Absence of shadowing in Drell-Yan production at finite transverse momentum exchange

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Within a perturbative scalar QED model recently considered by Brodsky *et al.*, we study how leading-twist Coulomb rescatterings affect the Drell-Yan cross section at small $x = x_{\text{target}}$, and compare to the case of deep inelastic scattering at small x_B. We show that in the range where the transverse momentum transferred to the target is large compared to its minimal value $\sim \mathcal{O}(x)$, Coulomb rescatterings affect the DIS cross section but not the Drell-Yan production rate. This illustrates that the leading-twist parton distribution functions become nonuniversal when cross sections which are differential in target-related particles are considered.

I. INTRODUCTION AND SUMMARY

Within the parton model, deep inelastic lepton-nucleon scattering structure functions have been shown to measure the probability to find in the target nucleon a parton with a longitudinal momentum fraction $x_{\text{Bjorken}} = x_B$ in the infinite momentum frame [1]. This result was obtained in a theory of pions and nucleons for the strong interaction. Since, the correct theory of the strong interaction has been established to be a gauge theory, QCD. According to QCD factorization theorems [2], at leading-twist the inclusive deep inelastic scattering (DIS) and Drell-Yan (DY) cross sections (in particular) can be factorized and expressed as convolutions between quark and gluon distributions in the incoming hadron(s) and the partonic subprocess cross sections. The predictive power of factorization theorems arises from the statement that parton distribution functions are *universal* quantities, i.e., independent of the collision. The universality of parton distributions appears to be supported by the data, at least up to some accuracy. Also, the quark distribution (in the nucleon N of momentum p) probed in DIS,

$$
f_{q/N}(x_B, Q^2) = \frac{1}{8\pi} \int dy^- \exp(-i x_B p^+ y^-)
$$
$$
\times \langle N(p) | \bar{q}(y^-) \gamma^+
$$
$$
\times \mathrm{P} \exp \left[ig \int_0^{y^-} dw^- A^+(w^-) \right] q(0) | N(p) \rangle,
$$

(1)

where all fields are evaluated at equal light-cone time $y^+ = 0$ and transverse position $\vec{y}_\perp = \vec{0}_\perp$, seems directly related to the nucleon light-cone wave function in $A^+ = 0$ gauge, supporting the probabilistic interpretation of the parton distribution functions (and hence of the DIS structure functions), as in the original parton model.

But the expression (1) is incorrect in $A^+ = 0$ gauge, i.e., the quark distribution is not given by the (squared) nucleon light-cone wave function [3]. Roughly speaking, this is because in the Bjorken $\nu \to \infty$ limit, the eikonal coupling of the struck quark of momentum p_1 to the target color field A^μ satisfies $p_1 \cdot A \propto \nu A^+ \to \infty$ in all gauges, *except* $A^+ = 0$. More

precisely, although the light-cone time y^+ between the absorption and emission of the virtual photon in the forward DIS amplitude vanishes as $1/\nu$, Coulomb interactions occurring in this short time interval actually modify the DIS cross section at leading-twist in all gauges, including the light-cone $A^+ = 0$ gauge [3]. Thus in a *gauge* theory, the simple identification between parton distribution and parton probability (defined as the square of the nucleon light-cone wave function) does not hold. Although not excluded by this observation, the universality of parton distributions becomes much less intuitive. In this respect it was recently shown that single transverse spin asymmetries in semi-inclusive DIS appear at leading-twist [4], correcting previous statements [5]. This is due to the nonuniversality of spin-dependent parton distributions (in other words of the Sivers asymmetry [6]), which originates from the subtle behavior of Wilson lines under time reversal [7]. A possible correct expression in light-cone gauge for the gauge link entering the definition of spin-dependent parton distributions has recently been suggested [8].

In this context, it is important to reconsider the question of universality of spin-independent parton distributions. In the present work, I compare in a simple model the spin-independent quark distributions probed in DIS and in the Drell-Yan process at small values of x. I show that in the range of the exchanged transverse momentum $k_\perp$ responsible for leading-twist shadowing in DIS, the Coulomb rescattering corrections *a priori* modifying the DY cross section are in fact unitary. This is similar to what Bethe and Maximon found in the case of high energy bremsstrahlung and pair production [9]. Before the advent of QCD, it was also found that corrections to the parton model Drell-Yan formula are actually absent [10,11]. In the context of gauge theories, our result is an example of the nonuniversality of the leading-twist parton distributions, which arises when considering a cross section which is *differential* in the target structure. However, according to the QCD factorization theorem for *inclusive* cross sections, we expect the $k_\perp$-*integrated* quark distributions probed in DIS and DY to be identical, even though the typical $k_\perp$ contributing in both cases is different. We check in Appendix A that this identity indeed occurs within a model where the scale $\sim \mathcal{O}(x)$ is screened by a finite photon mass. But the fact that this holds in general, for
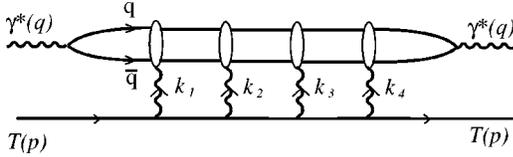
FIG. 1. Forward $\gamma^* T \to \gamma^* T$ amplitude in the DIS model of Ref. [3].

FIG. 2. Single gluon exchange DIS amplitude in scalar QED.

any target, is not obvious, and we think further studies are needed to settle (or disprove) the universality of parton distributions.

I briefly review in Sec. II the model of Brodsky *et al.* developed in Ref. [3] for DIS shadowing at small x_B. We recall that this model concentrates on the *leading-twist* shadowing correction to the DIS cross section (arising from the aligned-jet kinematic region), which can be interpreted as part of the target quark distribution function probed in DIS. The typical value $\langle k_\perp \rangle_{\rm DIS}$ of the exchanged transverse momentum is found to be of the order of a soft but x_B-independent scale $\langle k_\perp \rangle_{\rm DIS} \sim \mathcal{O}(m)$. The model is simply extended to DY production in Sec. III. Similarly to DIS, the leading-twist Coulomb corrections to the DY cross section arise from a kinematic region which we call the "aligned-photon" region (in analogy to the aligned-jet region of DIS) where the longitudinal momentum fraction taken from the incoming projectile (anti)quark by the radiated virtual photon approaches unity. Those corrections are interpreted as part of the quark distribution probed in the DY process. We find that for $Mx \ll m$, where M is the target mass and $x = x_{\rm target} \ll 1$, the DY cross section is unaffected by Coulomb rescattering at values $k_\perp \sim \mathcal{O}(m)$, contrary to the DIS cross section. This is the main result of the present paper.

