

Instability of spacelike and null orbifold singularities

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Time dependent orbifolds with spacelike or null singularities have recently been studied as simple models of cosmological singularities. We show that their apparent simplicity is an illusion: the introduction of a single particle causes the spacetime to collapse to a strong curvature singularity (a big crunch), even in regions arbitrarily far from the particle.

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I. INTRODUCTION

Understanding the physics at cosmological singularities has long been a challenge for string theory. One of the main open questions is whether time (as we know it) simply begins and ends, or do quantum effects produce a kind of bounce, with a well defined semiclassical spacetime on the other side. (One of the earliest attempts to apply string theory to cosmology, the pre-big bang scenario [1], requires a bounce.) In order to study this question, it is natural to start with the simplest examples of solutions with cosmological singularities: time dependent orbifolds. These are quotients of flat spacetime by boosts (or combinations of boosts and rotations). Since the spacetimes are locally flat, they are exact classical solutions to string theory and string propagation is relatively easy to study. Given that the singularities of ordinary orbifolds are harmless in string theory, one might expect the same would be true here. However, arguments were given many years ago that this would not be the case [2].

Recently, interest has been refocused on this question by the ekpyrotic cosmology [3], which requires [4] that the universe contract to a singularity and then reexpand. The ekpyrotic singularity is spacelike and corresponds to orbifolding by a simple boost. The cyclic universe model [5] has a similar orbifold singularity. It has been argued that from some points of view this singularity is rather gentle [6,7], while from other points of view it is not [8].¹ One difficulty with this orbifold is that the resulting spacetime has closed timelike curves. As a warmup, Liu, Moore, and Seiberg (LMS) [10] have studied the orbifold by a null boost [2,11]. This has no closed timelike curves, though it does have a submanifold with closed null curves, and a null singularity. It also has the advantage of being supersymmetric. LMS study four-string amplitudes in this space, and finds that they are nonsingular except for exceptional momenta. They do not study the back reaction and so are not able to conclude whether there is a bounce or not.

In this paper we will reexamine this question, and will

¹It was noted in [9] that metric perturbations become large near the orbifold singularity.

argue that in fact these orbifold singularities are highly unstable. Given their application to both M theory and string theory, we will consider general D dimensional orbifolds. For a simple boost, the identification acts on a two dimensional subspace, so there are $D-2$ flat transverse directions. For the null orbifold, the identification involves three dimensions, so there are $D-3$ transverse directions. We will show that the addition of even a *single* particle, localized in the transverse directions, causes the entire universe to collapse into a spacelike singularity. The basic mechanism is the amplification of the energy of particles in the collapsing universe, a point that has been emphasized by many others going back to Ref. [2]. We will analyze the effects of this first in the context of general relativity, finding evidence for the collapse, and then argue that this effect can be seen in the string amplitudes of Ref. [10]. Unfortunately, this does not resolve the question of what happens at cosmological singularities in string theory. It just shows that orbifolds are not really any simpler than other cosmological models. One must learn to deal with the strong curvature effects.

While this was being written, several related papers appeared. Reference [12] shows that a *homogeneous* energy distribution in the transverse directions causes the null orbifold singularity to become spacelike. References [13,14] extend the analysis of LMS to the nonsingular orbifold in which the null boost is accompanied by a spacelike shift, and include remarks about the back reaction that overlap with ours. We should also note other discussions of Lorentzian orbifolds, of flat and curved spacetimes [15].

II. GENERAL RELATIVITY ARGUMENT FOR THE BIG CRUNCH

In this section we use general relativity to study the effect of introducing a single particle into the orbifolds with timelike and null singularities. The basic idea is to go to the covering space, and view the single particle on the orbifold as an infinite collection of particles in Minkowski spacetime all related by the appropriate boost. If the gravitational interaction of these particles produces a singularity, the same must be true for a single particle on the orbifold. Because of the large boosts involved, the rest mass of the particle can be neglected. We will therefore consider massless particles.

