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Composite quarks and leptons in higher space-time dimensions

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A new approach towards the composite structure of quarks and leptons in the context of the higher dimensional unified theories is proposed. Because of certain strong dynamics much like ordinary QCD, every possible vectorlike set of composites appears in higher dimensional bulk space-time. However, through a proper Sherk-Schwarz compactification only chiral multiplets of composite quarks and leptons survive as the massless states in four dimensions. In this scenario restrictions related to 't Hooft's anomaly matching condition turn out to be avoided and, as a result, the composite models look rather simple and economical. We demonstrate our approach by an explicit construction of a model of preons and their composites unified in the supersymmetric $SU(5)$ GUT in five space-time dimensions. The model predicts precisely three families of composite quarks and leptons being the triplets of the chiral horizontal symmetry $SU(3)_h$ which automatically appears in the composite spectrum when going to ordinary four dimensions.

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I. INTRODUCTION

The observed replication of quark-lepton families and the hierarchy of their masses and mixings are one of the major puzzles of modern particle physics. In this respect it is conceivable to think that quark and lepton spectroscopy finds its ultimate explanation in terms of the subfermions (preons) and their interactions in analogy with an explanation of hadronic spectroscopy in the framework of the quark model. However a direct realization of this program seems to meet with some serious difficulties. Among the problems that have appeared the basic one is, of course, that related to the dynamics responsible for the production of composite quarks and leptons whose masses m_f turned out to be in fact much less than a compositeness scale Λ_C which must be located at least in a few TeV region to conform with observations [1]. Indeed, if, as usual, one considers the underlying preon theory to be QCD like, then one inevitably comes to the vectorlike bound state spectra where most naturally $m_f \sim\Lambda_C$. To overcome this difficulty one has to require the presence of some chiral symmetry which is respected by the strong preon dynamics and makes quark and lepton bound states massless. As 't Hooft first argued [2] such a chiral symmetry in order to be preserved in the spectrum of massless composite fermions must yield the same chiral anomalies as those appearing in the underlying preon theory. However, this anomaly matching condition has turned out to be too restrictive to drive at the physically interesting self-consistent models. As a result most existing models [3–5] are rather complex and controversial and often contain too many exotic composite states apart the ordinary quarks and leptons.

Supersymmetric preon models [4,5] follow a somewhat different pattern of the anomaly matching condition since in this case the physical composites quarks and leptons, appear as both the three-fermion ("baryons") and scalar-fermion ("mesons") bound states. More interestingly these models may provide a new dynamical alternative for obtaining light composite fermions which emerge as (quasi-)Goldstone fermions [4] when the starting global symmetry G of the superpotential is spontaneously broken down to some lower symmetry H. In recent years there has been a renewal of interest in supersymmetric preon models [5] based on a powerful technique developed within strongly interacting $N=1$ supersymmetric gauge theories [6]. However, despite these attractive features of supersymmetric theories the supersymmetric composite models generally suffer from the same drawbacks as the more traditional nonsupersymmetric ones.

In this paper we suggest a new approach towards the composite structure of quarks and leptons, proposing the presence of extra space-time dimensions at small distances comparable or a bit larger than the radius of compositeness, $R_C \sim 1/\Lambda_C$. It is well known that the compactification of extra space-time dimensions (depending on the details of dimensional reduction) has appeared to be successful in obtaining realistic four-dimensional models where supersymmetry [7–9], gauge symmetry [10,11], and certain discrete symmetries such as P and CP [12] are broken in an intrinsically geometric way. Following this line of argument, we find that due to a certain Scherk-Schwarz compactification [7] the composite quarks and leptons turn out to be massless in four dimensions while all unwanted states (residing in the bulk) are massive. In this way the restrictions related with the original 't Hooft anomaly matching condition can be avoided. Thereby the physical composite models look rather simple and economical as we will show here by a few examples of the elementary preons and their composites unified in the $SU(5)$ supersymmetry (SUSY) grand unified theory (GUT) initially appearing in five space-time dimensions (5D).

