

# **$T$ -odd correlations in $B \rightarrow K^* l^+ l^-$ decay beyond the standard model**

T. M. Aliev,<sup>1,\*</sup> A. Özpineci,<sup>2,†</sup> M. Savcı,<sup>1,‡</sup> and C. Yüce<sup>1</sup>

<sup>1</sup>*Physics Department, Middle East Technical University, 06531 Ankara, Turkey*

<sup>2</sup>*The Abdus Salam International Center for Theoretical Physics, I-34100, Trieste, Italy*

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$T$ -odd correlations as physical observables in the  $B \rightarrow K^* l^+ l^-$  decay are studied using the most general form of the effective Hamiltonian. It is observed that these quantities are very sensitive to the new physics. We estimate the potential of discovery of these quantities at future hadron colliders.

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## **I. INTRODUCTION**

Rare  $B$  decays, induced by flavor changing neutral current (FCNC)  $b \rightarrow s(d)$  transitions, provide potentially the stringiest testing ground in the standard model (SM) at the loop level. Moreover,  $b \rightarrow s(d) l^+ l^-$  decay is also very sensitive to new physics beyond the SM. New physics effects manifest themselves in rare  $B$  decays in two different ways, either through new contributions to the Wilson coefficients existing in the SM or through new structures in the effective Hamiltonian which are absent in the SM.

Recently, time-reversal ( $T$ ) violation has been measured in the  $K^0$  system [1]. Unfortunately, the origin of  $T$ , as well as  $CP$  violation which also has been obtained experimentally in the  $K^0$  system, remains unclear. In the SM, both violations come from a weak phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. The SM predicts also the violation of  $CP$  in the  $B^0$  system (see, for example, [3]). The study of  $CP$  violation constitutes one of the main research areas of the working  $B$  factories [4]. These factories have already reported evidence for  $CP$  violation in the  $B$  systems, namely  $\sin 2\beta = 0.741 \pm 0.067$  [5]. In this work we investigate  $T$ -violating effects in the  $B \rightarrow K^* l^+ l^-$  using the most general form of the effective Hamiltonian. It should be noted that  $T$ -violation effects in the  $\Lambda_b \rightarrow \Lambda l^+ l^-$  and  $B \rightarrow K^* l^+ l^-$  decays were studied in the framework of the supersymmetric model in [6] and [7] as well as in the  $\Lambda_b \rightarrow \Lambda l^+ l^-$  decay using the most general form of the effective Hamiltonian in [8], respectively.

It is known that for a general three-body decay, the triplet spin correlations  $\mathbf{s} \cdot (\mathbf{p}_i \times \mathbf{p}_j)$  are the  $T$ -odd observables, where  $\mathbf{s}$ ,  $\mathbf{p}_i$ , and  $\mathbf{p}_j$  are the spin and final momenta of the final particles. Thus in the  $B \rightarrow K^* l^+ l^-$  decay, the  $T$ -odd observables can be constructed in two different ways: either by choosing lepton polarization as the polarization of the final particles, or by choosing polarization of  $K^*$ .

The first possibility, i.e., the choice of the lepton polarization in the  $B \rightarrow K^* l^+ l^-$  decay, was studied in detail in [9]. For this reason in the present work, in investigating the  $T$ -violating effects, we choose the second possibility, namely,

we choose  $K^*$  polarization to represent the polarization of the final state.

The paper is organized as follows. In Sec. II, using the most general, model independent form of the decay amplitude for the  $b \rightarrow s l^+ l^-$  transition, we study  $T$  violation in the  $B \rightarrow K^* l^+ l^-$  decay. Section III is devoted to the numerical analysis and concluding remarks.

## **II. THEORETICAL BACKGROUND**

The matrix element of the  $B \rightarrow K^* l^+ l^-$  decay is described by the  $b \rightarrow s l^+ l^-$  transition at quark level. The decay amplitude for the  $b \rightarrow s l^+ l^-$  transition, in a general, model independent form can be written in the following form [9–11]:

$$\begin{aligned} \mathcal{M} = & \frac{G\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left\{ C_{SL} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \bar{l} \gamma^\mu l \right. \\ & + C_{BR} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \bar{l} \gamma^\mu l + C_{LL}^{\text{tot}} \bar{s}_L \gamma^\mu b_L \bar{l}_L \gamma_\mu l_L \\ & + C_{LR}^{\text{tot}} \bar{s}_L \gamma^\mu b_L \bar{l}_R \gamma_\mu l_R + C_{RL} \bar{s}_R \gamma^\mu b_R \bar{l}_L \gamma_\mu l_L \\ & + C_{RR} \bar{s}_R \gamma^\mu b_R \bar{l}_R \gamma_\mu l_R + C_{LRLR} \bar{s}_L b_R \bar{l}_R l_L \\ & + C_{RLLR} \bar{s}_R b_L \bar{l}_L l_R + C_{LRR L} \bar{s}_L b_R \bar{l}_R l_L \\ & \left. + C_{RLRL} \bar{s}_R b_L \bar{l}_R l_L \right\}, \end{aligned} \quad (1)$$