A. Preliminaries

We begin with the basic system of two massless particles with momenta P^μ and \tilde{P}^μ in D spacetime dimensions. Let b be the impact parameter in the center of mass frame. Since the center of mass energy squared is $s = -2P \cdot \tilde{P}$, the condition that the particles approach within their Schwarzschild radius and form a black hole is

$$G\sqrt{s} > b^{D-3} \quad (1)$$

up to coefficients of order 1. This heuristic condition can be justified in general relativity as follows. The gravitational field of a single massless particle is given by a shock wave called the Aichelberg-Sexl metric [16]. For $D > 4$, the metric is

$$ds^2 = -2dy^+ dy^- + dy^2 + \frac{\mu}{(\mathbf{y} \cdot \mathbf{y})^{(D-4)/2}} \delta(y^+) (dy^+)^2 \quad (2)$$

where μ is proportional to the energy of the particle. In $D = 4$, the metric is similar, with the power law dependence in the last term replaced by a log. The curvature is concentrated on a null plane, and falls off as one moves away from the particle. The high energy collision of two particles can be described in the center of mass frame by superposing two such shock waves in the past. This is possible, despite the nonlinearities of general relativity, because the spacetime is flat in front of each shock wave. After the particles collide, the exact solution is not known. But if the impact parameter is small enough, there are trapped surfaces in the spacetime when the particles collide [17]. It follows from the singularity theorems that a singularity must form. Assuming cosmic censorship, the singularity must be inside a black hole. In four spacetime dimensions, a trapped surface has been found [18] provided the impact parameter satisfies the bound (1). In higher dimensions, a trapped surface has been found for $b = 0$ [18]. The size of the trapped surface is of order the Schwarzschild radius for the given energy, so one again expects a black hole to form whenever b satisfies Eq. (1), but this has not yet been rigorously shown.

It will be useful to have a general formula for the impact parameter b in the center of mass frame, given two null geodesics

$$X^\mu(\lambda) = P^\mu \lambda + a^\mu, \quad \tilde{X}^\mu(\tilde{\lambda}) = \tilde{P}^\mu \tilde{\lambda} + \tilde{a}^\mu. \quad (3)$$

This is easily obtained as follows. Projecting each trajectory into the subspace orthogonal to $P^\mu + \tilde{P}^\mu$, which is at constant time in the center of mass frame, we get

$$X^\mu(\lambda) \rightarrow \frac{1}{2}(P - \tilde{P})^\mu \lambda + a^\mu - (P + \tilde{P})^\mu \frac{a \cdot (P + \tilde{P})}{2P \cdot \tilde{P}}, \quad (4)$$

$$\begin{aligned} \tilde{X}^\mu(\tilde{\lambda}) \rightarrow & -\frac{1}{2}(P - \tilde{P})^\mu \tilde{\lambda} + \tilde{a}^\mu \\ & - (P + \tilde{P})^\mu \frac{\tilde{a} \cdot (P + \tilde{P})}{2P \cdot \tilde{P}}. \end{aligned} \quad (5)$$

Now take the difference between these two trajectories and compute the norm. The norm depends on $\lambda + \tilde{\lambda}$, and minimizing with respect to this parameter yields

$$b^2 = Y^2 - \frac{2(P \cdot Y)(\tilde{P} \cdot Y)}{P \cdot \tilde{P}} \quad (6)$$

where $Y^\mu = a^\mu - \tilde{a}^\mu$. Since a^μ and \tilde{a}^μ are arbitrary points along the geodesics, one can set Y^μ equal to the difference between *any* pair of points on the respective trajectories, $Y^\mu = X^\mu(\lambda) - \tilde{X}^\mu(\tilde{\lambda})$. This does not change the impact parameter because Eq. (6) is invariant under shifting Y^μ by any multiple of P^μ or \tilde{P}^μ .