This result is obtained in a scalar QED model and in the limit $x \ll 1$, which allows great technical simplications in the loop calculations. Since we neglect the scale Mx compared to $k_\perp$ from the beginning, the $k_\perp$-integrated DY cross section is out of reach in the present model. Thus we cannot exclude that the *total* DY cross section receives a nonzero leading-twist shadowing correction. However, if this happens, the typical value of $k_\perp$ responsible for this effect must be, for $x \ll 1$,

$$\langle k_\perp \rangle_{\rm DY} \sim Mx \ll \langle k_\perp \rangle_{\rm DIS}. \qquad (2)$$

This might have some implications on the properties of momentum broadening and energy loss in the Drell-Yan process. We note that the observed difference between the nuclear broadening of the average transverse momentum in DY production and in dijet photoproduction is not understood [12]. The result (2) might give some hint to this problem.

But more importantly, it might question the universality of parton distributions at small x, as we will discuss in Sec. IV. In this respect, let us note that our result, namely, the fact that Coulomb rescatterings do not modify the leading-twist DY Born cross section in the region of transverse momentum exchange $k_\perp \gg Mx$, is similar to what was found in Ref. [13]. There it was shown, for transverse momenta being large
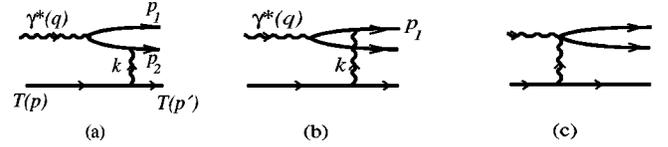
compared to infrared cutoffs (and much smaller than the collision energy), that long-distance contributions to the DY cross section cancel out at the two-loop order. This was argued to be a good indication for the validity of factorization. I stress that it makes on the contrary factorization much less evident, since in the same transverse momentum domain, Coulomb rescatterings *modify* the DIS cross section, resulting in the observed nuclear shadowing of the DIS parton distributions.

II. LEADING-TWIST SHADOWING IN DIS

A. Model for the quark distribution function

A perturbative model for leading-twist DIS shadowing has recently been studied in Ref. [3]. Before extending this model to the DY process in the next section, we recall its main features. A specific contribution to $\sigma_{\rm DIS}$ is evaluated, via the optical theorem, from the forward DIS amplitude shown in Fig. 1.

The model is perturbative and chosen to be scalar QED. One takes for the target a scalar "quark" of mass M and momentum p, and for the light "quark" and "antiquark" scalars of mass m and momenta p_1 and p_2. The couplings of the "gluons" of momenta k_i and of the incoming virtual photon of momentum q to the scalars are denoted by g and e, respectively. The forward amplitude of Fig. 1 contributes to $\sigma_{\rm DIS}$ through three different cuts between the Coulomb gluon exchanges. Calling A, B, and C the single, double, and three-gluon exchange amplitudes for the process $\gamma^*(q)T(p) \to q(p_1)\bar{q}(p_2)T(p')$, the rescattering correction of order $e^2 g^8$ to the Born term $\int |A|^2$ reads

$$\Delta \sigma_{\rm DIS} \sim \int \frac{d^2 \vec{p}_{2\perp}}{(2\pi)^2} \frac{d^2 \vec{k}_\perp}{(2\pi)^2} [|B|^2 + 2\,{\rm Re}(A^*C)]. \qquad (3)$$

Feynman diagrams contributing to A are shown in Fig. 2. The amplitudes B and C are obtained by adding to the Born amplitude A one or two gluon exchanges between the target and the light quarks. In the Bjorken limit[1] and at small x_B, $\Delta \sigma_{\rm DIS}$ receives a leading-twist contribution, arising from the aligned-jet configuration and presenting the features of a shadowing correction to the DIS Born cross section [3]. It was shown that the kinematic region where leading-twist shadowing appears reads

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[1] The Bjorken limit is defined as $q^- = 2\nu \to \infty$, $Q^2 = -q^2 \to \infty$ with $x_B = Q^2/2M\nu$ being fixed. We use the light-cone variables $q^\pm = q^0 \pm q^z$.

$$2\nu \sim p_1^- \gg p_2^- \gg k_\perp, \ k_{i\perp}, \ p_{2\perp}, \ k_i^-,$$

$$m, \ M \gg k_i^+, \ k^+, \ p_2^+ \sim \mathcal{O}(Mx_B), \quad (4)$$

where when $\nu \to \infty$ the total momentum transfer k satisfies

$$k^+ = Mx_B + p_2^+. \quad (5)$$

The kinematic limit (4) holds in the target rest frame, where in the four-momentum notation $k = (k^+, k^-, \vec{k}_\perp)$ we have

$$p = (M, M, \vec{0}_\perp),$$

$$q = (-Mx_B, 2\nu, \vec{0}_\perp),$$

$$\epsilon_L = \frac{Q}{\nu}(1, -1, \vec{0}_\perp). \quad (6)$$

In the case of scalar QED, the leading-twist contribution to σ_{DIS} arises from the light quarks coupling to a photon with longitudinal polarization ϵ_L.

The scale ν is the single hard scale in the problem, and the limit $\nu \to \infty$ is taken from the beginning. In the aligned-jet kinematics $q^- = 2\nu \simeq p_1^-$, Coulomb rescattering corrections contribute at leading-twist to the DIS cross section [14]. Compared to the scale ν, the antiquark has a soft momentum p_2^- and must be considered as part of the (soft) target dynamics [2]. (At small x_B, $p_2^- \propto 1/x_B$ can, however, become large enough, so that the physics of destructive interferences between diffractive amplitudes takes place, resulting in shadowing.)

In addition, the hard vertex $\gamma^* q \to q$ (as viewed in the infinite momentum frame) is taken at zeroth order in the strong coupling g. Hence the contribution to $\Delta\sigma_{\text{DIS}}$ arising from the domain (4) is a perturbative model for the scaling target quark distribution $f_{q/T}(x_B)$. The leading-twist contribution to $\Delta\sigma_{\text{DIS}}$ found in Ref. [3] is thus interpreted as shadowing of the quark distribution function in the target.

We stress that the momentum of the parton probed by the incoming virtual photon is $k - p_2$ [yielding $(k - p_2)^+/p^+ = x_B$]. As a consequence, in our notation the variable $k_\perp$ must be distinguished from the transverse momentum K_T of the probed parton. Our final result [see Eq. (33)] states that the universality between DIS and DY is broken when considering cross sections differential in $k_\perp$, in other words in target-related particles. We do not discuss K_{T^-} dependent parton distributions. Those could be obtained in the present context by integrating over $k_\perp$, *at fixed* $K_T = (k - p_2)_\perp$.

In order to compare the quark distributions probed in DIS and DY, we will apply the model described above to the DY process in the next section. Let us repeat before the results obtained in Ref. [3] for the DIS amplitudes *A, B, C* and for the shadowing correction $\Delta\sigma_{\text{DIS}}$.

