New approach to the complex-action problem and its application to a nonperturbative study of superstring theory

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Monte Carlo simulations of a system whose action has an imaginary part are considered to be extremely difficult. We propose a new approach to this ''complex-action problem,'' which utilizes a factorization property of distribution functions. The basic idea is quite general, and it removes the so-called overlap problem completely. Here we apply the method to a nonperturbative study of superstring theory using its matrix formulation. In this particular example, the distribution function turns out to be positive definite, which allows us to reduce the problem even further. Our numerical results suggest an intuitive explanation for the dynamical generation of 4D space-time.

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I. INTRODUCTION

It occurs in many interesting systems ranging from condensed matter physics to high-energy physics that their action has an imaginary part. Some examples for instance in high-energy physics are the finite density QCD, Chern-Simons theories, systems with topological terms (such as the θ term in QCD), and systems with chiral fermions. While this is not a conceptual problem, it poses a technical problem when one attempts to study these systems by Monte Carlo simulations, which would otherwise provide a powerful tool to understand their properties from first principles (see Refs. [1–3] for recent works).

In this paper we propose a new approach to this ''complex-action problem.'' Suppose we want to obtain an expectation value of some observable. Then, as a more fundamental object, we consider the distribution function associated with that observable. In general the distribution function has a factorization property, which relates it to the distribution function associated with the same observable but calculated *omitting* the imaginary part of the action. The effect of the imaginary part is represented by a correction factor which can be obtained by a constrained Monte Carlo simulation. One of the virtues of this method is that it removes the so-called overlap problem completely. This problem comes from the fact that the two distribution functions—one for the full model and the other for the model omitting the imaginary part—have little overlap in general. The method avoids this problem by ''forcing'' the simulation to sample the important region for the full model.

The determination of the correction factor becomes increasingly difficult as the system size increases. In this sense, our approach does not solve the complex action problem completely. This should be contrasted with the meron-cluster algorithm [1], with which one can study a special class of complex-action systems by computer efforts increasing at most by some power of the system size. The factorization method eliminates the overlap problem, which composes some portion of the complex action problem, but not the whole. However, the resolution of the overlap problem is in fact a substantial progress. For instance, Refs. [2] developed a new method to weaken the same problem in finite density QCD, and the critical point was successfully identified. Therefore we expect that the *complete* resolution of the overlap problem allows us to address various interesting questions related to complex-action systems with the present computer resources. Since our method is based on the general property of distribution functions, it can be applied to *any* complex-action systems.

In this article we are concerned with a nonperturbative study of superstring theory using its matrix formulation [4]. Eventually we would like to examine the possibility that our 4-dimensional space time appears dynamically in 10-dimensional string theory [5–10]. Monte Carlo simulation of the matrix model suffers from the complex action problem, and there are evidences that the imaginary part of the action plays a crucial role in the dynamical reduction of the space-time dimensionality [7]. We will discuss how we can study such an issue by Monte Carlo simulation using the new approach.

II. THE SUPERSTRING MATRIX MODEL

As a nonperturbative definition of type IIB superstring theory in 10 dimensions, Ishibashi, Kawai, Kitazawa and Tsuchiya [4] proposed a matrix model, which can be formally obtained by the zero-volume limit of $D=10$, $\mathcal{N}=1$, pure super Yang-Mills theory. The partition function of the type IIB matrix model (and its obvious generalizations to $D=4,6$) can be written as

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$$Z = \int dA e^{-S_b} Z_f[A], \tag{1}$$

where A_μ ($\mu = 1, \ldots, D$) are D bosonic $N \times N$ traceless Hermitian matrices, and $S_b = -(1/4g^2)\mathrm{Tr}([A_\mu, A_\nu]^2)$ is the bosonic part of the action. The factor $Z_f[A]$ represents the quantity obtained by integration over the fermionic matrices, and its explicit form is given for example in Refs. [7,11]. The convergence of the integral (1) for arbitrary $N \geq 2$ was first conjectured [12] and proved recently [13]. The only parameter g in the model can be absorbed by rescaling $A_\mu \mapsto \sqrt{g} A_\mu$, which means that g is merely a scale parameter rather than a coupling constant. Therefore, one can determine the g dependence of any quantities on dimensional grounds [16]. Throughout this paper, we make our statements in such a way that they do not depend on the choice of g.