B. Two examples

We now consider our first example: two dimensional Minkowski spacetime quotiented by a boost, times a flat transverse space. The spacetime is

$$ds^2 = -2dy^+ dy^- + d\mathbf{y}^2 \quad (7)$$

with the identification

$$(y^+, y^-, \mathbf{y}) \equiv (e^{n\alpha} y^+, e^{-n\alpha} y^-, \mathbf{y}) \quad (8)$$

for all integers n and some constant α . This orbifold has fixed points at $y^+ = 0$, $y^- = 0$ which is a spacelike singularity. It consists of four regions, two with closed timelike curves.² As mentioned above, we will study the effect of adding a single massless particle to this orbifold by going to the covering space where one has the original particle together with an infinite number of boosted images. The momentum of the n th image is related to the original momentum (p^+, p^-, \mathbf{p}) by $(e^{n\alpha} p^+, e^{-n\alpha} p^-, \mathbf{p})$. On the covering space, a sufficient condition for the formation of a black hole, and therefore a spacelike singularity, is that any pair of images satisfy the condition (1). Consider say the original particle and its n th image. Their center of mass energy grows like $s \sim \cosh n\alpha$. To compute the impact parameter, we use Eq. (6). It is convenient to choose the affine parameter λ to vanish at $X^+ = 0$, so $a^+ = 0$. Then Y^2 only depends on the transverse components of a which are independent of n . The n dependent contribution to $P \cdot Y$ is $-P^+ a^- (1 - e^{-n\alpha})$ for

²To avoid this, one might consider just the past and future wedges. Since this is not a true orbifold (it is a quasi-orbifold in the terminology of Ref. [19]) there are no methods as yet for studying such a space in string theory, or even for showing that it exists as a solution.

the original particle and $-P^+ a^- (e^{n\alpha} - 1)$ for the n th image. This implies that for large $|n|$, the impact parameter is independent of n .

It follows that for large n the condition (1) for formation of a black hole is always satisfied. Moreover, by taking large enough n , the Schwarzschild radius

$$R_s^{D-3} \sim G(p^+ p^-)^{1/2} e^{n\alpha/2} \quad (9)$$

becomes arbitrarily large, so that the black hole occupies all of space. In other words, the entire spacetime ends in a curvature singularity and one has a big crunch. It has often been noted that the spacetime inside a black hole is analogous to an (anisotropic) cosmology with a big crunch. When the black hole is arbitrarily large so there is no spacetime outside the horizon, this analogy becomes exact.

We now show that the same thing happens for the null orbifold, which is described by

$$ds^2 = -2dx^+ dx^- + dx^2 + d\mathbf{x}^2 \quad (10)$$

with the identification

$$\begin{aligned} X &\equiv (x^+, x^-, \mathbf{x}) \\ &\cong X_n \\ &\equiv (x^+, x^+ + nx^+, x^- + nx^- + n^2 x^+ / 2, \mathbf{x}). \end{aligned} \quad (11)$$

Note that the $D-3$ transverse coordinates \mathbf{x} are not affected by the identification. The fixed points are at $x^+ = 0$, $x^- = 0$ and form a null singularity. There is a similar identification of the momenta

$$\begin{aligned} P &\equiv (p^+, p^-, \mathbf{p}) \\ &\cong P_n \\ &\equiv (p^+, p^+ + np^+, p^- + np^- + n^2 p^+ / 2, \mathbf{p}). \end{aligned} \quad (12)$$

(Note $X_0 \equiv X$ and $P_0 \equiv P$ by definition.)

As before, a massless particle in this orbifold can be described by an infinite collection of particles on the covering space. If we focus on one particle with $X(\lambda) = P\lambda + a$ and its n th image, the center of mass energy is

$$s_n = -2P \cdot P_n = 2p^+ p_n^- + O(n) = n^2 (p^+)^2, \quad (13)$$

and so the center of mass energy grows as n . To compute the impact parameter, we again choose the affine parameter to vanish at $X^+ = 0$, so $a^+ = 0$. One quickly verifies that Y^2 is independent of n , and that in the second term of Eq. (6) both numerator and denominator are of order n^2 , so that the impact parameter does not grow with n . Once again, one produces black holes with arbitrarily large size by taking n sufficiently large.

Thus, in both cases, a single particle induces a gravitational collapse everywhere in spacetime. Essentially, the gravitational shock wave of the particle wraps infinitely many times through the identified space and intersects itself infinitely many times, and successive intersections are increasingly energetic.

