Global monopole in asymptotically dS/Ads spacetime

Xin-zhou Li* and Jian-gang Hao

Shanghai United Center for Astrophysics, Shanghai Normal University, Shanghai 200234, China

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In this paper, we investigate the global monopole in asymptotically dS or AdS spacetime and find that the mass of the monopole in the asymptotically dS spacetime can be positive if the cosmological constant is greater than a critical value. This shows that the gravitational field of the global monopole can be attractive or repulsive depending on the value of the cosmological constant.

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Various kinds of topological defect could be produced by the phase transition in the early Universe and their existence has important implications in cosmology [1]. The global monopole, which has a divergent mass in flat spacetime, is one of the most important of these defects. The properties of the global monopole in curved spacetime, or, equivalently, its gravitational effects, were first studied by Barriola and Vilenkin [2]. When one considers gravity, the linearly divergent mass of the global monopole has an effect analogous to that of a deficit solid angle plus a tiny mass at the origin. Harari and Loustò [3] and Shi and Li [4] have shown that this small gravitational potential is actually repulsive. A new class of cold stars, called *D* stars (defect stars) has been proposed by Li *et al.* [5–7]. One of the most important features of such stars, compared to *Q* stars, is that the theory has monopole solutions when the matter field is absent, which makes the *D* stars behave very differently from *Q* stars. The topological defects are also investigated in Friedmann-Robertson-Walker spacetime [8]. On the other hand, there has been renewed interest in AdS spacetime due to the theoretical speculation of AdS conformal field theory (CFT) correspondence, which states that string theory in anti–de Sitter space (usually with extra internal dimensions) is equivalent to conformal field theory in one fewer dimension [9,10]. Recently, holographic duality between quantum gravity on de Sitter (dS) spacetime and a quantum field theory living on the past boundary of dS spacetime was proposed [11], and vortices in dS spacetime were studied by Ghezelbash and Mann [12]. Many authors have conjectured that the dS/CFT correspondence has a lot of similarities with Ads/CFT correspondence, although some interpretive issues remain. Monopole and dyon solutions in gauge theories based on the various gauge groups have been found [13]. However, in flat space there cannot be a static soliton solution in pure Yang-Mills theory [14]. The presence of gravity can supply an attractive force that binds the non-Abelian gauge field into a soliton. The cosmological constant influence the behavior of the soliton solution significantly. In asymptotically Minkowski spacetime electric components are forbidden in the static solution [15]. If the spacetime includes a cosmological constant, forbidding the electric components of the non-Abelian gauge fields fails, thus allowing dyon solutions. A continuum of new dyon solutions in the Einstein-Yang-Mills theory in asymptotically AdS spacetime has been investigated [16]; they are regular everywhere and specified by their mass and non-Abelian electric and magnetic charges. Similarly, the presence of the cosmological constant affects the behavior of the global monopole remarkably. If the spacetime is modified to include a positive cosmological constant, the gravitation field of the global monopole can be attractive, in contrast to the same problem in asymptotically Minkowski or AdS spacetime.

In this paper, we study the global monopole in asymptotically AdS or dS spacetime and show that the mass of the monopole might be positive in asymptotically dS spacetime if the cosmological constant is greater than a critical value.

The Lagrangian for the global monopole is

$$L=\frac{1}{2}\partial_\mu\phi^a\partial^\nu\phi^a-\frac{1}{4}\lambda^2(\phi^a\phi^a-\sigma_0)^2 \tag{1}$$

where ϕ^a is the triplet of the Goldstone field and possesses internal O(3) symmetry. When the symmetry breaks down to U(1), there will exist topological defects known as monopoles. The configuration describing the monopole solution is

$$\phi^a=\sigma_0 f(\rho)\frac{x^a}{\rho} \tag{2}$$

where $x^a x^a=\rho^2$ and $a=1,2,3$.

When f approaches unity at infinity, we have a monopole solution. The static spherically symmetric metric is

$$ds^2=B(\rho)dt^2-A(\rho)d\rho^2-\rho^2(d\theta^2+\sin^2\theta d\varphi^2). \tag{3}$$

By introducing the dimensionless parameter $r=\sigma_0\rho$, we obtain the equations of motion for the Goldstone field as

$$\frac{1}{A}f''+\left[\frac{2}{Ar}+\frac{1}{2B}\left(\frac{B}{A}\right)'\right]f'-\frac{2}{r^2}f-\lambda^2(f^2-1)f=0 \tag{4}$$

where the prime denotes the derivative with respect to r.