In this model space-time is represented by A_μ, and hence treated dynamically [5]. It is Euclidean as a result of the Wick rotation, which is always necessary in path integral formalisms. Its dimensionality is dynamically determined and can be probed by the moment of inertia tensor defined by [6]

$$T_{\mu\nu} = \frac{1}{N}\mathrm{Tr}(A_\mu A_\nu). \tag{2}$$

Since $T_{\mu\nu}$ is a $D \times D$ real symmetric matrix, it has D real eigenvalues corresponding to the principal moments of inertia, which we denote as λ_i ($i = 1, \ldots, D$) with the ordering

$$\lambda_1 > \lambda_2 > \cdots > \lambda_D > 0. \tag{3}$$

Let us define the vacuum expectation value (VEV) $\langle \mathcal{O} \rangle$ with respect to the partition function (1). If we find that $\langle \lambda_i \rangle$ with $i = 1, \ldots, d$ is much larger than the others, we may conclude that the dimensionality of the dynamical space-time is d.

III. THE COMPLEX ACTION PROBLEM

The fermion integral $Z_f[A]$ in the partition function (1) is complex in general for $D = 10$, $N \geq 4$ and for $D = 6$, $N \geq 3$ [7]. Let us restrict ourselves to these cases in what follows. Parameterizing the fermion integral as $Z_f[A] = \exp(\Gamma_R + i\Gamma)$, the partition function (1) may be written as

$$Z = \int dA e^{-S_0} e^{i\Gamma}, \tag{4}$$

where $S_0 = S_b - \Gamma_R$ is real. According to the standard reweighting method, one evaluates the VEV $\langle \lambda_i \rangle$ as

$$\langle \lambda_i \rangle = \frac{\langle \lambda_i e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}, \tag{5}$$

where the symbol $\langle \cdot \rangle_0$ denotes a VEV with respect to the partition function

$$Z_0 = \int dA e^{-S_0}. \tag{6}$$

The VEV $\langle \cdot \rangle_0$ can be evaluated by standard Monte Carlo simulations. However, $\langle e^{i\Gamma} \rangle_0$ is nothing but the ratio of the two partition functions Z_0/Z, and therefore it behaves as $e^{-N^2 \Delta F}$ at large N, where $\Delta F > 0$ is the difference of the free energy density of the corresponding two systems. This enormous cancellation (note that $|e^{i\Gamma}| = 1$ for each configuration) is caused by the fluctuation of the phase Γ, which grows linearly with the number of fermionic degrees of freedom, which is of $O(N^2)$. As a result the number of configurations required to obtain the VEV $\langle e^{i\Gamma} \rangle_0$ with sufficient accuracy grows as $e^{\mathrm{const}\, N^2}$. The same is true for the numerator $\langle \lambda_i e^{i\Gamma} \rangle_0$ in Eq. (5). This is the notorious ''complex action problem'' (or rather the ''sign problem,'' as we see below), which occurs also in many other interesting systems.

In fact we may simplify the expression (5) by using a symmetry. We note that under parity transformation:

$$A_1^P = -A_1, \quad A_i^P = A_i \quad \text{for} \quad 2 \leq i \leq D, \tag{7}$$

the fermion integral $Z_f[A]$ becomes complex conjugate [7], while the bosonic action S_b is invariant. Since the observable λ_i is also invariant, we can rewrite Eq. (5) as

$$\langle \lambda_i \rangle = \frac{\langle \lambda_i \cos\Gamma \rangle_0}{\langle \cos\Gamma \rangle_0}. \tag{8}$$

Note, however, that the problem still remains, since $\cos\Gamma$ flips its sign violently as a function of A_μ.

