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Matching conditions in metric-affine gravity

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By using an isotropic field configuration for the *triplet* ansatz sector of the metric-affine theories of gravity (MAG), we find a class of harmonic solutions which represents the interior and exterior field of a distribution endowed with electric and strong gravitoelectric multipole moments. Moreover, the general matching and junction conditions in MAG and their reduction to the *triplet* ansatz sector are presented.

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I. INTRODUCTION

Even though Einstein's treatment of spacetime as a Riemannian manifold appears almost fully corroborated experimentally, there are several reasons to believe that its validity is limited to macroscopic structures and to the present cosmological era. The only available finite perturbative treatment of quantum gravity, namely, the theory of the quantum superstrings [1], suggests that non-Riemannian features are present on the scale of the Planck length. On the other hand, recent advances in the study of the early universe, as represented by the inflationary paradigm, involve, in addition to the metric tensor, at the very least a scalar field [2], induced by a Weyl geometry, i.e., the inflaton, an essential departure from Riemannian metricity [3]. Even at the classical cosmological level, a dilatonic field has recently been used to describe the presence of dark matter in the universe, as well as to explain certain cosmological observations which contradicted the fundamentals of the standard cosmological model [4].

There exist good experimental evidence that, at the present state of the universe, the geometrical structure of spacetime corresponds to a metric-compatible geometry in which nonmetricity vanishes. Consequently, a metric-affine geometry is irrelevant for the geometrical description of the universe today. However, during the early universe, when the energies of the cosmic matter were much higher than today, we expect scale invariance to prevail and, according to the metric affine theory of gravity (MAG), the canonical dilation (or scale) current of matter, the trace of the hypermomentum current $\Delta^{\gamma}{}_{\gamma}$, becomes coupled to the Weyl covector $Q^{\gamma}{}_{\gamma}$. Moreover, shear type excitations of the material multispinors, i.e., Regge trajectory type of constructs, are expected to arise, thereby liberating the metric-compatible Riemann-Cartan spacetime from its constraint of vanishing nonmetricity $Q_{\alpha\beta}=0$. It is therefore important to derive and investigate exact solutions of these theories which contain information about the new geometric objects like torsion and nonmetricity (for a survey of these theories see [5]).

For restricted irreducible pieces of torsion and nonmetricity there are similarities between the Einstein-Proca system and the vacuum MAG field equations [6–9]. This observation enables us to find new solutions for MAG theories [10]. Special electrovacuum solutions in MAG have already been found in Refs. [11–13]. Moreover, also static black hole configurations in MAG have been investigated in vacuum [14] and in the presence of Abelian and non-Abelian matter [15].

In this paper we use an isotropic field configuration and the underlying harmonic structure of the MAG theories, in their *triplet* ansatz sector, in order to generalize our previous vacuum results [16] by introducing additional electromagnetic fields. We obtain electric multipole solutions. These solutions are interesting because they imply the consideration of matching conditions between internal and external solutions. However, the necessary matching conditions between two different spacetimes in MAG have not been investigated so far. Arkuszewski *et al.* [17] advanced in this topic in the framework of the Einstein-Cartan theory. The matching conditions and their general implications in MAG are analyzed to some extent.

The plan of the paper is as follows: Section II contains a brief presentation of MAG theories and its triplet ansatz sector. In Sec. III the isotropic field configuration in MAG is discussed. In Sec. IV electrically charged multipole solutions in MAG are displayed and the physical interpretation of the parameters entering the solutions is analyzed. In Sec. V we discuss the discontinuities. In Sec. VI the matching conditions in MAG are considered. In Sec. VII the matching conditions for the specific Einstein-Proca-Maxwell system of the triplet ansatz sector of MAG are presented. In Sec. VIII the results are discussed.

II. MAG FIELD EQUATIONS AND THE TRIPLET ANSATZ

A. MAG in brief

Let us consider a frame field and a coframe field denoted by

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$$e_\alpha = e^\mu{}_\alpha \partial_\mu , \quad \vartheta^\beta = e_\mu{}^\beta dx^\mu , \tag{1}$$

respectively. The $GL(4,R)$-covariant derivative for a tensor valued p form reads

$$D = d + \Gamma_\alpha{}^\beta \rho(L^\alpha{}_\beta)\wedge , \tag{2}$$

where $\rho(L^\alpha{}_\beta)$ is the representation of $GL(4,R)$ and $L^\alpha{}_\beta$ are the generators; the connection one-form is $\Gamma_\alpha{}^\beta = \Gamma_{\mu\alpha}{}^\beta dx^\mu$. The nonmetricity one-form, the torsion, and curvature two-forms read

$$Q_{\alpha\beta} := -Dg_{\alpha\beta}, \quad T^\alpha := D\vartheta^\alpha ,$$
$$R_\alpha{}^\beta := d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta , \tag{3}$$

respectively, and the Bianchi identities are given by

$$DQ_{\alpha\beta} \equiv 2R_{(\alpha\beta)} , \quad DT^\alpha \equiv R_\gamma{}^\alpha \wedge \vartheta^\gamma , \quad DR_\alpha{}^\beta \equiv 0 . \tag{4}$$

It is worthwhile to stress the fact that the nonmetricity $Q_{\alpha\beta}$, the torsion T^α, and the curvature $R_\alpha{}^\beta$ play the role of field strengths.

We now turn to the *source currents* for the fields above, these will depend on the Lagrangian (Ψ is a matter manifield),

$$\mathcal{L} = \mathcal{L}(g_{\alpha\beta}, dg_{\alpha\beta}, \vartheta^\alpha, d\vartheta^\alpha, \Gamma_\alpha{}^\beta, d\Gamma_\alpha{}^\beta, \Psi, D\Psi), \tag{5}$$

which can be rewritten in a covariantized form as

$$\mathcal{L} = \mathcal{L}(g_{\alpha\beta}, Q_{\alpha\beta}, \vartheta^\alpha, T^\alpha, \Gamma_\alpha{}^\beta, R_\alpha{}^\beta, \Psi, D\Psi). \tag{6}$$