B. DIS rescattering amplitudes

Born amplitude. At small x_B the Born amplitude for the DIS process is obtained in Feynman gauge from the dominant diagrams of Figs. 2(a) and 2(b), and in light-cone $A^+ = 0$ gauge from the diagram of Fig. 2(a) only. The gauge invariant result reads in momentum space

$$A(p_2^-, \vec{p}_{2\perp}, \vec{k}_\perp) = \frac{2eg^2 MQp_2^-}{k_\perp^2}\left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_\perp)}\right],$$

$$(7)$$

where

$$D(\vec{p}_\perp) = p_\perp^2 + m_\parallel^2, \quad (8)$$

$$m_\parallel^2 = p_2^- Mx_B + m^2. \quad (9)$$

In transverse coordinate space we have

$$\tilde{A}(p_2^-, \vec{r}_\perp, \vec{R}_\perp) = \int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2}\frac{d^2\vec{k}_\perp}{(2\pi)^2}A(p_2^-, \vec{p}_{2\perp}, \vec{k}_\perp)$$

$$\times \exp(i\vec{r}_\perp \cdot \vec{p}_{2\perp} + i\vec{R}_\perp \cdot \vec{k}_\perp)$$

$$= 2eg^2 MQp_2^- V(m_\parallel r_\perp)W(\vec{r}_\perp, \vec{R}_\perp). \quad (10)$$

The functions V and W stand, respectively, for the incoming photon wave function describing its $q\bar{q}$ content and for the $q\bar{q}$ dipole scattering amplitude:

$$V(mr_\perp) \equiv \int \frac{d^2\vec{p}_\perp}{(2\pi)^2}\frac{e^{i\vec{r}_\perp \cdot \vec{p}_\perp}}{p_\perp^2 + m^2} = \frac{1}{2\pi}K_0(mr_\perp), \quad (11)$$

$$W(\vec{r}_\perp, \vec{R}_\perp) \equiv \int \frac{d^2\vec{k}_\perp}{(2\pi)^2}\frac{1 - e^{i\vec{r}_\perp \cdot \vec{k}_\perp}}{k_\perp^2}e^{i\vec{R}_\perp \cdot \vec{k}_\perp}$$

$$= \frac{1}{2\pi}\log\left(\frac{|\vec{R}_\perp + \vec{r}_\perp|}{R_\perp}\right). \quad (12)$$

Two-gluon exchange. The gauge invariant expression of the one-loop DIS amplitude B corresponding to two gluon exchanges between the target and the light quarks is [3]

$$B(p_2^-, \vec{p}_{2\perp}, \vec{k}_\perp) = -ieg^4 MQp_2^- \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2}\frac{1}{k_{1\perp}^2 k_{2\perp}^2}$$

$$\times \left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_{1\perp})}\right.$$

$$\left. - \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_{2\perp})} + \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_\perp)}\right],$$

$$(13)$$

where $\vec{k}_{2\perp} = \vec{k}_\perp - \vec{k}_{1\perp}$. In transverse coordinate space

$$\tilde{B}(p_2^-,\vec{r}_\perp,\vec{R}_\perp) = -ie\,g^4 M Q p_2^- V(m_\| r_\perp) W^2(\vec{r}_\perp,\vec{R}_\perp)$$

$$= \frac{-ig^2}{2!} W \tilde{A}. \qquad (14)$$

Three-gluon exchange. We give the expression of the three-gluon exchange amplitude C found in Ref. [3]

$$C(p_2^-,\vec{p}_{2\perp},\vec{k}_\perp) = -\frac{1}{3} e g^6 M Q p_2^- \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2} \frac{d^2\vec{k}_{2\perp}}{(2\pi)^2} \frac{1}{k_{1\perp}^2 k_{2\perp}^2 k_{3\perp}^2}$$

$$\times \left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{3}{D(\vec{p}_{2\perp}-\vec{k}_{1\perp})} \right.$$

$$\left. + \frac{3}{D(\vec{p}_{2\perp}-\vec{k}_{1\perp}-\vec{k}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp}-\vec{k}_\perp)} \right], \qquad (15)$$

where $\vec{k}_{3\perp} = \vec{k}_\perp - \vec{k}_{1\perp} - \vec{k}_{2\perp}$. In coordinate space

$$\tilde{C}(p_2^-,\vec{r}_\perp,\vec{R}_\perp) = -\frac{1}{3} e g^6 M Q p_2^- V(m_\| r_\perp) W^3(\vec{r}_\perp,\vec{R}_\perp)$$

$$= \frac{(-ig^2)^2}{3!} W^2 \tilde{A}. \qquad (16)$$

C. The $k_\perp$ range in DIS

We stress here that the amplitudes B and C are infrared finite. This is because the quark p_1 and antiquark p_2 form a dipole, whose scattering amplitude W vanishes with the separation $r_\perp$ between the two quarks [see Eq. (12)]. Thus in Eqs. (13) and (15) the typical values of $k_{i\perp}$ are $\sim \mathcal{O}(k_\perp, p_{2\perp})$. The only other (soft) scale present being $m_\|$ given in Eq. (9), the typical value of the total exchanged transverse momentum $k_\perp$ contributing to the $k_\perp$-integrated correction $\Delta\sigma_{\text{DIS}}$ is

$$\langle k_\perp \rangle_{\text{DIS}} \sim m_\| \sim \mathcal{O}(m). \qquad (17)$$

The rescattering correction can be obtained from Eqs. (14) and (16):

$$\Delta\sigma_{\text{DIS}} \sim \int d^2\vec{r}_\perp d^2\vec{R}_\perp [|\tilde{B}|^2 + 2\tilde{A}\tilde{C}]$$

$$= -\frac{1}{3} \int d^2\vec{r}_\perp d^2\vec{R}_\perp \frac{g^4}{4} \tilde{A}^2 W^2. \qquad (18)$$

This is the leading-twist shadowing correction to the Born DIS cross section found in Ref. [3], interpreted as part of the (scalar) quark distribution $f_{q/T}(x_B)$.

III. RESCATTERING EFFECTS IN DRELL-YAN PRODUCTION

A. Model for Drell-Yan production

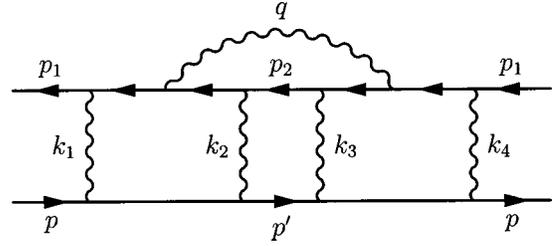We now extend the model presented previously for DIS to the Drell-Yan process. This can be done by simply exchang-

FIG. 3. Forward amplitude of order $e^2 g^8$ for the Drell-Yan process. Only one diagram is shown.

ing the virtual photon q and the quark p_1. We thus describe DY production in the target rest frame where the incoming antiquark has a large "minus" momentum component $p_1^- \simeq 2\nu$. As we will see the basic process for DY production in this frame corresponds to quark-antiquark annihilation in the infinite momentum frame.