One might wonder whether the intersections at smaller n might introduce nonlinear effects that would prevent the formation of larger black holes at larger n . This should not be the case, because the trapped surface corresponding to larger n is at larger radius, spacelike separated from the region of nonlinearity. In fact, the combined effect of multiple shocks goes the other way: if we consider the total center of mass energy of the first n images of each particle, then one finds that (in, e.g., the null orbifold)

$$s_{\text{tot}} \propto n^4, \quad b \propto n^0, \quad R_s^{D-3} \sim G n^2 p^+ \quad (14)$$

so R_s grows even more quickly with n .

The collision of two shock waves changes the spacetime only to the future of their intersection. We now ask whether the orbifold singularity indeed lies to the future of the intersection of two shocks. A null particle $P^\mu \lambda + a^\mu$ generates a shock wave consisting of all points x^μ satisfying $P \cdot (x - a) = 0$. The n th boosted image produces the shock wave $P_n \cdot (x - a_n) = 0$. Since $P \cdot a = P_n \cdot a_n$ by Lorentz invariance, the intersection of the two shocks must satisfy $(P_n - P) \cdot x = 0$. For the null orbifold, this implies

$$(np + n^2 p^+ / 2) x^+ = np^+ x. \quad (15)$$

Depending on the sign of x , one can take n either positive or negative to insure that $x^+ < 0$. Note that for large $|n|$, $x^+ = O(1/|n|)$. Since every point with $x^+ > 0$ lies to the future of every point with $x^+ < 0$ [10], it is clear that the entire region to the future of the orbifold singularity lies to the future of the intersection of the shocks.

For the boost orbifold (8) the condition $(P_n - P) \cdot y = 0$ yields

$$p^+ y^- = p^- y^+ e^{-n\alpha}. \quad (16)$$

Since y^+ and y^- always have the same sign, the intersection never occurs in the region of closed timelike curves. If we substitute Eq. (16) into the condition for a single shock, $P \cdot (y - a) = 0$, we obtain

$$(e^{-n\alpha} + 1) p^- y^+ = \mathbf{p} \cdot \mathbf{y} - P \cdot a. \quad (17)$$

If the right hand side is negative, the orbifold singularity lies to the future of the intersection of the shocks for all images n . If the right hand side is positive, it lies to the past. However even in this case, it is not possible for a particle to propagate into the future cone before it encounters strong curvature. If we ask when the trajectory $P^\mu \lambda + a^\mu$ collides with the image shock $P_n \cdot (y - a_n) = 0$, a short calculation shows that the affine parameter λ is negative (for positive n) and so the collision occurs in the past cone.

It is somewhat counterintuitive that a single particle can change the geometry at arbitrary distance. In particular, it might appear to violate causality. To see that this is not the case, and make the above result more plausible, we present another argument which leads to the same conclusion. For definiteness, we will consider the null orbifold. Suppose we start with a massive particle rather than a massless one (the above argument still goes though essentially unchanged).

This particle produces a gravitational perturbation which at large distances is described by the linearized Schwarzschild solution,

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{D-2}^2 + \left(\frac{r_0}{r}\right)^{D-3} \left[dt^2 + \frac{1}{D-3} (dr^2 + r^2 d\Omega_{D-2}^2) \right]. \quad (18)$$

On the covering space, we have an infinite number of particles which produce a field at large transverse distance which is just the sum of the Schwarzschild perturbation of each one. In terms of the above null coordinates, $t = (x^+ + x^-)/\sqrt{2}$, $r^2 = (x^+ - x^-)^2/2 + \mathbf{x}^2$. It is easy to show that this sum diverges for $x^+ = x^- = 0$ and any transverse separation \mathbf{x}^2 . Since we start with a particle at $r=0$, it clearly passes through the fixed point surface $x^+ = 0, x^- = 0$, so all of its images agree at this point. Thus the radial distance r from each is the same and one has an infinite number of copies of a perturbation of strength $(r_0/r)^{D-3}$. Actually the divergence is stronger than this, since the particles have a relative boost. If one takes the perturbation (18) in the rest frame of the n th image, and translates back into the original coordinates, one finds e.g. $(dx^-)^2 \rightarrow (dx^- + ndx + n^2 dx^+/2)^2$ so different components of the perturbation pick up extra powers of n . Of course, once the perturbation becomes large, one can no longer trust the linearized approximation, but this clearly shows why strong gravitational fields can arise from a single particle even at large transverse distance³.

