# $\overline{B^0}\to\pi^+X$ in the standard model

X.-G. He,[1] C.-P. Kao,[1] J. P. Ma,[2] and S. Pakvasa[3]
[1]*Department of Physics, National Taiwan University, Taipei, Taiwan*
[2]*Institute of Theoretical Physics, Academia Sinica, Beijing*
[3]*Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822*
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We investigate the possibility of studying the $B\to\pi$ form factor using the semi-inclusive decays $\overline{B^0}\to\pi^+ + X_q$. In general, $B\to PX$ semi-inclusive decays involve several hadronic parameters. However, for $\overline{B^0}\to\pi^+X_q$ decays, we find that in the factorization approximation, the only unknown hadronic parameters are the form factors $F_{0,1}^{B\to\pi}$. Therefore, these form factors can be studied in $\overline{B^0}\to\pi^+X_q$ decays. Using theoretical model calculations for the form factors, the branching ratios for $\overline{B^0}\to\pi^+X_d(\Delta S=0)$ and $\overline{B^0}\to\pi^+X_s(\Delta S=-1)$, with $E_\pi>2.1$ GeV, are estimated to be in the ranges of $(3.1\text{–}4.9)\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$ and $(2.5\text{–}4.2)\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$, respectively, depending on the value of the $CP$-violating phase $\gamma$. The combined branching ratio for $\overline{B^0}\to\pi^+(X_d+X_s)$ is about $7.4\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$ and is insensitive to $\gamma$. We also discuss $CP$ asymmetries in these decay modes.

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Recently, a number of exclusive rare charmless hadronic $B$ decays have been measured. These decays are sensitive to the $CP$-violating parameter $\gamma$ in the standard model (SM) and also to new physics [1–6]. While most studies have concentrated on exclusive $B$ decay modes, there are also some studies of semi-inclusive decays [3–5]; for example, the mode $B\to\eta' X$ has been studied in detail experimentally [7]. At present there are several other multibody rare $B$ decays that have been measured, such as $B\to\pi\pi K, KKK$ [8]. It can be expected that more $B\to PX$ decay modes, with $P$ being a light meson, will be experimentally studied. Theoretically, at the quark level, the effective Hamiltonian for $B$ decays is well understood in the SM. If quark-hadron duality were exact, it would allow one to have a good understanding of inclusive hadronic decays. For exclusive and semi-inclusive decays there are additional uncertainties from our poor understanding of long-distance strong interaction dynamics.

In exclusive charmless two-body $B$ decays of the type $B\to PP$, the operators which induce them in the SM, to the lowest order, are four quark operators $O_i$. In the factorization approximation, the four quark operators are factorized into biquark operators

$$\begin{aligned}\langle P_1P_1|O_i|B\rangle=&\langle P_1|j_1|0\rangle\langle P_2|j_2|B\rangle+\langle P_1|j_1'|0\rangle\langle P_2|j_2'|B\rangle\\&+(1\to2,2\to1)+\langle P_1P_2|j_1|0\rangle\langle 0|j_2|B\rangle\\&+\langle P_1P_2|j_1'|0\rangle\langle 0|j_2'|B\rangle,\qquad(1)\end{aligned}$$

where $j_1'\times j_2'$ is the Fierz transform of $j_1\times j_2$. The last two terms known as the annihilation contributions are usually assumed to be small and are neglected. The amplitudes $\langle P_i|j_1^{(\prime)}|0\rangle$ and $\langle P_i|j_2^{(\prime)}|B\rangle$ can be related to $P$ meson decay constants or $B\to P$ transition form factors. Several of the decay constants, such as $f_\pi$ and $f_K$, have been measured to good accuracy, but less is known about form factors involving $B$. Present experimental measurements are consistent with several model calculations. To better determine $CP$-violating parameters and constrain possible new physics, more accurate determination of the form factors is necessary.

