Structure of radiative interferences and $g=2$ for vector mesons

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(Received 23 July 2002; published 14 November 2002)

The result of Burnett and Kroll (BK) states that for radiative decays, the interference of $O(\omega^{-1})$ in the photon energy ω vanishes after the sum over polarizations of the involved particles. Using radiative decays of vector mesons, we show that if the vector meson is polarized, the $O(\omega^{-1})$ terms are null only for the canonical value of the magnetic dipole moment of the vector meson, namely, $\mathbf{g}=2$ in Bohr magneton units. A subtle cancellation of all $O(\omega^{-1})$ terms happens when summing over all polarizations to recover the Burnett-Kroll result. We also show the source of these terms and the corresponding cancellation for the unpolarized case and exhibit a global structure that can make them individually vanish in a particular kinematical region.

DOI: 10.1103/PhysRevD.66.097301 PACS number(s): 13.40.−f, 14.40.Cs

I. INTRODUCTION

The early work by Low [1] that relates the radiative process with the corresponding nonradiative and the electrostatic properties of the involved particles provided the grounds to develop bremsstrahlung studies in a model-independent basis. These processes have been used as a way to obtain information on the electromagnetic structure of particles [2] and the importance of off-shell effects [3]. Subsequent works [2,4,5] exploited Low's result to show explicit properties of the so-called Low amplitude. One of these is the work of Burnett and Kroll (BK) [5], which stated that the interference of the first and second terms of the amplitude expansion, in powers of the photon energy (ω), after the sum over polarizations, is null, namely, the $O(\omega^{-1})$ in the squared amplitude. This was simultaneously found by Zakharov *et al.* [2]. In general, those terms in the amplitude can be identified with the electric charge and magnetic dipole emissions, respectively, and thus the result can be seen to resemble the classical observation of the noninterference of these multipoles. These and other features are embodied in the so-called soft-photon approximation.

The subtle cancellation of the interferences in practice is not traced back and thus many interesting features are shadowed by just checking that the results satisfy the Burnett-Kroll theorem. One of these is the existence of additional structures that can help us to obtain more information about the decay and the properties of the involved particles. One may also wonder if the polarized case satisfies the Burnett-Kroll result, and if so, under which conditions. At the same time, it is interesting to know, in practice, how the sum over polarizations leads to the vanishing of $O(\omega^{-1})$ contributions.

Given the fact that the electric charge is completely determined by charge conservation, the magnetic dipole moment (MDM) value naturally plays a key role in radiative decays. For example, the W gauge boson MDM is predicted by the standard model to be $\mathbf{g}=2$ in Bohr magneton units (we will refer to the MDM by the giromagnetic ratio $\mathbf{g}$) and is a test of the gauge structure of the theory. Indeed, many authors [6] have shown that this value has many interesting features in the description of electromagnetic phenomena for particles of half and integer spin. Moreover, in theories in which the vector mesons are considered as gauge bosons of a hidden symmetry [7], the coupling to an electromagnetic field has $\mathbf{g}=2$ in a similar way to the W gauge boson. Thus this particular value is frequently assumed to be the canonical one. In this work, we offer an additional feature to favor it by observing the radiative decay interferences for polarized vector mesons, where the $O(\omega^{-1})$ terms are null only if $\mathbf{g}=2$.

In other works, in the soft-photon approximation it was observed [2,8,9] that for radiative decays involving vector mesons it is possible to suppress the electric charge contribution to the photon-energy spectrum by an appropriate choice of a kinematical configuration. In a more recent work [10], an exploration beyond that approximation showed that *the interference of the electric charge radiation with any gauge-invariant term* of the transition amplitude exhibits a typical structure that in particular is null by the same kinematical configuration, i.e., where the photon is collinearly emitted off the charged particle of the final state when in the initial particle rest frame. Although not explicitly mentioned, this is also true for each of the $O(\omega^{-1})$ terms whose total sum is null as stated by Burnett and Kroll. Here we apply that result to unpolarized vector meson radiative decays to explicitly exhibit this behavior and to show how the BK result is obtained, thus gaining insight into the destructive interferences. All over the interference contributions, the MDM value is involved and therefore its role can be inferred. These are the questions that we plan to address in this paper.

In the present work, we explore all this for radiative decays of vector mesons typically of the form $V^{+}\rightarrow P^{+}P'^{0}\gamma$, where V (P,P') is a vector (pseudoscalar) meson. We start by considering the vector meson polarization to study the effect of the MDM in the radiation probability structure. Then we state the features of the interferences in the unpolarized case, and we exhibit how the BK result is obtained. At the end, we discuss our results and their implications.