One can also show that the perturbation becomes large even when the initial particle has $x^- \neq 0$ at $x^+ = 0$. From our earlier argument we know that the minimum separation between the original particle and its n th image is independent of n . This minimum separation may be reached at different times for different images. Nevertheless, focusing on just the initial particle and one image, at the time of minimum separation, the field at a large transverse distance will include the sum of two Schwarzschild perturbations with essentially the same r . Since certain components of the perturbation are multiplied by powers of n , for any transverse distance, one can choose n large enough so that the perturbation is larger than 1.

In the above, we considered a generic particle with $p^+ \neq 0$. The situation is different if one introduces a particle with $p^+ = 0$, so the trajectory lies in a null surface of constant x^+ . Since the total momentum must be null, the only nonzero component can be p^- . In our first orbifold, obtained by quotienting by a simple boost, the description of this particle in the covering space is an infinite series of parallel particles converging to $x^+ = 0$ with increasing energy. The gravitational shock waves of these particles do not intersect, and the entire spacetime can be viewed as a time dependent pp wave. However, since the energy of the particles diverges as $x^+ \rightarrow 0$ the pp wave becomes singular there, much like the

singular plane waves studied in [21]. For the null orbifold (11), it is clear that the image trajectories in the covering space are simply translated in x direction and have exactly the same momentum. Parallel null particles never form a black hole. The gravitational back reaction is described by a single shock wave at one value of x^+ , with a profile that is a linear superposition of the shock wave for each particle. In this one nongeneric case there is no singularity.

C. Generalizations

There is a generalization of the null orbifold which regulates the singularity. If one adds a commuting spacelike shift to the null boost, then there are no fixed points. The identification is now

$$(x^+, x^-, \mathbf{x}) \cong (x^+, x^- + nx^+, x^- + nx + n^2 x^+/2, \mathbf{x} + n\mathbf{d}). \quad (19)$$

This has been called a ‘‘null brane’’ in the literature [22,11]. Geometrically, there is a compact direction which starts at infinite radius in the past, contracts down to a size given by \mathbf{d} and then expands out to infinity. What happens if one adds a single massless particle to a null brane? The extra shift does not affect the center of mass energy of two image particles. However it has the important effect that the impact parameter now grows linearly with n . The result depends⁴ on the total dimension D . Consider first a chain of identical point masses, each with mass M . If the separation $|\mathbf{d}|$ is smaller than R_s where $R_s^{D-3} \sim GM$, there will be a cylindrical event horizon surrounding the masses, i.e., a black string. The transverse size of the black string is $R_{tr}^{D-4} \sim GM/|\mathbf{d}|$. This transverse size can also be obtained as follows. The total mass clearly grows linearly with distance along the chain. But in $D > 4$ dimensions, the Schwarzschild radius grows more slowly. So at most a finite number of masses contribute to form a black hole. Setting $R_s^{D-3} = GMn$ equal to $(n\mathbf{d})^{D-3}$, solving for n and substituting back in, one finds that R_s agrees with the transverse size of the black string.

The only difference between this chain and the null brane is that, in the center of mass frame, the energy of each particle in the chain grows linearly with n , so the total mass grows like n^2 . It follows that for $D=5$, no black hole forms for large shift, and an infinite mass black hole forms for small shift, as before. In $D > 5$, the situation is different. A large shift again produces no singularities, but even a small shift will produce only a finite size black hole. Outside this black hole, there will not be a big crunch. The spacetime will approach the null brane at large distances. The size of the black hole can be estimated as follows. A black hole of size R_s will contain n images where $R_s = n|\mathbf{d}|$. Since the mass of the n images is of order $n^2 p^+$, we have

$$R_s^{D-3} = Gn^2 p^+ = (n\mathbf{d})^{D-3}. \quad (20)$$

³For $D=5$, the sum of the image perturbations diverges at all times, even long before the orbifold singularity. However, this is just a gauge artifact [20].