Some theoretical studies for semi-inclusive charmless hadronic decay modes $B\to PX$ have been carried out before [3–5]. In the factorization approximation, the decay amplitude contains several terms:

$$\begin{aligned}A(B\to PX)=&\langle X|j_1|0\rangle\langle P|j_2|B\rangle+\langle P|j_1|0\rangle\langle X|j_2|B\rangle\\&+\langle PX|j_1|0\rangle\langle 0|j_2|B\rangle+(\text{Fierz transformed terms}).\qquad(2)\end{aligned}$$

It may be possible to understand semi-inclusive decays better from a theoretical point of view than exclusive modes. Experimentally semi-inclusive decays may be more difficult to study than exclusive modes for a number of reasons. In order to make sure that the observed events are from rare charmless $B$ decays and not from other processes, such as $B\to D(D^*)X'\to PX''$, one needs to make a cut on the $P$ energy. It has been shown that with a cut of $E_P>2.1$ GeV, most of the unwanted events can be eliminated [3]. The resulting events will have a small invariant mass-squared $M_X^2$. With the cut $E_P>2.1$ GeV, $M_X^2$ is less than $5.7\ \text{GeV}^2$. Rare charmless hadronic $B$ semi-inclusive decays can be studied experimentally and useful information can be obtained. In this paper, we study the semi-inclusive decay $\overline{B^0}\to\pi^+X_q$ with emphasis on the possibility of using these decays to determine the form factor $F_{0,1}^{B\to\pi}$.

From Eq. (2) we see that in general there are three types of terms in the factorization approximation for semi-inclusive decays of the type $B\to PX$. Each of the terms involves different hadronic parameters, with the first, the second, and the third terms being proportional to the $B\to P$ transition form factor from $\langle P|j_2|B\rangle$, the $P$ decay constant from $\langle P|j_1|0\rangle$, and some other parameters from $\langle PX|j_1|0\rangle$ and $\langle 0|j_2|B\rangle$, respectively. If all three terms in Eq. (2) are of the same order of magnitude, the accumulated uncertainties will be substantial due to uncertainties in all the hadronic parameters involved, especially in the form factors.

 

If one or two terms in Eq. (2) can be eliminated, one can have a better estimate of the magnitude involved. Indeed this can be achieved by an appropriate choice of the initial meson $B$ and the final meson $P$. For example, in $\overline{B}^0 \to (K^- X),(\pi^- X)$, the term proportional to the form factors does not appear [4], and hence a reliable theoretical prediction is possible. We find that there is only one possible choice for $P$ where the second term is eliminated. This is the mode $\overline{B}^0 \to \pi^+ X_q$ with $X_q$ equal to $X_d$ or $X_s$. Here $X_d$ and $X_s$ indicate the states having $\Delta S=0$ and $-1$, respectively. These decays are directly related to the form factors $F_{0,1}^{B\to\pi}$. Therefore, $\overline{B}^0 \to \pi^+ X_q$ can be used to study these form factors and to test model calculations. The form factors can also be studied in semileptonic $B \to l \overline{\nu}_l \pi$ decays, probably more accurately than from $\overline{B}^0 \to \pi^+ X_q$. However, the final states are different, one in the leptonic environment and the other in the hadronic environment. They are complementary to each other.

In $\overline{B}^0 \to \pi^+ X_q$, the biquark operators can only be in the forms $j_1 = \overline{q}\Gamma_1 u$ and $j_2 = \overline{u}\Gamma_2 b$ and therefore

$$A(\overline{B}^0 \to \pi^+ X_q) = \langle X_q | j_1 | 0 \rangle \langle \pi^+ | j_2 | \overline{B}^0 \rangle + \langle X_q \pi^+ | j_1' | 0 \rangle \langle 0 | j_2' | \overline{B}^0 \rangle. \tag{3}$$

The second term, being of the annihilation type, is subleading and will be neglected. Note that for $q=s$, the annihilation term is automatically zero.

