Impact parameter dependent parton distributions and transverse single spin asymmetries

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Generalized parton distributions (GPDs) with a purely transverse momentum transfer can be interpreted as Fourier transforms of the distribution of partons in impact parameter space. The helicity-flip GPD $E(x,0,-\mathbf{\Delta}_\perp^2)$ is related to the distortion of parton distribution functions in impact parameter space if the target is not a helicity eigenstate, but has some transverse polarization. This transverse distortion can be used to develop an intuitive explanation for various transverse single spin asymmetries.

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I. INTRODUCTION

Deep-inelastic scattering experiments allow the determination of parton distribution functions (PDFs), which have the very physical interpretation as momentum (fraction) distributions in the infinite momentum frame (IMF). PDFs are defined as the forward matrix element of a lightlike correlation function, i.e.,

$$q(x)=\langle P,S|\hat{O}_q(x,\mathbf{0}_\perp)|P,S\rangle \tag{1.1}$$

$$\Delta q(x)S^+=P^+\langle P,S|\hat{O}_{q,5}(x,\mathbf{0}_\perp)|P,S\rangle$$

with

$$\hat{O}_q(x,\mathbf{0}_\perp)\equiv\int\frac{dx^-}{4\pi}\bar{q}\left(-\frac{x^-}{2},\mathbf{0}_\perp\right)\gamma^+q\left(\frac{x^-}{2},\mathbf{0}_\perp\right)e^{ixp^+x^-} \tag{1.2}$$

$$\hat{O}_{q,5}(x,\mathbf{0}_\perp)\equiv\int\frac{dx^-}{4\pi}\bar{q}\left(-\frac{x^-}{2},\mathbf{0}_\perp\right)\gamma^+\gamma_5q\left(\frac{x^-}{2},\mathbf{0}_\perp\right)e^{ixp^+x^-}.$$

When sandwiched between states that have the same light-cone momentum $p^+=1/\sqrt{2}(p^0+p^3)$, these operators act as a "filter" for quarks of flavor q with momentum fraction x. Throughout this work, we will use the light-cone gauge $A^+=0$. In all other gauges, a straight line gauge string connecting the quark field operators needs to be included in this definition (1.1). Obviously, since PDFs are expectation values taken in plane wave states, they contain no information about the position space distribution of quarks in the target.

Generalized parton distributions (GPDs) [1], which describe for example the scaling limit in real and virtual Compton scattering experiments, are defined very similar to PDFs except that one now takes a nonforward matrix element of the light-cone correlator

$$\langle P',S'|\hat{O}_q(x,\mathbf{0}_\perp)|P,S\rangle =\frac{1}{2\bar{p}^+}\bar{u}(p',s')\left(\gamma^+H_q(x,\xi,t)+i\frac{\sigma^{+\nu}\Delta_\nu}{2M}E_q(x,\xi,t)\right)\times u(p,s) \tag{1.3}$$

$$\langle P',S'|\hat{O}_{q,5}(x,\mathbf{0}_\perp)|P,S\rangle =\frac{1}{2\bar{p}^+}\bar{u}(p',s')\left(\gamma^+\gamma_5\tilde{H}_q(x,\xi,t)+i\frac{\gamma_5\Delta^+}{2M}\tilde{E}(x,\xi,t)\right)\times u(p,s) \tag{1.4}$$

with $\bar{p}^\mu=\frac{1}{2}(p^\mu+p'^\mu)$ being the mean momentum of the target, $\Delta^\mu=p'^\mu-p^\mu$ the four momentum transfer, and $t=\Delta^2$ the invariant momentum transfer. The skewedness parameter $\xi=-\Delta^+/2\bar{p}^+$ quantifies the change in light-cone momentum.

An important physical interpretation for GPDs derives from the fact that they are the form factors of the light-cone correlators $\hat{O}_q(x,\mathbf{0}_\perp)$ and $\hat{O}_{q,5}(x,\mathbf{0}_\perp)$. Because of that, and by analogy with ordinary form factors, one would therefore expect that GPDs can be interpreted as some kind of Fourier transform of parton distributions in position space. Indeed, as has been shown in Refs. [3–5], the helicity nonflip[1] GPD H for $\xi=0$ is the Fourier transform of the (unpolarized) impact parameter dependent parton distribution function $q(x,\mathbf{b}_\perp)$, i.e.

$$q(x,\mathbf{b}_\perp)=\int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}H(x,0,-\mathbf{\Delta}_\perp^2). \tag{1.5}$$

The reference point for the impact parameter in Eq. (1.5) is the (transverse) center of momentum (c.m.) of the target

$$\mathbf{R}_\perp\equiv\frac{1}{p^+}\int d^2\mathbf{x}_\perp\int dx^-T^{++}\mathbf{x}_\perp=\sum_{i\in q,g}x_i\mathbf{r}_{\perp,\mathbf{i}}, \tag{1.6}$$

where T^{++} is the light-cone momentum density component of the energy momentum tensor. The sum in the parton representation for $\mathbf{R}_\perp$ extends over the transverse positions $\mathbf{r}_{\perp,\mathbf{i}}$ of all quarks and gluons in the target, and the weight factor x_i is the momentum fraction carried by each parton. The impact parameter dependent PDFs are defined by introducing the $\mathbf{b}_\perp$-dependent light-cone correlation

[1]The "helicity" basis that we are using refers to the infinite momentum frame helicity [2].

$$q(x,\mathbf{b}_\perp)\equiv\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda|\hat{O}_q(x,\mathbf{b}_\perp)|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\rangle, \tag{1.7}$$

where

$$|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\rangle\equiv\mathcal{N}\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle \tag{1.8}$$

is a state whose transverse c.m. is localized at the origin and $\mathcal{N}$ is a normalization constant. They are simultaneous eigenstates of the light-cone momentum p^+, the transverse c.m. (with eigenvalue $\mathbf{0}_\perp$), and the angular momentum operator J_z, which is possible due to the Galilean subgroup of transverse boosts in the IMF [2].