where  $L = (1 - \gamma_5)/2$  and  $R = (1 + \gamma_5)/2$  are the chiral operators and  $C_X$  are the coefficients of the four-Fermi interaction. Note that this form of the decay amplitude is motivated by various extensions of the SM, such as the two Higgs doublet model and supersymmetric models. The first two of these coefficients,  $C_{SL}$  and  $C_{BR}$  describe the penguin contributions which correspond to  $-2m_s C_7^{eff}$  and  $-2m_b C_7^{eff}$  in the SM, respectively. The next four terms in Eq. (1) represent the vector type interactions, of whom the two with the coefficients  $C_{LL}^{\text{tot}}$  and  $C_{LR}^{\text{tot}}$  do exist in the SM in the forms  $(C_9^{eff} - C_{10})$  and  $(C_9^{eff} + C_{10})$ , respectively, i.e.,

$$\begin{aligned} C_{LL}^{\text{tot}} &= C_9^{eff} - C_{10} + C_{LL}, \\ C_{LR}^{\text{tot}} &= C_9^{eff} + C_{10} + C_{LR}. \end{aligned} \quad (2)$$

\*Email address: taliev@metu.edu.tr

†Email address: ozpineci@ictp.trieste.it

‡Email address: savci@metu.edu.tr

The remaining last four terms describe the scalar type interactions.

The effective Wilson coefficient  $C_9^{eff}$  is given by [12,13]

$$C_9^{eff} = C_9(\mu) + Y_{pert} + \frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i=J/\psi, \psi', \dots} \frac{\Gamma(V_i \rightarrow l^+ l^-) m_{V_i}}{\kappa_i m_{V_i}^2 - q^2 - i m_{V_i} \Gamma_{V_i}}, \quad (3)$$

where  $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$ ,  $m_{V_i}$  and  $\Gamma(V_i \rightarrow l^+ l^-)$  are the masses and the widths of the  $\psi$  family, and  $Y_{pert}(q^2/m_b^2)$  arises from the one-loop matrix element of the four-quark operators and can be found in [12,13]. The last term in Eq. (3) describes the long distance contribution from the real intermediate  $\bar{c}c$  states [14]. The factor  $\kappa_i$  for the lowest resonances are chosen as  $\kappa_{J/\psi} = 1.65$  and  $\kappa_{\psi'} = 2.36$  (see [15]) and for the higher resonances the average of the  $\kappa_{J/\psi}$  and  $\kappa_{\psi'}$  have been used.

Exclusive  $B \rightarrow K^* l^+ l^-$  decay is described in terms of matrix elements of the four-quark operators in Eq. (1) over meson states  $B$  and  $K^*$ , which are parametrized in terms of form factors. The decay amplitude for the  $B \rightarrow K^* l^+ l^-$  decays is found to be

$$\begin{aligned} \mathcal{M} = & \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \{ \bar{l} \gamma^\mu (1 - \gamma_5) l [ -2\mathcal{V}_{L_1} \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p^\lambda q^\sigma \\ & - i\mathcal{V}_{L_2} \varepsilon_\nu^* + i\mathcal{V}_{L_3} (\varepsilon^* q) P_\mu + i\mathcal{V}_{L_4} (\varepsilon^* q) q_\mu ] \\ & + \bar{l} \gamma^\mu (1 + \gamma_5) l [ -2\mathcal{V}_{R_1} \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p^\lambda q^\sigma - i\mathcal{V}_{R_2} \varepsilon_\nu^* \\ & + i\mathcal{V}_{R_3} (\varepsilon^* q) P_\mu + i\mathcal{V}_{R_4} (\varepsilon^* q) q_\mu ] \\ & + \bar{l} (1 - \gamma_5) l [ i\mathcal{S}_L (\varepsilon^* q) ] + \bar{l} (1 + \gamma_5) l [ i\mathcal{S}_R (\varepsilon^* q) ] \}, \end{aligned} \quad (4)$$

where  $P = p + p_B$ ,  $q = p_B - p$ , and  $p$  and  $\varepsilon$  are the  $K^*$  meson four-momentum and four-polarization vectors, and  $\mathcal{V}_{L_i}$  and  $\mathcal{V}_{R_i}$  are the coefficients of left- and right-handed leptonic currents with vector structure, and  $\mathcal{S}_{L,R}$  are the coefficients of the scalar currents with left and right chirality. Definitions of the form factors and functions  $\mathcal{V}_{L_i, R_i}$  can be found in [16].