II. SUPERSYMMETRY IN 5D AND SCHERK-SCHWARZ COMPACTIFICATION

Before turning to the construction of composite models let us recall some aspects of the $N=1$ 5D supersymmetry and Scherk-Schwarz compactification which are relevant to our subsequent discussion. Consider in 5D the $N=1$ supersymmetric gauge theory with a local symmetry group G under which the matter fields transform according to one of its irreducible representation R. The $N=1$ supersymmetric Yang-Mills supermultiplet $\mathcal{V}=(A^M,\ \lambda^i,\ \Sigma,\ X^a)$ in 5D contains a vector field $A^M=A^{M\alpha}T^\alpha$, a real scalar field $\Sigma=\Sigma^\alpha T^\alpha$, and two gauginos $\lambda^i=\lambda^{i\alpha}T^\alpha$, which form a doublet under the R-symmetry group $SU(2)_R$ and auxiliary fields $X^a=X^{a\alpha}T^\alpha$ being a triplet of $SU(2)_R$ (here $M=0,1,2,3,4$ are space-time indices; $i=1,2$ and $a=1,2,3$ are $SU(2)_R$-doublet and $SU(2)_R$-triplet indices, respectively; α runs over the G group index values and T^α are the generators of G algebra). These fields are combined into the $N=1$ 4D vector supermultiplet $V=(A^m,\ \lambda^1,\ X^3)$ $(m=0,1,2,3)$ and a chiral supermultiplet $\Phi=(\Sigma+iA^4,\ \lambda^2,\ X^1+iX^2)$. The matter fields are collected in the hypermultiplet $\mathcal{H}=(h^i,\ \Psi,\ F^i)$ which contains the scalar fields h^i being a doublet of $SU(2)_R$, the Dirac fermion $\Psi=(\psi_1,\ \psi_2^+)^T$ being the $SU(2)_R$ singlet, and also the $SU(2)_R$ doublet of auxiliary fields F^i. All these fields form two $N=1$ 4D chiral multiplets $H=(h^1,\ \psi_1,\ F^1)$ and $H^c=(h^2,\ \psi_2,\ F^2)$, transforming according to the representations R and anti-R of gauge group G, respectively. The 5D supersymmetric and G-symmetric action then can be written as (see, e.g., [13])

$$S=\int d^5x\int d^4\theta[H^c e^V H^{c\dagger}+H^\dagger e^V H]+\int d^5x\int d^2\theta\left[H^c\left(\partial_4-\frac{1}{\sqrt{2}}\Phi\right)H+\text{h.c.}\right]. \tag{1}$$

The above theory (1) is in fact vectorlike and, hence, anomaly free.

Now let us compactify the extra fifth dimension x^4 on a circle of radius R_C. Aside from the trivial (periodic) boundary conditions under the $2\pi R_C$ translation of extra dimension one can impose on the 5D fields the following nontrivial [$U(1)$-twisted] conditions:

$$H(x^m,x^4+2\pi R_C,\theta)=\exp(i2\pi q_H)H(x^m,x^4,e^{i\pi(q_H+q_{H^c})}\theta),$$

$$H^c(x^m,x^4+2\pi R_C,\theta)=\exp(i2\pi q_{H^c})H(x^m,x^4,e^{i\pi(q_H+q_{H^c})}\theta),$$

$$V(x^m,x^4+2\pi R_C,\theta,\overline{\theta})=V(x^m,x^4,e^{i\pi(q_H+q_{H^c})}\theta,e^{-i\pi(q_H+q_{H^c})}\overline{\theta}),$$

$$\Phi(x^m,x^4+2\pi R_C,\theta)=\Phi(x^m,x^4,e^{i\pi(q_H+q_{H^c})}\theta), \tag{2}$$

where q_H and q_{H^c} are the R charges of the superfields H and H^c, respectively. As a result of to the periodicity conditions (2), the component fields are Fourier expanded as

$$h^1(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp[ix^4(n+q_H)/R_C]h^{1(n)}(x^m),$$

$$h^2(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp[ix^4(n+q_{H^c})/R_C]h^{2(n)}(x^m),$$

