Role of lepton flavor violating muon decay in the seesaw model and LSND

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The aim of this work is to study lepton flavor violation in a newly proposed seesaw model of neutrino mass and to see whether it could explain the Liquid Scintillation Neutrino Detector excess. The motivation of this seesaw model is that there is no new physics beyond the TeV scale. By studying $\mu\to 3e$ in this model, it is shown that the upper bound on the branching ratio requires a Higgs boson mass m_h of a new scalar doublet with the lepton number $L=-1$ needed in this model to be about 9 TeV. The predicted branching ratio for $\mu\to e\nu_l\overline{\nu_l}$ is too small to explain the LSND.

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I. INTRODUCTION

The problem of physics beyond the standard model (SM) has been studied for a considerable length of time. In the past few years some progress has been made in understanding this new physics, among them lepton flavor violation (LFV) is the most promising candidate. At present, we have rigorous bounds on lepton flavor violating μ decay, e.g. [1],

$$\mathcal{B}(\mu\to 3e)\leqslant 10^{-12}. \tag{1}$$

Using experimental bounds on these three-body decays the corresponding bounds on two body decays are calculated in [2,3],

$$\mathcal{B}(Z\to \mu e)\leqslant 1.7\times 10^{-13} \tag{2}$$

and [3]

$$\mathcal{B}(J/\psi\to \mu e)\leqslant 4\times 10^{-13} \tag{3}$$

$$\mathcal{B}(\Upsilon\to \mu e)\leqslant 2\times 10^{-9} \tag{4}$$

$$\mathcal{B}(\Phi\to \mu e)\leqslant 4\times 10^{-17}. \tag{5}$$

At present, the best experimental limit on the branching ratio of $Z\to \mu e$ decay is (95% C.L.)

$$\mathcal{B}(Z\to \mu e)\leqslant 1.7\times 10^{-6}. \tag{6}$$

The possible source of the suppression of the bounds found in Eqs. (2)–(5) is discussed in [2–5].

The most alluring issue in present day physics is whether or not the neutrinos have nonzero mass. In the minimal standard model of particle interactions, neutrinos are massless. To generate a small neutrino mass in this model, there is an effective dimension five operator

$$\mathcal{L}_{eff}=\frac{f_{ij}}{\Lambda}L_iL_j\Phi\Phi, \tag{7}$$

where $L_i=(\nu_i,l_i)_L$ is the usual left-handed lepton doublet, $\Phi=(\phi^+,\phi^0)$ is the usual scalar Higgs doublet, and Λ is an effective large mass scale [6]. This operator has different tree-level realizations: (i) the canonical seesaw mechanism with a right-handed neutrino [7]; (ii) the model having a Higgs triplet [8]; and (iii) the model having a heavy Majorana ferimon triplet [9]. These new interactions exist at higher mass scale. In the usual seesaw mechanism, in order to have a very small mass for a left-handed neutrino, the corresponding mass for the right-handed neutrino has to be very large, i.e. of order 10^3 TeV. Recently a new seesaw model of neutrino mass is proposed with the motivation that there is no new physics beyond the TeV scale [10]. In this model the smallness of mass for the right-handed neutrino does not require a very heavy right-handed neutrino. This mechanism requires $m_N\sim 1$ TeV. However, in this model, a new Higgs doublet η with lepton number -1 is also necessary. The right-handed neutrino N and Higgs doublet η can give rise to LFV processes. We identify an effective operator in the standard model and show that the scale of new physics Λ must be $\Lambda\geqslant 5$ TeV. In Ma's model, $\mu\to 3e$ can proceed through N and η exchange at loop level. Using the experimental limit on this process, the box diagrams provide the most stringent limit on mass of Higgs doublet η ($m_h \geqslant 9$ TeV). We also show by constructing an effective $Z\to\mu e$ vertex and its realization through the η and N exchanges that no limit is put on m_h.

The experimental evidence of the neutrino masses comes, from three anomalous effects: the Liquid Scintillation Neutrino Detector (LSND) excess [11,12], atmospheric neutrino anomaly [13–15] and the solar neutrino deficit [16–19].