A similar connection exists between $\tilde{H}$ and impact parameter dependent polarized PDFs

$$\Delta q(x,\mathbf{b}_\perp)=\int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2), \tag{1.9}$$

where

$$\Delta q(x,\mathbf{b}_\perp)\equiv\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\uparrow|\hat{O}_{q,5}(x,\mathbf{b}_\perp)|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\uparrow\rangle. \tag{1.10}$$

It should be emphasized that impact parameter dependent parton distributions have an interpretation as a probability density. In fact

$$\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)=q(x)$$

$$q(x,\mathbf{b}_\perp)\geqslant 0 \quad (x>0)$$

$$q(x,\mathbf{b}_\perp)\leqslant 0 \quad (x<0)$$

$$\int d^2\mathbf{b}_\perp\Delta q(x,\mathbf{b}_\perp)=\Delta q(x)$$

$$|\Delta q(x,\mathbf{b}_\perp)|\leqslant|q(x,\mathbf{b}_\perp)|. \tag{1.11}$$

Equations (1.5) and (1.9) imply that GPDs for $\xi=0$ can be used to construct "tomographic images" [6] of the target nucleon, where one can study "slices" of the nucleon in impact parameter space for different values of the light-cone momentum fraction x, and one can learn how the size of the nucleon depends on x. Another useful piece of information that is contained in these 3-dimensional images is how the light-cone momentum distribution of the quarks varies with the distance from the c.m.

Amazingly, the transverse resolution in these images is not limited by relativistic effects, but only by the inverse momentum of the photon that is used to probe the GPDs, which determines the pixel size in these images.

II. GPDs WITH HELICITY FLIP

In order to develop a probabilistic interpretation for $E(x,0,t)$, it is necessary to consider helicity flip amplitudes because otherwise $E(x,\xi=0,t)$ does not contribute [7]:[2]

$$\langle p^+,\mathbf{p}_\perp+\mathbf{\Delta}_\perp,\uparrow|\hat{O}_q(x,\mathbf{0}_\perp)|p^+,\mathbf{p}_\perp,\uparrow\rangle =H(x,0,-\mathbf{\Delta}_\perp^2), \tag{2.1}$$

$$\langle p^+,\mathbf{p}_\perp+\mathbf{\Delta}_\perp,\uparrow|\hat{O}_{q,5}(x,\mathbf{0}_\perp)|p^+,\mathbf{p}_\perp,\downarrow\rangle =-\frac{\Delta_x-i\Delta_y}{2M}E(x,0,-\mathbf{\Delta}_\perp^2). \tag{2.2}$$

Therefore, if one wants to develop a density interpretation for $E(x,0,-\mathbf{\Delta}_\perp^2)$ one needs to consider states that are not helicity eigenstates. The superposition where the contribution from E is maximal corresponds to states where $\uparrow$ and $\downarrow$ contribute with equal magnitude. We thus consider the state

$$|X\rangle\equiv\frac{1}{\sqrt{2}}[|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\uparrow\rangle+|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\downarrow\rangle], \tag{2.3}$$

which one may interpret as a state that is "polarized in the x direction (in the IMF)." However, since the notion of a transverse polarization is somewhat tricky in the basis that we are using (states that are eigenstates of p^+ and $\mathbf{R}_\perp$), there may be some relativistic corrections to the actual interpretation of what this state corresponds to. In the following, we will keep this caveat in mind when studying the properties of this state even though we will refer to this state as a "transversely polarized nucleon (in the IMF)." The unpolarized impact parameter dependent PDF in this state will be denoted $q_X(x,\mathbf{b}_\perp)$.

Repeating the same steps that led to Eq. (1.5) and using Eqs. (2.1) and (2.2), one finds

$$\begin{aligned}q_X(x,\mathbf{b}_\perp)&\equiv\langle X|\hat{O}_q(x,\mathbf{b}_\perp)|X\rangle\\&=\int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}\left[H_q(x,0,-\mathbf{\Delta}_\perp^2)+\frac{i\Delta_y}{2M}E_q(x,0,-\mathbf{\Delta}_\perp^2)\right]\\&=q(x,\mathbf{b}_\perp)-\frac{1}{2M}\frac{\partial}{\partial b_y}\mathcal{E}_q(x,\mathbf{b}_\perp),\end{aligned} \tag{2.4}$$

where we denoted $\mathcal{E}_q$ the Fourier transform of E_q, i.e.

$$\mathcal{E}_q(x,\mathbf{b}_\perp)\equiv\int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp}E_q(x,0,-\mathbf{\Delta}_\perp^2). \tag{2.5}$$

Physically, what this result means is that for a nucleon that is transversely polarized and moves with a large momentum, an observer at rest sees parton distributions that are distorted sideways in the transverse plane. Obviously, for transversely polarized nucleons the axial symmetry of the problem is bro-

[2]The helicity labels $\uparrow$, $\downarrow$ in Eqs. (2.1) and (2.2) refer to helicity states in the IMF [2].

