Reanalysis of baryon magnetic moments using the effective mass and screened charge of quarks

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Magnetic moments of octet and Ω^- baryons have been calculated employing the concepts of effective mass and screened charge. The magnetic moments obtained compare well with their experimental counterparts.

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I. INTRODUCTION

There have been recent measurements [1] of baryon magnetic moments with good precision. But there are discrepancies between the known magnetic moments and the theoretical predictions given in column 2 of Table I [2]. Various efforts have been made to improve the situation by including effects such as symmetry breaking [3], configuration mixing [4], the chiral quark model [5], meson cloud corrections [6], the Skyrmion model [7], and the QCD based quark model with loop corrections [8]. Sogami and Oh'yamaguchi [9] proposed for the first time an interesting prescription for the baryon magnetic moment. They suggested that, when a quark is probed by a soft photon, the effective mass of the quark gets modified due to its interaction with other quarks inside the baryon. Further, Verma and Khanna [10] investigated the concept of the effective mass of a quark due to its interaction with the neighboring quarks by the single gluon exchange mechanism. Theoretical efforts have also been made to investigate baryon moments through expansions in relativistic field theory [11] and heavy baryon chiral perturbation theory (HBChPT) [12]. The calculation of baryon magnetic moments from QCD is hard at present. However, a general expansion for the mass and magnetic moment operator has been performed [11]. It has been shown that the resulting parametrization is approximated well by a nonrelativistic constituent quark model for both the baryon masses and magnetic moments. Further, it has been established that the success of the additive quark model results from the dominance of the one-body operator over the perturbative two- and three-body operators. The latter are proportional perturbatively to the relatively small spin-spin interaction [12]. In this paper, we reinvestigate the baryon magnetic moments using the concepts of effective mass and effective quark charge. We obtain improved agreement with experiment.

II. EFFECTIVE QUARK MASS AND BARYON MAGNETIC MOMENT

Following the formalism developed by Verma and Khanna [10], the baryon mass is taken to be the sum of the quark masses plus a spin-dependent hyperfine interaction:

$$M_B=\sum_i m_i+\sum_{i<j} b_{ij}\mathbf{s}_i\cdot\mathbf{s}_j, \tag{1}$$

where

$$b_{ij}=16\pi\alpha_s/(9m_im_j)\langle\Psi_0|\delta^3(\mathbf{r})|\Psi_0\rangle. \tag{2}$$

$\mathbf{s}_i$ and $\mathbf{s}_j$ are the spin operators of the ith quark and jth quarks and Ψ_0 is the baryon wave function. Any spin-independent interaction is approximated by the normalization of the quark masses. Thus the mass of the quark in the baryon B (123) gets modified due to its interaction with other quarks. When quarks 1 and 2 are identical, one can write

$$m_1^{\mathrm{eff}}=m_2^{\mathrm{eff}}=m+\alpha b_{12}+\beta b_{13},$$
$$m_3^{\mathrm{eff}}=m_3+2\beta b_{13}, \tag{3}$$

where

$$m_1=m_2=m,\quad b_{13}=b_{23}. \tag{4}$$

The α and β parameters are calculated through

$$M_B=\sum_i m_i^{\mathrm{eff}}=\sum_i m_i+\sum_{i<j} b_{ij}\mathbf{s}_i\cdot\mathbf{s}_j$$
$$=2m+m_3+b_{12}/4-b_{13}, \tag{5}$$

where

$$\mathbf{s}_1\cdot\mathbf{s}_2=1/4,\quad \mathbf{s}_1\cdot\mathbf{s}_3=\mathbf{s}_2\cdot\mathbf{s}_3=-1/2.$$

The parametrization used here seems to go beyond the leading order in quark mass splitting because the m_i term appears as $1/m_im_j$ through the hyperfine interaction. However, higher order effects are at least partially absorbed in the nonlinear fitting of the m_i. It has been shown [11,12] that contributions from new nonlinear terms must be small because the fitted masses satisfy the Gell-Mann–Okubo mass formula, which is exact to leading order in the quark mass splitting. Therefore, the effective quark mass defined here is equivalent to first order in baryon mass splitting to the leading order parametrization of the baryon masses in chiral perturbation theory [12]. Equation (5) gives

$$\alpha=1/8\quad\text{and}\quad\beta=-1/4, \tag{6}$$

so that

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TABLE I. Baryon magnetic moments.