IV. THE NEW METHOD

A. The factorization property of distributions

The model (6) omitting the phase Γ was studied up to $N = 768$ and $N = 512$ for $D = 6$ and $D = 10$ respectively using the low-energy effective theory [5]. There it was found that $\langle \lambda_i \rangle_0 / (g N^{1/2})$ approaches a universal constant independent of i as N increases. This means that the dynamical space-time becomes isotropic in D dimensions at $N = \infty$, and hence the absence of spontaneous symmetry breaking (SSB) of SO(D) symmetry, *if one omits the phase* Γ.

We normalize the principal moments of inertia λ_i as

$$\tilde{\lambda}_i \overset{\mathrm{def}}{=} \frac{\lambda_i}{\langle \lambda_i \rangle_0}. \tag{9}$$

Then the deviation of $\langle \tilde{\lambda}_i \rangle$ from 1 represents the effects of the phase. The relevant question is whether the deviation depends on i at large N. In order to obtain the expectation value $\langle \tilde{\lambda}_i \rangle$, we consider the distribution associated with the observable $\tilde{\lambda}_i$:

$$\rho_i(x) \overset{\mathrm{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle. \tag{10}$$

As an important property of the distribution $\rho_i(x)$, it factorizes as

$$\rho_i(x) = \frac{1}{C}\rho_i^{(0)}(x) w_i(x), \tag{11}$$

where C is a normalization constant given by

$$C \overset{\text{def}}{=} \langle e^{i\Gamma} \rangle_0 = \langle \cos\Gamma \rangle_0 . \tag{12}$$

The real positive function $\rho_i^{(0)}(x)$ is defined by

$$\rho_i^{(0)}(x) \overset{\text{def}}{=} \langle\, \delta(x - \tilde{\lambda}_i) \rangle_0 , \tag{13}$$

which is nothing but the distribution of $\tilde{\lambda}_i$ in the model (6) without Γ. The function $\rho_i^{(0)}(x)$ is peaked at $x=1$ due to the chosen normalization (9). The function $w_i(x)$ in Eq. (11) can be regarded as the correction factor representing the effect of Γ, and it is given explicitly as

$$w_i(x) \overset{\text{def}}{=} \langle e^{i\Gamma} \rangle_{i,x} = \langle \cos\Gamma \rangle_{i,x} , \tag{14}$$

where the symbol $\langle \cdot \rangle_{i,x}$ denotes a VEV with respect to yet another partition function

$$Z_{i,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i). \tag{15}$$

Given all these definitions, it is straightforward to prove the relation (11).

B. Monte Carlo evaluation of $\rho_i^{(0)}(x)$ and $w_i(x)$

In order to obtain the function $w_i(x)$, we have to simulate Eq. (15). In practice we simulate instead the system

$$Z_{i,V} = \int dA e^{-S_0} e^{-V(\lambda_i)}, \tag{16}$$

where $V(\lambda_i)$ is some potential introduced only for the i-th principal moment of inertia. The explicit form of the potential we used in the study is $V(z) = \frac{1}{2}\gamma(z-\xi)^2$, where γ and ξ are real parameters. The results are insensitive to γ as far as it is sufficiently large and we took $\gamma = 1000.0$. Let us denote the VEV associated with the partition function (16) as $\langle \mathcal{O} \rangle_{i,V}$. Then the expectation value $\langle \cos\Gamma \rangle_{i,V}$ provides the value of $w_i(x)$ at $x = \langle \lambda_i \rangle_{i,V}$.