We will consider a metric-affine theory described by the particular Lagrangian

$$\mathcal{L} = V_{\mathrm{MAG}} + \mathcal{L}_{\mathrm{MAT}} , \tag{7}$$

where $\mathcal{L}_{\mathrm{MAT}}$ represents the Lagrangian of the matter field. The matter current three-forms are then given by the Euler-Lagrange functional derivatives (denoted by δ) of the material piece L_{MAT}. We have the canonical energy-momentum current

$$\Sigma_\alpha := \delta L_{\mathrm{MAT}}/\delta\vartheta^\alpha = \partial L_{\mathrm{MAT}}/\partial\vartheta^\alpha + D(\partial L_{\mathrm{MAT}}/\partial T^\alpha), \tag{8}$$

the hypermomentum current

$$\begin{aligned}\Delta^\alpha{}_\beta &:= \delta L_{\mathrm{MAT}}/\delta\Gamma_\alpha{}^\beta \\ &= \rho(L^\alpha{}_\beta)\Psi\wedge[\partial L_{\mathrm{MAT}}/\partial(D\Psi)] + 2g_{\beta\gamma}(\partial L_{\mathrm{MAT}}/\partial Q_{\alpha\gamma}) \\ &\quad + \vartheta^\alpha \wedge (\partial L_{\mathrm{MAT}}/\partial T^\beta) \\ &\quad + D(\partial L_{\mathrm{MAT}}/\partial R_\alpha{}^\beta),\end{aligned} \tag{9}$$

and also a related current four-form, the (symmetric) metric energy-momentum, namely

$$\begin{aligned}\sigma^{\alpha\beta} &:= 2\,\delta L_{\mathrm{MAT}}/\delta g_{\alpha\beta} = 2\,\partial L_{\mathrm{MAT}}/\partial g_{\alpha\beta} \\ &\quad + 2D(\partial L_{\mathrm{MAT}}/\partial Q_{\alpha\beta}).\end{aligned} \tag{10}$$

Hence the MAG field equations turn out to be [5]

$$\delta L_{\mathrm{MAT}}/\delta\Psi = 0 \qquad (\text{matter}), \tag{11}$$

$$DM^{\alpha\beta} - m^{\alpha\beta} = \sigma^{\alpha\beta} \qquad (\text{zeroth}), \tag{12}$$

$$DH_\alpha - E_\alpha = \Sigma_\alpha \qquad (\text{first}), \tag{13}$$

$$DH^\alpha{}_\beta - E^\alpha{}_\beta = \Delta^\alpha{}_\beta \qquad (\text{second}), \tag{14}$$

where we have used the canonical momenta (‘‘excitations’’),

$$M^{\alpha\beta} := -2\,\partial V_{\mathrm{MAG}}/\partial Q_{\alpha\beta} , \tag{15}$$

a momentum three-form conjugate to the metric field,

$$H_\alpha := -\partial V_{\mathrm{MAG}}/\partial T^\alpha , \tag{16}$$

a momentum two-form conjugate to the coframe field, and

$$H^\alpha{}_\beta := -\partial V_{\mathrm{MAG}}/\partial R_\alpha{}^\beta , \tag{17}$$

the momentum two-form conjugate to the $GL(4,R)$ connection.

The currents $m^{\alpha\beta}, E_\alpha, E^\alpha{}_\beta$ are components of the metric energy momentum, of the canonical energy momentum, and of the hypermomentum currents, contributed by the gravitational fields themselves, respectively, in V_{MAG} the so-called vacuum contributions.

Diffeomorphisms and $GL(4,R)$ invariance yield two Noether identities [5] which, given in their ‘‘weak’’ form, i.e., after the application of the matter equation of motion (11), become

$$\begin{aligned}D\Sigma_\alpha &= (e_\alpha \rfloor T^\beta)\wedge\Sigma_\beta \\ &\quad + (e_\alpha \rfloor R_\beta{}^\gamma)\wedge\Delta^\beta{}_\gamma \\ &\quad - (1/2)(e_\alpha \rfloor Q_{\beta\gamma})\sigma^{\beta\gamma},\end{aligned} \tag{18}$$

$$D\Delta^\alpha{}_\beta + \vartheta^\alpha\wedge\Sigma_\beta - g_{\beta\gamma}\sigma^{\alpha\gamma} = 0. \tag{19}$$

It is worthwhile to stress the fact that given a solution of the second MAG field equation the zeroth and the first field equations are not independent, i.e., one of them is redundant.

In a metric-affine spacetime, the curvature has 11 irreducible pieces [5], whereas the nonmetricity has *four* and the torsion *three* irreducible pieces. The most general parity conserving Lagrangian V_{MAG} which has been constructed in terms of all irreducible pieces of the post-Riemannian components has been investigated previously [10] and reads

$$V_{\rm MAG}=\frac{1}{2\kappa}\bigg[-a_0R^{\alpha\beta}\wedge\eta_{\alpha\beta}-2\lambda\,\eta+T^\alpha\wedge{}^*\bigg(\sum_{I=1}^{3}a_I{}^{(I)}T_\alpha\bigg)+2\bigg(\sum_{I=2}^{4}c_I{}^{(I)}Q_{\alpha\beta}\bigg)\wedge\vartheta^\alpha\wedge{}^*T^\beta+Q_{\alpha\beta}\wedge{}^*\bigg(\sum_{I=1}^{4}b_I^{(I)}Q^{\alpha\beta}\bigg)$$
$$+b_5({}^{(3)}Q_{\alpha\gamma}\wedge\vartheta^\alpha)\wedge{}^*({}^{(4)}Q^{\beta\gamma}\wedge\vartheta_\beta)\bigg]-\frac{1}{2\rho}R^{\alpha\beta}\wedge{}^*\bigg(\sum_{I=1}^{6}w_I^{(I)}W_{\alpha\beta}+w_7\vartheta_\alpha\wedge(e_\gamma\rfloor{}^{(5)}W^\gamma{}_\beta)+\sum_{I=1}^{5}z_I^{(I)}Z_{\alpha\beta}+z_6\vartheta_\gamma$$
$$\wedge(e_\alpha\rfloor{}^{(2)}Z^\gamma{}_\beta)+\sum_{I=7}^{9}z_I\vartheta_\alpha\wedge(e_\gamma\rfloor{}^{(I-4)}Z^\gamma{}_\beta)\bigg). \tag{20}$$

The Minkowski metric is $o_{\alpha\beta}=\mathrm{diag}(-+++)$, $*$ is the Hodge dual, $\eta:={}^*1$ is the volume four-form, the constant λ is the cosmological constant, ρ the strong gravity coupling constant, the constants and $a_0,\ldots,a_3$, $b_1,\ldots,b_5$, c_2,c_3,c_4, $w_1,\ldots,w_7$, $z_1,\ldots,z_9$ are dimensionless. We have introduced in the curvature square term the irreducible pieces of the antisymmetric part $W_{\alpha\beta}:=R_{[\alpha\beta]}$ and the symmetric part $Z_{\alpha\beta}:=R_{(\alpha\beta)}$ of the curvature two-form. In $Z_{\alpha\beta}$, we have the purely *post* Riemannian part of the curvature. Note the peculiar cross terms with c_I and b_5.