ken and the impact parameter dependent PDFs no longer need to be axially symmetric. The direction of the distortion is perpendicular to both the spin and the momentum of the nucleon.[3] Although the distortion is mathematically described by Eq. (2.4) in a model-independent way, it is instructive to consider a semi-classical picture for the effect where the physical origin of this distortion results from a superposition of translatory and orbital motion of the partons when the nucleon is polarized perpendicular to its direction of motion. If the spin of the nucleon is "up" (looking into the direction of motion of the nucleon) and the orbital angular momentum of the quarks is parallel to the nucleon spin then the orbital motion adds to the momentum on the right side of the nucleon and subtracts on the left side, i.e. partons on the right side get boosted to larger momentum fractions x and on the left they get decelerated to smaller x (compared to longitudinally polarized nucleons). Since parton distributions decrease with x (at large momenta they drop like a power of x and at small x they grow like an inverse power of x), boosting all partons on one side of the nucleon results in an increase of the number of partons at a fixed value of x on that side, while the opposite effect occurs on the other side. Therefore, the acceleration/deceleration due to the superposition of the orbital with the translatory motion results in an increase of partons on the right and a decrease on the left, i.e. the net result is that the parton distribution in the transverse plane has been shifted or distorted to the right. Of course, for quarks with orbital angular momentum antiparallel to the nucleon spin the direction of the distortion is reversed (to the left). In Ref. [8] it has been shown that the helicity flip GPD E is related to the angular momentum carried by the quark. This result, together with the above semiclassical description about the physical origin of the distortion, provides an intuitive explanation for the fact that this distortion is described by E.

It should be emphasized that transverse asymmetries in impact parameter dependent PDFs are consistent with time-reversal invariance since $\vec{b}\cdot(\vec{p}\times\vec{S})$ is invariant under T. In contrast, $\vec{k}\cdot(\vec{p}\times\vec{S})$ is *not* invariant under T, and therefore transverse asymmetries in unintegrated parton densities $q(x,\mathbf{k}_\perp)$ are only permitted if final state interaction effects are incorporated into the definition of unintegrated parton densities [9].

Unfortunately, little is known about generalized parton distributions and it is therefore in general difficult to make predictions without making model assumptions. However, it *is* possible to make a model independent statement about the resulting transverse flavor dipole moment

$$
\begin{aligned}
d_q^y &\equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y \\
&= -\frac{1}{2M}\int dx \int d^2\mathbf{b}_\perp b_y \frac{\partial}{\partial b_y}\mathcal{E}_q(x,\mathbf{b}_\perp) \\
&= \frac{1}{2M}\int dx \int d^2\mathbf{b}_\perp \mathcal{E}_q(x,\mathbf{b}_\perp) \\
&= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{F_{2,q}(0)}{2M},
\end{aligned} \tag{2.6}
$$

where we used that the integral of E_q yields the Pauli form factor $F_{2,q}$ for flavor q [8]. For u and d quarks, $F_{2,q}(0) \equiv \kappa_{q/p}$ in the proton is of the order of $|\kappa_{q/p}| \sim 1-2$ (for a more detailed estimate see Appendix A), i.e. the resulting transverse flavor dipole moments are on the order of

$$d_q^y \sim 0.1-0.2 \ \text{fm}. \tag{2.7}$$

In fact, using only isospin symmetry, one finds for a transversely polarized proton (A4)

$$d_u^y - d_d^y = \frac{\kappa_{u/p}-\kappa_{d/p}}{2M} \approx 0.4 \ \text{fm}, \tag{2.8}$$

i.e. the flavor center for u and d quarks gets separated in opposite directions to the point where the separation is of the same order as the expected size of the valence quark distribution.[4]

In order to illustrate the magnitude of the distortion graphically, we make a simple model for the $\mathbf{\Delta}_\perp$ dependence of GPDs [4]

$$H_q(x,0,-\mathbf{\Delta}_\perp^2) = q(x) e^{-a\mathbf{\Delta}_\perp^2(1-x)\ln(1/x)}. \tag{2.9}$$

This ansatz incorporates both the expected large x behavior (H_q should become x-independent as $x\to 1$) and the small x behavior (Regge behavior). Furthermore, in the forward limit ($\mathbf{\Delta}_\perp=0$), H_q reduces to the unpolarized PDF $q(x)$. In impact parameter space this ansatz implies

$$q(x,\mathbf{b}_\perp^2) = q(x)\frac{1}{4\pi a(1-x)\ln\frac{1}{x}} \exp\left(-\frac{\mathbf{b}_\perp^2}{4a(1-x)\ln\frac{1}{x}}\right). \tag{2.10}$$

For the helicity flip distributions E_q we assume that the $\mathbf{\Delta}_\perp$ dependence is the same as for H_q and we fix the overall normalization by demanding that the integral of $E_q(x,0,0)$ yields the anomalous magnetic moments

$$
\begin{aligned}
E_u(x,0,t) &= \frac{1}{2}\kappa_u H_u(x,0,t) \\
E_d(x,0,t) &= \kappa_d H_d(x,0,t).
\end{aligned} \tag{2.11}
$$

We should emphasize that this is not intended to be a realistic model and we only use it to illustrate the typical size of effects that one might anticipate.

The resulting parton distributions in impact parameter space for u and d quarks are shown in Figs. 1 and 2 respectively. Note that PDFs as well as GPDs decrease significantly

[3] Note that $\vec{S}\times\vec{p}$ transforms like a position space vector $\vec{r}$ under P and T transformations.

[4] It should be emphasized that the transverse center of momentum of the whole nucleon does not shift since $\sum_{i\in q,g}\int dx x E_i(x,0,0) = 0$ if one sums over the contributions from all flavors as well as from the glue [10].