One diagram contributing to the DY forward amplitude is represented in Fig. 3. All diagrams are simply obtained by taking into account all possible permutations of the lower and upper vertices.

The Born DY cross section will get a rescattering correction

$$\Delta\sigma_{\text{DY}} \sim \int \frac{d^2\vec{p}_{2\perp}}{(2\pi)^2} \frac{d^2\vec{k}_\perp}{(2\pi)^2} [|B_{\text{DY}}|^2 + 2\,\text{Re}(A_{\text{DY}}^* C_{\text{DY}})], \qquad (19)$$

where A_{DY}, B_{DY}, C_{DY} are the amplitudes for the process $\bar{q}(p_1) T(p) \to \gamma^*(q) \bar{q}(p_2) T(p')$ corresponding to one, two, and three-gluon exchange. In the following we will evaluate these amplitudes in the small x limit.

In the present DY case the photon momentum q is time-like, $q^2 = Q^2 > 0$ is the final lepton pair invariant mass squared, and the momenta are chosen as ($q^+ > 0$):

$$q = (+Mx, q^-, \vec{q}_\perp),$$

$$p_1 = (p_1^+, 2\nu, \vec{0}_\perp),$$

$$p = (M, M, \vec{0}_\perp), \qquad (20)$$

where

$$x = \frac{Q^2}{2M\nu}. \qquad (21)$$

It is easy to check that the configuration $p_1^- = 2\nu \simeq q^- \to \infty$, which we call the "aligned-photon" configuration by analogy to the DIS aligned-jet region, gives a leading-twist contribution to $\Delta\sigma_{\text{DY}}$. In the DY calculation the same longitudinal photon polarization vector as for DIS can be used.

In the $\nu \to \infty$ limit the total momentum transfer k still satisfies [see Eq. (5)]

$$k^+ = Mx + p_2^+. \qquad (22)$$

The relevant kinematics in the target rest frame is similar to Eq. (4),
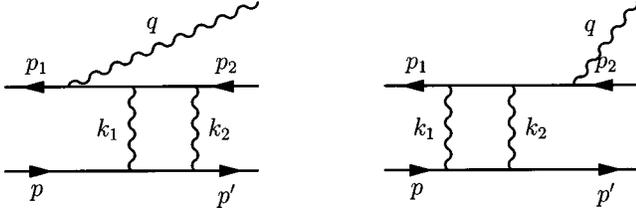
FIG. 4. Diagrams contributing dominantly to the two-gluon exchange DY amplitude in Feynman gauge and in the small x limit. Crossed diagrams, obtained by permuting the lower vertices, are included.

$$2\nu = p_1^- \gg p_2^- \gg k_\perp, \; k_{i\perp}, \; p_{2\perp}, \; k_i^-,$$

$$m, \; M, \; q_\perp \gg k_i^+, \; k^+ = Mx + p_2^+, \qquad (23)$$

where one just added the soft $q_\perp$ scale.

As in DIS, the antiquark p_2 is part of the soft target dynamics. The incoming "hadron" is modeled as a single antiquark, whose energy ν is transferred totally to the virtual photon. Thus in the present model the colliding partons from the projectile and target carry, respectively, the momentum fractions $x_1 = 1 \simeq x_F$ and $x_2 = (k^+ - p_2^+)/p^+ = x$. In the infinite momentum frame, we recover quark-antiquark annihilation as the basic partonic process for DY production.

The hard $q\bar{q} \to \gamma^*$ vertex is still taken at zeroth order in g, thus all the soft dynamics should be interpreted as part of the target quark distribution, probed at a value x of the longitudinal momentum fraction. Since the shadowing contribution found in DIS describes the target quark distribution probed at $x_B = x$, one would naively expect, assuming parton distributions to be universal, to find a rescattering correction to the DY Born cross section originating from the domain (23) equal to that of DIS.

As we will show, in the region (23) the rescattering corrections to σ_{DY} are unitary, i.e., do not modify the Born DY cross section, contrary to the DIS case. In this sense the effect of shifting the *outgoing* quark of DIS to an *incoming* antiquark in DY is drastic.

B. DY rescattering amplitudes

I now give the DY amplitudes in the small x limit. The calculation has been performed both in Feynman and light-cone $A^+ = 0$ gauge, yielding gauge-invariant results. Since different diagrams can contribute in these two gauges, for simplicity the following discussion refers to the Feynman gauge calculation.

Born amplitude. The Born amplitude for the DY process is given in Feynman gauge by the diagrams obtained by exchanging q and p_1 in Figs. 2(a) and 2(b). The result in the small x limit reads

$$A_{\text{DY}}(p_2^-, \vec{p}_{2\perp}, \vec{k}_\perp) = -\frac{2eg^2 MQp_2^-}{k_\perp^2}$$

$$\times \left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_\perp)} \right]. \quad (24)$$
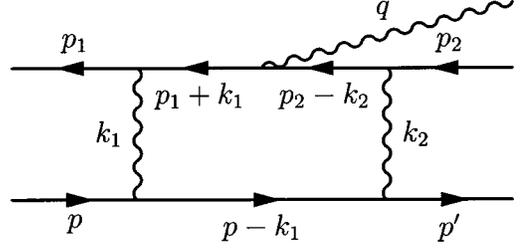

FIG. 5. A diagram (together with the contribution from the crossed one) for the two-gluon exchange DY amplitude which vanishes in the small x limit.

This is equal to the Born amplitude obtained for DIS [see Eq. (7)], up to an irrelevant sign. This sign arises since the coupling of the photon brings a factor $\epsilon_L \cdot (2p_1 - q) = Q$ for the DIS amplitude, and $\epsilon_L \cdot (q - 2p_1)$ for the DY amplitude. This is due to the fact we consider for DY an incoming *anti*quark of momentum p_1.

Two-gluon exchange. In Feynman gauge, the one-loop diagrams which dominate in the small x limit are shown in Fig. 4. The "crossed" diagrams, obtained by permuting the gluon coupling vertices to the target line, are also taken into account. We found that the diagrams where the virtual photon emission occurs between the two gluon exchanges are suppressed in this limit (see one example in Fig. 5, where the crossed diagram is also implicitly included). This suppression of radiation in DY production has been mentioned previously [15], but we stress here that it occurs only when the transferred momenta $k_{i\perp}$ are large compared to Mx, which is precisely the limit studied here [see Eq. (23)].