In addition to the three neutrinos, a sterile neutrino is needed to explain the three effects in terms of the neutrino flavor oscillations. But still the problem is unresolved [20]. Attempts to explain all the data in terms of the three massive neutrinos are excluded by the latest data [21].

An atmospheric anomaly and the solar neutrino deficit can be explained in terms of neutrino flavor oscillations, but LSND excess cannot be explained along these lines because of its small transition probability [$P_{\overline{\nu}_\mu\to\overline{\nu}_e}=(2.5\pm 0.6\pm 0.4)\times 10^{-3}$] [22]. It either requires a sterile neutrino or some mechanism other than neutrino flavor oscillation and LFV is one of the candidates [23]. We analyze the consquences of small LFV interactions to explain LSND excess and show that the branching ratio for $\mu^+\to e^+\nu_l\overline{\nu_l}$ turns out to be too small to explain the LSND excess.

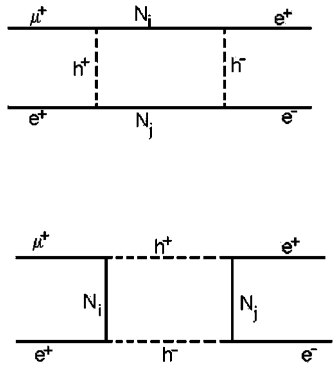

FIG. 1. Box diagrams for $\mu\to 3e$.

II. LFV IN SEESAW MODEL OF NEUTRINO MASS AND BOUNDS ON NEW HIGGS MESON MASS

A new seesaw model of neutrino mass has been proposed [10], where right handed fermion singlets N_i with lepton number $L=0$ are added to the minimal SM together with a second scalar doublt (η^+,η^0) with lepton number $L=-1$. The fermion singlet is allowed to have Majorana mass m_N with the effective interaction $f_{ij}\overline{N}_{iR}(\nu_{jL}\eta^0-l_{jL}\eta^+)$. The smallness of seesaw neutrino mass $(m_\nu=m_D^2/m_N)$ can be explained by a rather small value of m_N, if m_D comes from $\langle\eta^0\rangle$ instead of $\langle\phi^0\rangle$ because $\langle\eta^0\rangle\ll\langle\phi^0\rangle$, and is of the order 1 TeV and as such can be observed experimentlly. As the motivativation of the model is that there is no new physics beyond the TeV scale, therefore masses of the new Higgs scalar doublet, which are necessary in this model, should not be larger than a few TeV. We study this question in relation to the experimental bounds on $\mu\to 3e$ and $Z\to\mu e$.

First we note that an effective operator invariant under the symmetries of the standard model that can induce the decay $\mu\to 3e$ is

$$\frac{1}{\Lambda^4}(\overline{L}_\mu e_R\Phi)(\overline{L}_e e_R\Phi) \tag{8}$$

which is of dimension 8 and here Φ is the standard model Higgs doublet. On inserting the vacuum expectation value $\langle\phi^0\rangle=v$ it would generate the four-fermion interaction

$$\frac{v^2}{\Lambda^4}(\overline{\mu}_L e_R)(e_L e_R). \tag{9}$$

This operator leads to $\mu\to 3e$ with the branching ratio

$$\mathcal{B}(\mu\to 3e)=\left(\frac{v^2}{4\sqrt{2}G_F\Lambda^4}\right)^2. \tag{10}$$

Inserting $v=174$ GeV, the branching ratio (10) gives $\Lambda\geqslant 4.7$ TeV. However, the above effective operator cannot be induced in Ma's model at the tree level. In this model $\mu\to 3e$ occurs in the one loop shown in Fig. 1. The couplings at each vertex can be taken from [10].