$$\psi_1(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp\left[ix^4\left(n+\frac{q_H-q_{H^c}}{2}\right)\Big/R_C\right]\times\psi_1^{(n)}(x^m),$$

$$\psi_2(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp\left[ix^4\left(n-\frac{q_H-q_{H^c}}{2}\right)\Big/R_C\right]\times\psi_2^{(n)}(x^m),$$

$$\lambda^1(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp\left[ix^4\left(n-\frac{q_H+q_{H^c}}{2}\right)\Big/R_C\right]\times\lambda^{1(n)}(x^m),$$

$$\lambda^2(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp\left[ix^4\left(n+\frac{q_H+q_{H^c}}{2}\right)\Big/R_C\right]\times\lambda^{2(n)}(x^m),$$

$$A^m(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp\left(ix^4\frac{n}{R_C}\right)A^{m(n)}(x^m),$$

$$(\Sigma+iA^4)(x^m,x^4)=\sum_{n=-\infty}^{\infty}\exp\left(ix^4\frac{n}{R_C}\right)(\Sigma+iA^4)^{(n)}(x^m). \tag{3}$$

Let us note now that all the fields with the nontrivial R charges necessarily turn out to be massive when reducing the theory from 5D to 4D. Particularly zero modes of all fermionic fields in Eqs. (3) and those of the scalars h^1 and h^2 have the masses q/R_C where q are the corresponding R charges while the zero modes of the gauge fields A^m and adjoint scalar $(\Sigma+iA^4)$ are massless. Nevetheless, the latter picks up mass of the order of $\sim 1/R_C$ radiatively since supersymmetry is broken by the above Scherk-Schwarz compactification so that in general only gauge fields A^m are left to be massless. However, if the R charges of the superfields H and H^c are equal ($q_H = q_{H^c}$), then as one can easily see from Eqs. (3) the zero modes of ψ_1 and ψ_2 happen to be massless as well. Note also that for composite operators containing the above superfields the R charge assignment and thus the spectrum of the massless zero modes can be rather different. This is the key point we will use below in the construction of composite models of quarks and leptons.

III. ONE-GENERATION COMPOSITE MODEL

Let us consider $N=1$ supersymmetric $G\otimes SU(N)_{HC}$ gauge theory in 5D where G is the gauged part of some hyperflavor symmetry G_{HF}, which includes all observed symmetries [color $SU(3)_C$ and electroweak $SU(2)_W \otimes U(1)_Y$ or grand unified symmetry $SU(5)$, etc.], and $SU(N)_{HC}$, which describes hypercolor interactions responsible for the formation of hypercolorless bound states from preons. We assume that the preons and antipreons reside in the 5D hypermultiplets $\mathcal{P}=(P,P^c)$ and transform under the hypercolor gauge group $SU(N)_{HC}$ as its fundamental ($P \sim N$) and antifundamental ($P^c \sim \overline{N}$) representations, respectively. The preons should carry also the quantum numbers related to the hyperflavor symmetry group G_{HF}. The hypercolor gauge group $SU(N)_{HC}$ has to be asymptotically free as in the case of ordinary QCD. Otherwise, the theory will not be well defined as an interacting quantum field theory (because of the Landau pole problem) and can be consistently treated only as a low energy limit of some other theory. Thus the asymptotic freedom of the $SU(N)_{HC}$ restricts the number of the allowed hyperflavors N_{HF} to be

$$\frac{N_{HF}}{2} \leqslant N. \tag{4}$$

Now let us take G the gauged part of the total hyperflavor symmetry G_{HF} to be the minimal grand unified group, i.e., $G\equiv SU(5)$, so that the preons transform under the $SU(5)\otimes SU(N)_{HC}$ as

$$\begin{aligned} &P_{(5)} \sim (5,N),\\ &P^c_{(\overline{5})} \sim (\overline{5},\overline{N}),\\ &P_{(s)i} \sim (1,N),\\ &P^{ci}_{(\overline{s})} \sim (1,\overline{N}), \end{aligned} \tag{5}$$

where $i=1,\ldots,N_g$. Therefore, the total number of flavors is $N_{HF}=5+N_g$. The $SU(5)$ singlet preons (antipreons) $P_{(s)i}(P^{ci}_{(\overline{s})})$ in Eqs. (5) are indeed necessary in order to produce the entire set of composite quark and leptons transforming as $\overline{5}+10$ representations of $SU(5)$. We call them "generation" preons. Thus the preons carry all "basic" quantum numbers presently observed in quark-lepton phenomenology at low energies such as three colors two weak isospin components [being unified within the $SU(5)$], and the generation numbers as well.

