*0}$	$T_s^+\|\|T_s^{*+}$	$T_c^0\|\|T_c^{*0}$	$T_b^+\|\|T_b^{*+}$	$\boldsymbol{\eta_t\|\|\theta}$

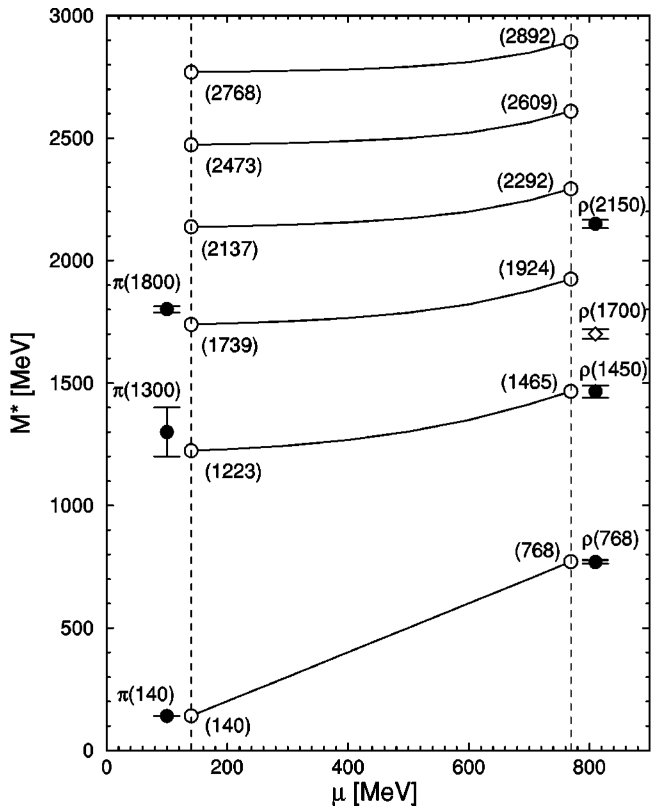

FIG. 1. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=1$. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. $\rho(1700)$ (labeled with empty diamond) might be D-wave dominant [5,11]. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

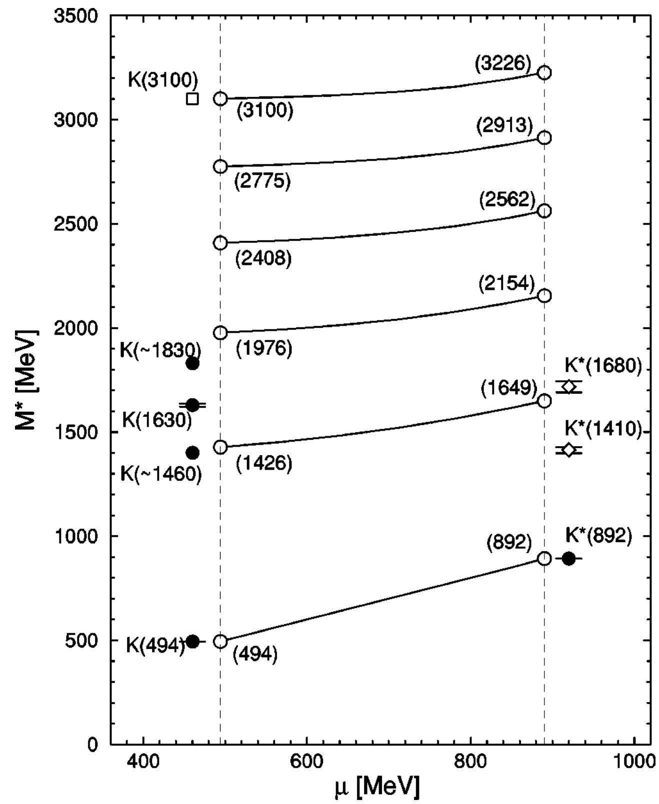

FIG. 2. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=1/2$ of the strange mesons. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. Errors are not available for $K(1460)$ and $K(1830)$. There is ambiguity about the radial quantum number of $K^*(1410)$ and $K^*(1680)$ [6] (labeled with empty diamond). $K(3100)$ is not confirmed and represented by empty square. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