The amplitude can be written as follows [24]:

$$iT(\mu\to 3e)=2\sum_{i,j}(f^*_{\mu i}f_{ei}f^*_{ej}f_{ej})\int\frac{d^4k}{(2\pi)^4}\overline{v}(p_1)\not{k}\times\left(\frac{1-\gamma_5}{2}\right)v(p_2)\times\overline{u}(p_3)\not{k}\left(\frac{1-\gamma_5}{2}\right)v(p_4)\times\left[\frac{1}{k^2-m_h^2}\right]^2\left[\frac{1}{k^2-m_i^2}\right]\left[\frac{1}{k^2-m_j^2}\right]. \tag{11}$$

It is assumed that the loop momenta is very high, i.e. $k\to\infty$ so that the external momenta are neglected. After calculating the loop integration Eq. (11) becomes (we take $m_i=m_j=m$)

$$T(\mu\to 3e)=\frac{1}{(4\pi)^2\times 4m_h^2}\times\sum_{i,j}\xi_i\xi_j\{\overline{v}(p_1)\gamma^\alpha\times(1-\gamma_5)v(p_2)\overline{u}(p_3)\gamma_\alpha(1-\gamma_5)v(p_4)\}A(x) \tag{12}$$

where $x=(m^2/'m_h^2)$ and $f^*_{\mu i}f_{ei}=\xi_i$, $f^*_{ej}f_{ej}=\xi_j$, giving the decay width

$$\Gamma(\mu\to 3e)=\left[\frac{1}{(4\pi)^2\times 4m_h^2}\right]^2 2\sum_{i,k}\xi_i\xi_k^*\sum_{j,l}\xi_j\xi_l^*A(x)A(x)\times\frac{m_\mu^5}{192\times\pi^3}, \tag{13}$$

where

$$A(x)=\left\{\frac{1-x^2+x\ln(x^2)}{2(x-1)^3}\right\}. \tag{14}$$

This gives the branching ratio

$$\mathcal{B}(\mu\to 3e)=\left[\frac{1}{4(4\pi)^2}\right]^2\left(\frac{2}{G_F^2}\right)\sum_{i,k}\xi_i\xi_k^*\sum_{j,l}\xi_j\xi_l^*\left(\frac{x^2}{m^4}\right)\times[A(x)]^2. \tag{15}$$

Assuming that all the Yukawa couplings are of the order unity and taking $m=1$ TeV the experimental bound (1) is obtained for $x=1.12\times 10^{-2}$ giving $m_h=9$ TeV.

We now consider $Z\to\mu e$ for which the effective operator is

$$g_{Z\mu e}\overline{L}_\mu\gamma^\mu D_\mu L_\mu \tag{16}$$

or

$$\tilde{g}_{Z\mu e}\overline{\mu}_R\gamma^\mu D_\mu e_R \tag{17}$$

which are renormalizable operators of dimension 4 so that the effective coupling constants are dimensionless. D_μ is the covariant derivative. The effective operator (16) is induced in

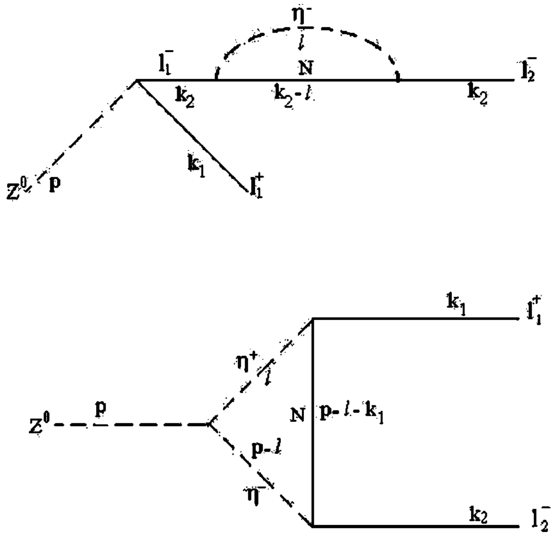

FIG. 2. One-loop diagrams for $Z\to\mu e$.