II. POLARIZED RADIATION DISTRIBUTION

To make clear the structure of the Burnet-Kroll terms, in the following we write the explicit gauge-invariant Low amplitude for the decay of a vector meson that we choose to be $\rho^+ \to \pi^+ \pi^0 \gamma$, although the results are not restricted to this particular case. We will use the 4-momentum notation $q \to p\ p'\ k$ in the respective order, and ϵ (η) corresponds to the polarization 4-vector of the photon (vector meson). Thus, the Low amplitude can be written as [8]

$$\begin{aligned}\mathcal{M}_L = ieg_{\rho\pi\pi}\Bigg[&(p-p')\cdot\eta L\cdot\epsilon^* + L\cdot\epsilon^* k\cdot\eta \\ &-\frac{\mathbf{g}p\cdot k}{q\cdot k}\left(\frac{p\cdot\epsilon^*}{p\cdot k}k\cdot\eta-\epsilon^*\cdot\eta\right) \\ &+\left[2-\left(1-\frac{\mathbf{g}}{2}\right)\left(1+\frac{\Delta^2}{m_\rho^2}\right)\right]\left(\frac{q\cdot\epsilon^*}{q\cdot k}k\cdot\eta-\epsilon^*\cdot\eta\right)\Bigg],\end{aligned} \tag{1}$$

where $g_{\rho\pi\pi}$ denotes the $\rho\pi\pi$ coupling, e is the electric charge of the positron, m_ρ is the ρ meson mass, and m_π (m_{π^0}) is the mass of the charged (neutral) pion. The magnetic dipole moment is given by $\mathbf{g}$ in Bohr magneton units, $\Delta^2 \equiv m_\pi^2 - m_{\pi^0}^2$ and $L^\mu \equiv \{[p^\mu/p\cdot k]-[q^\mu/q\cdot k]\}$.

Let us now study the interferences behavior by considering the polarization of the vector meson. The magnetic-dipole moment $\mathbf{g}$ is not restricted to a particular value and the masses of the charged (m_π) and neutral (m_{π^0}) pions are different. We consider, for simplicity, the rest frame of the decaying particle and the radiation gauge ($\epsilon_0=0$). The first condition implies that the vector meson polarization tensor η has the form $\eta_\mu^{(j)}=(0,\vec{\eta}^{(j)})$ with $j=1,2,3$. For definiteness we choose the following base:

$$\vec{\eta}^{(1)}=\frac{1}{\sqrt{2}}(1,i,0),\ \vec{\eta}^{(2)}=\frac{1}{\sqrt{2}}(1,-i,0),\ \vec{\eta}^{(3)}=(0,0,1). \tag{2}$$

The coordinate system is taken in such a way that the photon vector momentum is along the $\hat{z}$ direction; this means $k=(\omega,0,0,\omega)$, which, in the gauge radiation, implies that $\vec{k}\cdot\vec{\epsilon}=\omega\epsilon_3=0$ and therefore the photon polarization tensor ϵ_μ can be written as

$$\epsilon_\mu=(0,\epsilon_1,\epsilon_2,0).$$

Once we have established our conventions, we proceed to compute the polarized amplitudes from Eq. (1). The explicit expressions after simplifications, for each of the vector meson polarizations (2), are the following, respectively:

$$\begin{aligned}\mathcal{M}^{(1)}=ie\frac{g_{\rho\pi\pi}}{\sqrt{2}}&\left[\frac{\mathbf{g}p\cdot k}{q\cdot k}-2+\left(1+\frac{\Delta^2}{m_\rho^2}\right)\left(1-\frac{\mathbf{g}}{2}\right)\right](\epsilon_1^*+i\epsilon_2^*) \\ &+\sqrt{2}ieg_{\rho\pi\pi}\frac{p\cdot\epsilon^*}{p\cdot k}(p_1+ip_2),\end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{M}^{(2)}=ie\frac{g_{\rho\pi\pi}}{\sqrt{2}}&\left[\frac{\mathbf{g}p\cdot k}{q\cdot k}-2+\left(1+\frac{\Delta^2}{m_\rho^2}\right)\left(1-\frac{\mathbf{g}}{2}\right)\right](\epsilon_1^*-i\epsilon_2^*) \\ &+\sqrt{2}ieg_{\rho\pi\pi}\frac{p\cdot\epsilon^*}{p\cdot k}(p_1-ip_2),\end{aligned} \tag{4}$$

$$\mathcal{M}^{(3)}=2ieg_{\rho\pi\pi}\frac{p\cdot\epsilon^*}{p\cdot k}\left[p_3+\omega\left(1-\frac{\mathbf{g}p\cdot k}{2q\cdot k}\right)\right]. \tag{5}$$