We would like to point out that $\overline{B}^0 \to \pi^+ X_q$ is a multibody decay, and is different from two-body decays. There are several ways of factorization for such a decay, such as $\langle X_1 | j_1 | 0 \rangle \langle X_1' \pi^+ | j_2 | \overline{B}^0 \rangle$ and $\langle X_2 \pi^+ | j_1' | 0 \rangle \langle X_2' | j_2' | \overline{B}^0 \rangle$, with $X_q = X_1 + X_1' = X_2 + X_2'$. The two terms in Eq. (3) correspond to the cases $\langle X_1' | = \langle 0 |$ and $\langle X_2' | = \langle 0 |$, respectively. For $\overline{B}^0 \to \pi^+ X_q$ with a cut $E_\pi > 2.1$ GeV, the final state $X_q$ has a small invariant mass. This is a quasi-two-body decay, with $\pi^+$ and $X_q$ moving rapidly apart in opposite directions. The probability of forming the final state $\langle X_1' \pi^+ |$ with $\langle X_1' | \neq \langle 0 |$ is less than the probability of forming the simple final state $\langle \pi^+ |$. The contribution of the configuration $\langle X_2 \pi^+ | j_1' | 0 \rangle \langle X_2' | j_2' | B \rangle$ is dominated by $\langle X_q \pi^+ | j_1' | 0 \rangle \langle 0 | j_2' | \overline{B}^0 \rangle$ when it is nonvanishing. The cases with $|X_1'\rangle$ and $|X_2'\rangle$ not equal to $|0\rangle$ are higher order in $\alpha_s$ and therefore suppressed.

We now present the detailed calculations. The effective Hamiltonian for rare charmless hadronic $B$ decays at the quark level is given by

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{uq}^* (c_1 O_1 + c_2 O_2) - \sum_{i=u,c,t} \sum_{n=3}^{10} V_{ib} V_{iq}^* c_n^i O_n \right\}. \tag{4}$$

Here $O_n$ are operators given by

$$\begin{aligned}
O_1 &= (\overline{s}_i u_j)_{V-A} (\overline{u}_j b_i)_{V-A}, \quad O_2 = (\overline{s}_i u_i)_{V-A} (\overline{u}_j b_j)_{V-A}, \\
O_{3(5)} &= (\overline{s}_i b_i)_{V-A} \sum_{q'} (\overline{q}_j' q_j')_{V-(+)A}, \\
O_{4(6)} &= (\overline{s}_i b_j)_{V-A} \sum_{q'} (\overline{q}_j' q_i')_{V-(+)A}, \\
O_{7(9)} &= \frac{3}{2} (\overline{s}_i b_i)_{V-A} \sum_{q'} e_{q'} (\overline{q}_j' q_j')_{V+(-)A}, \\
O_{8(10)} &= \frac{3}{2} (\overline{s}_i b_j)_{V-A} \sum_{q'} e_{q'} (\overline{q}_j' q_i')_{V+(-)A},
\end{aligned} \tag{5}$$

where $(V \pm A)(V \pm A) = \gamma^\mu (1 \pm \gamma_5) \gamma_\mu (1 \pm \gamma_5)$, $q' = u,d,s,c,b$, $e_{q'}$ is the electric charge number of the $q'$ quark, and $i$ and $j$ are color indices.

The Wilson coefficients $c_n^i$ have been calculated in different schemes [9,10]. In this paper, we will use consistently the regularization scheme independent results. The values of $c_n$ at $\mu \approx m_b$ with the next-to-leading-order (NLO) QCD corrections are given by [10]

$$\begin{aligned}
&c_1 = -0.307, \quad c_2 = 1.147, \quad c_3^t = 0.017, \quad c_4^t = -0.037, \\
&c_5^t = 0.010, \quad c_6^t = -0.045, \quad c_7^t = -0.0017\alpha_{\rm em}, \\
&c_8^t = 0.052\alpha_{\rm em}, \quad c_9^t = -1.37\alpha_{\rm em}, \quad c_{10}^t = -0.282\alpha_{\rm em}, \\
&c_{3,5}^{u,c} = -\frac{1}{N_c}, \quad c_{4,6}^{u,c} = \frac{1}{N_c} P_s^{u,c}, \quad c_{7,9}^{u,c} = P_e^{u,c}, \quad c_{8,10}^{u,c} = 0,
\end{aligned} \tag{6}$$

where $N_c = 3$ is the number of colors and $\alpha_{\rm em} = 1/137$ is the electromagnetic fine-structure constant. The functions $P_{s,e}^i$ are given by

$$\begin{aligned}
P_s^i &= (\alpha_s / 8\pi)\, c_2 [\tfrac{10}{8} + G(m_I, \mu, k^2)], \\
P_e^i &= (\alpha_{\rm em} / 9\pi)\, (N_c c_1 + c_2) [\tfrac{10}{9} + G(m_i, \mu, k^2)], \\
G(m, \mu, k^2) &= 4 \int_0^1 x(1-x) \ln \frac{m^2 - x(1-x)k^2}{\mu^2} dx.
\end{aligned} \tag{7}$$