the present model by renormalizable interaction represented by the Feynman diagrams shown in Fig. 2. Each of the above diagrams is logarithmically divergent; but this divergence cancels in the sum if we note that the first diagram involves Z-lepton coupling in the form

$$(g_V+g_A)\frac{g_2}{2\cos\theta_w}=\left\{-\frac{1}{2}(1-4\sin^2\theta_w)+\left(-\frac{1}{2}\right)\right\}\frac{g_2}{2\cos\theta_w}$$
$$=-(1-2\sin^2\theta_w)\frac{g_2}{2\cos\theta_w} \tag{18}$$

while in the second diagram $Z\to\eta\bar{\eta}$ gauge coupling is $(g_2/2\cos\theta_w)(1-2\,sin^2\theta_w)$. In fact the two diagrams exactly cancel in the limit $m_Z=0$ and lepton mass $=0$. Using the dimensional regularization these diagrams together give

$$iT=\frac{g_2(1-2\sin^2\theta_w)}{2\cos\theta_w}\frac{f^2}{(4\pi)^2}\varepsilon_\mu\bar{u}(k_1)\gamma^\mu(1-\gamma_5)v(k)$$
$$\times\left(\frac{m_Z^2}{m_h^2}\right)I(x) \tag{19}$$

where with $x=m_i^2/m_h^2$ and where we have kept only the linear term in m_Z^2/m_h^2; $I(x)$ is given by

$$I(x)=-\frac{1}{36(1-x)}\left\{2-\frac{3x}{1-x}+\frac{6x^2}{(1-x)^2}+\frac{6x^3}{(1-x)^3}\ln x\right\}. \tag{20}$$

In obtaining the final result we have neglected a convergent contribution from the second diagram which is proportional to the lepton mass. Using $\sin^2\theta_w\simeq\frac{1}{4}$, Eq. (19) gives

$$\Gamma(Z\to\mu e)=\frac{G_F}{\sqrt{2}}\frac{m_Z^3}{6\pi}\frac{1}{2}\left(\frac{1}{4\pi}\right)^2 f^4\left(\frac{m_Z^2}{m_h^2}\right)^2 I^2 \tag{21}$$

while

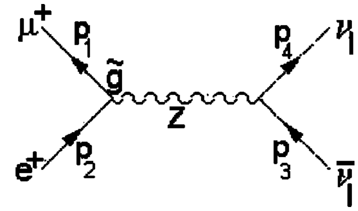

FIG. 3. Anomalous muon decay.

$$\Gamma_Z^{\text{tot}}=8\frac{G_F}{\sqrt{2}}\frac{m_Z^3}{6\pi}. \tag{22}$$

Thus the branching ratio is given by

$$\mathcal{B}(Z\to\mu e)=\frac{1}{16}f^4\left(\frac{1}{16\pi^2}\right)^2\left(\frac{m_Z^2}{m_h^2}\right)^2 I^2$$
$$=2.5\times10^{-6}\left(\frac{m_Z^2}{m_h^2}\right)^2 I^2 \tag{23}$$

where we have taken the Yukawa couplings $f\simeq1$. Taking the two extreme limits $x\to1$ and $x\to0$, $I(x)$ is, respectively, $\frac{1}{24}$ and $\frac{1}{18}$; the branching ratio bound (2) can be satisfied for $m_h\geqslant m_i\simeq500$ GeV. Thus no limit is put on m_h.

III. THE ANOMALOUS MUON DECAY AND LSND

The standard muon decay $\mu^+\to e^+\nu_e\bar{\nu}_\mu$ has no $\bar{\nu}_e$ which is found at LSND. Instead of the neutrino flavor oscillation this excess can be found through the LFV muon decay $\mu^+\to e^+\nu_l\bar{\nu}_l$. The anomalous muon decay $\mu^+\to e^+\nu_l\bar{\nu}_l$ (where $l=e$, μ, or τ) can occur via the Z-exchange as shown in Fig. 3, where the effective Lagrangian $Z\mu e$ vertex can be written as [3]

$$\mathcal{L}_{eff}=g_{Z\mu e}\bar{\mu}_L\gamma^\alpha e_L Z^\alpha+\text{H.c.} \tag{24}$$