In order to obtain  $T$ -odd terms  $\epsilon_{\mu\nu\alpha\beta} q^\mu \varepsilon^{*\nu} p_l^\alpha P^\beta$ , we study the  $B \rightarrow K^* l^+ l^- \rightarrow (K\pi) l^+ l^-$  process. The helicity amplitude  $M_{\lambda}^{\lambda_l \bar{\lambda}_l}$  of the  $B \rightarrow K^* l^+ l^-$  decay can be written as

$$M_{\lambda_i}^{\lambda_l \bar{\lambda}_l} = \sum_{\lambda_{V^*}} \eta_{\lambda V^*} L_{\lambda V^*}^{\lambda_l \bar{\lambda}_l} H_{\lambda V^*}^{\lambda_i}, \quad (5)$$

where

$$L_{\lambda V^*}^{\lambda_l \bar{\lambda}_l} = \varepsilon_{V^*}^\mu \langle l^-(p_{l^-}, \lambda_i) l^+(p_{l^+}, \bar{\lambda}_j) | J_\mu^l | 0 \rangle, \quad (6)$$

$$H_{\lambda V^*}^{\lambda_i} = \varepsilon_{V^*}^\mu \langle K^*(p, \lambda_i) | J_\mu^h | B(p_B) \rangle,$$

where  $\varepsilon_{V^*}$  is the polarization vector of the virtual intermediate vector boson ( $\gamma$  or  $Z$ ), satisfying the relation

$$-g^{\mu\nu} = \sum_{\lambda_{V^*}} \eta_{\lambda V^*} \varepsilon_{\lambda V^*}^\mu \varepsilon_{\lambda V^*}^\nu,$$

where the summation is over the helicities  $\lambda_{V^*} = \pm 1, 0, s$  of the virtual intermediate vector boson, with the metric defined as  $\eta_+ = \eta_0 = -\eta_s = 1$  (see [17,18]). In Eq. (6),  $J_\mu^l$  and  $J_\mu^h$  represent the leptonic and hadronic currents, respectively.

Using Eqs. (4)–(6), we get, for the helicity amplitudes,

$$\begin{aligned} \mathcal{M}_{\pm}^{++} &= \sin \theta_l A_{\pm}^{++}, \\ \mathcal{M}_{\pm}^{+-} &= (-1 \pm \cos \theta_l) A_{\pm}^{+-}, \\ \mathcal{M}_{\pm}^{-+} &= (1 \pm \cos \theta_l) A_{\pm}^{-+}, \\ \mathcal{M}_{\pm}^{--} &= \sin \theta_l A_{\pm}^{--}, \\ \mathcal{M}_0^{++} &= \cos \theta_l A_0^{++} + B_0^{++}, \\ \mathcal{M}_0^{+-} &= \sin \theta_l A_0^{+-}, \\ \mathcal{M}_0^{-+} &= \sin \theta_l A_0^{-+}, \\ \mathcal{M}_0^{--} &= \cos \theta_l A_0^{--} + B_0^{--}, \end{aligned} \quad (7)$$

where  $\theta_l$  is the polar angle of position in the rest frame of the intermediate boson with respect to its helicity axis. Explicit expressions of the functions  $A$  and  $B$  are presented in the Appendix (see also [16]).

Using the helicity amplitudes given in Eq. (7), the angular distribution in  $B \rightarrow K^* (\rightarrow K\pi) l^+ l^-$  is given by the following expression:

$$\begin{aligned} d\Gamma = & \frac{3G^2\alpha^2}{2^{17}\pi^6 m_B^3 m_q^2} |V_{tb} V_{ts}^*|^2 |B(K^* \rightarrow K\pi) dq^2 d \cos \theta_K d \cos \theta_l d\varphi|^{1/2} (m_B^2, m_{K^*}^2, q^2) \lambda^{1/2}(m_{K^*}^2, m_K^2, m_\pi^2) \lambda^{1/2}(q^2, m_l^2, m_l^2) \\ & \times \{ 2 \cos^2 \theta_K [ \cos^2 \theta_l N_1 + \sin^2 \theta_l N_2 + 2 \cos \theta_l \text{Re}(N_3) + N_4 ] + \sin^2 \theta_K [ \sin^2 \theta_l N_5 + (1 + \cos^2 \theta_l) N_6 + 2 \cos \theta_l N_7 \\ & + 2 \sin^2 \theta_l \sin 2\varphi \text{Im}(N_8) - 2 \sin^2 \theta_l \cos 2\varphi \text{Re}(N_8) ] + \sqrt{2} \sin 2\theta_K \sin \theta_l \cos \varphi \text{Re}(\cos \theta_l N_9 + N_{10}) \\ & - \sqrt{2} \sin 2\theta_K \sin \theta_l \sin \varphi \text{Im}(\cos \theta_l N_{11} + N_{12}) \}. \end{aligned} \quad (8)$$

TABLE I. The  $B \rightarrow K^*$  transition form factors in a three-parameter fit. The values of the form factors are taken from [20].