B. Triplet ansatz sector of MAG and equivalence theorem

The Lagrangian (20) is very complicated and difficult to manage. Therefore we will consider here only the simplest nontrivial case of torsion and nonmetricity with shear. Then, for the nonmetricity we use the ansatz

$$Q_{\alpha\beta}={}^{(3)}Q_{\alpha\beta}+{}^{(4)}Q_{\alpha\beta}, \tag{21}$$

where

$${}^{(3)}Q_{\alpha\beta}=\frac{4}{9}\left(\vartheta_{(\alpha}e_{\beta)}\rfloor\Lambda-\frac{1}{4}g_{\alpha\beta}\Lambda\right), \tag{22}$$

with

$$\Lambda:=\vartheta^\alpha e^\beta\rfloor\not{Q}_{\alpha\beta}$$

is the proper shear piece and ${}^{(4)}Q_{\alpha\beta}=Q\,g_{\alpha\beta}$ represents the dilation piece, where $Q:=(1/4)\,Q_\gamma{}^\gamma$ is the Weyl one-form, and $\not{Q}_{\alpha\beta}:=Q_{\alpha\beta}-Q\,g_{\alpha\beta}$ is the traceless piece of the nonmetricity. Other pieces of the irreducible decomposition of the nonmetricity [5] are taken to be zero.

Let us choose for the torsion only the covector piece as nonvanishing:

$$T^\alpha={}^{(2)}T^\alpha=\frac{1}{3}\vartheta^\alpha\wedge T,\quad\text{with}\quad T:=e_\alpha\rfloor T^\alpha. \tag{23}$$

Thus we are left with a triplet of nontrivial one-forms Q, Λ, and T for which we make the following ansatz (for details, see [10])

$$Q=\frac{k_0}{k_1}\Lambda=\frac{k_0}{k_2}T, \tag{24}$$

where k_0, k_1, and k_2 are given in terms of the gravitational coupling constants, i.e., $k_0\equiv4\alpha_2\beta_3-3\gamma_3^2$, $k_1\equiv9(\alpha_2\beta_5/2-\gamma_3\gamma_4)$, $k_2\equiv3(4\beta_3\gamma_4-3\beta_5\gamma_3/2)$, and $\alpha_2=a_2-2a_0$, $\beta_3=b_3+a_0/8$, $\beta_4=b_4-3a_0/8$, $\beta_5=b_5-a_0$, $\gamma_3=c_3+a_0$, and $\gamma_4=c_4+a_0$. In other words, we assume that the triplet of one-forms are proportional to each other [6,8–10,18]. This is the so-called *triplet* ansatz sector of MAG theories [6,8,9].

Consequently, here we limit ourselves to the special case in which the only surviving strong gravity piece is the square of the segmental curvature (with vanishing cosmological constant), i.e.,

$$V_{\rm MAG}=\frac{1}{2\kappa}[-a_0R^{\alpha\beta}\wedge\eta_{\alpha\beta}+a_2T^\alpha\wedge{}^{*(2)}T_\alpha+2(c_3{}^{(3)}Q_{\alpha\beta}$$
$$+c_4{}^{(4)}Q_{\alpha\beta})\wedge\vartheta^\alpha\wedge{}^*T^\beta+Q_{\alpha\beta}\wedge{}^*(b_3{}^{(3)}Q^{\alpha\beta}$$
$$+b_4{}^{(4)}Q^{\alpha\beta})]-\frac{z_4}{2\rho}R^{\alpha\beta}\wedge{}^{*(4)}Z_{\alpha\beta}, \tag{25}$$

where the segmental curvature ${}^{(4)}Z_{\alpha\beta}:=R_\gamma{}^\gamma g_{\alpha\beta}/4=g_{\alpha\beta}dQ$, with the Weyl covector $Q:=Q_\gamma{}^\gamma/4$. Therefore

$$-\frac{z_4}{2\rho}R^{\alpha\beta}\wedge{}^{*(4)}Z_{\alpha\beta}=-\frac{z_4}{2\rho}R_\alpha{}^\alpha\wedge{}^*R_\beta{}^\beta$$
$$=-\frac{2z_4}{\rho}dQ\wedge{}^*dQ \tag{26}$$

is the kinetic term for the Weyl one-form.

Under the above given assumptions it is now straightforward to apply Obukhov's equivalence theorem [6,7,9] according to which the field equations following from the pure geometrical part of the Lagrangian (7), i.e., $V_{\rm MAG}$, are equivalent to Einstein's equations with an energy-momentum tensor determined by a Proca field. In the case investigated here we have an additional term due to the presence of the matter field in Eq. (7). Thus the field equations reduce to

$$\frac{a_0}{2}\eta_{\alpha\beta\gamma}\wedge\tilde{R}^{\beta\gamma}=\kappa\Sigma_\alpha, \tag{27}$$

$$d{}^*\omega+m^2{}^*\phi=0, \tag{28}$$

where ϕ represents the Proca one-form, $\omega\equiv d\phi$ is the corresponding field strength, m is completely given in terms of the coupling constants, and a tilde denotes the Riemannian part of the curvature. The energy-momentum current entering the right-hand side of the Einstein equations is given by

$$\Sigma_\alpha = \Sigma_\alpha^{(\phi)} + \Sigma_\alpha^{(\mathrm{MAT})}, \tag{29}$$

where

$$\Sigma_\alpha^{(\phi)} := \frac{z_4 k_0^2}{2\rho}\{(e_\alpha \rfloor d\phi)\wedge {}^*d\phi - (e_\alpha\rfloor {}^*d\phi)\wedge d\phi + m^2[(e_\alpha\rfloor\phi) \wedge {}^*\phi + (e_\alpha\rfloor {}^*\phi)\wedge\phi]\} \tag{30}$$

is the energy-momentum current of the Proca field, and $\Sigma_\alpha^{(\mathrm{MAT})}$ is the energy-momentum current of the additional matter field which also satisfies the corresponding Euler-Lagrange equations.