The function $\rho_i^{(0)}(x)$ can be obtained from the same simulation (16). Note that the distribution function for $\tilde{\lambda}_i$ in the system (16) is given by

$$\rho_{i,V}(x) \overset{\text{def}}{=} \langle\, \delta(x - \tilde{\lambda}_i) \rangle_{i,V} \propto \rho_i^{(0)}(x) e^{-V(\langle \lambda_i \rangle_0 x)}. \tag{17}$$

The position of the peak $x_{\rm p}$ is given by the solution to

$$0 = \frac{\partial}{\partial x} \ln \rho_{i,V}(x) = f_i^{(0)}(x) - \langle \lambda_i \rangle_0 V'(\langle \lambda_i \rangle_0 x), \tag{18}$$

where we have defined

$$f_i^{(0)}(x) \overset{\text{def}}{=} \frac{\partial}{\partial x} \ln \rho_i^{(0)}(x). \tag{19}$$

This implies that $\langle \lambda_i \rangle_0 V'(\langle \lambda_i \rangle_0 x_{\rm p})$ gives the value of $f_i^{(0)}(x)$ at $x = x_{\rm p}$. Since we take γ sufficiently large, the distribution $\rho_{i,V}(x)$ has a sharp peak, and we can safely replace the position of the peak $x_{\rm p}$ by the expectation value $\langle \tilde{\lambda}_i \rangle_{i,V}$. Once we obtain $f_i^{(0)}(x)$, we can obtain $\rho_i^{(0)}(x)$ by

$$\rho_i^{(0)}(x) = \exp\left[\int_0^x dz\, f_i^{(0)}(z) + \text{const} \right], \tag{20}$$

where the integration constant can be determined by the normalization of $\rho_i^{(0)}(x)$.

C. Resolution of the overlap problem

From $\rho_i^{(0)}(x)$ and $w_i(x)$, we may obtain the VEV $\langle \tilde{\lambda}_i \rangle$ by

$$\langle \tilde{\lambda}_i \rangle = \int_0^\infty dx\, x \rho_i(x) = \frac{\displaystyle\int_0^\infty dx\, x\, \rho_i^{(0)}(x) w_i(x)}{\displaystyle\int_0^\infty dx\, \rho_i^{(0)}(x) w_i(x)}. \tag{21}$$

Actually this simply amounts to using the reweighting formula (8) but calculating the VEVs on the right-hand side (RHS) by

$$\langle \tilde{\lambda}_i \cos\Gamma \rangle_0 = \int_0^\infty dx\, x\, \rho_i^{(0)}(x) w_i(x) \tag{22}$$

$$\langle \cos\Gamma \rangle_0 = \int_0^\infty dx\, \rho_i^{(0)}(x) w_i(x). \tag{23}$$

This reveals one of the virtues of our approach as compared with the standard reweighting method using the formula (8) directly. Suppose we are to obtain the LHS of Eqs. (22) and (23) by simulating the system (6). Then for most of the time, $\tilde{\lambda}_i$ takes the value at the peak of $\rho_i^{(0)}(x)$. However, in order to obtain the VEVs accurately we have to sample configurations whose $\tilde{\lambda}_i$ takes a value where $|\rho_i^{(0)}(x) w_i(x)|$ becomes large. In general the overlap of the two functions becomes exponentially small with the system size, and this makes the important sampling ineffective. Therefore, this "overlap problem" composes some portion of the complex-action problem. The new approach eliminates this problem by "forcing" the simulation to sample the important region.

D. Further improvement in the case $w_i(x) > 0$

So far, we have been discussing the general properties of the new method. In the case at hand, we can actually further reduce the problem by using the fact that the correction factors $w_i(x)$ are actually *positive definite*, and so is the full distribution function $\rho_i(x)$. (Note that this is not guaranteed *in general*.) This allows us to obtain the VEV $\langle \tilde{\lambda}_i \rangle$ by minimizing the "free energy density" $F_i(x) = -(1/N^2)\log \rho_i(x)$,