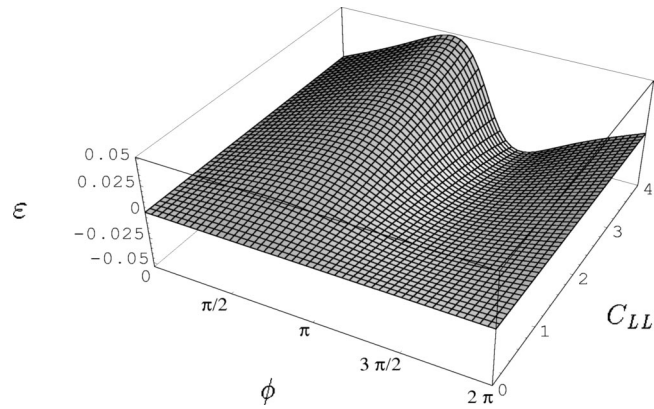
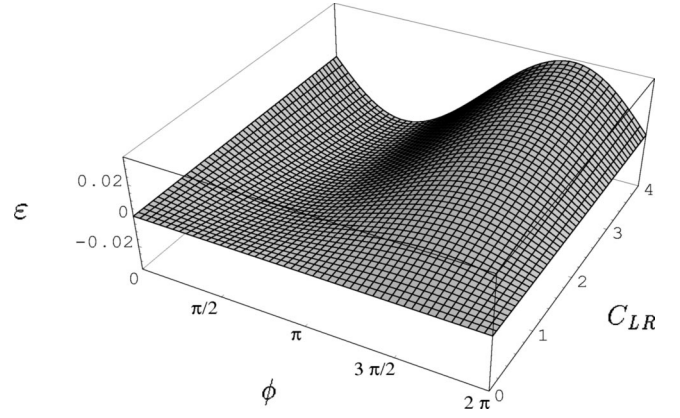
	$F(0)$	$a_F$	$b_F$
$A_1$	$0.34 \pm 0.05$	0.60	$-0.023$
$A_2$	$0.28 \pm 0.04$	1.18	0.281
$V$	$0.46 \pm 0.07$	1.55	0.575
$T_1$	$0.19 \pm 0.03$	1.59	0.615
$T_2$	$0.19 \pm 0.03$	1.49	$-0.241$
$T_3$	$0.13 \pm 0.02$	1.20	0.008

Various angles in Eq. (8) are defined as follows:  $\theta_K$  is the polar angle of the  $K$  meson in the rest frame of the  $K^*$  meson, measured with respect to the helicity axis, i.e., the outgoing direction of the  $K^*$  meson.  $\theta_l$  is the polar angle of the  $l^+$  in the dilepton rest frame, measured with respect to the helicity axis of the dilepton, and  $\varphi$  is the azimuthal angle between the two planes defined by the momenta of the decay products  $K^* \rightarrow K\pi$  and  $V \rightarrow l^+ l^-$ . Also, explicit expressions of the functions  $N_i$  are given in the Appendix.

It follows from Eq. (8) that terms with  $\sim N_8$ ,  $N_{11}$ , and  $N_{12}$  contain an imaginary part. If we rewrite Eq. (8) for the SM case, we immediately see that there are two possible sources for  $T$  violation:  $T$  violation coming from  $\text{Im } C_9^{\text{eff}} C_7^*$ , and  $T$  violation coming from  $\text{Im } C_{10} C_7^*$ .

In SM only  $C_9^{\text{eff}}$  has an imaginary part [see Eq. (3)]. Therefore we can conclude that  $T$ -odd observables could be nonzero in the processes involving strong phases or absorptive parts even without weak  $CP$  violating phase. In this work we explore the possibility of the existence of  $T$  violation due to the new weak  $CP$ -violating phases. It follows from Eq. (8) that in order to have nonvanishing  $T$  violation interactions of a new type must exist, and contributions of different new Wilson coefficients must have weak  $CP$ -violating phases.

In order to discard terms  $\sim \text{Im } C_9^{\text{eff}} C_7^*$  which give rise to  $T$  violation in the SM, we consider the following  $T$ -odd observable

FIG. 1. The dependence of the statistical significance  $\varepsilon$  on the new Wilson coefficient  $C_{LL}$  and on the weak phase  $\phi$  for the  $B \rightarrow K^* \mu^+ \mu^-$  decay.FIG. 2. The same as in Fig. 1 but for the Wilson coefficient  $C_{LR}$ .