Within the framework described above the minimal possible hypercolor group is $SU(3)_{HC}$ which admits a single ($N_g=1$) "generation" preon and thus in total only six hyperflavors of preons, $N_{HF}=6$. This hypercolor interaction is assumed to be responsible for the formation of hypercolorless "baryons"

$$\begin{aligned} &\overline{D}_1 \sim P_{(5)}P_{(5)}P_{(5)} \sim \overline{10}, \quad D_1 \sim P^c_{(\overline{5})}P^c_{(\overline{5})}P^c_{(\overline{5})} \sim 10,\\ &D_2 \sim P_{(5)}P_{(5)}P_{(s)} \sim 10, \quad \overline{D}_2 \sim P^c_{(\overline{5})}P^c_{(\overline{5})}P^c_{(\overline{s})} \sim \overline{10}, \end{aligned} \tag{6}$$

and "mesons,"

$$\begin{aligned} &\overline{Q} \sim P^c_{(\overline{5})}P_{(s)} \sim \overline{5}, \qquad Q \sim P_{(5)}P^c_{(\overline{s})} \sim 5,\\ &M \sim P^c_{(\overline{5})}P_{(5)} \sim 24+1, \quad S \sim P^c_{(\overline{s})}P_{(s)} \sim 1, \end{aligned} \tag{7}$$

at the compositeness scale Λ_C [in Eq. (6) antisymmetrized products are meant]. All these bound states come out in vectorlike $SU(5)$ representations and they are in fact the $N=1$ 4D superfields.

As in the previous section compactifying the extra dimensions on a circle of radius R_C (and assuming that $R_C > 1/\Lambda_C$) we impose the Scherk-Schwarz boundary conditions to the preonic superfields (5) of type

$$\begin{aligned} &P_{(5)}(x^m,x^4+2\pi R_C,\theta)=e^{i2\pi q_5}P_{(5)}(x^m,x^4,e^{i\pi(q_5+q_{\overline{5}})}\theta),\\ &P_{(5)}(x^m,x^4+2\pi R_C,\theta)=e^{i2\pi q_5}P_{(5)}(x^m,x^4,e^{i\pi(q_5+q_{\overline{5}})}\theta),\\ &P_{(s)}(x^m,x^4+2\pi R_C,\theta)=e^{i2\pi q_s}P_{(s)}(x^m,x^4,e^{i\pi(q_s+q_{\overline{s}})}\theta),\\ &P^c_{(\overline{s})}(x^m,x^4+2\pi R_C,\theta)=e^{i2\pi q_{\overline{s}}}P^c_{(\overline{s})}(x^m,x^4,e^{i\pi(q_s+q_{\overline{s}})}\theta), \end{aligned} \tag{8}$$

where

$$q_5+q_{\overline{5}}=q_s+q_{\overline{s}}. \tag{9}$$

The vector supermultiplets and the adjoint superfields are periodic as in Eqs. (2). Expanding the 5D preonic fields as in Eqs. (3) one can see that all fermionic preons are massive in 4D and thus the low energy preonic theory can be treated as a consistent quantum theory since the gauge anomalies are absent. Obviously supersymmetry is broken by the above boundary conditions (8). Specifying the boundary conditions for the preonic fields one can easily obtain the R charges for the composite states (6) and (7) as

$$\begin{aligned} &\overline{D}_1 \sim 3q_5, \quad D_1 \sim 3q_{\overline{5}}, \quad D_2 \sim 2q_5+q_s, \quad \overline{D}_2 \sim 2q_{\overline{5}}+q_{\overline{s}},\\ &\overline{Q} \sim q_{\overline{5}}+q_s, \quad Q \sim q_5+q_{\overline{s}}, \quad M \sim q_{\overline{5}}+q_5,\\ &S \sim q_{\overline{s}}+q_s. \end{aligned} \tag{10}$$