Baryon	Naive quark model	Effective mass scheme	Screening effect SU(3) symmetry	Ansatz Eq. (17)	Experimental values (nuclear magneton)
P	2.79	2.86	2.79	2.79	2.793
N	-1.86	-2.01	-1.91	-1.86	-1.913
Λ	-0.61	-0.58	-0.61	-0.61	-0.613 ± 0.004
Σ^+	2.68	2.56	2.56	2.46	2.458 ± 0.010
Σ^-	-1.04	-0.96	-0.97	-1.10	-1.160 ± 0.025
Ξ^0	-1.44	-1.50	-1.52	-1.25	-1.250 ± 0.014
Ξ^-	-0.51	-0.46	-0.57	-0.65	-0.6507 ± 0.0025
$\Sigma^0\rightarrow\Lambda^0\gamma$	-1.61	-1.68	-1.65	-1.61	-1.61 ± 0.08
Ω	-1.84	-1.67	-1.94	-2.08	-2.02 ± 0.05

$$m_1^{\rm eff}=m_2^{\rm eff}=m+b_{12}/8-b_{13}/4,$$

$$m_3^{\rm eff}=m_3-b_{13}/2. \tag{7}$$

For the Λ^0 baryon it turns out that

$$m_s^{\rm eff}=m_s, \quad m_u^{\rm eff}=m_d^{\rm eff}=m_u-(3/8)b_{uu}. \tag{8}$$

The values of m, m_3, b_{12}, and b_{13} are obtained from the known physical baryon masses. With the help of Eq. (1), we get

$$m_u=m_d=362 \ \text{MeV}, \quad m_s=538 \ \text{MeV},$$

$$b_{uu}=b_{dd}=b_{ud}=195 \ \text{MeV},$$

$$b_{us}=b_{ds}=(m_u/m_s)b_{uu}=131 \ \text{MeV},$$

$$b_{ss}=(m_u/m_s)^2b_{uu}=88 \ \text{MeV}. \tag{9}$$

Substituting these into Eqs. (7) and (8), we obtain effective masses as follows:

$$m_u^p=m_d^n=338 \ \text{MeV}, \quad m_d^p=m_u^n=265 \ \text{MeV},$$

$$m_s^{\Lambda^0}=538 \ \text{MeV}, \quad m_u^{\Lambda^0}=m_d^{\Lambda^0}=290 \ \text{MeV},$$

$$m_u^{\Sigma^+}=m_d^{\Sigma^-}=356 \ \text{MeV},$$

$$m_s^{\Sigma^+}=m_s^{\Sigma^-}=473 \ \text{MeV},$$

$$m_u^{\Xi^0}=m_d^{\Xi^0}=297 \ \text{MeV},$$

$$m_s^{\Xi^-}=m_s^{\Xi^-}=516 \ \text{MeV}. \tag{10a}$$

For the $\Sigma^0\rightarrow\Lambda^0\gamma$ transition, we take

$$m_u^{\Sigma^0\Lambda^0}=m_d^{\Sigma^0\Lambda^0}=(m_u^{\Sigma^0}+m_u^{\Lambda^0})/2$$
$$=322 \ \text{MeV}.$$

For the Ω^- hyperon

$$m_s^{\Omega^-}=m_s+1/4b_{ss}. \tag{10b}$$

With these effective masses, the magnetic moments are obtained using the magnetic moment operator

$$\mu=\sum_i e_i/2m_i^{\rm eff}\sigma_i. \tag{11}$$

We wish to remark here that we use the nonrelativistic magnetic moment operator for the sake of simplicity. It has been shown by Morpurgo [11] that the nonrelativistic constituent quark model for the static properties of baryons is completely equivalent to a parametrization of the relativistic field theory of strong interactions in a spin-flavor basis. A similar connection between the constituent quark model and HBChPT was discovered by Durand, Ha, and Jaczko [12]. In fact, the spin-flavor description of the constituent quark model clarifies the structure and dynamical interpretation of the chiral expansion in effective field theory. A similar kinematic correspondence has also been extended to large N_c QCD by Carone, Georgi, and Osofsky [13]. Practically speaking, the remarkable success of the usual additive quark model is a consequence of the relative smallness of the nonadditive two- and three-body operators arising from spin-dependent interactions. So the use of the one-body operator is justified in light of the decoupling of the spatial and spin parts of the ground state baryon wave functions [11,12]. The calculated baryon moments are given in column 3 of Table I.

III. MAGNETIC MOMENT WITH EFFECTIVE MASS AND SHIELDED CHARGE

In addition to the variation of the quark mass caused by its environment, its charge may also be affected [14]. For example, when a soft photon probes a quark inside a baryon, its charge may be screened due to the presence of neighboring quarks. This situation is similar to the shielding of the nuclear charge of the helium atom due to its outer electron cloud. We take the effective charge to depend linearly on the charge of the shielding quarks. Thus, the effective charge e_a of quark a in the baryon B (a,b,c) is taken as [14]

$$e_a^B = e_a + \alpha_{ab} e_b + \alpha_{ac} e_c , \tag{12}$$

where e_a is the bare charge of quark a. Taking $\alpha_{ab} = \alpha_{ba}$ and invoking isospin symmetry, we have

$$\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \beta,$$

$$\alpha_{us} = \alpha_{ds} = \alpha,$$

and

$$\alpha_{ss} = \gamma. \tag{13}$$

We express the screened quark charges for various baryons in terms of the three parameters α, β, γ:

$$e_u^p = \frac{2}{3}(1+\beta/2), \quad e_d^p = -\frac{1}{3}(1-4\beta),$$

$$e_u^n = \frac{2}{3}(1-\beta), \quad e_d^n = -\frac{1}{3}(1-\beta),$$

$$e_s^{\Lambda^0} = -\frac{1}{3}(1-\alpha),$$

$$e_u^{\Sigma^+} = \frac{2}{3}(1-\alpha/2+\beta), \quad e_s^{\Sigma^+} = -\frac{1}{3}(1-4\alpha),$$

$$e_d^{\Sigma^-} = -\frac{1}{3}(1+\alpha+\beta),$$

$$e_s^{\Sigma^-} = -\frac{1}{3}(1+2\alpha),$$

$$e_u^{\Xi^0} = \frac{2}{3}(1-\alpha), \quad e_s^{\Xi^0} = -\frac{1}{3}(1-2\alpha+\gamma),$$

$$e_d^{\Xi^-} = -\frac{1}{3}(1+2\alpha), \quad e_s^{\Xi^-} = -\frac{1}{3}(1+\alpha+\gamma),$$

$$e_u^{\Sigma^0\Lambda^0} = \frac{2}{3}, \quad e_d^{\Sigma^0\Lambda^0} = -\frac{1}{3}, \quad e_s^{\Sigma^0\Lambda^0} = -\frac{1}{3},$$

$$e_s^{\Omega^-} = -\frac{1}{3}(1+2\gamma). \tag{14}$$

Substituting these values of effective charge and sandwiching the magnetic moment operator

$$\mu = \sum_i e_i^B / 2 m_i^{\mathrm{eff}} \sigma_i , \tag{15}$$

we determine the baryon magnetic moments. These are expressed as given below:

$$\mu(p) = -[(-1+4\beta)m_n]/9(-b_{ud}/2+m_d) + 8(1+\beta/2)m_n/9(-b_{ud}/4+b_{uu}/8+m_u),$$

$$\mu(n) = 4(-1+\beta)m_n/9(b_{dd}/8-b_{ud}/4+m_d) - 2(1-\beta)m_n/9(-b_{ud}/2+m_u),$$

$$\mu(\Lambda^0) = (-1+\alpha)m_n/3m_s ,$$

$$\mu(\Sigma^+) = -[(-1+4\alpha)m_n]/9(-b_{us}/2+m_s) + 8(1-\alpha/2+\beta)m_n/9(-b_{us}/4+b_{uu}/8+m_u),$$

$$\mu(\Sigma^-) = 4(-1-\alpha-\beta)m_n/9(b_{dd}/8-b_{ds}/4+m_d) - (-1-2\alpha)m_n/9(-b_{ds}/2+m_s),$$

$$\mu(\Xi^0) = 4(-1+2\alpha-\gamma)m_n/9(b_{ss}/8-b_{us}/4+m_s) - 2(1-\alpha)m_n/9(-b_{us}/2+m_u),$$

$$\mu(\Xi^-) = -[(-1-2\alpha)m_n]/9(-b_{ds}/2+m_d) + 4(-1-\alpha-\gamma)m_n/9(-b_{ds}/4+b_{ss}/8+m_s),$$

$$\mu(\Sigma^0\Lambda^0) = -2m_n/\sqrt{3}(-3b_{ud}/8-b_{us}/4+b_{uu}/8+2m_u),$$

$$\mu(\Omega^-) = (-1-2\gamma)m_n/(b_{ss}/4+m_s), \tag{16}$$

where m_n denotes the nucleon mass.

IV. NUMERICAL RESULTS

First, to reduce the number of screening parameters, we assume SU(3) flavor symmetry for the shielding effect $\alpha = \beta = \gamma$, and we use the hyperfine splitting parameters given in Eq. (9). Using the p, n, and Λ moments as input, we obtain $m_u = m_d = 336$ MeV, $m_s = 494$ MeV, and $\alpha = 0.033$, which in turn predict the remaining baryon moments. The results are shown in column 4 of Table I. However, the results show further improvement when the following ansatz is taken for the screening parameters α_{ij} appearing in Eq. (12):

$$\alpha_{ij} = |(m_i - m_j)|\,\delta/(m_i + m_j), \quad i,j = u,d, \text{ or } s. \tag{17}$$

Taking the p, n, and Ξ^0 moments as input, we then get

$$m_u = m_d = 336 \text{ MeV}, \quad m_s = 450 \text{ MeV}, \quad \delta = 0.81.$$

The results obtained are presented in column 5 of Table I. The obtained agreement with experiment is quite good. In the present scheme, the quark masses obtained from the moments are different from those obtained from the baryon masses given in Eq. (9). This is expected, as the shielding of quark charges, occurring through certain two-body interactions, may also involve correction to quark mass terms. Due to the lack of exact strong interaction dynamics at low energy, we feel unable to give a dynamical basis for the present model. We expect that two-body effects arising through spin-spin interactions have been included through the effective quark mass and charge parametrization in the present scheme. However, such effects essentially present small deviations from the conventional magnetic moment operator.

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