In a three-body decay, besides the masses, only two Lorentz invariants are independent. We choose them to be $p\cdot k$ and $q\cdot k$ to exhibit the dependence on the photon energy. The total probability transition for each of the three directions of the polarization can then be computed and expressed in terms of those kinematical variables as follows:

$$\begin{aligned}|\mathcal{M}^{(1)}|^2=&\left(eg_{\rho\pi\pi}\frac{p\cdot\epsilon^*}{p\cdot k}\right)^2\Bigg[2m_\rho^2\frac{p\cdot k}{q\cdot k}\left(1+\frac{\Delta^2}{m_\rho^2}-\frac{p\cdot k}{q\cdot k}\right) \\ &-2m_\pi^2-2p\cdot k\left(1-\frac{\mathbf{g}}{2}\right)\left(1+\frac{\Delta^2}{m_\rho^2}-2\frac{p\cdot k}{q\cdot k}\right)\Bigg], \\ &-\frac{(eg_{\rho\pi\pi})^2}{2}\epsilon^*\cdot\epsilon\left[\mathbf{g}\frac{p\cdot k}{q\cdot k}-2+\left(1+\frac{\Delta^2}{m_\rho^2}\right)\left(1-\frac{\mathbf{g}}{2}\right)\right]^2,\end{aligned}$$

$$|\mathcal{M}^{(2)}|^2=|\mathcal{M}^{(1)}|^2,$$

$$\begin{aligned}|\mathcal{M}^{(3)}|^2=&\left(eg_{\rho\pi\pi}\frac{p\cdot\epsilon^*}{p\cdot k}\right)^2\Bigg[m_\rho^2\left(1+\frac{\Delta^2}{m_\rho^2}-2\frac{p\cdot k}{q\cdot k}\right)^2 \\ &+4\left(\frac{p\cdot k}{m_\rho}\right)^2\left(1-\frac{\mathbf{g}}{2}\right)^2+4p\cdot k\left(1-\frac{\mathbf{g}}{2}\right) \\ &\times\left(1+\frac{\Delta^2}{m_\rho^2}-2\frac{p\cdot k}{q\cdot k}\right)\Bigg].\end{aligned} \tag{6}$$

These equations exhibit clearly the structure $|\mathcal{M}^{(i)}|^2 = A^{(i)}/\omega^2 + B^{(i)}/\omega + C^{(i)}\omega^0$, and it can be observed that the $O(\omega^{-1})$ terms are not null unless the magnetic-dipole moment takes the canonical value $\mathbf{g}=2$. This result is very interesting because this simplifying feature was also found in Ref. [11] when describing the equation of motion of the polarization tensor for a particle in a homogeneous external electromagnetic field and reinforces the observation by many authors [6] that the choice $\mathbf{g}=2$ for charged vector mesons leads to richer properties of the radiative process description. We can also notice that the $O(\omega^{-1})$ terms are proportional to the kinematical factor $[(p\cdot\epsilon^*)/(p\cdot k)]^2$, whose importance will be clarified in the next section.

Adding the three polarized equations (6), we render to the unpolarized case

TABLE I. Interferences for the decay $\rho^+\to\pi^+\pi^0\gamma$.

$\sum_\eta \mathcal{M}_e\mathcal{M}^e_*$	$(eg_{\rho\pi\pi}L\cdot\epsilon)^2\left[m_\rho^2-4m_\pi^2+\frac{1}{m_\rho^2}(2p\cdot k-q\cdot k)^2-2q\cdot k\right]$
$2\ \mathrm{Re}\sum_\eta \mathcal{M}_e\mathcal{M}_0^*$	$(eg_{\rho\pi\pi}L\cdot\epsilon)^2\left[-\frac{2}{m_\rho^2}(2p\cdot k-q\cdot k)^2+2q\cdot k\right]$
$\sum_\eta \mathcal{M}_0\mathcal{M}^0_*$	$-4(eg_{\rho\pi\pi})^2\epsilon\cdot\epsilon^*\left(1-\frac{p\cdot k}{q\cdot k}\right)^2+(eg_{\rho\pi\pi}L\cdot\epsilon)^2\frac{1}{m_\rho^2}(2p\cdot k-q\cdot k)^2$
$\sum_\eta \mathcal{M}_L\mathcal{M}^L_*$	$(eg_{\rho\pi\pi}L\cdot\epsilon)^2(m_\rho^2-4m_\pi^2)-4(eg_{\rho\pi\pi})^2\epsilon\cdot\epsilon^*\left(1-\frac{p\cdot k}{q\cdot k}\right)^2$