Since the R charges (10) for the composite states differ from those of preons (8), one can expect different spectram of composite zero modes. Particularly we are looking for such an assignment of preonic R charges (8) which lead to massless composite fermions in 4D in $(\overline{5}+10)$ representation of $SU(5)$ which are nothing but composite quarks and leptons. It is evident from Eqs. (6) and (7) that we should identify the fermionic components of $\overline{Q}$ superfield with an antiquintet of $SU(5)$ where the down-type antiquark and the lepton doublet reside. The $SU(5)$ decuplet where the quark doublet, the up-type antiquark, and the charged antilepton reside can be identified with the fermionic components of either D_1 or D_2 superfields. The fermionic zero modes of $\overline{Q}$ and D_1 will be massless if the R charges (8) along with Eq. (9) satisfy also the following equations:

$$q_{\overline{5}}+q_s=\frac{q_{\overline{5}}+q_5}{2}, \tag{11}$$

$$3q_{\overline{5}}=\frac{q_{\overline{5}}+q_5}{2}. \tag{12}$$

In solving Eqs. (9), (11), and (12), one has to remember that due to periodicity the R charges q are defined up to an arbitrary integer number $q=q+k,\ k\in\mathbf{Z}$. To ensure that only the desired set of fermionic zero modes are massless in 4D we restrict general $U(1)$-twisted boundary conditions to some discrete Z_K ones. It is easy to verify then that any $K\neq 2,3,4,6,9,12$ will provide the desired solutions of Eqs. (9), (11), and (12):

$$q_5=\frac{5}{K},\quad q_{\overline{5}}=\frac{1}{K},\quad q_s=\frac{2}{K},\quad q_{\overline{s}}=\frac{4}{K}. \tag{13}$$

The minimal choice is Z_5-twisted boundary conditions with R charges $q_5=0,\ q_{\overline{5}}=\frac{1}{5},\ q_s=\frac{2}{5}$, and $q_{\overline{s}}=-\frac{1}{5}$ so that only one generation of composite quarks and leptons is massless in 4D at low energies. All extra composite states are massive with masses of the order of $1/R_C$.

If one identifies the quark-lepton decuplet of $SU(5)$ with fermionic components of the D_2 superfield, then one has to determine the R charges from Eqs. (9) and (11) and the equation

$$2q_5+q_s=\frac{q_{\overline{5}}+q_5}{2}, \tag{14}$$

which appears instead of Eq. (12). Any Z_K-twisted boundary conditions with $K\neq 2,3,4,6,9,12$ and

$$q_5=-\frac{2}{K},\quad q_{\overline{5}}=-\frac{4}{K},\quad q_s=\frac{1}{K},\quad q_{\overline{s}}=-\frac{7}{K} \tag{15}$$

will lead to the desired solutions. The minimal possibility is again Z_5 but now with the following R charges: $q_5=-\frac{2}{5}$, $q_{\overline{5}}=\frac{1}{5}$, $q_s=\frac{1}{5}$, and $q_{\overline{s}}=-\frac{2}{5}$. It is quite intriguing that just the composite quarks and leptons (without any extra states) unified within the $SU(5)$ gauge theory emerge at low energies in 4D from a simple and economical preon dynamics discussed above.