Here, $g_{Z\mu e}$ is the effective coupling of the LFV vertex which is constrained from the experimental bound on the branching ratio of $\mu\to3e$. The coupling through the diagram of Fig. 3 contributes to the $A(\mu\to e\nu_l\bar{\nu}_l)$ amplitude a term

$$A(\mu\to e\nu_l\bar{\nu}_l)=\frac{g_{Z\mu e}g_{Z\nu\bar{\nu}}}{M_Z^2-s}[\bar{v}(p_1)\gamma^\alpha v(p_2)\bar{u}(p_4)\gamma_\alpha$$
$$\times(1-\gamma_5)v(p_3)] \tag{25}$$

The corresponding decay width becomes

$$\Gamma(\mu\to e\nu_l\bar{\nu}_l)=\left(\frac{g_{Z\mu e}g_{Z\nu\bar{\nu}}}{M_Z^2}\right)^2\frac{m_\mu^5}{192\pi^3}2$$
$$=g_{Z\mu e}^2\left(\frac{G_F}{\sqrt{2}}\right)^2\frac{m_\mu^5}{192\pi^3} \tag{26}$$

where we have used that $g_{Z\nu\bar{\nu}}=(g_2/2\cos\theta_w)\frac{1}{2}$. This gives the branching ratio

$$\mathcal{B}(\mu\to e\nu_l\overline{\nu_l})_{Z-exch.}=g_{Z\mu e}^2 =(4\sqrt{2}G_F m_Z^2)\mathcal{B}(Z\to\mu e) =(0.55)\mathcal{B}(Z\to\mu e) \quad (27)$$

where we have used Eq. (22). If we use the bound (2), we obtain

$$\mathcal{B}(\mu\to e\nu_l\overline{\nu_l})_{Z-exch.}\leqslant 10^{-13}, \quad (28)$$

which is much too small compared to the needed branching ratio for $\mu\to e\nu_l\overline{\nu_l}$ implied by

$$P_{\overline{\nu}_\mu\to\overline{\nu}_e}=(2.5\pm0.6\pm0.4)\times10^{-3}. \quad (29)$$

Even if one uses the direct limit (6) it still remains small $\simeq 10^{-6}$.

Since there is no ν_R in the standard model, one cannot write an operator of the form (8) with e_R replaced by ν_R. One can, however, write two dimension 9 operators as in [25] which would generate $\Delta L=2$ decay $\mu^+\to e^+\overline{\nu_e}\overline{\nu_l}$ with a branching ratio which could explain LSND excess without any conflict with $\Delta L=0$ processes like $\mu\to 3e$ with a scale of new physics at a rather low value $\Lambda\simeq 360$ GeV. However, such operators cannot be induced in the present model at tree level. If one considers the box diagrams for $\mu\to e\nu_l\overline{\nu_l}$, which gives the same result as in Eq. (15), namely

$$\mathcal{B}(\mu\to e\nu_l\overline{\nu_l})=\left(\frac{1}{64\pi^2\times m_i^2}\right)^2\frac{2}{G_F^2}(81f^8)[xA(x)]^2 \quad (30)$$

which gives the branching ratio to be $\approx 10^{-12}$ for $f\approx 1$ and $m_h\approx 9$ TeV as previously found. This is much too small compared to the required value $\simeq 2.5\times10^{-3}$.

IV. CONCLUSION

By studying process $\mu\to 3e$ at the loop level, we have put a bound on the mass of the new Higgs boson m_h needed in the seesaw model of neutrino masses [10]. Taking the Yukawa couplings f to be order 1 and the mass of the heavy neutrino to be 1 TeV as in [10], we found the bound on $m_h \geqslant 9$ TeV. No limit is put on m_h from the experimental bound on $Z\to\mu e$. The LSND neutrino anomaly in terms of the decay $\mu^+\to e^+\nu_l\overline{\nu_l}$ which requires the branching ratio of the new decay to be about $(1.5-3)\times10^{-3}$ cannot be explained in the present model. Lastly we wish to discuss the senstivity of the above bound on f. First of all we note that atmospheric anomaly required $m_\nu\geqslant 5\times10^{-2}$ eV which in Ma's model implies that $f^2\geqslant 5\times10^{-2}$ keeping $m_i\simeq 1$ TeV. Then one obtains no bound on m_h.

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