$$\sum_\eta |\mathcal{M}|^2=(eg_{\rho\pi\pi})^2\left(\frac{p\cdot\epsilon^*}{p\cdot k}\right)^2\left[\left(1+\frac{\Delta^2}{m_\rho^2}\right)^2 m_\rho^2 -4m_\pi^2+4\left(\frac{p\cdot k}{m_\rho}\right)^2\left(1-\frac{\mathbf{g}}{2}\right)^2\right] -(eg_{\rho\pi\pi})^2\epsilon^*\cdot\epsilon\left[\mathbf{g}\frac{p\cdot k}{q\cdot k}-2 +\left(1+\frac{\Delta^2}{m_\rho^2}\right)\left(1-\frac{\mathbf{g}}{2}\right)\right]^2, \tag{7}$$

which is free of $O(\omega^{-1})$ terms, independently of the MDM value, in accordance with the BK theorem.

III. UNPOLARIZED RADIATION DISTRIBUTION

In a recent work [10] we showed that in a radiative three-body decay involving vector mesons, the interference between the electric charge and gauge-invariant terms has a typical structure regardless of the order in the photon energy. This is a result of two features of the total amplitude, which can be written as follows:

$$\mathcal{M}\propto\epsilon^*\cdot L+M\cdot\epsilon^*. \tag{8}$$

(i) *The radiation from electric charges is gauge-invariant through the tensor* $L^\mu\equiv\{[p^\mu/(p\cdot k)]-[q^\mu/(q\cdot k)]\}$, with q and p the four-momenta of the initial and final charged particles, respectively, and $k(\epsilon)$ the four-momentum (polarization) of the photon.

(ii) *The electromagnetic gauge invariance of the other terms* summarized in M, which can be of any order in the photon energy. After summing over the vector meson polarizations, the interference between these two terms can be written as [10]

$$\sum_\eta (L\cdot\epsilon^*)(\epsilon\cdot M)=a_2(L\cdot\epsilon^*)^2, \tag{9}$$

with a_2 a Lorentz invariant function. The corresponding limit for the soft-photon approximation is the Burnett-Kroll term, which must be null after summing *all* $O(\omega^{-1})$ contributions, i.e., because $(L\cdot\epsilon^*)^2$ is of $O(\omega^{-2})$, then the $O(\omega)$ term from a_2 does not contribute to the squared amplitude. Equation (9) suggests that kinematical advantages can be gained by considering the rest frame of the decaying particle. After summing over the photon polarizations, we have that

$$L^2=L\cdot L=-\frac{|\vec{p}|^2}{(p\cdot k)^2}\sin^2\theta, \tag{10}$$

with θ the angle between the photon three-momentum and $\vec{p}$. Thus the interference shows an important angular dependence, namely the cancellations are less crucial as we approach the collinear case up to becoming null and vice versa. The $O(\omega)$ part of the a_2 function should vanish as prescribed by BK and therefore a game between the two conditions can be played.

In the following, we show this structure for the decay under consideration $(\rho\to\pi\pi\gamma)$ and how the BK cancellation happens. For the sake of clarity, in this section we assume isospin symmetry $(m_\pi=m_{\pi^0})$ and the magnetic dipole moment to be $\mathbf{g}=2$, in Bohr magneton units. Based upon these conditions in the Low amplitude (1), we identify the contributions. The first term of the amplitude is of $O(\omega^{-1})$ (hereafter $\mathcal{M}_e$) and can be identified with the electric charge radiation. The remaining terms, which are of order ω^0, are explicitly gauge-invariant. We will refer to them as $\mathcal{M}_0$. The interference of these gauge-invariant amplitudes are summarized in Table I. We have included the result of the electric charge interference with itself[1] and the corresponding result for $\mathcal{M}_0$ because the cancellation of the Burnett-Kroll terms is linked, as we show below.

It is interesting to note that the cancellation of $O(\omega^{-1})$ terms is a result of a combination between the electric charge radiation itself, the interference term, and the nonelectric one. After summing over the photon polarization, owing to

[1]Examples of higher-order interferences can be found in [10].