$$M_n^2 = 2m_s\left[\left(2n+\frac{3}{2}\right)\sqrt{c_2/m_r}+\frac{1}{2}m_s - c_0\right]$$
$$= nw + m_s^2 + 3m_s\sqrt{c_2/m_r} - 2m_s c_0, \tag{8}$$

where n $(0,1,2,\ldots)$ is the radial quantum number and $w=4m_s\sqrt{c_2/m_r}$. Note that here n begins from 0 for convenience, while in the discussions in Sec. III it begins from 1 in accordance with the literature. In the present model, Eq. (8) gives the vector meson spectrum since the Dirac delta interaction acts only on pseudoscalar mesons. The mass squared of the ground state $(n=0)$ of the $^3S_1$ meson can be written as

$$M_{\rm gs}^2 = m_s^2 + 3m_s\sqrt{c_2/m_r} - 2m_s c_0. \tag{9}$$

With the subtraction point μ taken to be the mass of the pseudoscalar meson ground state, the reduced T matrix was derived in Ref. [4] as

$$t_{\mathcal{R}}^{-1}(M^2) = (2\pi)^3\sum_n |\Psi_n(0)|^2\left(\frac{1}{\mu^2 - M_n^2} - \frac{1}{M^2 - M_n^2}\right). \tag{10}$$

The value of the S-wave eigenfunction at the origin is given by [10]

$$\Psi_n(0) = \alpha^{3/2}\left[\frac{2^{2-n}}{\sqrt{\pi}}\frac{(2n+1)!!}{n!}\right]^{1/2}, \tag{11}$$

where α^{-1} is the oscillator length. The final form the reduced T matrix is

$$t_{\mathcal{R}}^{-1}(M^2) = (2\pi\alpha)^3\sum_{n=0}^{\infty}\frac{2^{2-n}}{\sqrt{\pi}}\frac{(2n+1)!!}{n!} \times\left(\frac{1}{\mu^2 - nw - M_{\rm gs}^2} - \frac{1}{M^2 - nw - M_{\rm gs}^2}\right). \tag{12}$$

The zeros of Eq. (12) give the eigenvalues of the the mass squared operator of Eq. (3). In Ref. [4], μ was changed continuously to study the splitting between the $^1S_0$ and $^3S_1$ spectra of the π-ρ mesons. In the following section, we are

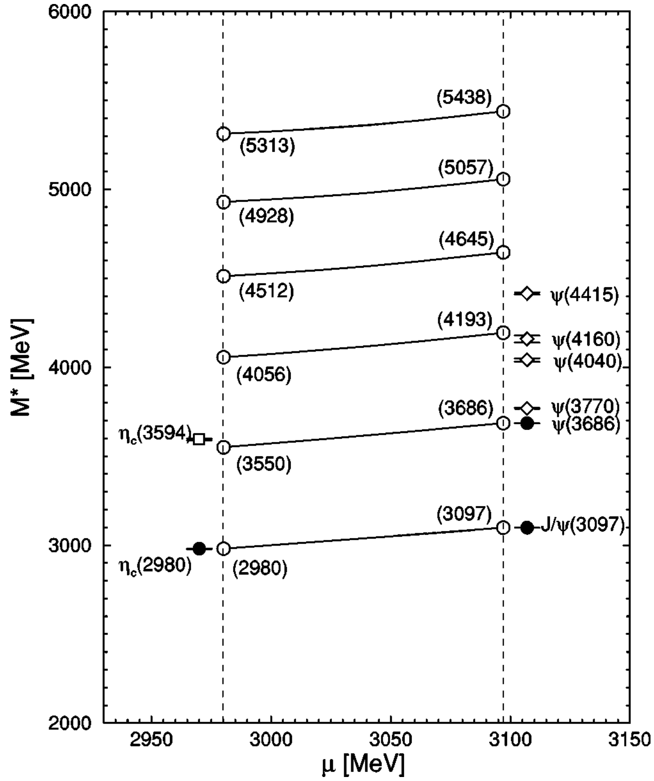

FIG. 3. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=0$ of the charmed mesons. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. No radial quantum numbers assigned to $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ (labeled with empty diamond). $\eta_c(3594)$ is not confirmed and represented by empty square. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