Thus the triplet ansatz sector of a MAG theory coupled to a matter field has been reduced to the effective Einstein-Proca system of differential equations coupled to a matter field. Moreover, by setting $m=0$ the system acquires the constraint $\beta_4 = (k_1\beta_5/2 + k_2\gamma_4)/4k_0$ among the coupling constants of the Lagrangian (20), and it reduces to an Einstein-Maxwell system, cf. Ref. [10].

III. ISOTROPIC FIELD CONFIGURATION IN MAG

We will look for exact multipole solutions of the field equations arising from the Lagrangian

$$\mathcal{L} = V_{\mathrm{MAG}} + L_{\mathrm{Max}}, \quad \text{with } L_{\mathrm{Max}} = -(1/2)F\wedge {}^\star F \tag{31}$$

the Lagrangian of the Maxwell field, and $F=dA$. Since the matter part L_{Max} does not depend on the connection $\Gamma_\alpha{}^\beta$, the hypermomentum $\Delta^\alpha{}_\beta := \delta L_{\mathrm{Max}}/\delta\Gamma_\alpha{}^\beta$ vanishes, i.e. $\Delta^\alpha{}_\beta = 0$, and the only external current is the electromagnetic energy-momentum current Σ_α, given by

$$\Sigma_\alpha = e_\alpha \rfloor L_{\mathrm{Max}} + (e_\alpha\rfloor F)\wedge H. \tag{32}$$

We will consider a static axially symmetric configuration for the metric-affine spacetime as well as for the Maxwell field. Using the invariance of a metric-affine spacetime under diffeomorphisms, it can be shown that the conditions of staticity and axial symmetry lead to a metric with only three independent components [19] which, for instance in spherical coordinates (t,r,θ,ϕ), coordinates depend on the two spatial coordinates r and θ only. For the sake of simplicity, in this work we will assume that the spacetime can be completely described by only one independent function, say $f=f(r,\theta)$. Furthermore, we limit ourselves to the *isotropic* coframe

$$\vartheta^{\hat 0} = \frac{1}{f}dt, \quad \vartheta^{\hat 1} = f\,d\,r, \quad \vartheta^{\hat 2} = f\,r\,d\,\theta, \quad \vartheta^{\hat 3} = f\,r\sin\theta d\,\phi, \tag{33}$$

with the unknown function $f=f(r,\theta)$. Since the coframe is assumed to be *orthonormal* with the local Minkowski metric $o_{\alpha\beta} := \mathrm{diag}(-1,1,1,1) = o^{\alpha\beta}$, we have the metric in *isotropic* form

$$ds^2 = o_{\alpha\beta}\vartheta^\alpha\otimes\vartheta^\beta = -\frac{1}{f^2}dt^2 + f^2[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{34}$$

Let us make the following ansatz for the triplet

$$Q = k_0 N_e u(r,\theta)\vartheta^{\hat 0} = \frac{k_0}{k_1}\Lambda = \frac{k_0}{k_2}T, \tag{35}$$

with N_e the gravito-electric charge. This is the so-called *triplet* ansatz sector of MAG theories [9].

The electromagnetic potential appropriate for this configuration reads [12,13]

$$\mathbf{A} = e_0 u(r,\theta)\vartheta^{\hat 0}, \tag{36}$$

where e_o is the electric charge. Here we have introduced a second function $u(r,\theta)$ which together with $f(r,\theta)$ has to be determined by the field equations.

IV. HARMONIC SOLUTIONS IN MAG

In order to solve the equations arising from the MAG Lagrangian (25), we substitute the local metric $o_{\alpha\beta}$, the coframe (33), and the ansatz (35) of the nonmetricity and torsion into the field equations (13), (14) of the Lagrangian (25), we find that the functions $f=f(r,\theta)$ and $u=u(r,\theta)$ have to satisfy the two-dimensional Laplace equation $\Delta f = 0 = \Delta u$ with

$$\Delta = \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right). \tag{37}$$

Furthermore, the coefficients k_0, k_1, k_2 in the ansatz (24) are determined by the dimensionless coupling constants of the Lagrangian:

$$k_0 = \left(\frac{a_2}{2} - a_0\right)(8b_3 + a_0) - 3(c_3+a_0)^2, \tag{38}$$

$$k_1 = -9\left[a_0\left(\frac{a_2}{2} - a_0\right) + (c_3+a_0)(c_4+a_0)\right], \tag{39}$$

$$k_2 = \frac{3}{2}[3a_0(c_3+a_0) + (8b_3+a_0)(c_4+a_0)]. \tag{40}$$

A rather weak condition, which must be imposed on these coefficients, prescribes the value

$$b_4 = \frac{a_0 k + 2c_4 k_2}{8k_0}, \quad \text{with } k := 3k_0 - k_1 + 2k_2, \tag{41}$$

for the coupling constant b_4, and leads to the following relation for z_4 and the effective gravitational constant q, the electric charge e_0, and the gravitoelectric charge N_e:

$$q^2=\kappa\left[e_0^2+z_4\frac{(k_0N_e)^2}{2a_0}\right]. \tag{42}$$

Thus we see that the assumption of an isotropic field configuration for the *triplet* ansatz sector of MAG leads to a single linear differential equation for both unknowns f and u and, therefore, the corresponding solution can be given in terms of harmonic functions with arbitrary constant parameters.