$$\langle \mathcal{O} \rangle = \int \mathcal{O} d\Gamma, \quad (9)$$

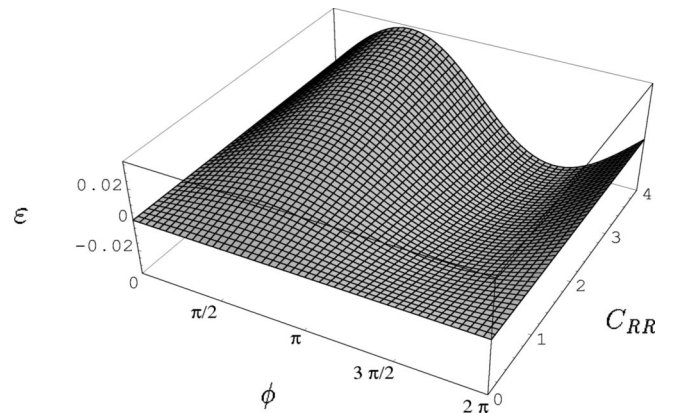
where  $\mathcal{O}$  is the  $T$ -odd correlation, given by

$$\mathcal{O} = \frac{(\mathbf{p}_B \cdot \mathbf{p}_K)[\mathbf{p}_B \cdot (\mathbf{p}_K \times \mathbf{p}_{l^+})]}{|\mathbf{p}_B|^2 |\mathbf{p}_K|^2 (q p_l + \sqrt{2})}. \quad (10)$$

In the  $K^*$  rest frame,  $\mathcal{O} = \cos \theta_K \sin \theta_K \sin \theta_l \sin \varphi$ . The statistical significance of the  $T$ -odd observable in Eq. (8) is determined by [7]

$$\varepsilon = \frac{\int \mathcal{O} d\Gamma}{\sqrt{\int d\Gamma} \sqrt{\int \mathcal{O}^2 d\Gamma}}. \quad (11)$$

It should be noted that in Eq. (11), integration over  $q^2$  is carried out in order to eliminate the  $q^2$  dependence of  $\varepsilon$ . Our final remark in this section is that  $T$ -odd effects that are related with the  $CP$  violation and  $CP$  violating asymmetry between the decay rates of  $B \rightarrow K^* l^+ l^-$  and  $\bar{B} \rightarrow \bar{K}^* l^+ l^-$  are discussed in the second reference in [10].

FIG. 3. The same as in Fig. 1 but for the Wilson coefficient  $C_{RR}$ .



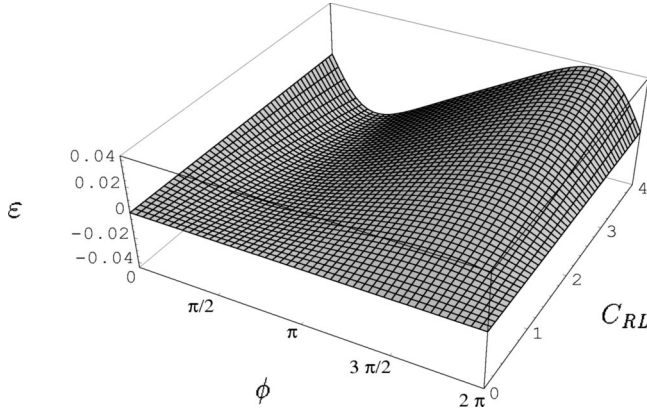


FIG. 4. The same as in Fig. 1 but for the Wilson coefficient  $C_{RL}$ .

### III. NUMERICAL ANALYSIS

In this section we will study the dependence of the statistical significance  $\varepsilon$  on the new Wilson coefficients. For the  $B \rightarrow K^*$  transition form factors, which are the main input parameters in  $\varepsilon$ , we use the light cone QCD sum rules method prediction [19–21]. The dependence of the form factors on  $q^2$  can be written in terms of the three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F(q^2/m_B^2) + b_F(q^2/m_B^2)^2}. \quad (12)$$

The value of the parameters  $F_i(0)$ ,  $a$ , and  $b$  for various form factors are presented in Table I.

In further numerical analysis, we use next-to-leading logarithmic approximation results for the values of the Wilson coefficients  $C_7$ ,  $C_9^{eff}$ , and  $C_{10}$  at  $\mu = m_b$  [12,13]. As has already been noted, in the process under consideration, that only short distance contributions are taken into account in the Wilson coefficients  $C_9^{eff}$  [see the expression for  $C_9^{eff}$  given in Eq. (3)]. The new Wilson coefficients vary in the range  $-|C_{10}| \leq C_X \leq |C_{10}|$ . The experimental bounds on the branching ratio of the  $B \rightarrow K^* \mu^+ \mu^-$  [21]<sup>1</sup> and  $B \rightarrow \mu^+ \mu^-$  decays [22] suggest that this is the right order of magnitude range for the vector and scalar interaction coefficients. The present experimental values on the branching ratio  $\mathcal{B}(B \rightarrow K l^+ l^-) = (0.78^{+0.24+0.11}_{-0.24-0.11}) \times 10^{-6}$  lead to stronger restrictions on some of the new Wilson coefficients, namely,  $-1.5 \leq C_T \leq 1.5$ ,  $-3.3 \leq C_{TE} \leq 2.6$ ,  $-2 \leq C_{LL}$ ;  $C_{RL} \leq 2.3$  while for all remaining coefficients  $-4 \leq C_X \leq 4$ . Note that if the latest results for the branching ratio for the  $B \rightarrow K^* l^+ l^-$  decay are taken into account (see the footnote below), the allowed regions of the new coefficients are  $-2.5 \leq C_{LL} \leq 0$ ,  $0 \leq C_{RL} \leq 4$ , and all remaining coefficients vary in the region  $-4 \leq C_X \leq 4$ . As has already been noted, in order to obtain considerable statistical significance  $\varepsilon$ , the new Wilson coefficients must have a new weak phase. For

<sup>1</sup>The latest result released by the BaBar Collaboration for the branching ratio of the  $B \rightarrow K^* l^+ l^-$  decay is  $\mathcal{B}(B \rightarrow K^* l^+ l^-) = (1.68^{+0.68+0.18}_{-0.58-0.18}) \times 10^{-6}$ .