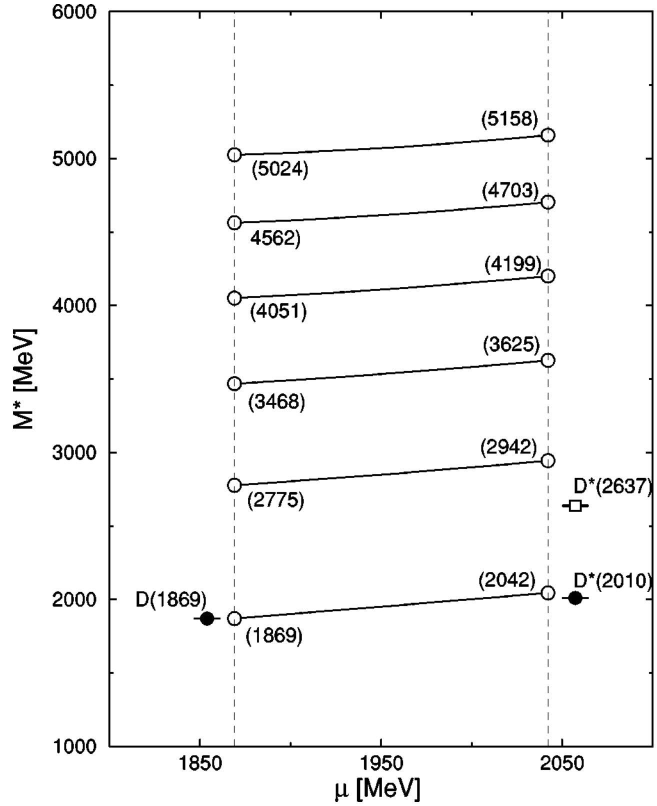

FIG. 4. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=1/2$ of the charmed mesons. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. $D^*(2637)$ is not confirmed and represented by empty square. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

going to discuss this splitting in more detail and apply the model to mesons with heavy quarks as well.

III. RESULTS AND DISCUSSION

A. Parameters and nomenclature

The masses of $\rho(770)$, $\rho(1450)$, $J/\psi(1S)$, and $\psi(2S)$ [6] and Eqs. (8),(9) are used to fix c_0, c_2, and up, down, and charm quark masses with the assumption of $m_u=m_d$. Then the masses of strange and bottom quarks are determined by the masses of K^* and B^* [6] and Eq. (9). In order to predict the t-quark meson spectrum, we adopt an estimate from Godfrey and Isgur [7], $m_t=35$ GeV. The parameters used in the present model are listed in Table I.

The physical nomenclature of the mesons is given in Table II to facilitate the following discussion. Since we assume that up and down quarks are the same, and since mixing between different flavors cannot be dealt with within this simple model, we investigate only $I=1$ states among the diagonal meson sectors containing u, d, and s quarks. $m_u=m_d$ also means that in our model the spectrum of π^0 (ρ^0, K^0, $K^{0}*$, D^0, $D^{0}*$, B^0, $B^{0}*$, T^0, and $T^{0}*$) is the same as that of $\pi^\pm$ ($\rho^\pm$, $K^\pm$, $K^{\pm}*$, $D^\pm$, $D^{\pm}*$, $B^\pm$, $B^{\pm}*$, $T^\pm$, and $T^{\pm}*$). In the following discussions, we will omit the charge signs of mesons for simplicity.