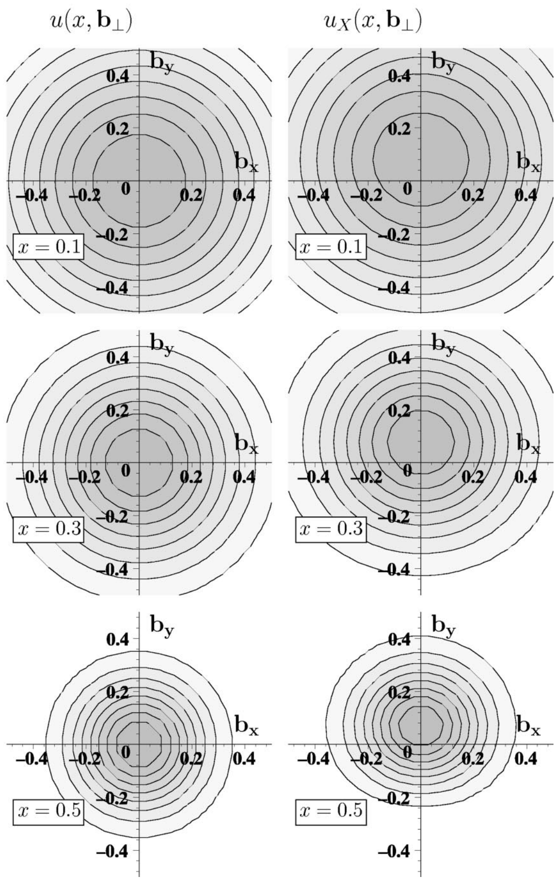

FIG. 1. u quark distribution in the transverse plane for $x=0.1$, 0.3, and 0.5 (2.10). Left column: $u(x,\mathbf{b}_\perp)$, i.e. the u quark distribution for unpolarized protons; right column: $u_X(x,\mathbf{b}_\perp)$, i.e., the unpolarized u quark distribution for ''transversely polarized'' protons $|X\rangle=|\uparrow\rangle+|\downarrow\rangle$. The distributions are normalized to the central (undistorted) value $u(x,\mathbf{0}_\perp)$.

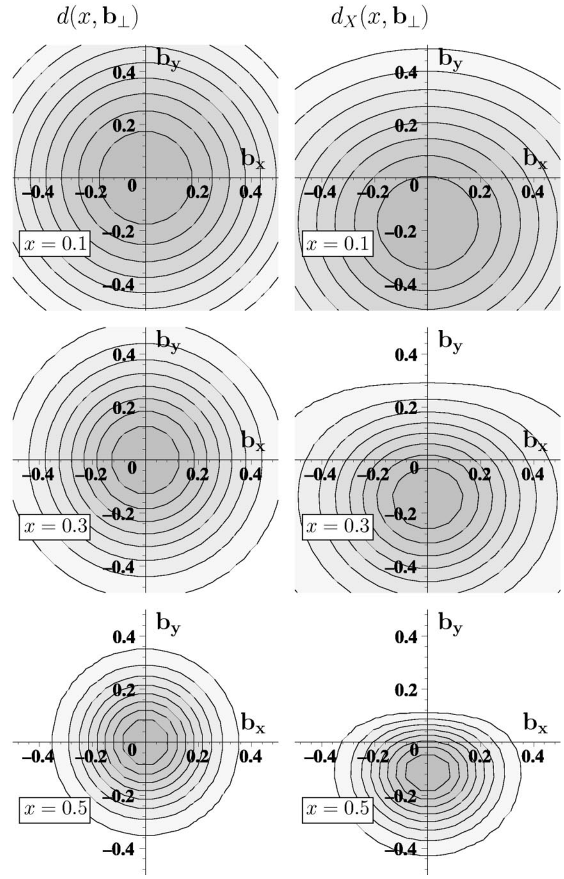

FIG. 2. Same as Fig. 1, but for d quarks.

from $x=0.1$ to $x=0.5$. In order to be able to plot the impact parameter dependence we normalized the distributions for each value of x and both u and d quark distributions to the value of the longitudinally polarized distribution at $\mathbf{b}_\perp=0$.

The ''tomographic slices,'' i.e. the impact parameter dependences for a few fixed values of x, that are shown in Figs. 1 and 2 clearly demonstrate what should have been clear already from our model-independent result above Eq. (2.6): at larger values of x, the u and d quark distributions in a transversely polarized proton are shifted to opposite sides and the magnitude of the distortion is such that there is a significant lack of overlap between the two. Other models for $E(x,0,t)$ [11] yield very similar results since the overall magnitude of the effect is constrained by the model independent relation Eq. (2.6).

Such a large separation between quarks of different flavor, which is both perpendicular to the momentum and spin of the proton, must have some observable effects. For example, in semi-inclusive photo-production of pions off transversely polarized nucleons, the u quarks are knocked out predominantly on one side of the nucleon. Therefore the final state interaction will be different for pions produced going to the right compared to those going to the left, which in turn may lead to a transverse asymmetry of produced pions. Other examples are flavor exchange reactions, and for given transverse polarization the added quarks might be picked up predominantly one on particular side of the hadron, suggesting a transverse asymmetry of the hadron production relative to the nucleon spin. In the next section, we will present a simple model for these final state interactions, which together with the transverse asymmetries in the position space distribution of partons leads to predictions for the signs of the transverse asymmetries in various hadron production reactions.

III. SINGLE TRANSVERSE SPIN ASYMMETRIES

Many semi-inclusive hadron production experiments show surprisingly large transverse polarizations or asymmetries [12]. Moreover, the signs of these polarizations are usually not dependent on the energy. This very stable polarization pattern suggests that there is a simple mechanism that underlies these polarization effects. In the following, an attempt is made to link the large transverse distortions of parton distributions in impact parameter space for transversely polarized nucleons (baryons) with these transverse single spin asymmetries.

We will make the following model assumptions for flavor transitions in high energy scattering events: In a flavor changing process, as many quarks as possible (hereafter referred to as ''spectators'') originate from the impacting hadron. Any additional quarks are produced from the breaking of a string that connects the spectators with the target right after the impact. Since this string exerts an attractive force on the "spectators" before it breaks, this picture suggests that the transverse momentum of the final state hadron will point in the direction given by the side on which the additional quarks were produced.