⁴We thank H. Liu and N. Seiberg for a discussion on this point.

Solving for n yields

$$R_s = (Gp^+/\mathbf{d}^2)^{1/(D-5)}. \quad (21)$$

One sees clearly that as the shift \mathbf{d} goes to zero, the size of the black hole grows to infinity.

One can do the same thing for the orbifold with a space-like singularity (8). By adding a commuting shift to the standard boost, one avoids the singularity [23]. In this case, since the center of mass energy grows exponentially and the separation only grows linearly, a single particle will still produce a big crunch for any finite shift.⁵

The orbifold (8) can be written in the form

$$ds^2 = -dt^2 + t^2 d\phi^2 + d\mathbf{y}^2 \quad (22)$$

where ϕ is periodic. If we identify ϕ with $-\phi$, we obtain a model of two “end of the world” branes colliding. This is the geometry of the cyclic universe model [5]. This clearly has the same instability as we discussed above. However, in the cyclic universe, one might expect quantum fluctuations to stop the branes from hitting at exactly the same time everywhere [24]. Can this also regulate the singularity and avoid the instability? This is very unlikely for two reasons. First, our argument is local. We considered just a neighborhood of a single particle and showed that its interaction with its images produces black holes of unbounded size. We did not have to assume that the circle was shrinking down to zero size everywhere at exactly the same time. Second, if the quantum fluctuations lead to classical perturbations, then they will classically grow and produce curvature singularities even without introducing extra particles. For example, one can easily verify that any metric of the form

$$ds^2 = -dt^2 + [t + f(\mathbf{y})]^2 d\phi^2 + d\mathbf{y}^2 \quad (23)$$

has a curvature singularity when $t + f(\mathbf{y}) = 0$, unless f is a linear function.

III. STRING THEORY ARGUMENT FOR THE BIG CRUNCH

So far, our analysis has been strictly in the context of general relativity. One might hope that the situation would be better in string theory—that stringy physics in these Lorentzian orbifolds would be nonsingular, just as it is in Euclidean orbifolds. However, this is unlikely. The singularity involves the formation of an arbitrarily large black hole, with Schwarzschild radius much larger than the string scale. At these distances string theory should go over to general relativity. LMS [10] have calculated string scattering amplitudes in the null orbifold, and so we can look for the expected breakdown of perturbation theory in these.

Let us first ask how one would detect the onset of black hole formation in the tree level $2 \rightarrow 2$ string amplitude. First

⁵We thank Liu and Seiberg, and Cornalba and Costa for pointing out an incorrect statement in an earlier version of this paper.

in general relativity, the covariant scattering amplitude is of order

$$\mathcal{A} \sim Gs^2/t \quad (24)$$

with s the center-of-mass energy squared and t the momentum transfer squared. Fourier transforming with respect to the $D-2$ transverse dimensions, and including a factor of s^{-1} to convert from covariant to canonical normalization of states, yields a dimensionless amplitude

$$\delta \sim \frac{Gs}{b^{D-4}} \quad (25)$$

where b is the impact parameter. In string theory, this is modified at $b^2 \gtrsim \alpha' \ln s$ by the logarithmic spreading of the string [25], but this is much smaller than the Schwarzschild radius. There is also an amplitude for the strings to become excited, but this is again small at large radius. Thus the general relativistic result (25) extends to string theory.