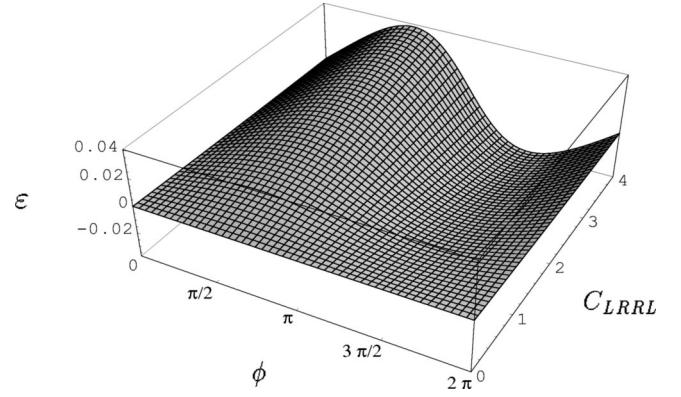


FIG. 5. The dependence of the statistical significance  $\varepsilon$  on the new Wilson coefficient  $C_{LRRL}$  and on the weak phase  $\phi$  for the  $B \rightarrow K^* \tau^+ \tau^-$  decay.

simplicity we assume that all new Wilson coefficients have a common weak phase  $\phi$ . The dependence of the  $\varepsilon$  on the Wilson coefficients  $C_{LL}$ ,  $C_{LR}$ ,  $C_{RL}$ , and  $C_{RR}$  and on the weak phase  $\phi$  for the  $B \rightarrow K^* \mu^+ \mu^-$  decay is presented in Figs. 1–4. Note that the dependence of  $\varepsilon$  on the Wilson coefficients for scalar interactions for the  $B \rightarrow K^* \mu^+ \mu^-$  decay is not presented since for all their values  $|\varepsilon|$  is very small ( $\leq 0.2\%$ ).

From these figures we see that  $\varepsilon$  gets its largest value for  $C_{LL}$  about 5%, for  $C_{LR}$  and  $C_{RR}$  about 3%, and  $C_{RL}$  about 4%, for the  $B \rightarrow K^* \mu^+ \mu^-$  decay.

The situation is quite different from the previous case for the  $B \rightarrow K^* \tau^+ \tau^-$  decay. In this case contributions coming from the scalar type interactions are dominant (see Figs. 5–8), while vector type interactions give negligibly small contributions to  $\varepsilon$ . We observe from these figures that  $\varepsilon$  gets its maximum value  $\sim 4\%$  for  $C_{LRRL}$  and  $C_{RLLR}$ . We also note that in the present work  $C_{BR}$  and  $C_{SL}$  are assumed to be identical, as is the case in the SM, since the experimentally measured branching ratio of  $B \rightarrow X_s \gamma$  decay is very close to the SM prediction [23–25].

Finally we would like to discuss the detectability of  $\varepsilon$  in the experiments. In order to observe this effect at the  $n\sigma$  level, the required number of  $B$  mesons are  $\mathcal{N}_B = n^2/(\mathcal{B}\varepsilon^2)$ . If the branching ratio takes on the following values

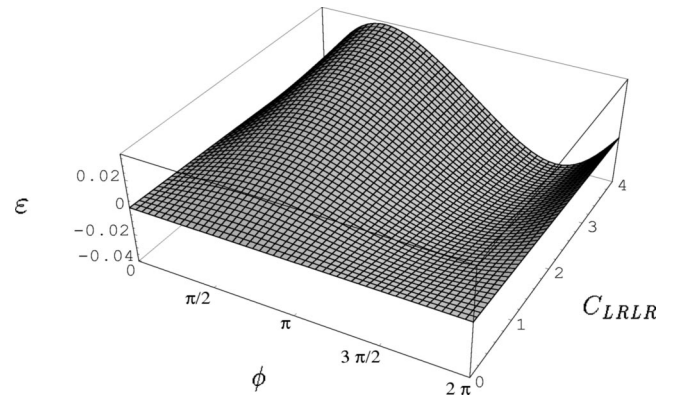


FIG. 6. The same as in Fig. 5 but for the Wilson coefficient  $C_{LRRL}$ .