Note that this model implicitly focuses on more peripheral scattering events for describing the signs of baryon polarizations at large x_F. Although these may not be the only possible events, we expect that central collisions are less likely to produce the observed pattern of large and only weakly energy dependent polarizations. This is supported for example by the observation that the polarization of the produced Λ hyperons is particularly large in diffractive production [13].

These simple model assumptions, together with the transverse distortion of quarks in transversely polarized hadrons, provide an intuitive explanation for the large observed transverse polarization in inclusive hyperon production as we will demonstrate in the following. For this purpose, let us consider for example a Λ that is produced moving to the left of the incident proton beam.

Using our model assumptions above, this implies that the s quark was produced on the left side of the Λ. Since $\kappa_{s/\Lambda} > 0$, such a state with an s-quark produced on the left side has a much better overlap with a Λ that has spin down (when one looks into the beam direction) rather than spin up (see Figs. 3 and 4).

Therefore, for a Λ that has been deflected to the left one would expect a polarization that points downward. Following the usual convention, where the polarization direction is defined with respect to the normal vector $\vec{n} \equiv \vec{p}_{beam} \times \vec{p}_{final} / |\vec{p}_{beam} \times \vec{p}_{final}|$, the Λ should have negative polarization, which is also what is observed experimentally [12]. Likewise, since $\kappa_{s/\Sigma} < 0$ and $\kappa_{s/\Xi} > 0$ (Appendix), one would expect that Σ and Ξ hyperons are produced with polarizations "up" and "down" respectively when one starts from an incident proton beam and the hyperon is produced to the left of the beam.

If the incident beam consists of Λ or Σ hyperons, then the polarization of produced Ξ hyperons is of course the same as in the case of incident nucleons since it is still only s quarks that need to be substituted. However, the situation changes if one considers $\Lambda \rightarrow \Sigma$ and $\Sigma \rightarrow \Lambda$ production reactions, because there it is a u or d quark that needs to be substituted. If we now use that $\kappa_{u/\Lambda} = \kappa_{d/\Lambda} < 0$ and for example $\kappa_{u/\Sigma} > 0$, one finds that the sign of the polarization of Λ/Σ produced from a Σ/Λ beam is reversed compared to the respective polarizations that arise when one starts from a nucleon beam (Fig. 5). However, we should emphasize that $|\kappa_{u/\Lambda}|$ is only about half as large as $\kappa_{s/\Lambda}$ and therefore the transverse distortion of the u/d quarks in a transversely polarized Λ is expected to be smaller than the one of the s quarks. We therefore expect that the polarization of Λ produced from an incident Σ beam is not only reversed but also significantly smaller in magnitude than those produced from a proton beam.

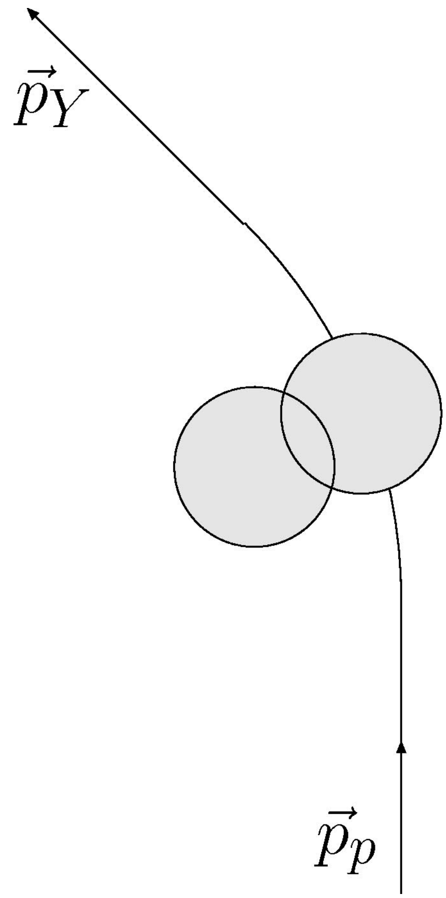

FIG. 3. Inclusive $p \rightarrow Y$ scattering where the incoming p (from bottom) diffractively hits the right side of the target and is therefore, according to the model assumptions, deflected to the left during the reaction. The $s\bar{s}$ pair is assumed to be produced roughly in the overlap region, i.e. on the left "side" of the Y.

For neutron production the spin in the final state is not self-analyzing. However, our model also predicts interesting asymmetries with respect to the spin of the initial state.

In order to be converted into a neutron, the proton must strip off one of its u quarks. A proton that is polarized "down" has its u quarks shifted to the left of its center of momentum, i.e. it can strip off a u quark more easily when it passes the target on the right and, at least within our model,

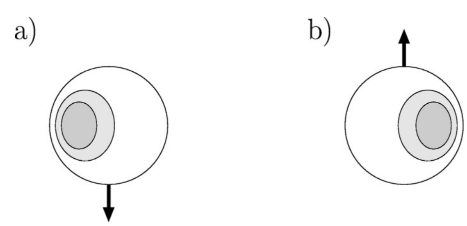

FIG. 4. Schematic view of the transverse distortion of the s quark distribution (in gray scale) in the transverse plane for a transversely polarized hyperon with $\kappa_s^Y > 0$. The view is (from the rest frame) into the direction of motion (i.e. momentum into plane) for a hyperon that moves with a large momentum. In the case of spin down (a), the s-quarks get distorted toward the left, while the distortion is to the right for the case of spin up (b).