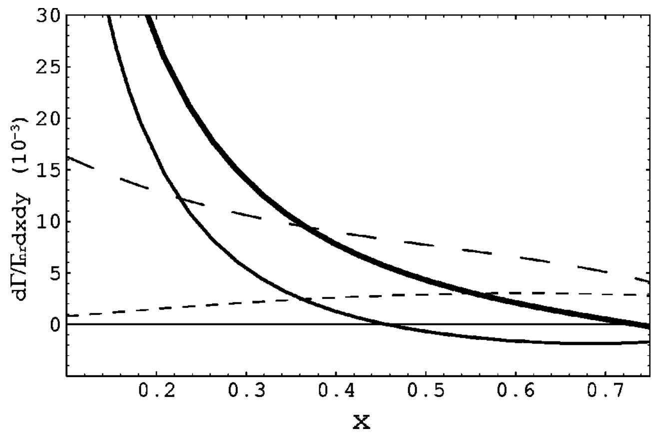

FIG. 1. Angular and energy distributions of photons in the process $\rho^+ \to \pi^+ \pi^0 \gamma$, normalized to the nonradiative rate, as a function of the photon energy ($X \equiv 2\omega/m_\rho$) for an angle $\theta = 10°$ ($y = \cos\theta$) between the photon and the charged pion 3-momenta. The long-dashed, short-dashed, and light solid line are $\Sigma_\eta \mathcal{M}_e \mathcal{M}_e^*$, $2\,\mathrm{Re}\Sigma_\eta \mathcal{M}_e \mathcal{M}_0^*$, and $\Sigma_\eta \mathcal{M}_0 \mathcal{M}_0^*$, respectively. The bold solid line corresponds to the total spectrum.

L^2, their proportionality to $\sin^2(\theta)$ (θ is the angle between the photon and charged pion 3-momenta) is a global factor, and although the $O(\omega^{-1})$ terms vanish after summation, they individually do not do so and therefore can be used to understand how important these cancellations are; in particular it can be seen that they are suppressed until they vanish as we approach the collinear case and conversely.

It is straightforward to show that the result in the last line of the table coincides with the unpolarized case (7) obtained previously, with $\mathbf{g} = 2$ and $\Delta^2 = 0$.

Let us now show how their relative magnitudes are in the photon-energy angular distribution. Figure 1 shows the photon-energy spectrum of each term for an angle $\theta = 10°$. The long-dashed, short-dashed, and light solid lines are $\Sigma_\eta \mathcal{M}_e \mathcal{M}_e^*$, $2\,\mathrm{Re}\Sigma_\eta \mathcal{M}_e \mathcal{M}_0^*$, and $\Sigma_\eta \mathcal{M}_0 \mathcal{M}_0^*$, respectively. The bold solid line corresponds to the total spectrum. We can observe that for a long part of the region, the interference contribution surpasses the total spectrum, thus demonstrating its relevance. This is crucial in the final region. As expected, the electric radiation is the leading one at low photon energies.

IV. DISCUSSION

We have computed the transition probability for the process $\rho^+ \to \pi^+ \pi^0 \gamma$, as a typical radiative vector meson decay, by considering each of the three degrees of polarization of the vector meson. We found that unlike the BK result, the $O(\omega^{-1})$ contributions, which partially come from the interference between the electric charge and the magnetic dipole moment radiation, are not null for each polarization state, unless the MDM takes the canonical value $\mathbf{g} = 2$. This result is important if we consider that no further restriction on its value is required elsewhere and that this is the same one that has been found for other authors, leading to particular properties of the radiative processes. The presence of $O(\omega^{-1})$ terms is due to the effect of the vector meson spin; disappear after summing over polarizations, satisfying the BK theorem. Our results are important in the sense that qualitatively different behavior arises for the radiation as a consequence of the electromagnetic properties of the vector mesons.

In addition, we studied the interferences of the unpolarized radiative decay amplitude of vector mesons, using the fact that the gauge invariance requirements for the amplitude yield a typical structure to the interferences between the charge and gauge-invariant terms. In particular, we showed this for the interference with the magnetic contribution, which, as stated by Burnett and Kroll, vanishes after summation of all $O(\omega^{-1})$ terms of the squared amplitude. There we exhibit the subtle cancellation and exploit the kinematical facility coming from the global factor L^2 to explore the importance of the interferences by looking at their relative magnitudes. In our analysis, we have only exploited the gauge invariance properties of the contributions and the known form of the radiation off electric charges.

Finally, we want to point out that the commonly shadowed radiation interferences can offer further interesting information about the particle properties.

The author is grateful to G. Lopez Castro for useful conversations and acknowledges financial support from CONACYT.

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