We obtain the decay amplitudes in the factorization approximation as

$$\begin{aligned}
A(\overline{B}^0 \to \pi^+ X_q) &= [\alpha_q \overline{q} \gamma_\mu (1-\gamma_5) u + \beta_q \overline{q} \gamma_\mu (1+\gamma_5) u] \\
&\quad \times \left[ F_1(q^2)(p_B^\mu + p_\pi^\mu) + [F_0(q^2) - F_1(q^2)] \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right. \\
&\quad \left. + \gamma_q \overline{q} (1+\gamma_5) u \frac{m_B^2 - m_\pi^2}{m_b - m_u} F_0(q^2) \right],
\end{aligned} \tag{8}$$

where $q = p_B - p_\pi$, and

$$\begin{aligned}
\alpha_q = \frac{G_F}{\sqrt{2}} \Bigg[ & V_{ub} V_{uq}^* \left( \frac{1}{N_c} c_1 + c_2 + \frac{1}{N_c} c_3^{tu} + c_4^{tu} + \frac{1}{N_c} c_9^{tu} + c_{10}^{tu} \right) \\
& + V_{cb} V_{cq}^* \left( \frac{1}{N_c} c_3^{tc} + c_4^{tc} + \frac{1}{N_c} c_9^{tc} + c_{10}^{tc} \right) \Bigg],
\end{aligned}$$

$$\beta_q = \frac{G_F}{\sqrt{2}}[V_{ub}V_{uq}^*(c_6^{tu}+c_8^{tu})+V_{cb}V_{cq}^*(c_6^{tc}+c_8^{tc})],$$

$$\gamma_q = \frac{G_F}{\sqrt{2}}[V_{ub}V_{uq}^*(c_5^{tu}+c_7^{tu})+V_{cb}V_{cq}^*(c_5^{tc}+c_7^{tc})]\left(-\frac{2}{N_c}\right), \tag{9}$$

where $c^{ij}=c^i-c^j$. The above coefficients depend on the momentum exchange $k^2$. In the heavy $b$ quark limit, $k^2 = m_B^2(1-2E_q/m_B)$.

From the decay amplitudes obtained in the previous section, we obtain the differential branching ratio,

$$\frac{d\Gamma}{dx\,dy} = \frac{m_B^5}{16\pi^3}[(|\alpha_q|^2+|\beta_q|^2)F_1^2(1-x)(x+y-1) + \tfrac{1}{4}|\gamma_q|^2F_0^2(1-x)], \tag{10}$$

where $y=2E_q/m_B$ and $x=2E_\pi/m_B$. The physical integration intervals are $0<x<1$ and $1-x<y<1$. The branching ratios with the appropriate cut on the $\pi^+$ energy $E_\pi > E_{\rm cut}$ are given by

$$\Gamma(E_\pi > E_{\rm cut}) = \int_{2E_{\rm cut}/m_B}^1 dx \int_{1-x}^1 dy \frac{d\Gamma}{dx\,dy}. \tag{11}$$

There are several estimates of the form factors with values in the range of 0.3–0.4 [11,12]. We will use $F_{0,1}^{B\to\pi}(0)=0.33$ for illustration. For $E_\pi > 2.1$ GeV, the dependence on $q^2$ is small, and we will use a single pole form as an approximation. For the KM matrix elements we will use the following independent variables: $V_{us}=\lambda$, $V_{ub}=|V_{ub}|\exp(-i\gamma)$, and $V_{cb}=A\lambda^2$, with $\lambda=0.2196$, $A=0.835$, and $|V_{ub}|=0.08|V_{cb}|$. The $CP$-violating phase $\gamma$ is treated as a free parameter. The results for the branching ratios are shown in Fig. 1. We see that the branching ratios for $\bar{B}^0\to\pi^+X_d$ and $\bar{B}^0\to\pi^+X_s$ are in the ranges of $(3.1\text{–}4.9)\times10^{-5}$ and $(2.5\text{–}4.2)\times10^{-5}$, respectively. These can be reached at $B$ factories.