It is instructive to note the mathematical origin of this suppression, as it occurs in the Feynman gauge calculation. The diagram of Fig. 5 is suppressed because the poles in the (arbitrarily chosen) integration variable k_2^+ arising from the internal propagators $p_1 + k_1$ and $p_2 - k_2$ lie on the same half plane.[2] Note that the associated Feynman gauge DIS diagram (obtained by the $q \leftrightarrow p_1$ exchange) is not suppressed in the small x_B limit because the corresponding propagators read $p_1 - k_1$ and $p_2 - k_2$, yielding poles lying on different sides of the real axis. Shifting the p_1 line from final (DIS) to initial (DY) state has nontrivial analytical consequences [13].

The result for the full DY one-loop amplitude is [compare to the one-loop DIS amplitude (13)]

$$B_{\text{DY}}(p_2^-, \vec{p}_{2\perp}, \vec{k}_\perp) = -ieg^4 MQp_2^- \left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_\perp)} \right]$$

$$\times \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2} \frac{1}{k_{1\perp}^2 k_{2\perp}^2}. \quad (25)$$

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[2] In $A^+ = 0$ gauge the diagram of Fig. 5 can contribute, depending on the prescription which is used to regularize the spurious $k_i^+ = 0$ pole of the gluon propagator in this gauge. The present discussion concerning the location of *physical* poles holds in any gauge, but in $A^+ = 0$ gauge, a finite value for the diagram of Fig. 5 arises when one uses for instance the principal value prescription, because this prescription involves spurious poles on both sides of the real axis.

The infrared sensitivity of the amplitude B_{DY} will be discussed below.

Three-gluon exchange. Similarly to the one-loop case, radiation within the target is suppressed in the region (23), where $k_{i\perp} \gg Mx$. In Feynman gauge only two diagrams (including obvious permutations) contribute to the two-loop amplitude, corresponding to the three exchanges occurring all before or all after the virtual photon emission. The result reads [compare to the DIS amplitude (15)]

$$C_{DY}(p_2^-,\vec{p}_{2\perp},\vec{k}_\perp) = \frac{1}{3}eg^6MQp_2^-\left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp}-\vec{k}_\perp)}\right]$$

$$\times \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^2}\frac{1}{k_{1\perp}^2 k_{2\perp}^2 k_{3\perp}^2}. \quad (26)$$

C. Absence of DY shadowing for $k_\perp \gg Mx$

We now discuss the expressions (25) and (26) for the DY loop amplitudes. Contrary to the case of DIS, they show an infrared sensitivity when $k_{i\perp} \to 0$. This infrared singularity is absent in the cross section, the Coulomb phase originating from scattering between charged particles cancelling between the production amplitude and its conjugate. However the *total* DY cross section is out of reach within the present approximation (23). Indeed, the amplitudes have been evaluated with the assumption $k_{i\perp}$, $k_\perp \gg Mx$, and their precise infrared behavior can thus not be inferred. However, as we will see now the *partial* contribution to the cross section originating from $k_\perp \gg Mx$ can be obtained. We will show that this contribution actually vanishes (at order e^2g^8).

For a finite $k_\perp \gg Mx$ the expression (24) for the Born amplitude is valid and Eqs. (25) and (26) can be written as

$$B_{DY} = i\frac{g^2}{2}A_{DY}\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2}\frac{k_\perp^2}{k_{1\perp}^2 k_{2\perp}^2}, \quad (27)$$

$$C_{DY} = -\frac{g^4}{6}A_{DY}\int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2}\frac{d^2\vec{k}_{2\perp}}{(2\pi)^2}\frac{k_\perp^2}{k_{1\perp}^2 k_{2\perp}^2 k_{3\perp}^2}. \quad (28)$$

One gets for the rescattering correction to the Born term

$$\frac{d\Delta\sigma_{DY}}{d^2\vec{p}_{2\perp}d^2\vec{k}_\perp} \propto |B_{DY}|^2 + 2A_{DY}C_{DY} = |A_{DY}|^2 F(k_\perp^2), \quad (29)$$

$$F(k_\perp^2) \equiv \frac{g^4}{4(2\pi)^4}k_\perp^4\left\{[R_2(k_\perp)]^2 - \frac{4}{3}R_{13}(k_\perp)\right\}, \quad (30)$$

where

$$R_2(k_\perp) = \int \frac{d^2\vec{k}_{1\perp}}{k_{1\perp}^2(\vec{k}_\perp-\vec{k}_{1\perp})^2},$$

$$R_{13}(k_\perp) = \frac{1}{k_\perp^2}\int \frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}}{k_{1\perp}^2 k_{2\perp}^2(\vec{k}_\perp-\vec{k}_{1\perp}-\vec{k}_{2\perp})^2}. \quad (31)$$

Let us stress that in the small x limit, Eq. (29) is correct for any finite $k_\perp$, since in Eq. (29) the momenta $k_{i\perp}$ flowing in the loops are also large, $k_{i\perp} \gg Mx$. Indeed, although the individual amplitudes are infrared singular, in dimensional regularization one obtains the nontrivial result (see, for instance, Ref. [3], where the same expression appeared in another context)

$$k_\perp \neq 0 \Rightarrow F(k_\perp^2) \propto [R_2(k_\perp)]^2 - \frac{4}{3}R_{13}(k_\perp) = 0. \quad (32)$$

The fact that $F(k_\perp^2)$ is infrared finite shows that the typical values of $k_{i\perp}$ in the loop integrals of Eq. (31) are of order $k_\perp$, the only scale at disposal. This justifies the approximation $k_{i\perp} \gg Mx$ used to evaluate the loop amplitudes. But since moreover $F(k_\perp^2)=0$ for any finite $k_\perp$, only small $k_\perp \sim Mx \to 0$ may contribute to the $k_\perp$-integrated correction $\Delta\sigma_{DY}$.

We obtain here the main result of this paper. For a fixed $k_\perp$ satisfying $k_\perp \gg Mx$, the rescattering correction (of relative order g^4) to the DY Born cross section vanishes:

$$k_\perp \gg Mx \Rightarrow \frac{d\Delta\sigma_{DY}}{dk_\perp^2} = 0. \quad (33)$$

This is in contrast with the DIS situation, where $k_\perp \sim m_\| \gg Mx$ contributes to the $\mathcal{O}(g^4)$ correction to σ_{DIS}. These features are similar to what was found by Bethe and Maximon for high energy pair production and bremsstrahlung [9]. At momentum transfers much larger than their minimal value, Coulomb rescatterings modify the Born cross section for pair production, but not for bremsstrahlung. The absence of corrections to the parton model DY formula was also found in a pre-QCD context [10,11]. As emphasized in the end of Sec. II A, our result (33) [or Eq. (2)] does not mean that K_T-dependent parton distributions are nonuniversal [since here $K_T = (k-p_2)_\perp \neq k_\perp$], but that universality is broken when considering cross sections which are differential in $k_\perp$, i.e., differential in the target substructure.