Therefore the general solution can be written as

$$f(r,\theta)=1+q\sum_{n=0}^{\infty}\left(\tilde{f}_n r^n+\frac{\hat{f}_{n+1}}{r^{n+1}}\right)P_n(\cos\theta), \tag{43}$$

$$u(r,\theta)=\sum_{n=0}^{\infty}\left(\tilde{u}_n r^n+\frac{\hat{u}_{n+1}}{r^{n+1}}\right)P_n(\cos\theta), \tag{44}$$

where $\tilde{f}_n$, $\hat{f}_{n+1}$, $\tilde{u}_n$, and $\hat{u}_{n+1}$ are arbitrary *integration constants*, and P_n are the Legendre polynomials of order n. The constant q is related with the parameter z_4 of the Lagrangians (20)–(42). Consequently, the electromagnetic potential can be written as

$$\mathbf{A}=e_0\sum_{n=0}^{\infty}\left(\tilde{u}_n r^n+\frac{\hat{u}_{n+1}}{r^{n+1}}\right)P_n(\cos\theta)\,\vartheta^{\hat{0}}. \tag{45}$$

If we collect our results, then the nonmetricity and the torsion reads as follows:

$$Q^{\alpha\beta}=\sum_{n=0}^{\infty}\left(\tilde{u}_n r^n+\frac{\hat{u}_{n+1}}{r^{n+1}}\right)P_n(\cos\theta)\times\left[k_0N_e\,o^{\alpha\beta}+\frac{4}{9}k_1N_e\left(\vartheta^{(\alpha}e^{\beta)}\rfloor-\frac{1}{4}o^{\alpha\beta}\right)\right]\vartheta^{\hat{0}}, \tag{46}$$

$$T^{\alpha}=\frac{k_2N_e}{3}\sum_{n=0}^{\infty}\left(\tilde{u}_n r^n+\frac{\hat{u}_{n+1}}{r^{n+1}}\right)P_n(\cos\theta)\,\vartheta^{\alpha}\wedge\vartheta^{\hat{0}}. \tag{47}$$

The harmonic structure of the metric as well as nonmetricity and torsion allows us to interpret this solution in terms of multipole moments. In fact, the functions $f(r,\theta)$ and $u(r,\theta)$ can be written as the sum of two infinite series, $f=\tilde{f}+\hat{f}$ and $u=\tilde{u}+\hat{u}$, where $\tilde{f}=\tilde{f}(r,\theta)$ and $\tilde{u}=\tilde{u}(r,\theta)$ contain only terms with positive powers of r, whereas $\hat{f}=\hat{f}(r,\theta)$ and $\hat{u}=\hat{u}(r,\theta)$ include all terms with negative powers of r. The function $\hat{f}$ also contains a constant term necessary in order to obtain the Minkowski metric in the limiting case of vanishing curvature, torsion and, nonmetricity. This resembles the standard decomposition of the field generated by a nonspherically symmetric distribution of matter.

Consider first the internal part of the matter distribution. In order to avoid singularities on the origin of coordinates, which we suppose to lay inside the matter distribution, we must demand the vanishing of the functions $\hat{f}$ and $\hat{u}$. This can be achieved by putting the corresponding coefficients $\hat{f}_n=0$ and $\hat{u}_n=0$ $(n=0,1,2,\ldots)$. Accordingly, the interior solution is given as

$$f=q\sum_{n=0}^{\infty}\tilde{f}_n r^n P_n(\cos\theta), \tag{48}$$

$$\mathbf{A}=e_0\sum_{n=0}^{\infty}\tilde{u}_n r^n P_n(\cos\theta)\,\vartheta^{\hat{0}}, \tag{49}$$

$$Q^{\alpha\beta}=\sum_{n=0}^{\infty}\tilde{u}_n r^n P_n(\cos\theta)\left[k_0N_e\,o^{\alpha\beta}+\frac{4}{9}k_1N_e\left(\vartheta^{(\alpha}e^{\beta)}\rfloor-\frac{1}{4}o^{\alpha\beta}\right)\right]\vartheta^{\hat{0}}, \tag{50}$$

$$T^{\alpha}=\frac{k_2N_e}{3}\sum_{n=0}^{\infty}\tilde{u}_n r^n P_n(\cos\theta)\,\vartheta^{\alpha}\wedge\vartheta^{\hat{0}}. \tag{51}$$

This interior solution is valid inside a three-dimensional hypersurface $\Sigma(r,\theta)=0$ which corresponds to the surface of the matter distribution. From the form of the metric function f we can see that the coefficients $\tilde{f}_n$ determine the interior multipole moments of the gravito-electric field. On the other hand, the Maxwell field is defined by the electromagnetic potential $\mathbf{A}$ which in this case contains only interior electric multipole moments of the form $e_0\tilde{u}_n$ $(n=0,1,2,\ldots)$. Nonmetricity and torsion also present a multipole structure where the interior multipole moments are all proportional to $\tilde{u}_n$ due to the *triplet* ansatz we are using here.

Consider now the exterior part of the matter distribution. Since nonmetricity and torsion are geometric objects like curvature, divergences in their components at spatial infinity are nonphysical. For example, torsion can be measured by spin precession [20,21]. An infinite torsion will then give rise to a spin precession with an infinite angular velocity at spatial infinity. Moreover, the electric potential must tend to a constant value or has to vanish at spatial infinity because it represents the field of a finite charge distribution. For simplicity, here we assume that the electromagnetic potential vanishes at spatial infinity. In order to avoid such unphysical singularities in the exterior region of the distribution, we have to demand the vanishing of the corresponding coefficients $\tilde{f}_n=0$ and $\tilde{u}_n=0$. Therefore the class of solutions for this case reads:

$$f=1+q\sum_{n=0}^{\infty}\frac{\hat{f}_{n+1}}{r^{n+1}}P_n(\cos\theta), \tag{52}$$

$$\mathbf{A}=e_0\sum_{n=0}^{\infty}\frac{\hat{u}_{n+1}}{r^{n+1}}P_n(\cos\theta)\,\vartheta^{\hat{0}}, \tag{53}$$

$$Q^{\alpha\beta}=\sum_{n=0}^{\infty}\frac{\hat{u}_{n+1}}{r^{n+1}}P_n(\cos\theta)\left[k_0N_e\,o^{\alpha\beta}+\frac{4}{9}k_1N_e\left(\vartheta^{(\alpha}e^{\beta)}\rfloor-\frac{1}{4}o^{\alpha\beta}\right)\right]\vartheta^{\hat{0}},\tag{54}$$

$$T^{\alpha}=\frac{k_2N_e}{3}\sum_{n=0}^{\infty}\frac{\hat{u}_{n+1}}{r^{n+1}}P_n(\cos\theta)\,\vartheta^{\alpha}\wedge\vartheta^{\hat{0}},\tag{55}$$

with free coefficients $\hat{f}_{n+1}$ and $\hat{u}_{n+1}$ representing the exterior multipole structure of the solutions and q the effective gravitational constant. To make a more specific interpretation of these coefficients, we have to consider some properties of the geometric objects entering the solution.