We point out an interesting prediction regarding rate differences. Due to the unitarity property of the KM matrix elements, $\mathrm{Im}(V_{ub}V_{ud}^*V_{cd}V_{cb}^*) = -\mathrm{Im}(V_{ub}V_{us}^*V_{cs}V_{cb}^*)$, the rate difference $\Delta_d = \Gamma(\bar{B}^0\to\pi^+X_d) - \bar{\Gamma}(B^0\to\pi^+\bar{X}_d)$ and the corresponding rate difference $\Delta_s$ have the same magnitude [in the $SU(3)$ limit] but opposite sign. When the final states $X_d$ and $X_s$ are not distinguished, one would get a vanishing value for the asymmetry, $A$,

$$A = \frac{\Gamma[\bar{B}^0\to\pi^+(X_d+X_s)] - \bar{\Gamma}[B^0\to\pi^-(\bar{X}_d+\bar{X}_s)]}{\Gamma[\bar{B}^0\to\pi^+(X_d+X_s)] + \bar{\Gamma}[B^0\to\pi^-(\bar{X}_d+\bar{X}_s)]}. \tag{12}$$

This can provide a test for the standard model.

We also studied $CP$ asymmetries, with the energy cut $E_\pi > 2.1$ GeV, defined by

$$A_{CP} = \frac{\Gamma(\bar{B}^0\to\pi^+X_q) - \bar{\Gamma}(B^0\to\pi^-\bar{X}_q)}{\Gamma(\bar{B}^0\to\pi^+X_q) + \bar{\Gamma}(B^0\to\pi^-\bar{X}_q)}. \tag{13}$$

The results for $A_{CP}$ are shown in Fig. 2. Since $\Delta_d=-\Delta_s$, when $\mathrm{Br}(\bar{B}^0\to\pi^+X_d)$ is smaller than $\mathrm{Br}(\bar{B}^0\to\pi^+X_s)$, $A_{CP}(\pi^+X_d)$ is larger than $A_{CP}(\pi^+X_s)$. This behavior is clearly shown in Fig. 2. We also see that the asymmetry in $\bar{B}^0\to\pi^+X_d(X_s)$ can be as large as 5% (6%).

From discussions in the previous sections, it can be seen that the measurements of the branching ratios for $\bar{B}^0\to\pi^+X_q$ can yield information about the form factors $F_{1,0}^{B\to\pi}$. If the form factor is known, the branching ratios can be predicted. The numerical values are obtained using $F_1(0)=F_0(0)=0.33$. In general, since $F_1(q^2)$ and $F_0(q^2)$ have different dependences on $q^2$, one would expect that several hadronic parameters are needed. However, since $F_1(0)=F_0(0)$ by current conservation, for small $q^2$ (which is the case with $E_\pi>2.1$ GeV), $F_1^{B\to\pi}(q^2)\approx F_0^{B\to\pi}(q^2)\approx F_1^{B\to\pi}(0)$, the branching ratios are proportional to $F_1^2(0)$. The branching ratios obtained can be normalized as $\mathrm{Br}(F_1)=\mathrm{Br}[F_1(0)=0.33)][F_1(0)/0.33]^2$. We have checked numerically using another set of realistic form factors having a different $q^2$ dependence from Ref. [12]. Indeed we find that in the kinematic region of interest, the results change little.

We have argued previously that experimental measurements for $\bar{B}^0\to\pi^+X_d$, although difficult, can be carried out with an appropriate cut on the energy of the pion $E_\pi$. Combining the measurements with $X_q=X_d$ and $X_q=X_s$ can further enhance the statistical significance. We find that the combined branching ratio for $\bar{B}^0\to\pi^+(X_d+X_s)$ is $\sim 7.4\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$, which is insensitive to the phase angle $\gamma$. This implies that even without a good determination of $\gamma$, one can have useful information about the form factors. The combined branching ratio also makes the task easier in that one does not need to know the strangeness of the final state. To ensure that the $\pi^+$ is from $\bar{B}^0\to\pi^+X_d$, and not from $B^0\to\pi^+\bar{X}_u$, tagging of the $\bar{B}^0$ or $B^0$ is necessary, and this can be carried out at $B$ factories. Measurements of $\bar{B}^0\to\pi^+X_q$ are therefore possible.