IV. DISCUSSION

We showed within a simple abelian model that whereas for $k_\perp \sim m_\|$ the DIS cross section gets a shadowing correction, Coulomb rescatterings do not modify the DY Born cross section at similar $k_\perp$. It is still possible that the $k_\perp$-*integrated* DY rescattering correction (29) could equal the result (18) found in DIS, in agreement with universality. But since the approximation (23) we used breaks down for $k_\perp \sim Mx$, we cannot integrate Eq. (29) down to such small $k_\perp$ values and thus cannot answer this question. Calculating the DY amplitudes beyond the small x limit would be much more involved. In particular, for $k_\perp \sim Mx$ radiation in between Coulomb scatterings is not suppressed.

Since $k_\perp \gg Mx$ induces unitary Coulomb corrections to the Drell-Yan cross section, any nonvanishing contribution of order e^2g^8 to $\Delta\sigma_{DY}$ must arise from the domain $k_\perp \sim Mx$, as stated in Eq. (2). The fact that different $k_\perp$ ranges in DIS and DY could then sum up to identical total cross sections for *any* target is not obvious. In the case of a totally

screened target, with inverse screening length Λ, the values $k_\perp \sim Mx$ are forbidden if $Mx \ll \Lambda$. One thus expects, for such values of x, Coulomb rescatterings to affect the DIS cross section but not the DY one (in the leading-twist regions of interest). Relying on Eq. (2), we thus suggest that the nucleon quark distribution functions probed in DIS and DY might become nonuniversal when $M_N x \ll \Lambda$, with M_N the nucleon mass and $\Lambda \sim \Lambda_{\text{QCD}}$. If the above argument based on the presence of screening in a realistic target is valid, we roughly estimate that the violation of universality could set in when the Ioffe time of the photon $\nu/Q^2 = 1/(2M_N x)$ becomes larger than $1/\Lambda_{\text{QCD}}$, i.e., when $x < 0.1$.

We found instructive to supplement our scalar QED model with a mass term for the exchanged Coulomb photons. Calling λ the photon mass, and considering the limit $\lambda \gg Mx$, the DY production amplitudes in this modified model are simply obtained by the replacements $k_\perp^2 \rightarrow k_\perp^2 + \lambda^2$ in Eq. (24) and $k_{i\perp}^2 \rightarrow k_{i\perp}^2 + \lambda^2$ in Eqs. (25) and (26). Then Eq. (29) can be integrated over the whole $k_\perp$ range, the photon mass λ acting effectively as an infrared cutoff. We show in Appendix A that in this specific case the integrated DY cross section $\Delta\sigma_{\text{DY}}$ is identical [see Eq. (A3)] to that of DIS given in Eq. (18). We also show that the typical value of $k_\perp$ is of order λ. This illustrates that when $k_\perp$ can reach its minimal value (λ in the present case), the two different $k_\perp$ ranges in DIS and DY *might* give equal total contributions. Let us mention that a similar result was found by Bethe and Maximon for pair production and bremsstrahlung, in the case of an unscreened target [9].

This somewhat academic calculation may help understanding why the universality of the quark distribution was claimed to hold in Refs. [15,16]. In these papers the DIS and DY cross sections depend on the same nonperturbative parameter (to be interpreted as the quark distribution, see in particular Ref. [15]), namely, the quark pair dipole cross section in the target, expressed in impact parameter space. It is what we find here [see the comments following Eq. (A3)], but in the very particular case of an unscreened pointlike target and for a finite photon mass $\lambda \gg Mx$. The fact that only small $k_\perp \sim \lambda$ contributes [see Eq. (A7)] would appear difficult to infer in a coordinate space approach. One indeed finds that the typical value of the impact parameter in Eq. (A3) is $\langle R_\perp \rangle \sim 1/m_\parallel$. However the dominance of small $k_\perp \ll m_\parallel$ for DY can be seen in our momentum space calculation, as expressed in Eq. (A7). We explain in Appendix A why the relation $\langle k_\perp \rangle_{\text{DY}} \ll 1/\langle R_\perp \rangle$ is possible. (In particular we do not contradict the uncertainty principle.) This point may have been overlooked in previous coordinate space approaches. We show in Appendix B that the derivation of the color dipole formulation of the Drell-Yan process [15–17] relies implicitly on the particular limit studied in Appendix A, namely, $\lambda \gg Mx$. Apparently no general proof, valid in the realistic limit $\lambda \rightarrow 0$ at fixed Mx, is known.

The result (2) is demonstrated in the present paper by comparing leading-twist Coulomb rescattering corrections in DIS and DY in a model with a pointlike target, but which, however, contains the relevant features of nuclear shadowing [3]. Our arguments indicate that for a realistic target, leading-

twist nuclear shadowing in DY might be reduced compared to shadowing in DIS. The data on DIS [18,14] and DY [19,20] shadowing seem to be reasonably consistent with the assumption that nuclear leading-twist quark distributions are universal, and any possible violation of universality can thus not be too large. But the difficulty to disentangle valence and sea quark shadowing, as well as quark energy loss effects [21] makes phenomenological analyses particularly intricate. We think that a possible violation of universality at small x is not ruled out by the existing data.

APPENDIX A: THE PARTICULAR LIMIT $\lambda \gg Mx$

Here we show that in the particular case where the Coulomb photons are given a finite mass $\lambda \gg Mx$, Coulomb rescatterings affect identically the *total* DIS and DY cross sections, in agreement with universality.[3] In this modified scalar QED model, the DY production amplitudes (24), (25), and (26) are regularized in the infrared by $k_{i\perp}^2 \rightarrow k_{i\perp}^2 + \lambda^2$ (and denoted by the subscript λ), and become in transverse coordinate space

$$\widetilde{A}_{\text{DY}}^\lambda(p_2^-, \vec{r}_\perp, \vec{R}_\perp) = -2eg^2 MQp_2^- V(m_\parallel r_\perp)$$
$$\times [G(R_\perp) - G(|\vec{R}_\perp + \vec{r}_\perp|)],$$

$$\widetilde{B}_{\text{DY}}^\lambda(p_2^-, \vec{r}_\perp, \vec{R}_\perp) = -ieg^4 MQp_2^- V(m_\parallel r_\perp)$$
$$\times [G(R_\perp)^2 - G(|\vec{R}_\perp + \vec{r}_\perp|)^2],$$

$$\widetilde{C}_{\text{DY}}^\lambda(p_2^-, \vec{r}_\perp, \vec{R}_\perp) = +\frac{eg^6}{3} MQp_2^- V(m_\parallel r_\perp)$$
$$\times [G(R_\perp)^3 - G(|\vec{R}_\perp + \vec{r}_\perp|)^3],$$

$$G(R_\perp) \equiv \int \frac{d^2\vec{k}_\perp}{(2\pi)^2} \frac{e^{i\vec{R}_\perp \cdot \vec{k}_\perp}}{k_\perp^2 + \lambda^2} = \frac{1}{2\pi} K_0(\lambda R_\perp). \tag{A1}$$