As it is well known [5], a propagating nonmetricity $Q_{\alpha\beta}$ generates *one dilation charge* related to the trace $Q := Q_{\gamma}{}^{\gamma}/4$ of the nonmetricity, called the Weyl covector $Q = Q_i dx^i$, and *nine* types of *shear charge* are related to the remaining traceless piece $\not{Q}_{\alpha\beta}$ of the nonmetricity. Therefore we should find $4+4+1$ shear charges and 1 dilation charge.

Besides the multipole charges $e_0\hat{u}_{n+1}$, the exterior solution carries dilation, shear, and spin charges, each of them of the covectorial type. We have then the following assignments:

$$q \rightarrow \text{effective gravitational constant},\tag{56}$$

$$e_0 \rightarrow \text{electric charge},\tag{57}$$

$$N_e \rightarrow \text{gravito-electric charge},\tag{58}$$

$$e_0\hat{u}_{n+1} \rightarrow \text{higher electric multipole moments},\tag{59}$$

$$k_0N_e\hat{u}_{n+1} \rightarrow \text{dilation (``Weyl'') multipole charges of type}\,{}^{(4)}Q^{\alpha\beta},\tag{60}$$

$$k_1N_e\hat{u}_{n+1} \rightarrow \text{shear multipole charges of type}\,{}^{(3)}Q^{\alpha\beta},\tag{61}$$

$$k_2N_e\hat{u}_{n+1} \rightarrow \text{spin multipole charges of type}\,{}^{(2)}T^{\alpha}.\tag{62}$$

In principle, however, these ‘‘charge’’ assignments need to be justified by integrating locally conserved Noether currents in MAG [5].

The solutions presented in this work have been checked using REDUCE [22] with its EXCALC package [23,24] for treating exterior differential forms [25] and the REDUCE-based GRG computer algebra system [26].

V. DISCONTINUITIES

The matching of inner and outer solutions is in general accompanied with discontinuities of certain fields under consideration. Therefore we have to formulate appropriate matching conditions in our MAG framework. In doing so, we take as a starting point the formalism developed in [17] where the same notions are introduced in the framework of Einstein-Cartan theory of gravitation. For the convenience of the reader we shortly summarize their results.

We consider a hypersurface Σ in the manifold $\mathcal{M}$ which is assumed to be of class C^2 everywhere and of C^4 in $\mathcal{M}\backslash\Sigma$. We choose coordinates $x=(\tau,\mathrm{x})$ such that the hypersurface is given by $\tau=0$. Therefore, $\mathrm{x}=(\mathrm{x}^{\alpha},\alpha=1,2,3)$ define a local coordinate system on Σ. We assume that this hypersurface is timelike. That is, there is a normal n of Σ with $g(n,n)=-1$. There is a corresponding projection operator $P=1+ng(n,\cdot)$ (in components: $P^{\mu}_{\nu}=\delta^{\mu}_{\nu}+n_{\nu}n^{\nu}$ with $n^{\nu}=g^{\nu\rho}n_{\rho}$). If T is a geometric object, then $\overline{T}$ denotes its projection $\overline{T}=PT$. We call $h=\overline{g}=Pg$.

A function $f:\mathcal{M}\mapsto\mathbf{R}$ is of class C^p_q if f is C^q in $\mathcal{M}\backslash\Sigma$ $(q\leqslant 4)$ and C^p everywhere $(p\leqslant 2)$. A function f of the class C^p_q defines on Σ the functions f_+, f_-, and $[f]$ which are of class C^4:

$$f_+(\mathrm{x})=\lim_{\tau\to{}^+0}f(\tau,\mathrm{x}),\tag{63}$$

$$f_-(\mathrm{x})=\lim_{\tau\to{}^-0}f(\tau,\mathrm{x}),\tag{64}$$

$$[f]=f_+-f_-.\tag{65}$$

For $f\in C^{-1}_q$ with $q\geqslant 1$ the derivative $\partial f/\partial\mathrm{x}^{\alpha}$ is a regular function and we have

$$[\nabla_{\mathrm{x}}f]=\nabla_{\mathrm{x}}[f].\tag{66}$$

For the derivative with respect to τ one has that

$$\partial_{\tau}f=[f](\mathrm{x})\,\delta(\tau)+g(\tau,\mathrm{x}),\tag{67}$$

where g is of class C^{-1}_{q-1}.

All these notions can be transcribed to any geometrical object if one applies the conditions above to the corresponding coefficients. As a consequence, if ϕ is a differential form of class C^{-1}_q, then the properties (66) and (67) are given by

$$dx\wedge[d\phi]=dx\wedge d[\phi],\tag{68}$$

$$d\phi(\tau,x^{\alpha})=dx\wedge[\phi](x^{\alpha})\,\delta(x)+\psi(\tau,x^{\alpha}),\tag{69}$$

where ψ is a regular distribution of the class C^{-1}_{q-1}. Therefore $dx\wedge[\phi]$ is necessary and sufficient for the regularity of $d\phi$.

VI. MATCHING CONDITIONS IN MAG

We assume the following matching conditions [17]: (1) The coframe and the metric tensor are of class $C^0{}_3$; (2) the spin tensor $\tau^{\alpha}{}_{\beta}$ is of class $C^{-1}{}_2$; and (3) the energy-momentum tensor is of class $C^{-1}{}_2$.