B. *S*-wave meson spectra

The splitting of the light mesons, π-ρ, and K-K^* spectra, due to the Dirac delta interaction acting in the pseudoscalar $^1S_0$ channel, were studied in previous work [4]. There the empirical slope (w) from Ref. [5] or Ref. [6] is used directly. In the present paper, π-ρ and K-K^* are reinvestigated in the same framework as other S-state mesons. The results are presented in Fig. 1 and Fig. 2, respectively. Similar agreement with the data as that in Ref. [4] is found in Fig. 1, simply because the present model gives $w=1.55\ \mathrm{GeV}^2$ which is very close to $w=1.39\ \mathrm{GeV}^2$ used in Ref. [4]. For K^*, the present model gives a large value of w, 1.92 GeV^2 (due to the fact that the strange quark mass is larger than the up-down quark mass, w for K^* must be larger than that for ρ from the present model), compared to that extracted from the data, 1.19 GeV^2. However, as pointed out in Refs. [5,6], there are ambiguities about the quantum number assignment of exited states of K and K^*. If we follow the identification of the quark model [6] for $K(1460)$ (2^1S_0), $K(1830)$ (3^1S_0), and $K^*(1410)$ (2^3S_1), the spectra of K-K^* from the present model are in good agreement with the data.

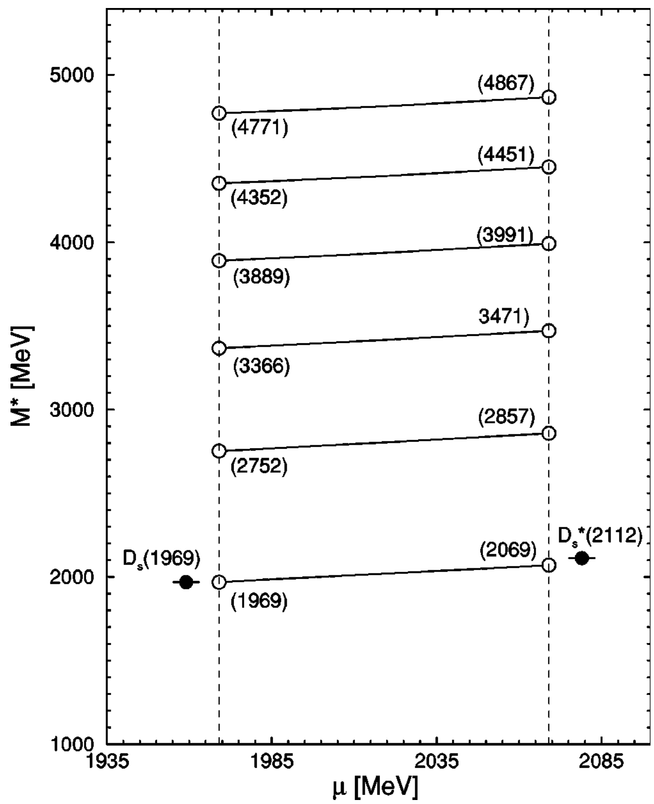

FIG. 5. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=0$ of the charmed strange mesons. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

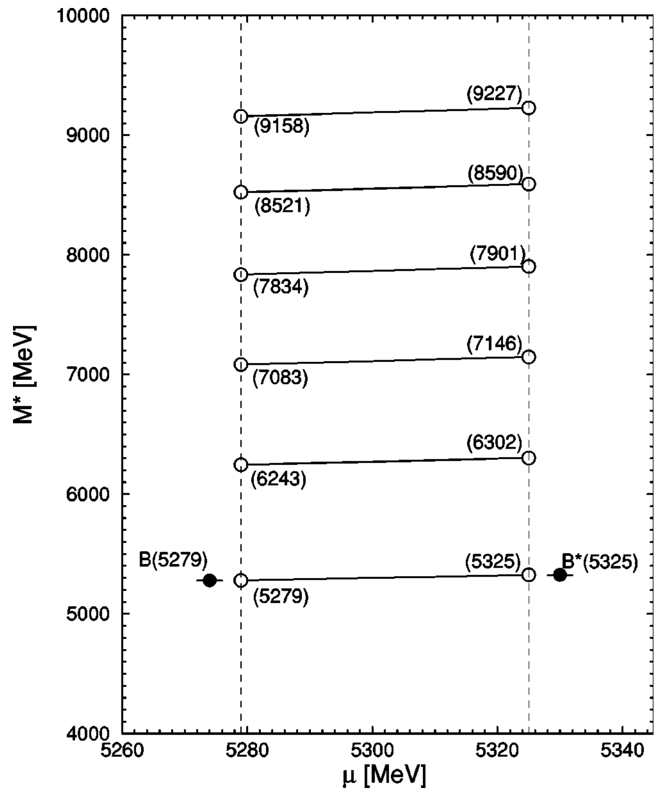

FIG. 6. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=1/2$ of the bottom mesons. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