Using the above expressions one obtains

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[3]We stress that the target, being a scalar *charged* quark, is still unscreened. For a *neutral* target, the screening scale Λ (intrinsic to the target form factor) and the photon mass λ would have *a priori* no reason to be identical.

$$\Delta\sigma_{\mathrm{DY}}=\int d^2\vec{r}_\perp d^2\vec{R}_\perp[|\tilde{B}^\lambda_{\mathrm{DY}}|^2+2\tilde{A}^\lambda_{\mathrm{DY}}\tilde{C}^\lambda_{\mathrm{DY}}]$$

$$=-\frac{(eg^4MQp_2^-)^2}{3}\int d^2\vec{r}_\perp d^2\vec{R}_\perp V(m_\| r_\perp)^2$$

$$\times[G(R_\perp)-G(|\vec{R}_\perp+\vec{r}_\perp|)]^4. \qquad (A2)$$

Assuming $\lambda\ll m_\|$ the typical value of $R_\perp$ contributing to Eq. (A2) is $\langle R_\perp\rangle\sim 1/m_\|\sim\langle r_\perp\rangle$. Using $K_0(x)\simeq\log(1/x)$ for $x\ll 1$ we get

$$\Delta\sigma_{\mathrm{DY}}=-\frac{1}{3}\int d^2\vec{r}_\perp d^2\vec{R}_\perp\frac{g^4}{4}\tilde{A}^2W^2 \qquad (A3)$$

where $\tilde{A}$ and W are given in Eqs. (10) and (12). Comparing Eq. (A3) to Eq. (18) one sees that in this particular case (finite photon mass $\lambda\gg Mx$) the leading-twist Coulomb corrections to the *total* DIS and DY cross sections are identical. Note also that similarly to Refs. [15,16], the same quantity W^2 (the quark pair dipole rescattering cross section) appears in both DIS and DY results.

We now show that whereas $\langle k_\perp\rangle_{\mathrm{DIS}}\sim m_\|$ contributes to the DIS cross section (18), $\langle k_\perp\rangle_{\mathrm{DY}}\sim\lambda\ll m_\|$ contributes to Eq. (A3). Equations (29), (30), and (31) are modified according to

$$\frac{d\Delta\sigma_{\mathrm{DY}}}{d^2\vec{p}_{2\perp}d^2\vec{k}_\perp}\propto|B^\lambda_{\mathrm{DY}}|^2+2A^\lambda_{\mathrm{DY}}C^\lambda_{\mathrm{DY}}=|A^\lambda_{\mathrm{DY}}|^2F^\lambda(k_\perp^2), \qquad (A4)$$

$$F^\lambda(k_\perp^2)\equiv\frac{g^4}{4(2\pi)^4}(k_\perp^2+\lambda^2)^2\left\{[R^\lambda_2(k_\perp)]^2-\frac{4}{3}R^\lambda_{13}(k_\perp)\right\}, \qquad (A5)$$

where

$$R^\lambda_2(k_\perp)=\int\frac{d^2\vec{k}_{1\perp}}{(k_{1\perp}^2+\lambda^2)[(\vec{k}_\perp-\vec{k}_{1\perp})^2+\lambda^2]},$$

$$R^\lambda_{13}(k_\perp)=\frac{1}{(k_\perp^2+\lambda^2)}\int$$

$$\times\frac{d^2\vec{k}_{1\perp}d^2\vec{k}_{2\perp}}{(k_{1\perp}^2+\lambda^2)(k_{2\perp}^2+\lambda^2)[(\vec{k}_\perp-\vec{k}_{1\perp}-\vec{k}_{2\perp})^2+\lambda^2]}. \qquad (A6)$$

For $k_\perp\gg\lambda$ a difficult calculation yields

$$k_\perp\gg\lambda\Rightarrow[R^\lambda_2(k_\perp)]^2-\frac{4}{3}R^\lambda_{13}(k_\perp)\sim\mathcal{O}\left[\frac{\lambda^2}{k_\perp^6}\log^2\left(\frac{k_\perp}{\lambda}\right)\right]. \qquad (A7)$$

This latter equation expresses within a mass regularization scheme the result (32) obtained in dimensional regularization. Thus for $\lambda\ll k_\perp\ll p_{2\perp}$, the integrand (A4) behaves as $\sim\lambda^2/k_\perp^4$ (since $A^\lambda_{\mathrm{DY}}\propto\vec{k}_\perp/k_\perp^2$ in this range) and the contribu-

tion from $k_\perp\gg\lambda$ is suppressed in the $k_\perp$-integrated quantity $\Delta\sigma_{\mathrm{DY}}$. We conclude that $\langle k_\perp\rangle\sim\lambda\ll m_\|$ dominates in $\Delta\sigma_{\mathrm{DY}}$. In the present particular case ($\lambda\neq 0$), this is reminiscent from the result $\langle k_\perp\rangle_{\mathrm{DY}}\sim Mx\ll m_\|$ we derived in Sec. III.

The fact that $\langle R_\perp\rangle\sim 1/m_\|$ and $\langle k_\perp\rangle\sim\lambda\ll m_\|$ does not contradict the uncertainty principle. One can easily see that the one-loop amplitude B^λ_{DY} appearing in Eq. (A4) behaves as $(\vec{k}_\perp/k_\perp^2)\log(k_\perp^2/\lambda^2)$ for $\lambda\ll k_\perp\ll m_\|$. Hence $\int d^2\vec{k}_\perp|B^\lambda_{\mathrm{DY}}(\vec{k}_\perp)|^2$ is dominated by the *logarithmic* range $\lambda\ll k_\perp\ll m_\|$. In impact parameter space the expression $\int d^2\vec{R}_\perp|\tilde{B}^\lambda_{\mathrm{DY}}(\vec{R}_\perp)|^2$ is dominated by the logarithmic interval $1/m_\|\ll R_\perp\ll 1/\lambda$, as can be seen from the expression of $\tilde{B}^\lambda_{\mathrm{DY}}$ given in Eq. (A1), and as expected from the uncertainty principle. A similar conclusion is obtained for the term $\sim A^\lambda_{\mathrm{DY}}C^\lambda_{\mathrm{DY}}$ in Eq. (A4). However, performing the sum $|B|^2+2AC$ suppresses the regions $k_\perp\gg\lambda$ in momentum space [see Eq. (A7)], and $R_\perp\gg 1/m_\|$ in coordinate space [see Eqs. (A2) and (A3)]. As a result $\langle k_\perp\rangle\sim\lambda$ and $\langle R_\perp\rangle\sim 1/m_\|$. This does not contradict the uncertainty principle because the function $\sqrt{|\tilde{B}|^2+2\tilde{A}\tilde{C}}$ is obviously not the Fourier transform of $\sqrt{|B|^2+2AC}$. We note that $\langle k_\perp\rangle\ll 1/\langle R_\perp\rangle$ is possible thanks to the presence of two different scales $m_\|$ and λ and to the logarithmic spread in the separate contributions from $|B|^2$ and $2AC$ to the DY cross section. This feature is absent in DIS. The DIS amplitudes are not infrared sensitive and thus only the scale $m_\|$ is relevant.