From the first assumption, it is clear that the Riemannian connection $\tilde{\Gamma}^{\alpha}{}_{\beta}$ is $C^{-1}{}_2$ and that the projection operator P is of $C^0{}_3$. The Riemannian connection,

$$\bar{\tilde{\Gamma}}^{\alpha}{}_{\beta}=h^{\alpha}{}_{\rho}h^{\mu}{}_{\beta}h^{\nu}{}_{\kappa}\tilde{\Gamma}^{\rho}{}_{\mu\nu}dx^{\kappa}, \tag{70}$$

on each of these hypersurfaces is associated with $\overline{g}_{\alpha\beta}=h_{\alpha\beta}$ and is algebraically dependent on $\partial\overline{g}_{\alpha\beta}/\partial x^{\gamma}$ and $\overline{g}_{\alpha\beta}$ is of class C_2^0, so that

$$\overline{[\tilde{\Gamma}^{\alpha}{}_{\beta}]}=0. \tag{71}$$

The conditions (1) and (2) imply that the contorsion $K^{\alpha}{}_{\beta}=\Gamma^{\alpha}{}_{\beta}-\tilde{\Gamma}^{\alpha}{}_{\beta}$, the torsion $T^{\alpha}{}_{\beta\kappa}$, and the connection $\Gamma^{\alpha}{}_{\beta}$ are of class C_2^{-1}. It is obvious from Eq. (71) and the definition of the contorsion that

$$\overline{[\Gamma^{\alpha}{}_{\beta}]}=\overline{[K^{\alpha}{}_{\beta}]}. \tag{72}$$

The quantity

$$N_{\alpha}=\overline{\nabla_{\beta}n_{\alpha}}dx^{\beta}\sim\overline{dn}_{\alpha}-\overline{\Gamma^{\kappa}{}_{\alpha}n_{\kappa}} \tag{73}$$

defines the second fundamental form $N_{\alpha}=N_{\alpha\beta}dx^{\beta}$ of any hypersurface $x=\mathrm{const}$ with respect to the connection $\Gamma^{\alpha}{}_{\beta}$. The form N_{α} is related to the Riemannian, symmetric second fundamental form $\tilde{N}_{\alpha}=\tilde{N}_{\alpha\beta}dx^{\beta}$ by

$$N_{\alpha}=\tilde{N}_{\alpha}-\overline{\kappa^{k}{}_{\alpha}n_{\kappa}}. \tag{74}$$

The third matching condition is equivalent to

$$\frac{1}{2}\,\eta_{\alpha\beta}{}^{\kappa}\wedge dx\wedge[\Gamma^{\beta}{}_{\kappa}]=0. \tag{75}$$

Let us decompose the form $dx\wedge\Gamma^{\beta}{}_{\kappa}$ as

$$dx\wedge\Gamma^{\beta}{}_{\kappa}=dx\wedge(\overline{\Gamma}^{\beta}{}_{\kappa}-\overline{\Gamma^{\mu}{}_{\kappa}n_{\mu}n^{\beta}}-\overline{\Gamma^{\beta}{}_{\mu}n^{\mu}n_{\kappa}}+\Gamma^{\mu}{}_{\rho}n_{\mu}n^{\rho}n^{\beta}n_{\kappa}), \tag{76}$$

and notice that the form

$$dx\wedge(\Gamma_{\alpha\beta}-\Gamma_{\beta\alpha})=dx\wedge dg_{\alpha\beta}, \tag{77}$$

is continuous. As a consequence of this Eq. (75) reduces to the following conditions:

$$\frac{1}{2}\,\eta_{\alpha\beta}{}^{\kappa}\wedge dx\wedge[\overline{\Gamma}^{\beta}{}_{\kappa}]=0,$$

$$\frac{1}{2}\,\eta_{\alpha\beta}{}^{\kappa}\wedge dx\wedge[\overline{\Gamma^{\rho}{}_{\kappa}n_{\rho}n^{\beta}}]=0, \tag{78}$$

Taking into account Eqs. (72) and (73), the above conditions can be written as

$$\frac{1}{2}\,\eta_{\alpha\beta}{}^{\kappa}\wedge dx\wedge[\overline{K}^{\beta}{}_{\kappa}]=0,$$

$$\frac{1}{2}\,\eta_{\alpha\beta}{}^{\kappa}\wedge dx\wedge[N_{\kappa}]n^{\beta}=0, \tag{79}$$

or equivalently

$$[\overline{K}^{\rho}{}_{\alpha\rho}]=0, \tag{80}$$

$$[N_{\alpha\beta}]=0. \tag{81}$$

If the spin vanishes, condition (80) becomes trivial, whereas condition (81) assures the regularity of the curvature $R^{\alpha}{}_{\beta}$. However, in general, regularity of the energy-momentum tensor is insufficient to assure regularity $R^{\alpha}{}_{\beta}$, because $dx\wedge\Gamma^{\alpha}{}_{\beta}$ is continuous, if and only if the equation (81) and the relation $[\overline{K}^{\rho}{}_{\alpha\beta}]=0$, instead of Eq. (80), are satisfied.

In a special coordinate system (τ,x^{α}), the continuity condition of the symmetric part $N_{(\alpha\beta)}$ of the second fundamental form reduces to

$$[\partial g_{\alpha\beta}/\partial\tau]=(2/g^{00})[K^{0}{}_{(\alpha\beta)}]. \tag{82}$$

Because the derivatives $\partial g_{\alpha\beta}/\partial\tau$ are continuous and the derivatives $\partial g_{0i}/\partial\tau$ can be made continuous by a convenient and admissible choice of a coordinate system, Eq. (82) determines the discontinuities of the first derivatives of the metric tensor.

By using Eq. (74), the continuity condition of the antisymmetric part $N_{[\alpha\beta]}$ of the second fundamental form reduces to

$$[\overline{n_{\rho}K^{\rho}{}_{[\alpha\beta]}}]=0. \tag{83}$$

It is interesting to notice that conditions (80) and (83) are equivalent to

$$[n_{\kappa}T^{\kappa}{}_{\alpha\beta}]=0. \tag{84}$$

We have reduced assumption (3) to the relations (82) and (84). This last equation gives a restriction on possible spin (torsion) discontinuities, while Eq. (82) expresses discontinuities of the metric tensor derivatives (nonmetricity) by spin discontinuities. When the spin tensor is continuous, one obtains Lichnerowicz's matching conditions [27]. If Eq. (82) is satisfied, the symmetric part $\sigma^{(\alpha\beta)}$ of the energy-momentum tensor is a regular distribution, while the antisymmetric part $\sigma_{[\alpha\beta]}$ is regular if Eq. (84) is fulfilled.