In Fig. 3 the results are shown for the η_c-ψ mass splitting as a function of the ground state pseudoscalar mass μ, which interpolates from η_c to the J/ψ meson spectrum. Compared to the light π-ρ and K-K^* mesons, the splitting is smaller even for the ground state (~ 100 MeV) and becomes weaker in the excited states, although the model attributes consistently smaller masses for the $^1S_0$ states compared to the respective $^3S_1$ ones. An excited state of η_c is observed without definite spin and parity assignment [6]. From our model, this state might be $\eta_c(2S)$, which is consistent with the assignment of the quark model [6]. There are many exited states for ψ, such as $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$; all of them are assigned to $J^\pi=1^-$. Considering that $\psi(3770)$ is $\psi(1^3D_1)$ [6,7], $\psi(4040)$ or $\psi(4160)$ seems to be $\psi(3^3S_1)$ from the present model.

From Eqs. (8),(9) and the quark masses listed in Table I, one obtains the spectrum of D^*. The splitting of D-D^* and the spectrum of the D meson can be calculated from Eq. (10) with the renormalized strength of the δ potential fixed by the mass of the corresponding pseudoscalar ground state D; it is shown in Fig. 4. One finds good agreement for with the data $D^*(1S)$. The predicted mass of $D(2S)$ is about 10% larger than the unconfirmed data.

Similarly, the present model gives $D_s^*(1S)$ in good agreement with the data as is seen from Fig. 5 in which the spectra for D_s-D_s^* are presented.

As mentioned before, the bottom quark mass is fixed by the mass of $B^*(1S)$ and Eq. (9). There are no other data for b-quark mesons. The predicted spectra for B and B^* are presented in Fig. 6.

The spectrum of Υ can be calculated from our model with the parameters listed in Table I and is shown in Fig. 7. We should note that the agreement with the data is good, from $\Upsilon(1S)$ to $\Upsilon(4S)$, considering that no parameter is adjusted specially for Υ, the heaviest meson states observed up to now. The unconfirmed bottomonium state $\eta_b(9300)$ is used to predict the spectrum of η_b and also presented in the same figure.

No confirmed data are available for the triplet states of B_s and B_c. With the bottom quark mass $m_b=5068$ MeV, the present model predicts the triplet ground state masses for B_s and B_c, as 5342.1 MeV and 6.346 GeV, respectively. Although the predicted mass for $B_s(1S)$ is comparable to the unconfirmed data 5416.6 MeV, both predictions are smaller than their corresponding singlet ground state masses, 5369.6 MeV and 6.4 GeV. Therefore we cannot calculate the singlet spectra B_s and B_c within our model. However, for completeness and in order to give a reference to the reader, we list the calculated values for triplets of the bottom strange and bottom charged mesons in Table III and Table IV, respectively.