In dS or AdS spacetime, the Einstein equation is

$$G_{\mu\nu}+\beta g_{\mu\nu}=\kappa T_{\mu\nu} \tag{5}$$

where β is the cosmological constant and $\kappa=8\pi G$. dS and AdS spacetimes correspond to the cases that β is positive and negative, respectively. The Einstein equations in dS or AdS spacetime now are ready to be written as

*Electronic address: kychz@shtu.edu.cn

$$-\frac{1}{A}\left(\frac{1}{r^2}-\frac{1}{r}\frac{A'}{A}\right)+\frac{1}{r^2}=\epsilon^2 T_0^0-\frac{\beta}{\sigma_0^2}, \tag{6}$$

$$-\frac{1}{A}\left(\frac{1}{r^2}+\frac{1}{r}\frac{B'}{B}\right)+\frac{1}{r^2}=\epsilon^2 T_1^1-\frac{\beta}{\sigma_0^2}, \tag{7}$$

where

$$T_0^0=D+U+V, \tag{8}$$

$$T_1^1=D+U-V \tag{9}$$

are energy-momentum tensors and

$$D=\frac{f^2}{r^2},$$

$$U=\frac{\lambda^2}{4}(f^2-1)^2, \tag{10}$$

$$V=\frac{f'^2}{2A},$$

and $\epsilon^2=\kappa\sigma_0^2$ is a dimensionless parameter. Solving Eqs. (6) and (7), one can obtain

$$A^{-1}(r)=1-\epsilon^2+\frac{\beta}{3\sigma_0^2}r^2-\frac{2G\sigma_0 M_A(r)}{r}, \tag{11}$$

$$B(r)=1-\epsilon^2+\frac{\beta}{3\sigma_0^2}r^2-\frac{2G\sigma_0 M_B(r)}{r}, \tag{12}$$

where

$$M_A(r)=4\pi\sigma_0\exp[-\Delta(r)]\int_0^r dy\,\exp[\Delta(y)]\times\left\{f^2-1+y^2\left[U+\left(1-\epsilon^2+\frac{\beta}{3\sigma_0^2}y^2\right)f'^2\right]\right\} \tag{13}$$

and

$$M_B(r)=M_A(r)\exp[\tilde{\Delta}(r)]+\frac{r[1-\epsilon^2+(\beta/3\sigma_0^2)r^2]}{2}\{1-\exp[\tilde{\Delta}(r)]\}, \tag{14}$$

in which

$$\Delta(r)=\frac{\epsilon^2}{2}\int_0^r dy(yf'^2) \tag{15}$$

and

$$\tilde{\Delta}(r)=\epsilon^2\int_\infty^r dy(yf'^2). \tag{16}$$

Next, we discuss the behavior of these functions in asymptotically dS or AdS spacetime. A global monopole solution f should approach unity when $r\gg 1$. If this convergence is fast enough then $M_A(r)$ and $M_B(r)$ will also quickly converge to finite values. Therefore, we can find the asymptotic expansions:

$$f(r)=1-\frac{3\sigma_0^2}{\beta+3\lambda^2\sigma_0^2}\frac{1}{r^2}-\frac{9[2\beta\epsilon^2\sigma_0^4+3(2\epsilon^2-3)\lambda^2\sigma_0^6]}{2(2\beta-3\lambda^2\sigma_0^2)(\beta+3\lambda^2\sigma_0^2)^2}\frac{1}{r^4}+O\left(\frac{1}{r^6}\right), \tag{17}$$

$$M_A(r)=M_A(\beta,\epsilon^2)+\frac{4\pi\sigma_0}{r}+O\left(\frac{1}{r^3}\right), \tag{18}$$

$$M_B(r)=M_A(r)\left(1-\frac{\epsilon^2}{r^4}\right)+\frac{4\pi\sigma_0(1-\epsilon^2)}{r^3}+O\left(\frac{1}{r^7}\right) \tag{19}$$

where $M_A(\beta,\epsilon^2)\equiv\lim_{r\to\infty}M_A(r)$, which is a function dependent on β and ϵ^2. The dependence on ϵ of the asymptotic behavior is quite weak for the global monopole solution. However, the asymptotic behavior is evidently dependent on the parameters β, σ_0, and λ. Equations (17)–(19) in the limit of small cosmological constant reduce to the well known ones in flat spacetime. The solution Eqs. (11), (12) induces a deficit angle in asymptotically dS or AdS spacetime. Numerical calculations for $f(r)$ show that its shape is quite insensitive to ϵ in the range $0\leqslant\epsilon\leqslant 1$, not only asymptotically, but also close to the origin. We also find that an increasing positive cosmological constant tends to make a thicker monopole solution and a decreasing negative cosmological constant tends to make a thinner monopole solution.