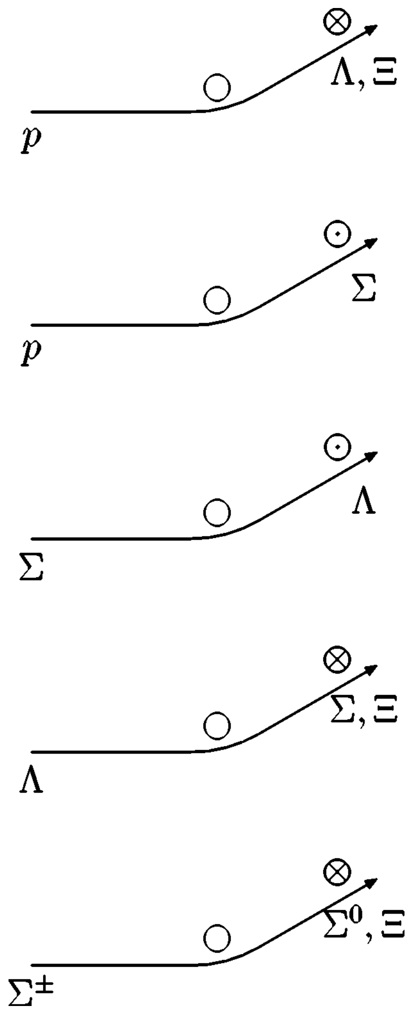

FIG. 5. Transverse polarizations of hyperons that are produced from an unpolarized beam and target (represented by an empty circle). According to the model assumptions, the final state hadron is deflected in the direction given by the side on which the missing quarks were produced. $\odot$ and $\otimes$ represent hyperons with spin pointing out of the plane and into the plane respectively.

will be more likely to result in a neutron that is deflected to the left (Fig. 6). In summary, we therefore expect neutrons to be more likely to be produced to the left of the beam if the proton spin is downward and to the right if its spin is upward, corresponding to a negative analyzing power. This result agrees with a recent measurement at RHIC [14].

We should emphasize that similar reasoning for inclusive hyperon production also implies a spin asymmetry with respect to the incident proton spin. If we define again a positive analyzing power A_N if protons with spin up give rise to a final state hadron that is deflected to the left, then $p \rightarrow \Lambda$ should also have $A_N < 0$ since there one also needs to substitute a u quark in the proton. The situation is similar for $p \rightarrow \Xi^-$, where both u quarks need to be substituted. In the case of $p \rightarrow \Sigma^+$, it is the d quark that is substituted and therefore $A_N > 0$.

The beam asymmetries in semi-inclusive meson production can be explained similarly. In order for a proton to convert into a π^+, one of its u quarks needs to "go through."

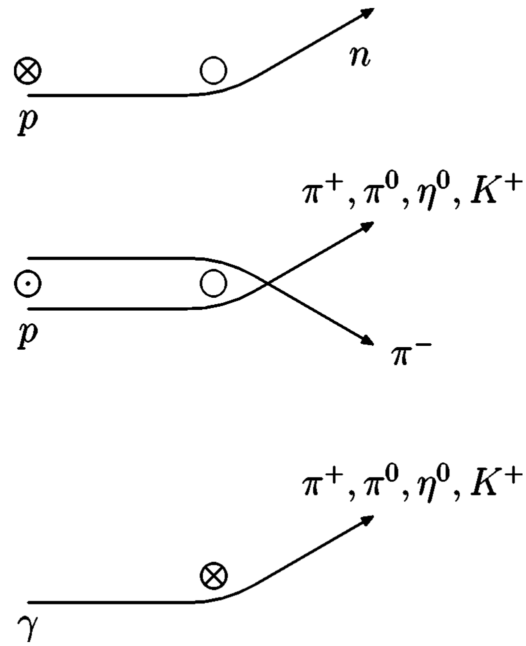

FIG. 6. Beam and target spin asymmetries for $p \rightarrow n/meson$ and semi-inclusive $\gamma \rightarrow meson$ respectively.

This is most likely to happen if the u quarks are on the "far side" of the interaction zone. This favors protons with spin up when the proton passes the target on the right and spin down when it passes on the left side of the target. If we assume again that the final state interaction that leads to string breaking is attractive (until the string breaks) then protons with spin up result in π^+ that are more likely deflected to the right, while protons with spin down are more likely resulting in π^+ that are deflected to the left, i.e. we expect a positive analyzing power for $p \rightarrow \pi^+$ and the same for $p \rightarrow K^+$. For π^- we expect a negative analyzing power since there the leading quark is a d quark, which would be more likely on the side opposite to the u quarks for a transversely polarized nucleon and one expects a negative analyzing power. For $p \rightarrow \pi^0, \eta^0$ the leading quark could be both u or d, but since valence u quarks outnumber the d quarks in a proton, one expects that the net analyzing power is again positive, but smaller than for π^+. These results seem to be consistent with the pattern that is observed experimentally [15].

In order to understand target spin asymmetries, it is useful to analyze the process in the c.m. frame where the projectile and the target have initially opposite momenta. As an example, let us consider the target spin asymmetry in semi-inclusive electro-production of pions on a transversely polarized proton target (Fig. 7).

For a target polarization that is into the plane, and applying the results from Sec. II, the u quark distribution in the c.m. frame is shifted down, while the d quark distribution is shifted up. Semi-inclusive photo-production of mesons with a u valence quark (e.g. $\pi^+, \pi^0, \eta^0, K^+$) occurs dominantly through photons that initially interact with a u quark in the target, which later fragments into the meson. Applying again our model assumption from above, i.e. using that the QCD string deflects the u quark toward the center, we conclude that the mesons with a valence u quark are produced prefer-