On the other hand, from the generalized energy-momentum conservation law the following equality can be deduced:

$$8\pi G[n_{\beta}T^{\beta}{}_{\alpha}]+[\overline{K}_{\beta\kappa\alpha}]n_{\rho}K^{\rho\beta\kappa}+\frac{1}{2}n_{\alpha}[\overline{K}_{\beta\kappa\rho}\overline{K}^{\rho\kappa\beta}]=0. \tag{85}$$

One says that a gravitational field given in the region $x<0$ matches a gravitational field in the region $x>0$, if conditions (1), (2), and (3) are satisfied in any admissible coordinate system. These conditions imply that the matter tensors $\sigma^{\alpha}{}_{\beta}$ and $T^{\kappa}{}_{\alpha\beta}$, being sources of those fields, satisfy the junction conditions (84) and (85). One can prove that the junction conditions are the only independent restrictions imposed on the jumps $[\sigma^{\alpha}{}_{\beta}]$ and $[T^{\kappa}{}_{\alpha\beta}]$ of the matter tensors by conditions (1), (2), and (3).

VII. EINSTEIN-PROCA-MAXWELL CASE

As mentioned above, in the triplet ansatz sector the MAG field equations reduce to an effective Einstein-Proca system of differential equations, and to an Einstein-Proca-Maxwell system in the case where one considers also electrodynamics.

Therefore let us consider a boundary surface $x^4=\text{const}$, the matching conditions of the last section reduce to the following conditions on this surface.

(1) $g^{I}_{\alpha,\beta}=g^{II}_{\alpha,\beta}$, the metric should be continuous. This ensures the equality of the intrinsic curvature on both sides of the boundary surface.

(2) $\kappa^{I}_{\alpha\beta}=\kappa^{II}_{\alpha\beta}$, the extrinsic curvature should also be continuous.

(3) The time components of the energy-momentum tensor $T^{4}{}_{4}$ and $T^{4}{}_{\alpha}$ are also continuous.

The above conditions for a timelike surface $x^4=0$, and $r=R=\text{const}$ imply:

$$\hat{f}_{n+1}=\frac{1}{q}R^{n+1}[q\tilde{f}_nR^n-1], \tag{86}$$

$$\hat{u}_{n+1}=R^{2n+1}\tilde{u}_n. \tag{87}$$

Hence the interior solution is given as

$$f_I=q\sum_{n=0}^{\infty}\tilde{f}_nr^n P_n(\cos\theta), \tag{88}$$

$$A_I=e_0\sum_{n=0}^{\infty}\tilde{u}_nr^nP_n(\cos\theta)\,\vartheta^{\hat{0}}, \tag{89}$$

$$Q_I^{\alpha\beta}=\sum_{n=0}^{\infty}\tilde{u}_nr^nP_n(\cos\theta)\bigg[k_0N_e\,o^{\alpha\beta}+\frac{4}{9}k_1N_e\bigg(\vartheta^{(\alpha}e^{\beta)}\rfloor-\frac{1}{4}o^{\alpha\beta}\bigg)\bigg]\vartheta^{\hat{0}}, \tag{90}$$

$$T_I^{\alpha}=\frac{k_2N_e}{3}\sum_{n=0}^{\infty}\tilde{u}_nr^nP_n(\cos\theta)\,\vartheta^{\alpha}\wedge\vartheta^{\hat{0}}, \tag{91}$$

and the exterior solution becomes

$$f_{II}=1+\sum_{n=0}^{\infty}\frac{[q\tilde{f}_nR^n-1]}{r^{n+1}}P_n(\cos\theta), \tag{92}$$

$$A_{II}=e_0\sum_{n=0}^{\infty}\frac{R^{2n+1}\tilde{u}_n}{r^{n+1}}P_n(\cos\theta)\,\vartheta^{\hat{0}}, \tag{93}$$

$$Q_{II}^{\alpha\beta}=\sum_{n=0}^{\infty}\frac{R^{2n+1}\tilde{u}_n}{r^{n+1}}P_n(\cos\theta)\bigg[k_0N_e\,o^{\alpha\beta}+\frac{4}{9}k_1N_e\bigg(\vartheta^{(\alpha}e^{\beta)}\rfloor-\frac{1}{4}o^{\alpha\beta}\bigg)\bigg]\vartheta^{\hat{0}}, \tag{94}$$

$$T_{II}^{\alpha}=\frac{k_2N_e}{3}\sum_{n=0}^{\infty}\frac{R^{2n+1}\tilde{u}_n}{r^{n+1}}P_n(\cos\theta)\,\vartheta^{\alpha}\wedge\vartheta^{\hat{0}}. \tag{95}$$

VIII. DISCUSSION

In this paper, we developed the matching and junction conditions between two different spacetimes in MAG. A gravitational field given in the region $x<0$ matches a gravitational field in the region $x>0$, if the following conditions are satisfied in any admissible coordinate system: (1) The coframe and the metric tensor are of class $C^0{}_3$; (2) The spin tensor $\tau^{\alpha}{}_{\beta}$ is of class $C^{-1}{}_2$; and (3) the energy-momentum tensor is of class $C^{-1}{}_2$. These conditions imply that the matter tensors $\sigma^{\alpha}{}_{\beta}$ and $T^{\kappa}{}_{\alpha\beta}$, being sources of those fields, satisfy the junction conditions (84) and (85). Moreover, in the framework of the triplet ansatz sector of MAG external and internal charged multipole solutions are presented. It is clear that one may start with any solution of the Einstein-Maxwell equations and then, after imposing a suitable constraint on the coupling constants, replace the electric and/or magnetic charge by strong gravitoelectric and/or gravitomagnetic charges, thereby arriving at the post-Riemannian triplet. It is important to stress that in the triplet ansatz sector of MAG there exists an underlying harmonic structure. This structure is reflected by the fact that the coframe function *f*, the triplet, and the electromagnetic one-forms satisfy the two-dimensional Laplace equation.

For the Einstein-Proca-Maxwell system of the triplet ansatz sector the matching conditions of MAG reduce to the following.

(1) $g^{I}_{\alpha,\beta}=g^{II}_{\alpha,\beta}$, the metric should be continuous. This ensures the equality of the intrinsic curvature on both sides of the boundary surface.

(2) $\kappa^{I}_{\alpha\beta}=\kappa^{II}_{\alpha\beta}$, the extrinsic curvature should also be continuous.

(3) The time components of the energy-momentum tensor $T^{4}{}_{4}$ and $T^{4}{}_{\alpha}$ are also continuous.

The study of the matching conditions in MAG presented here and its reduction in the triplet ansatz sector contributes to consider the structure of such a sector of MAG as fully understood.

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