In the following, we present a numerical analysis to the system, and the results are shown in Fig. 1.

From Fig. 1, one can see that the mass decreases to a negative asymptotic value when r approaches infinity in AdS spacetime. But in dS spacetime the mass is positive if the cosmological constant is large enough. The critical value for the cosmological constant is 0.0003 in our setup. The asymptotic mass for the above curves (a), (b), and (c) is 0.1415, 0.0000, and -0.1760, respectively. It is clear that the presence of the cosmological constant affects the behavior of the global monopole significantly. If the spacetime is modified to include a positive cosmological constant, the gravitational field of the global monopole can be attractive, in contrast to the same problem in asymptotically flat or AdS spacetime.

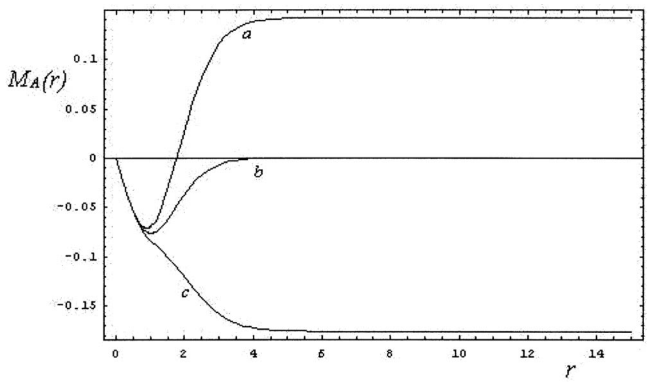

FIG. 1. The plot of mass as a function of r. Here we choose $\lambda=1$, $\sigma_0=0.01$, $G=1$. Curves (a), (b), and (c) are plotted when $\beta=0.0010$, 0.0003, and -0.0005, respectively.

Finally, we want to discuss why the tiny mass of the global monopole is defined as the limiting value of the function $M_A(r)$ given by Eq. (13). The standard definition of the Arnowitt-Deser-Misner (ADM) mass is different: It is defined by the $g_{rr}^{-1}=1-2GM(r)/r-(\lambda/3)r^2$, where λ is the cosmological constant. The mass M is then the limiting value of $M(r)$, and it is always positive. This agrees with the well known positive mass theorem for dS or AdS space first proved in [17]. However, in the case of a global monopole, $M(r)$ is linearly divergent for large r, which leads to a deficit solid angle plus a residual effect of the gravitational field. When a test particle moves in the gravitational field of the global monopole, the repulsive or attractive nature of the residual gravitational effect can be perceived by this particle [3,4]. That is, the standard definition of the mass gives in the case of a global monopole a linearly divergent expression plus a constant term. Now, it turns out that the divergent term does not produce any gravitational effect on the matter interacting with the monopole, and the whole interaction is entirely determined by the subleading finite term. This is why it is customary to subtract the divergent term from the definition of mass, since it is the resulting finite ''effective'' mass that determines the gravitational interaction with the monopole. Furthermore, the attractive or repulsive property of the residual gravitational field is determined by the positiveness or negativeness of $M_A(r)$. For the relation between the positive mass conjecture and the global monopole, one may refer to Ref. [18], in which Cvetič and Soleng pointed out that ''In Ref. [19], a positive mass conjecture was formulated saying that there is no singularity free solution of Einstein's field equations with matter sources (not including the vacuum) obeying the weak energy condition equations for which an exterior observer can see a negative mass object. A global monopole Refs. [2–4] would appear to be a counter example, but in this case the Goldstone fields extended to infinity, which means that these objects are extended sources and all observers must be inside the system.'' Therefore, although the effective mass M_A is negative under certain circumstances, it will not contradict the positive mass conjecture [19] and the positive mass theorem in dS or AdS spacetime [17].

When one studies the motion of test particles around a global monopole, it is an excellent approximation to take $M_A(r)$ as the constant $M_A(\beta,\epsilon^2)$ since the effective mass approaches its asymptotic value very quickly. Apart from academic interest in the global monopole configuration, the D stars [5–7] seem to make it relevant to the astronomical situation. Work on the generalization of D stars to asymptotically dS or AdS spacetime is in preparation.

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