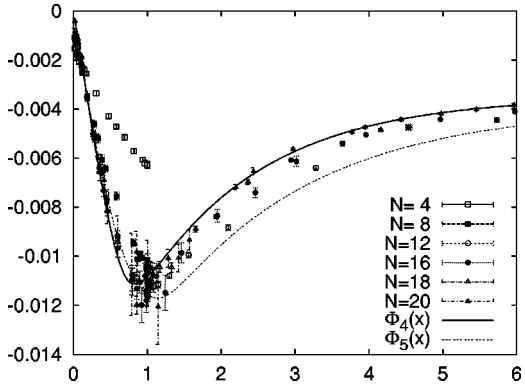

FIG. 1. The function $(1/N^2)\ln w_4(x)$ is plotted for $N=12,16,18,20$. For $x<1$ we also plot data for $N=4,8$ to clarify the convergence. We extract the scaling function $\Phi_4(x)$ by fitting the data to some analytic function, which is represented by the solid line. The dashed line represents $\Phi_5(x)$, which is obtained similarly from the scaling behavior of $(1/N^2)\ln w_5(x)$.

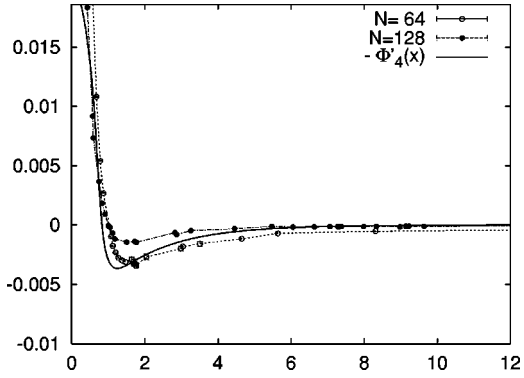

FIG. 2. The function $(1/N^2)f_4^{(0)}(x)$ is plotted for $N=64,128$. The solid line represents $-\Phi_4'(x)$, which we calculate from the scaling function $\Phi_4(x)$ extracted in Fig. 1.

instead of using Eq. (21). For that we simply need to solve $F_i'(x)=0$, which is equivalent to

$$\frac{1}{N^2}f_i^{(0)}(x)=-\frac{d}{dx}\left[\frac{1}{N^2}\ln w_i(x)\right]. \tag{24}$$

The function in the bracket $[\cdot]$ is expected to approach a well-defined function as N increases:

$$\frac{1}{N^2}\ln w_i(x)\to\Phi_i(x). \tag{25}$$

Let us note that $w_i(x)$ is nothing but the expectation value of $e^{i\Gamma}$ in the system (15). According to the argument below Eq. (6), $w_i(x)$ for fixed x decreases as $e^{-\alpha N^2}$ at large N. The constant α may depend on x, hence the assertion. Indeed our numerical results in Fig. 1 (although the achieved values of N are not very large) seem to support this argument. Once we extract the scaling function $\Phi_i(x)$, we may use it instead of $(1/N^2)\ln w_i(x)$ in Eq. (24) for larger N. Thus we are able to obtain the VEV $\langle\tilde{\lambda}_i\rangle$ for much larger N than those allowing the direct Monte Carlo evaluation of the correction factor $w_i(x)$.

The positive definiteness of $w_i(x)$ is crucial for such an extrapolation technique to work. If we were to calculate the VEV $\langle\tilde{\lambda}_i\rangle$ by Eq. (21), we would need to calculate the correction factor for larger N by $w_i(x)=e^{N^2\Phi_i(x)}$, where the multiplication by N^2 and the exponentiation would magnify the errors in $\Phi_i(x)$ considerably. This does *not* occur when we obtain the VEV $\langle\tilde{\lambda}_i\rangle$ by solving Eq. (24).

V. RESULTS

Monte Carlo simulation of Eq. (16) can be performed by using the algorithm developed for the model (6) in Ref. [11]. The required computational effort is $O(N^6)$. In this work, we use instead the low-energy effective theory proposed in Ref. [5] and further developed in Ref. [8]. The required computational effort becomes $O(N^3)$. For the definition of the low-energy effective theory as well as all the technical details including parameters used in the simulations, we follow Ref. [8]. The validity of the low-energy effective theory in studying the extent of the dynamical space time is discussed in Ref. [11]. We also note that the complex-action problem survives in passing from the full theory to the low-energy effective theory, and hence we expect that the effects of the phase on the reduction of space-time dimensionality should be visible also in the low-energy effective theory, if it is there at all. Here we study the $D=6$ case (instead of $D=10$, which corresponds to the type IIB matrix model) to decrease the computational efforts further.