APPENDIX B: DIPOLE FORMULATION OF THE DRELL-YAN PROCESS

In this appendix we discuss more precisely the correspondence between our model calculation and the dipole formulation of the Drell-Yan process [15–17]. We first note that this formulation was originally proposed in Ref. [16], on the basis of a *Born* calculation. The Born DY diagrams obtained from Figs. 2(a) and 2(b) by the $q\leftrightarrow p_1$ exchange were calculated in impact parameter space in Ref. [16]. In the present scalar QED model the Fourier transform of Eq. (24) reads

$$\tilde{A}_{\mathrm{DY}}(p_2^-,\vec{r}_\perp,\vec{R}_\perp)=-2eg^2MQp_2^-V(m_\| r_\perp)W(\vec{r}_\perp,\vec{R}_\perp). \qquad (B1)$$

The Born DY cross section is thus

$$\sigma^{\mathrm{Born}}_{\mathrm{DY}}\sim\int d^2\vec{r}_\perp d^2\vec{R}_\perp|\tilde{A}_{\mathrm{DY}}|^2$$

$$=4(eg^2MQp_2^-)^2\int d^2\vec{r}_\perp d^2\vec{R}_\perp V^2W^2. \qquad (B2)$$

At the Born level, the DIS and DY cross sections are identical because the DIS and DY Born amplitudes given in Eqs. (7) and (24) are the same (up to a sign). Thus the DIS quark pair dipole scattering cross section W^2 appears in Eq. (B2). We thus recover in a simple framework the dipole formulation (obtained at the Born level) of the Drell-Yan process proposed in Ref. [16].

In order to see what happens at higher orders, we sum over any number of Coulomb rescatterings. An obvious generalization of Eqs. (25) and (26) yields

$$\mathcal{M}_{DY} = A_{DY} + B_{DY} + C_{DY} + \cdots \tag{B3}$$

$$= 2ieMQp_2^- \left[\frac{1}{D(\vec{p}_{2\perp})} - \frac{1}{D(\vec{p}_{2\perp} - \vec{k}_\perp)} \right] \Delta(\vec{k}_\perp), \tag{B4}$$

$$\Delta(\vec{k}_\perp) = \frac{ig^2}{k_\perp^2} - \frac{g^4}{2!} \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2} \frac{1}{k_{1\perp}^2 k_{2\perp}^2}$$

$$- \frac{ig^6}{3!} \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2} \frac{d^2\vec{k}_{2\perp}}{(2\pi)^2} \frac{1}{k_{1\perp}^2 k_{2\perp}^2 k_{3\perp}^2} + \cdots, \tag{B5}$$

where $\vec{k}_{1\perp} + \cdots + \vec{k}_{n\perp} = \vec{k}_\perp$ in the denominators $k_{1\perp}^2 \cdots k_{n\perp}^2$ appearing in Eq. (B5). As already discussed in Sec. III C, the expression (B5) arises from the assumption $k_\perp$, $k_{i\perp} \gg Mx$ used to evaluate the Born and loop amplitudes, and contains infrared singularities when $k_{i\perp} \to 0$. Thus integrating over $\vec{k}_\perp$, as well as Fourier transforming Eq. (B4) to impact parameter space cannot be done, since the physical infrared regulators $\sim Mx$ have been neglected. Regularizing Eq. (B5) by $k_{i\perp}^2 \to k_{i\perp}^2 + \lambda^2$ with the *same* parameter λ in all denominators amounts to implicitly assume $\lambda \gg Mx$, as was done in Appendix A. After this regularization is done in Eq. (B5), Fourier transforming Eq. (B4) gives

$$\tilde{\mathcal{M}}_{DY}(p_2^-, \vec{r}_\perp, \vec{R}_\perp) = 2ieMQp_2^- V(m_\parallel r_\perp)\{\exp[ig^2 G(R_\perp)]$$

$$- \exp[ig^2 G(|\vec{R}_\perp + \vec{r}_\perp|)]\}, \tag{B6}$$

where $G(R_\perp)$ is given in Eq. (A1). Equation (B6) is similar to what was obtained in Refs. [15,17]. The first and second terms arise from photon radiation respectively after and before the Coulomb exchanges, and involve quark impact parameters which are shifted by the amount $\vec{r}_\perp$. We stress here that these terms are infrared singular when $\lambda \to 0$. Squaring Eq. (B6) one gets

$$\int d^2\vec{r}_\perp d^2\vec{R}_\perp |\tilde{\mathcal{M}}_{DY}|^2 = (4eMQp_2^-)^2 \int d^2\vec{r}_\perp d^2\vec{R}_\perp V(m_\parallel r_\perp)^2$$

$$\times \sin^2\left[\frac{g^2}{2} [G(R_\perp) - G(|\vec{R}_\perp + \vec{r}_\perp|)] \right]. \tag{B7}$$

This is infrared finite when $\lambda \to 0$, and equals the result obtained in DIS to all orders in g [3]:

$$\int d^2\vec{r}_\perp d^2\vec{R}_\perp |\tilde{\mathcal{M}}_{DY}|^2 \xrightarrow[\lambda \to 0]{} \int d^2\vec{r}_\perp d^2\vec{R}_\perp |\tilde{A}_{DY}|^2$$

$$\times \left[\frac{\sin(g^2 W/2)}{(g^2 W/2)} \right]^2. \tag{B8}$$

In particular the term of order g^8 in the expansion of Eq. (B8) reproduces Eq. (18) [and Eq. (A2)]. The identity of the DY and DIS cross sections found here arises from first assuming $\lambda \gg Mx$, and then taking the $\lambda \to 0$ limit. This procedure was implicitly used in Refs. [15,17]. In these works the replacement $G(R_\perp) - G(|\vec{R}_\perp + \vec{r}_\perp|) \xrightarrow[\lambda \to 0]{} W(\vec{r}_\perp, \vec{R}_\perp)$ [see Eqs. (B7) and (B8)] is made without mentioning that $G(R_\perp)$ is a well-defined quantity only in the presence of $\lambda \neq 0$. Thus the dipole formulation of the DY process [15–17] is in fact established beyond the Born approximation within the limit $\lambda \gg Mx$. Independently of the question whether this formulation holds also in the limit $\lambda \to 0$ at fixed Mx, we stress that the very expression (B6) is obtained from an integration over *all* $\vec{k}_\perp$'s down to very small values $\sim \lambda$, as shown in Appendix A. That such an expression can be obtained in general, for any realistic neutral target, has to our knowledge not been proven.

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