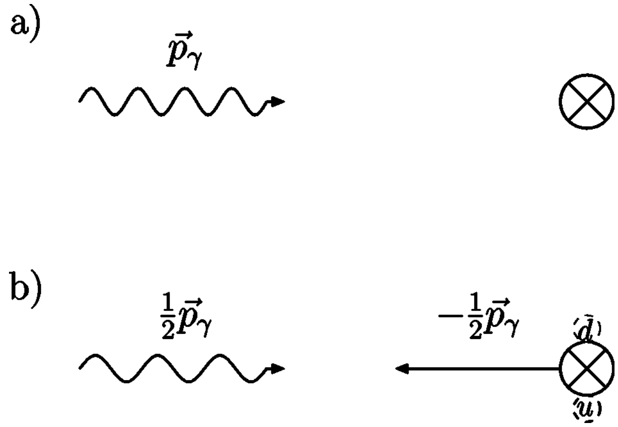

FIG. 7. Photon hitting proton target. (a) Laboratory frame, (b) c.m. frame. The polarization of the proton is into the plane. According to the results from Sec. II, the u quarks (schematically indicated by a dashed circle) are shifted down.

entially in the up direction (Fig. 6) within this model, i.e. to the left if one looks into the direction of the photon momentum and the spin of the proton is down. For mesons without valence u quarks, such as the π^-, there are two competing effects: when the photon hits the d quark first then our argumentation above would favor π deflected in the direction opposite to π^+, since the d quarks are, for a given polarization of the proton, shifted in the direction opposite to the u quarks. However, the contribution from ''disfavored'' fragmentation $u \to \pi^-$ is enhanced due to the fact that the photon is much more likely to hit a u than a d quark in the proton and therefore the resulting asymmetry is not immediately obvious.

IV. DISCUSSION

Our model for generating the polarizations and spin asymmetries is much too crude to make detailed quantitative predictions about the size of the effects. However, the model matches the observed signs and provides a natural explanation for the fact that the observed effects are very large. We not only obtain a unified description for polarization and single spin asymmetry experiments but at the same time develop a link between these spin observables and parton distributions in impact parameter space.

There have been a number of models attempting to explain polarizations observed in hyperon production experiments and it would be beyond the intended scope of this article to provide a detailed comparison with all of them,[5] but we would still like to point out a few similarities and differences.

It is interesting to compare our attempt to link asymmetries of parton distributions in impact parameter space with single spin asymmetries with attempts to link asymmetries of unintegrated parton densities with the single spin asymmetries [17]. The main difference between these two approaches is that we start from a transverse asymmetry in position space. The final state interaction of the outgoing quark converts the position space asymmetry into an asymmetry for the transverse momentum of the final state hadron. The Sivers effect is complementary in that it starts already from an asymmetry in the transverse momenta of the unintegrated parton densities. Of course, since $\vec{S}\cdot(\vec{k}_q\times\vec{p})$ is not a Lorentz scalar under time-reversal, such an asymmetry in the unintegrated parton densities appears only if the final state interaction is included into their definition via appropriate Wilson lines [9], i.e. in a sense the final state interactions are already included in the definition of these unintegrated parton densities. From that point of view, these two approaches are complementary attempts to explain single transverse spin asymmetries, which both have in common that they rely on final state interactions, although the technical details are very different and it remains to be seen whether the Sivers model and this work describe the same physics but only from different angles or whether they actually describe different physical mechanisms.

The pattern of signs that we predict resembles very much that of other semi-classical models. This should not come as a surprise since the orbital angular momentum of quarks plays an important role in many of these models. In our model the connection with quark orbital angular momentum appears because the same GPD that describes the transverse distortion of PDFs in impact parameter space [namely $E_q(x,0,\mathbf{\Delta}_\perp^2)$] also appears in a sum-rule for the angular momentum carried by the quarks [8]. Nevertheless there are few differences to these models. For example, in a model where the interaction is assumed to happen at the front of the hadron, the left-right asymmetries are generated by the transverse momentum of quarks with orbital angular momentum at the front side [19]. Such a model would in general predict exactly the same polarization/asymmetry pattern as our model, with the exception of reactions where the incoming projectile is a photon. In that case the absorption is weak and it is not legitimate to argue that the interaction of the photon with the target should be a surface effect. Therefore, models where the polarization results as a combination between the initial state interaction and the quark orbital angular momentum would only predict a very small transverse single spin asymmetry in semi-inclusive photo-production experiments. In our model, the impact parameter space asymmetry is translated into a momentum asymmetry of the outgoing hadron as a result of the final state interaction and therefore the expected asymmetries in semi-inclusive photo-production experiments are of the same order of magnitude as in hadroproduction experiments.

Like in Ref. [18], the physical mechanism that eventually leads to polarization/asymmetries in our model is the final state interaction of the fragmenting quark(s). It would be interesting to see if the similarity between these two mechanisms goes beyond this simple observation.

It is conceivable that studying spin transfers, i.e. the correlation D_{NN} between the transverse polarization of the produced baryon and the transverse polarization of the beam, leads to further insights about the mechanism for transverse polarizations because it may help to differentiate between various models. In our model a correlation between the spins

[5] A nice recent review on the subject can be found in Ref. [16].

of the initial and final baryon arises because the transverse distortion of impact parameter dependent PDFs in transversely polarized hadrons leads to both polarizations as well as transverse single spin asymmetries. The correlation between the initial and final state transverse spin is such that the removed valence quark should be on the same side of the initial state baryon as the substituted valence quark in the final state baryon. Therefore the sign of D_{NN} is determined by the sign of the product of the κ_q for the valence quark that stripped off and the quark that is substituted for it. For example, in the $p\rightarrow\Lambda$ transition, a u quark needs to be substituted by an s quark. Since $\kappa^*_{u/p}\kappa_{s/\Lambda}>0$ we would expect a positive spin transfer in this case.