In Fig. 1, we plot $(1/N^2)\ln w_4(x)$. The correction factor $w_4(x)$ has a minimum at $x\sim 1$ and it becomes larger for both $x<1$ and $x>1$. This can be understood as follows. Let us recall again that $w_i(x)$ is the expectation value of $e^{i\Gamma}$ in the system (15), where $\tilde{\lambda}_i$ is constrained to a given value of x. At $x=1$, the system (15) is almost equivalent to the system (6), because $\tilde{\lambda}_i$ would be close to 1 even without the constraint. [From this, it also follows that $w_i(1)$ takes almost the same value for all i.] Therefore, the dominant configurations of the system (15) at $x=1$ is isotropic at large N [8]. On the other hand, the dominant configurations of the system (15) at small x are $(i-1)$-dimensional, since the constraint forces $\tilde{\lambda}_i$ to be small, and due to the ordering (3), all the $\tilde{\lambda}_j$ with $j\geqslant i$ become small. Similarly the dominant configurations of the system (15) at large x are almost i-dimensional, since the constraint forces $\tilde{\lambda}_i$ to be large, and due to the ordering (3), all the $\tilde{\lambda}_j$ with $j\leqslant i$ become large. Now let us recall that the phase Γ vanishes when the configuration A has the dimensionality $d\leqslant d_{\rm cr}$, where $d_{\rm cr}=4,6$ for $D=6,10$, respectively [7]. As a consequence, $w_4(x)$ gets larger in both $x<1$ and $x>1$ regimes.

As mentioned already, Fig. 1 supports the scaling behavior (25) with increasing N. The scaling function $\Phi_4(x)$ can be extracted by fitting the data to some analytic function. We find that $\Phi_4(x)$ approaches 0 linearly as $x\to 0$, and it approaches some negative constant exponentially as $x\to\infty$. We observe a similar scaling behavior for $(1/N^2)\ln w_5(x)$. The

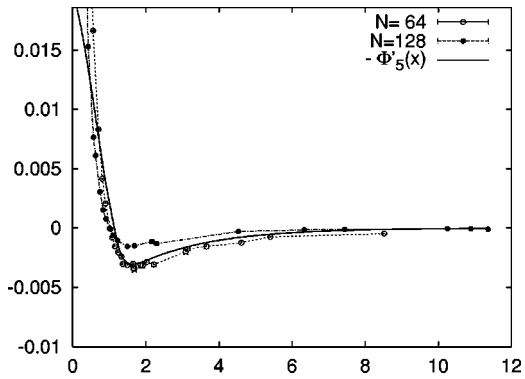

FIG. 3. The function $(1/N^2)f_5^{(0)}(x)$ is plotted for $N=64,128$. The solid line represents $-\Phi_5'(x)$, which we calculate from the scaling function $\Phi_5(x)$ shown in Fig. 1.

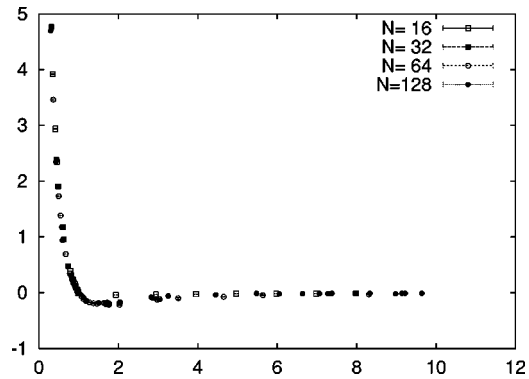

FIG. 4. The function $(1/N)f_4^{(0)}(x)$ is plotted for $N=16,32,64,128$. A clear scaling behavior is observed.

corresponding scaling function $\Phi_5(x)$ is plotted in Fig. 1 for comparison.