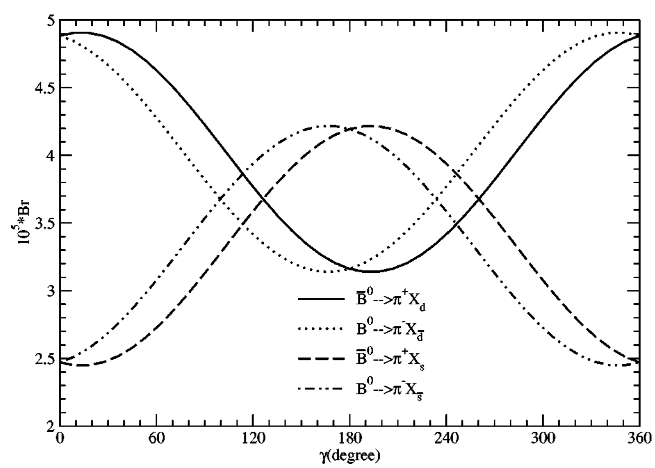


FIG. 1. The branching ratios as a function of $\gamma$.

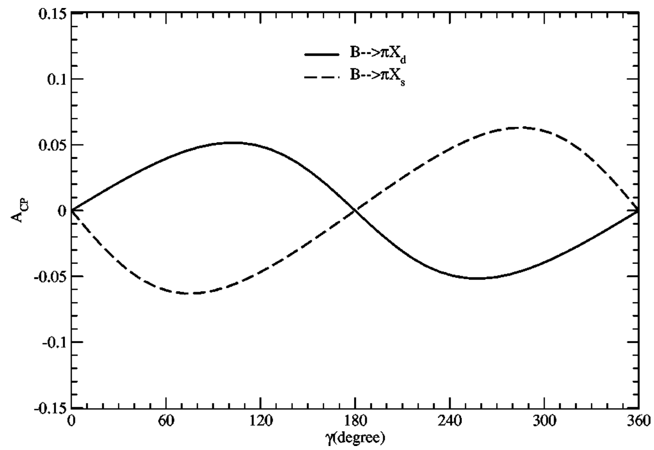


FIG. 2. The $CP$ asymmetries as a function of $\gamma$.

There are several uncertainties involved. The largest uncertainty is from the KM matrix elements, especially the phase $\gamma$. At present the best-fit value for $\gamma$ is around 60° [2]. If one uses the modes $\overline{B^0}\to\pi^+X_d$ and $\overline{B^0}\to\pi^+X_s$ individually, one needs a good knowledge of $\gamma$ to obtain precise information about the form factors. This can be eliminated by using the combined branching ratio. We have studied the sensitivity of the branching ratios to the magnitudes of the KM matrix elements. The largest one comes from the magnitude of $V_{ub}$. The branching ratio of $\overline{B^0}\to\pi^+X_d$ is almost proportional to $|V_{ub}|^2$, which can be easily rescaled. The branching ratio of $\overline{B^0}\to\pi^+X_s$ is less sensitive to $|V_{ub}|$. For an accurate determination of the form factors, a good knowledge of the magnitude of the KM matrix elements, especially $|V_{ub}|$, is important.

In this paper, we have studied a class of semi-inclusive charmless hadronic $B$ decays $\overline{B^0}\to\pi^+ + X_q$. We find that for these decays, in the factorization approximation, the only unknown hadronic parameters are the form factors $F_{0,1}^{B\to\pi}$. Accurate measurement of these decays can provide important information about form factors. Using theoretical model calculations for the form factors, the branching ratios for $\overline{B^0}\to\pi^+X_d(\Delta S=0)$ and $\overline{B^0}\to\pi^+X_s(\Delta S=-1)$, with the cut $E_\pi>2.1$ GeV, are estimated to be in the ranges of $(3.1\text{–}4.9)\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$ and $(2.5\text{–}4.2)\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$, respectively. The combined branching ratio is $7.4\times10^{-5}[F_1^{B\to\pi}(0)/0.33]^2$ and is almost independent of $\gamma$, and is within the reach of $B$ factories. $\overline{B^0}\to\pi^+X_q$ can provide interesting information about the form factors. $CP$-violating asymmetries in these decays can be studied, and with the current knowledge of the KM phase $\gamma$, we expect the asymmetries to be around 5%.

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