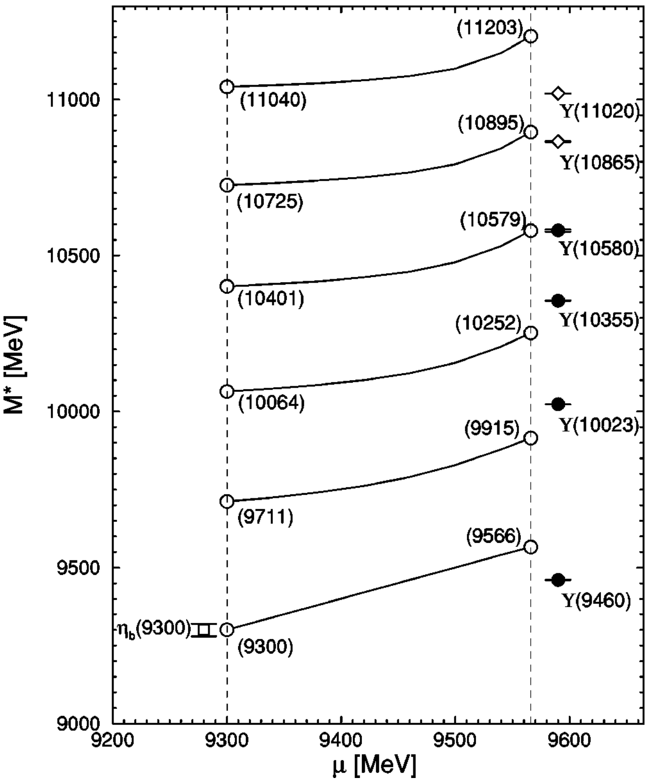

FIG. 7. Mass of the excited $q\bar{q}$ states (M^*) as a function of the mass (μ) of the pseudoscalar meson ground state for $I=0$ of the bottom mesons. The data are taken from Hagiwara *et al.* [6] and are shown by solid circles on the left and right of the figure for $^1S_0$ and $^3S_1$ mesons, respectively. No radial quantum numbers assigned to $\Upsilon(10865)$ and $\Upsilon(11020)$ (labeled with empty diamond). $\eta_b(9300)$ is not confirmed and represented by empty square. Calculated $^1S_0$ and $^3S_1$ spectra are given in parentheses within the dashed lines.

The vector top meson spectra from the present model with the top quark mass $m_t=35$ GeV [7] and $m_t=175$ GeV [6] are given in Table V. The top quark mass $m_t=35$ GeV is used in order to compare the present spectra with predictions of Ref. [7]. Qualitative agreement between the two models is found.

C. Further discussion

From the above results, it is clear that our model, with the mass squared operator consisting of a harmonic oscillator confining potential and a Dirac delta interaction, could be used to describe universally and satisfactorily both singlet and triplet states of *S*-wave mesons as well as radial excitations. However, lattice QCD calculations predict that the quark-antiquark potential increases linearly with the distance between quark and antiquark *r* when *r* is large. One may ask, does this contradict our model? The answer is no. In the following, we justify roughly that the two interactions are consistent with each other for large *r* disregarding the case for small *r* which is not important in the present work.

In the front form of QCD, the mass squared $M^2=m_s^2+2m_sE=m_s^2+2m_s(T_{\rm FF}+V_{\rm FF})$ (m_s is the sum of the quark and antiquark masses, T is the kinetic energy, and $V_{\rm FF}$ is the interaction) [9], while in the instant form of QCD, $M=m_s+E=m_s+(T_{\rm IF}+V_{\rm IF})$. The relation between the interactions $V_{\rm FF}$ and $V_{\rm IF}$ is found for large r (where $T\ll V\ll V^2$) as $V_{\rm FF}\sim V_{\rm IF}^2$, from which the harmoniclike potential for the mass squared operator in the front form can be derived from the linear confinement potential in the instant form of QCD.

Very recently the radial excitations of light mesons were studied in detail in the framework of the QCD string approach [12]. There the spin-averaged meson masses were calculated with a modified confining potential and the calculated slopes of the radial Regge trajectories are in agreement with Ref. [5]. In Ref. [12], the linear relation between mass squared M_n^2 and the radial quantum number n comes mainly from properties of the approximated eigenvalues of the spinless Salpeter equation with a linear confining potential. Since in our light-cone QCD model the harmonic oscillator potential is included in the mass squared operator, one arrives naturally at the same linear relation between M_n^2 and n for vector mesons. This gives an explicit explanation of the radial Regge trajectories found for light vector mesons in Ref. [5]. This relation is still valid even for heavy vector mesons as shown in the previous subsection. In addition, an extension of the present model with orbital excitation included could also be used to describe the orbital Regge trajectory.

For pseudoscalar mesons, particularly for the light ones, the present model does not support the simple linear relation because the Dirac delta interaction plays an important role now. However, this is not in contradiction with the data because light pseudoscalar mesons follow the radial Regge trajectories poorly. Let us take the pion as an example. The slope $w=1.67$ GeV^2 is derived from the masses of π (M

TABLE III. The present spectra of vector bottom strange mesons $B_s^*(nS)$ compared with available data from Hagiwara *et al.* [6] and predictions of Godfrey and Isgur [7]. Note that the data for $B_s^*(1S)$ are not confirmed [7]. Masses are in MeV.