V. SUMMARY

Generalized parton distributions for purely transverse momentum transfer can be related to the distribution of partons in the transverse plane. When the nucleon is polarized in the transverse direction (e.g. transverse with respect to its momentum in the infinite momentum frame) then the distribution of partons in the transverse plane is no longer axially symmetric. The direction of the transverse distortion is perpendicular to both the spin and the momentum of the nucleon. Classically the effect can be understood as a superposition of the translatory motion of the partons along the momentum of the nucleon with the orbital angular motion of partons in the nucleon. The sign and magnitude of the distortion of (unpolarized) PDF in impact parameter space can be expressed in terms of the helicity-flip generalized parton distribution $E_q(x,0,-\mathbf{\Delta}_\perp^2)$. Since $\int dx E_q$ can be related to the Pauli form factor $F_{2,q}$ for flavor q, one can thus relate the resulting transverse flavor dipole moment of the distorted parton distributions to the anomalous flavor-magnetic moment $\kappa_{q/p}$ in the proton. We are thus able to link the transverse distortion of partons to the magnetic properties of the nucleon which leads to a model-independent prediction for the resulting transverse flavor dipole moments that are on the order of $0.1-0.2$ fm.

Such a large transverse dipole polarization for quarks of different flavor should also have observable effects in semi-inclusive hadron production experiments. We introduced a simple model to translate the transverse asymmetry of the parton distributions in impact parameter space into transverse asymmetries of the produced hadrons. The basic idea of the model is that the leading quark(s),[6] before they fragment into the observed hadron, experience an attractive force from the QCD string before the string breaks. This attractive force between the produced outgoing hadron and the target remnant leads to the left-right asymmetry in the observed hadron distributions.

We use this model to explain or predict a number of *baryon* → *baryon′* experiments, where the transverse distortion of transversely polarized baryons favors certain final polarization states and therefore leads to transversely polarized baryons in the final state. We argue that the large transverse hyperon polarization at high energies that is observed in these experiments is naturally explained due to the fact that the transverse flavor dipole moment of transversely polarized baryons in the infinite momentum frame is also very large. A similar mechanism is used to explain the asymmetry in semi-inclusive meson production using either a transversely polarized proton beam or incident virtual photons hitting a transversely polarized target.

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APPENDIX: $SU(3)$ ANALYSIS OF BARYON MAGNETIC MOMENTS

We use a notation, where $F_2^{q/B}$ denotes the Pauli form factor F_2 defined as the matrix element of a vector current with flavor q, i.e. $\overline{q}\gamma^\mu q$ between states of the baryon B. It is related to the usual electromagnetic form factor for the baryons using

$$F_2^B(Q^2)=\frac{2}{3}F_2^{u/B}(Q^2)-\frac{1}{3}F_2^{d/B}(Q^2)-\frac{1}{3}F_2^{s/B}(Q^2). \tag{A1}$$

For the transverse flavor dipole moments, we need to know the anomalous magnetic moment contributions for each quark flavor and each baryon

$$\kappa_{q/B}\equiv F_2^{q/B}(0). \tag{A2}$$

Experimentally, little is known beyond the electro-magnetic linear combination $\Sigma_q e_q \kappa_{q/B}$ for a few baryons. For our purposes, namely explaining the signs of various asymmetries, it will be sufficient to know the sign and order of magnitude of the $\kappa_{q/B}$. Therefore, we will use $SU(3)$-flavor symmetry which should be sufficient for an accuracy of a couple of 10% to estimate the $\kappa_{q/B}$. The only input that we use is the anomalous magnetic moments of the proton and neutron

$$\kappa^p=\frac{2}{3}\kappa_{u/p}-\frac{1}{3}\kappa_{d/p}-\frac{1}{3}\kappa_{s/p}=1.79$$

$$\kappa^n=\frac{2}{3}\kappa_{u/n}-\frac{1}{3}\kappa_{d/n}-\frac{1}{3}\kappa_{s/n}=-1.91 \tag{A3}$$

and we will assume that $\kappa_{s/p}\approx 0$.[7] Using isospin symmetry, this implies

[6] In photo-production experiments, the ''leading quark'' in the model is simply the struck quark, while in hadro-production experiments the ''leading quarks'' are spectator quarks from the incident hadron.

[7] Although $\kappa_{s/p}$ is not known very accurately, it is nevertheless clear that its numerical value is significantly smaller than $\kappa_{u/p}$ and $\kappa_{d/p}$ and it should therefore be justified to neglect its contribution for the kind of estimate that we are interested in.

$$\kappa_{u/p}=2\kappa_p+\kappa_n+\kappa_{s/p}\approx 1.67$$
$$\kappa_{d/p}=2\kappa_n+\kappa_p+\kappa_{s/p}\approx -2.03. \quad \text{(A4)}$$

If one assumes $SU(3)$ symmetry, then the flavor magnetic moments for baryons of type aab are trivially related to the ones in the proton, using $\kappa_{a/B}=\kappa_{u/p}$, $\kappa_{b/B}=\kappa_{d/p}$, and $\kappa_{c/B}=\kappa_{s/p}$, which implies for example

$$\kappa_{s/\Sigma}=\kappa_{d/p}\approx -2.03$$
$$\kappa_{s/\Xi}=\kappa_{u/p}\approx 1.67. \quad \text{(A5)}$$

The Λ is less trivial, but a straightforward $SU(3)$ analysis yields

$$\kappa_{s/\Lambda}=\frac{2}{3}\kappa_{u/p}-\frac{1}{3}\kappa_{d/p}+\frac{2}{3}\kappa_{s/p}\approx 1.79. \quad \text{(A6)}$$

For flavor changing transitions among hyperons, we also need the u/d moments

$$\kappa_{u/\Sigma^+}=\kappa_{u/p}\approx 1.67$$

$$\kappa_{u/\Lambda}=\kappa_{d/\Lambda}=\frac{1}{6}\kappa_{u/p}+\frac{2}{3}\kappa_{d/p}+\kappa_{s/p}\approx -0.98. \quad \text{(A7)}$$

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