The dimensionless amplitude (25) becomes of order 1 at $b \propto s^{1/(D-4)}$, which at high energy is much larger than the Schwarzschild radius $b \propto s^{1/(2D-6)}$. Thus perturbation theory breaks down long before black holes form. There is a simple reason for this. At macroscopic distances and energies, a classical description of the gravitational field is valid. One can think of this as the exchange of many gravitons, which is a high order ladder graph, so indeed perturbation theory in this sense has broken down. Since there is a classical description, there should be a way to sum the large terms in perturbation theory. This is the eikonal approximation [26,27]. Essentially, the large amplitude exponentiates to give the S -matrix

$$S = \exp(2i\delta + \dots). \quad (26)$$

The phase is large, but the nontrivial physical effect comes only through the dependence of the phase on b . A measure of the magnitude of this is the scattering angle

$$\theta \sim s^{-1/2} \frac{d\delta}{db} \sim \frac{Gs^{1/2}}{b^{D-3}}. \quad (27)$$

We see that $\theta \sim 1$ is the criterion for black hole formation. This agrees with the classical analysis of scattering of ultrarelativistic particles: the energy at which a black hole forms is of the same order as that where the scattering angle becomes large.⁶

Now let us look for this effect in the string amplitudes in the null orbifold geometry. To compare with the $2 \rightarrow 2$ tree amplitude in LMS, we consider a slightly different situation from before—the interaction of one particle with the images of another, rather than with its own images (to see the latter

⁶The reader might be concerned that the signature for black hole formation is a scattering angle of order 1, which is highly suppressed at high energy at string tree level, but what the angle (27) actually represents is the effect of many soft scatterings.

effect, we would need to look at string loop amplitudes). In general relativity the argument in the previous section still goes through; for the n th image, the center of mass energy grows as n , while the minimum separation does not, and so a black hole of arbitrarily large radius forms.

For simplicity let us analyze the kinematics in the case that P and \tilde{P} are purely in the $+$ direction; one can check that the analysis extends directly to more generic momenta, and to massive external particles. Then

$$P_0 = (p^+, 0, 0, \mathbf{0}), \quad \tilde{P}_n = (\tilde{p}^+, n\tilde{p}^+, n^2\tilde{p}^+/2, \mathbf{0}). \quad (28)$$

If these exchange a momentum

$$K = (k^+, 0, k^-, \mathbf{k}), \quad (29)$$

then the mass shell conditions $(P_0 + K)^2 = (\tilde{P}_n - K)^2 = 0$ imply that for large n

$$k^- = \frac{\mathbf{k}^2}{2p^+}, \quad k^+ = -\frac{2\mathbf{k}^2}{n^2\tilde{p}^+}. \quad (30)$$

The key point is that k^+ is very small, of order $1/n^2$. This is just the region where LMS noted that their amplitude diverges. Thus we interpret this divergence as an indication of the breakdown of perturbation theory due to the onset of black hole formation. The kinematics above corresponds, in the notation of LMS ($p_1 \equiv p, p_2 \equiv \tilde{p}$) to $L_s = n^2 p_1^+ p_2^+ \Rightarrow q_+^2 = n^2 p_1^+ p_2^+ / (p_1^+ + p_2^+)$; $L_t - 4/\alpha' \sim \mathbf{k}^2$; $p_3^+ - p_1^+ = k^+$. In this regime the amplitude A in LMS 6.16 reduces to the general relativistic form (24), and for $\Delta J = 0$ the phase factor in LMS 6.16 is negligible. Thus the general relativistic analysis of this regime is not altered.

Our general relativistic analysis was in the spirit of the inheritance principle: for untwisted states, tree level amplitudes descend from amplitudes on the covering space. It is not obvious that this is valid here. Multiple graviton exchange is a multiloop process, even though the eikonal approximation allows it to be summed up in terms of classical general relativity. What our analysis has ignored is the exchange of winding states (which also would not be seen in the tree level string amplitude considered above). These states become light near $x^+ = 0$ where the black hole is forming, and so it is conceivable that they qualitatively change the process. Note that their effect is limited by causality, because they are heavy until just before the instant $x^+ = 0$.

This is hardly the last word on this subject, but we can summarize our conclusions as follows. The best reason for believing that a bounce occurs in this context is the resemblance of these spacetimes to Euclidean orbifolds. However, an application of orbifold technology shows that in fact these singularities are unstable toward the formation of singularities of a more terminal sort. The orbifold singularities are no better (or worse) than the spacelike curvature singularities of black holes, and so we must still understand the physics of these in string theory.

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