Figure 2 represents a graphical solution of Eq. (24) for $i=4$. The open and closed circles describe the function $(1/N^2)f_4^{(0)}(x)$ for $N=64,128$ respectively. It is positive at $x<1$ and turns negative at $x>1$, which reflects the fact that $\rho_i^{(0)}(x)$ is peaked at $x=1$. The solid line represents $-\Phi_4'(x)$. The intersections of the two curves provide the solutions to Eq. (24). At $N=128$, we find that the distribution $\rho_4(x)$ has two peaks; one at $x=x_s<1$ and the other at $x=x_l>1$. The ratio of the peak height $R=\rho_4(x_s)/\rho_4(x_l)$ can be written as $R=\exp\{N^2(\mathcal{A}_s-\mathcal{A}_l)\}$, where $\mathcal{A}_s$ and $\mathcal{A}_l$ are the area of the regions surrounded by the two curves. We obtain $\mathcal{A}_s\sim 5.0\times 10^{-4}$ and $\mathcal{A}_l\sim 4.5\times 10^{-3}$, from which we conclude that the peak at $x>1$ is dominant. In Fig. 3 we show the results of a similar analysis for $\rho_5(x)$. We find that the distribution $\rho_5(x)$ at $N=128$ also has two peaks; one at $x<1$ and the other at $x>1$. However, here we obtain $\mathcal{A}_s\sim 2.0\times 10^{-3}$ and $\mathcal{A}_l\sim 3.8\times 10^{-3}$, which are comparable.

VI. SUMMARY AND DISCUSSIONS

We have proposed a new method to study complex-action systems by Monte Carlo simulations. In particular we discussed how we can use the method to investigate the possibility that four-dimensional space time is dynamically generated in the type IIB matrix model. The space-time dimensionality is probed by the eigenvalues λ_i of the moment of inertia tensor and we study the distribution of each eigenvalue. The distribution $\rho_i^{(0)}(x)$ obtained without the phase Γ has a single peak, which is located at $x=1$. The effect of the phase Γ on the distribution function is represented by the multiplication of the correction factor $w_i(x)$ as stated in Eq. (11).

Our results for the 4-th and 5-th eigenvalues ($i=4,5$) in the $D=6$ case show that the correction factor $w_i(x)$ strongly suppresses the peak of $\rho_i^{(0)}(x)$ at $x=1$ and favors *both* smaller x and larger x. As a result, we observe that the distribution $\rho_i(x)$ including the effects of the phase, in fact, has a *double peak structure*. Moreover, the two peaks tend to move away from $x=1$ as N is increased. It is important to determine which of the two peaks becomes dominant in the large N limit. At $N=128$, we observe that the peak at $x>1$ is dominant for both $\rho_4(x)$ and $\rho_5(x)$. We note, however, that it is much more dominant for $\rho_4(x)$ than for $\rho_5(x)$.

From Figs. 2 and 3, we observe that the function $(1/N^2)f_i^{(0)}(x)$ changes drastically as we go from $N=64$ to $N=128$. In fact we find that $(1/N)f_i^{(0)}(x)$ scales [notice the normalization factor $(1/N)$], as shown in Fig. 4 for $i=4$. The scaling region extends from $x\sim 1$, where $(1/N)f_i^{(0)}(x)$ crosses zero, namely the place where $\rho_i^{(0)}(x)$ has a peak. A similar scaling behavior is observed for $i=5$. This scaling behavior is understandable if we recall that the long-distance property of the system is controlled by a branched-polymer-like system [5], which is essentially a system with N degrees of freedom. If we *naively* extrapolate this scaling behavior of $(1/N)f_i^{(0)}(x)$ to larger N, the LHS of Eq. (24) becomes negligible. It follows that the peak at $x<1$ eventually dominates for both $i=4,5$, considering the asymptotic behaviors of $\Phi_i(x)$ as $x\to 0$ and $x\to\infty$. This means that the space-time dimensionality becomes $d\leqslant 3$. However, it is well-known that the Hausdorff dimension of a branched polymer is $d_{\rm H}=4$, which implies that such a system is not easy to collapse into a configuration with dimensions $\leqslant 3$. The consequence would be that $\rho_4^{(0)}(x)$ is much more suppressed in the small x regime than $\rho_5^{(0)}(x)$ at large N. We consider that this prevents the peak at $x<1$ from dominating for $\rho_4(x)$, and as a result we obtain 4D space-time. Since the above argument is based only on the scaling behaviors and the branched polymer description, it is expected to be valid also in the $D=10$ case. (While this paper was being revised, an analytic evidence for the dominance of 4D space-time was also reported [14].)