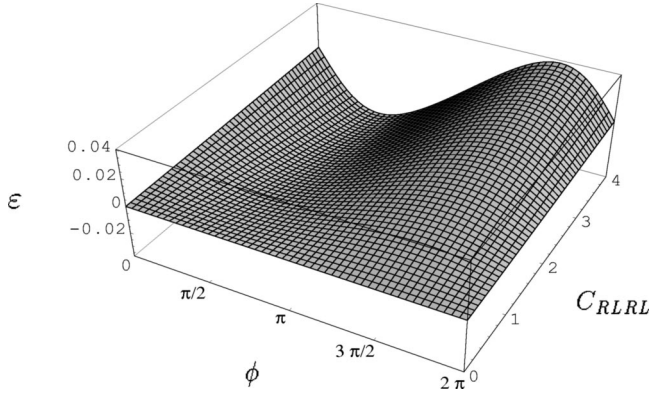


FIG. 7. The same as in Fig. 5 but for the Wilson coefficient  $C_{RLRL}$ .

$$\mathcal{B}(B \rightarrow K^* l^+ l^-) = \begin{cases} 2.0 \times 10^{-6} & \text{for } \mu \text{ mode} \\ 2.0 \times 10^{-7} & \text{for } \tau \text{ mode,} \end{cases}$$

then to be able to observe  $T$ -violating effects of  $\mathcal{O}$  in the  $B \rightarrow K^* l^+ l^-$  decay at the  $3\sigma$  level, with  $\varepsilon \sim 3\%$ , at least

$$\mathcal{N}_B = \begin{cases} 5 \times 10^9 & \text{for } \mu \text{ mode} \\ 5 \times 10^{10} & \text{for } \tau \text{ mode,} \end{cases}$$

$B$  mesons are needed. Since at LHC and BTeV machines  $10^{12} b\bar{b}$  pairs are expected to be produced per year [26], the observation of  $T$ -violating effects in the  $B \rightarrow K^* l^+ l^-$  decay is quite possible.

## APPENDIX

In this appendix we present the explicit expressions of the functions  $A$ ,  $B$ , and  $N_i$  entering into Eqs. (7) and (8):

$$A_{\pm}^{++} = \pm \sqrt{2} m_l \left\{ (C_{LL}^{tot} + C_{LR}^{tot}) H_{\pm} + \frac{2}{q^2} (C_{BR} G_{\pm} + C_{SL} g_{\pm}) + (C_{RR} + C_{RL}) h_{\pm} \right\},$$

$$A_{\pm}^{--} = -A_{\pm}^{++},$$

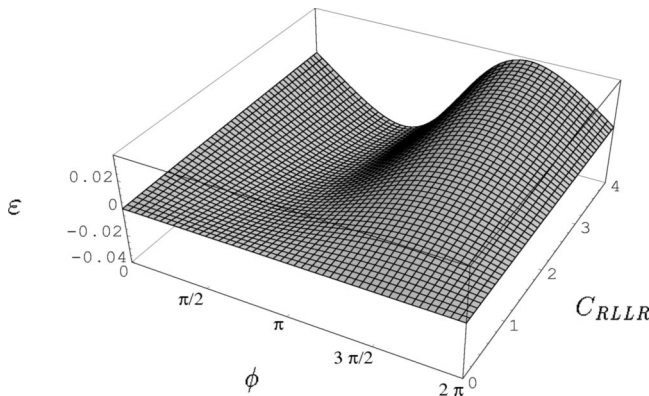


FIG. 8. The same as in Fig. 5 but for the Wilson coefficient  $C_{RLLR}$ .

$$A_{\pm}^{+-} = \sqrt{\frac{q^2}{2}} \left\{ [C_{LL}^{tot}(1-v) + C_{LR}^{tot}(1+v)] H_{\pm} + [C_{RL}(1-v) + C_{RR}(1+v)] h_{\pm} + \frac{2}{q^2} (C_{BR} G_{\pm} + C_{SL} g_{\pm}) \right\},$$

$$A_{\pm}^{-+} = A_{\pm}^{+-}(v \rightarrow -v),$$

$$A_0^{++} = 2m_l \left[ (C_{LL}^{tot} + C_{LR}^{tot}) H_0 + (C_{RL} + C_{RR}) h_0 + \frac{2}{q^2} (C_{BR} G_0 + C_{SL} g_0) \right], \quad (A1)$$

$$A_0^{--} = -A_0^{++},$$

$$B_0^{++} = -2m_l \left\{ (C_{LR}^{tot} - C_{LL}^{tot}) H_S^0 + (C_{RR} - C_{RL}) h_S^0 - \frac{2}{m_b} q^2 [(1-v)(C_{LRLR} - C_{RLLR}) - (1+v)(C_{LRRL} - C_{RLRL})] H_S^0 \right\},$$