B_s^*	This work	Data [6]	Godfrey and Isgur [7]
1^3S_1	5342.2	5416.6	5450
2^3S_1	6123.8	—	6010
3^3S_1	6816.4	—	—
4^3S_1	7444.7	—	—
5^3S_1	8024.2	—	—
6^3S_1	8564.6	—	—

TABLE IV. The present spectra of vector bottom charmed mesons $B_c^*(nS)$ compared with predictions of Godfrey and Isgur [7] (no data available). Masses are in MeV.

B_c^*	This work	Godfrey and Isgur [7]
1^3S_1	6345.8	6340
2^3S_1	6830.4	6890
3^3S_1	7282.9	—
4^3S_1	7709.0	—
5^3S_1	8112.6	—
6^3S_1	8497.1	—

TABLE V. The present spectra of vector *t*-quark mesons compared with predictions (in parentheses) of Godfrey and Isgur [7]. Results with $m_t = 35$ GeV [7] and $m_t = 175$ GeV [6] are given in the first and the third row of each sector. The masses are in GeV. The spectra of T^{*0} and $T^{*\pm}$ are the same since $m_u = m_d$ is assumed.

Mesons	$M(1\,^3S_1)$	$M(2\,^3S_1)$	$M(3\,^3S_1)$	$M(4\,^3S_1)$	$M(5\,^3S_1)$	$M(6\,^3S_1)$
T^*	35.24	36.27	37.27	38.24	39.19	40.12
	(35.39)					
	175.24	176.27	177.30	178.33	179.34	180.36
T_s^*	35.25	36.03	36.79	37.53	38.26	38.97
	(35.46)					
	175.25	176.02	176.79	177.56	178.32	179.08
T_c^*	36.25	36.67	37.08	37.49	37.89	38.29
	(36.30)					
	176.25	176.65	177.06	177.46	177.87	178.27
T_b^*	39.45	39.71	39.96	40.21	40.47	40.72
	(39.31)					
	179.44	179.68	179.92	180.16	180.40	180.64
θ	69.29	69.42	69.54	69.67	69.80	69.93
	(68.70)	(69.39)	(69.71)	(69.93)	(70.10)	(70.40)
	349.24	349.29	349.35	349.41	349.46	349.52

$=0.14$ GeV) and its first radial excited state ($M = 1.3$ GeV), while a slope smaller by $\sim 10\%$, $w = 1.55\ \mathrm{GeV}^2$, can be calculated from its first and second excited states ($M = 1.8$ GeV).

IV. SUMMARY

We applied the renormalized light-cone QCD-inspired effective theory with confinement in the mass squared operator to study mesons with heavy quarks. For the confinement, the harmonic oscillator potential is used, which allows an analytic solution of our model. The Coulomb-like potential is omitted in the present work while the Dirac delta interaction is kept; it acts on the singlet *S*-wave states and plays an important role in the splitting of the singlet-triplet spectra. The two parameters of the harmonic potential and the quark masses are fixed by the masses of $\rho(770)$, $\rho(1450)$, J/ψ, $\psi(2S)$, $K^*(892)$, and B^*. A T matrix renormalization method is used to renormalize the model, in which the pseudoscalar ground state mass fixes the renormalized strength of the Dirac delta interaction.

The model is applied to study the *S*-wave meson spectra from π-ρ to η_b-Υ and is also used to predict top quark meson spectra. The linear relationship between the mass squared of excited states and the radial quantum number [5]—the radial Regge trajectory [12]—is apparent from our model and is found to be qualitatively valid even for heavy mesons like Υ.

The simple model presents satisfactory agreement with available data and/or with the meson mass spectra given by Godfrey and Isgur [7]. Therefore, the recently proposed extension of the light-cone QCD-inspired model, which includes confinement while keeping simplicity and renormalizability, gives a reasonable picture of the spectra of both light and heavy mesons. An extension of the present model with orbital excitations included could also be used to describe the orbital Regge trajectories.

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