Our new approach to complex-action systems is based on the factorization property (11) of distribution functions, which is quite general. As we discussed in Sec. IV C, it resolves the overlap problem completely. In a separate paper we will report on a test of the new method in a random matrix theory for finite density QCD, where exact results in the thermodynamic limit are successfully obtained [15]. We hope that the ''factorization method'' allows us to study interesting complex-action systems in various branches of physics.

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[1] W. Bietenholz, A. Pochinsky, and U.J. Wiese, Phys. Rev. Lett. **75**, 4524 (1995); S. Chandrasekharan and U.J. Wiese, *ibid.* **83**, 3116 (1999); M.G. Alford, S. Chandrasekharan, J. Cox, and U.J. Wiese, Nucl. Phys. **B602**, 61 (2001); S. Chandrasekharan, J. Cox, J.C. Osborn, and U.J. Wiese, cond-mat/0201360.

[2] Z. Fodor and S.D. Katz, Phys. Lett. B **534**, 87 (2002); J. High Energy Phys. **03**, 014 (2002).

[3] P. Henelius and A.W. Sandvik, Phys. Rev. B **62**, 1102 (2000); C. Adami and S.E. Koonin, Phys. Rev. C **63**, 034319 (2001); B.E. Baaquie and Y.S. Seng, Int. J. Mod. Phys. C **13**, 587 (2002); E.B. Gregory, S.H. Guo, H. Kroger, and X.Q. Luo, Phys. Rev. D **62**, 054508 (2000).

[4] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, Nucl. Phys. **B498**, 467 (1997).

[5] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, and T. Tada, Prog. Theor. Phys. **99**, 713 (1998).

[6] T. Hotta, J. Nishimura, and A. Tsuchiya, Nucl. Phys. **B545**, 543 (1999).

[7] J. Nishimura and G. Vernizzi, J. High Energy Phys. **04**, 015 (2000); Phys. Rev. Lett. **85**, 4664 (2000).

[8] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, J. High Energy Phys. **07**, 011 (2000).

[9] P. Bialas, Z. Burda, B. Petersson, and J. Tabaczek, Nucl. Phys. **B592**, 391 (2001); Z. Burda, B. Petersson, and J. Tabaczek, *ibid.* **B602**, 399 (2001).

[10] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, F. Hofheinz, and J. Nishimura, Phys. Rev. D **65**, 086001 (2002).

[11] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, J. High Energy Phys. **07**, 013 (2000).

[12] W. Krauth, H. Nicolai, and M. Staudacher, Phys. Lett. B **431**, 31 (1998).

[13] P. Austing and J.F. Wheater, J. High Energy Phys. **02**, 028 (2001); **04**, 019 (2001).

[14] J. Nishimura and F. Sugino, J. High Energy Phys. **05**, 001 (2002); H. Kawai, S. Kawamoto, T. Kuroki, T. Matsuo, and S. Shinohara, hep-th/0204240.

[15] J. Ambjørn, K. Anagnostopoulos, J. Nishimura, and J.J.M. Verbaarschot, J. High Energy Phys. **10**, 062 (2002).

[16] It was found in the $D=4$ case that operators related to string theory possess a well-defined large N limit when g^2N is kept fixed [11].