$$B_0^{--} = B_0^{++}(v \rightarrow -v),$$

$$A_0^{+-} = -\sqrt{q^2} \left\{ [C_{LL}^{tot}(1-v) + C_{LR}^{tot}(1+v)] H_0 + [C_{RL}(1-v) + C_{RR}(1+v)] h_0 + \frac{2}{q^2} (C_{BR} G_0 + C_{SL} g_0) \right\},$$

$$A_0^{-+} = A_0^{+-}(v \rightarrow -v),$$

where  $v = \sqrt{1 - 4m_l^2/q^2}$  is the lepton velocity, superscripts denote helicities of the lepton and antilepton, and subscripts correspond to the helicities of the  $K^*$  meson, and furthermore,

$$H_{\pm} = \pm \lambda^{1/2}(m_B^2, s_M, q^2) \frac{V(q^2)}{m_B + m_{K^*}} + (m_B + m_{K^*}) A_1(q^2),$$

$$H_0 = \frac{1}{2\sqrt{s_M} q^2} \left[ -(m_B^2 - s_M - q^2)(m_B + m_{K^*}) A_1(q^2) + \lambda(m_B^2, s_M, q^2) \frac{A_2(q^2)}{m_B + m_{K^*}} \right],$$

$$H_S^0 = \frac{\lambda^{1/2}(m_B^2, s_M, q^2)}{2\sqrt{s_M} q^2} \left[ -(m_B + m_{K^*}) A_1(q^2) + \frac{A_2(q^2)}{m_B + m_{K^*}} (m_B^2 - s_M) + 2\sqrt{s_M} (A_3 - A_0) \right],$$

$$G_{\pm} = -2[\pm \lambda^{1/2}(m_B^2, s_M, q^2) T_1(q^2) + (m_B^2 - s_M) T_2(q^2)],$$

$$\begin{aligned}
G_0 &= \frac{1}{\sqrt{s_M q^2}} \left[ (m_B^2 - s_M)(s_B^2 - s_M - q^2) T_2(q^2) \right. \\
&\quad \left. - \lambda(m_B^2, s_M, q^2) \left( T_2(q^2) + \frac{q^2}{m_B^2 - s_M} T_3(q^2) \right) \right], \\
h_{\pm} &= H_{\pm}(A_1 \rightarrow -A_1), \\
h_S^0 &= H_S^0(A_1 \rightarrow -A_1, A_2 \rightarrow -A_2), \\
h_0 &= H_0(A_1 \rightarrow -A_1, A_2 \rightarrow -A_2), \\
g_{\pm} &= G_{\pm}(T_2 \rightarrow -T_2), \\
g_0 &= -G_0. \\
N_1 &= |A_0^{++}|^2 + |A_0^{--}|^2, \\
N_2 &= |A_0^{+-}|^2 + |A_0^{-+}|^2, \\
N_3 &= A_0^{++}(B_0^{++})^* + A_0^{--}(B_0^{--})^*, \\
N_4 &= |B_0^{++}|^2 + |B_0^{--}|^2,
\end{aligned} \tag{A2}$$

$$\begin{aligned}
N_5 &= |A_+^{++}|^2 + |A_+^{+-}|^2 + |A_+^{-+}|^2 + |A_+^{--}|^2, \\
N_6 &= |A_+^{+-}|^2 + |A_+^{-+}|^2 + |A_+^{++}|^2 + |A_+^{--}|^2, \\
N_7 &= |A_+^{+-}|^2 + |A_+^{-+}|^2 - |A_+^{++}|^2 - |A_+^{--}|^2, \\
N_8 &= A_+^{++}(A_+^{++})^* + A_+^{+-}(A_+^{+-})^* + A_+^{-+}(A_+^{-+})^* \\
&\quad + A_+^{--}(A_+^{--})^*, \\
N_9 &= A_0^{++}(A_+^{++} - A_+^{+-})^* - A_0^{+-}(A_+^{+-} + A_+^{++})^* \\
&\quad - A_0^{-+}(A_+^{-+} + A_+^{-+})^* + A_0^{--}(A_+^{--} - A_+^{-+})^*, \\
N_{10} &= B_0^{++}(A_+^{++} - A_+^{+-})^* + A_0^{+-}(-A_+^{+-} + A_+^{++})^* \\
&\quad + A_0^{-+}(A_+^{-+} - A_+^{-+})^* + B_0^{--}(A_+^{--} - A_+^{-+})^*, \\
N_{11} &= N_9(A_+^{++} \rightarrow -A_+^{++}, A_+^{+-} \rightarrow -A_+^{+-}, A_+^{-+} \\
&\quad \rightarrow -A_+^{-+}, A_+^{--} \rightarrow -A_+^{--}), \\
N_{12} &= N_{10}(A_+^{++} \rightarrow -A_+^{++}, A_+^{+-} \rightarrow -A_+^{+-}, A_+^{-+} \\
&\quad \rightarrow -A_+^{-+}, A_+^{--} \rightarrow -A_+^{--}).
\end{aligned} \tag{A3}$$

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