IV. THREE-GENERATION COMPOSITE MODEL

One can easily extend the above model with one generation of composite quarks and leptons to the case of three composite generations by simply copying the above structure thrice, resulting thus in a model with hypercolor group $SU(N)_1\otimes SU(N)_2\otimes SU(N)_3$. However, a more interesting way is based on treating the $SU(5)$-singlet preons in Eqs. (5) as the carriers of quantum numbers associated with quark-lepton generations. Thus we will take three- "generation" preons (antipreons) $P_{(s)i}(P^{ci}_{\overline{(s)}})(i=12,3)$ and the global $SU(3)_{P_{(s)}}$ symmetry of the 5D preonic Lagrangian will be interpreted as a "horizontal" hyperflavor symmetry $SU(3)_h$ for quark-lepton families (see below). Therefore we will also require this symmetry to survive upon the Scherk-Schwarz compactification; that is to say, the R charges for all three "generation" preons are the same. Now altogether there are $N_{HF}=8$ hyperflavors of preons and thus as a result of the asymptotic freedom constraint (4) the minimal hypercolor group is $SU(4)_{HC}$. While by following the arguments used in the previous section such a composite model leading to three quark-lepton generations can easily be constructed, it seems still more interesting to take the $SU(5)_{HC}$ as the hypercolor group. Apart from the possibility to treat all massles composites in the same way as the pure baryonic composites this case may be of special interest as the case suggesting some starting extra hypercolor-hyperflavor symmetry (HC$\leftrightarrow$HF) in 5D. The composite "baryons" and "mesons" are then

$$\begin{aligned}
\overline{D}_1&\sim P_{(5)}P_{(5)}P_{(5)}P_{(s)}P_{(s)}\sim(\overline{10,3}),\\
D_1&\sim P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(s)}}P^c_{\overline{(s)}}\sim(10,3),\\
D_2&\sim P_{(5)}P_{(5)}P_{(s)}P_{(s)}P_{(s)}\sim(10,1),\\
\overline{D}_2&\sim P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(s)}}P^c_{\overline{(s)}}P^c_{\overline{(s)}}\sim(\overline{10},1),\\
\overline{Q}&\sim P_{(5)}P_{(5)}P_{(5)}P_{(5)}P_{(s)}\sim(\overline{5},3)\\
Q&\sim P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(s)}}\sim(5,\overline{3}),\\
S&\sim P_{(5)}P_{(5)}P_{(5)}P_{(5)}P_{(5)}\sim(1,1),\\
\overline{S}&\sim P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(5)}}P^c_{\overline{(5)}}\sim(1,1),
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
\overline{Q}'&\sim P^c_{\overline{(5)}}P_{(s)}\sim(\overline{5},3), &\quad Q'&\sim P_{(5)}P^c_{\overline{(s)}}\sim(5,\overline{3}),\\
M&\sim P^c_{\overline{(5)}}P_{(5)}\sim(24+1,1), &\quad I&\sim P^c_{\overline{(s)}}P_{(s)}\sim(1,8+1),
\end{aligned} \tag{17}$$

respectively, transforming under $SU(5)\otimes SU(3)_h$ as indicated in brackets [antisymmetrization of all $SU(5)$ and

$SU(3)_h$ indices are implied in Eqs. (16)]. One can see that the $SU(5)$ decuplets (antidecuplets) in Eqs. (16) being the triplets (antitriplets) and singlets of the global family symmetry $SU(3)_h$ are pure baryonic composites. As to the $SU(5)$ antiquintets (quintets) being triplets (antitriplets) of the $SU(3)_h$ they appear as both baryonic (16) and mesonic (17) composites. Also some other states singlets and adjoints of $SU(5)$ and $SU(3)_h$ appear in the composite spectrum (16),(17). Now, as soon as the fermionic zero modes proposed for the D_1 supermultiplet in Eqs. (16) are massless one has to ensure that either the zero modes of fermionic components of the baryonic antiquintet $\overline{Q}$ in Eqs. (16) or mesonic antiquintet $\overline{Q}'$ in Eqs. (17) (but not of both) are also massless in order for the low energy composite model to be anomaly free, thus giving unique assignment of massless composites to the representation $(\overline{5}+10\,3)$ of the $SU(5)\otimes SU(3)_h$. Remarkably one can come to this basic consequence even if one starts with an arbitrary number of generation preons, $P_{(s)i}$ $(P^{ci}_{\overline{(s)}})$ $(i=1,2,\ldots,N_g)$. Since according to the above construction (16),(17) the number of composite $SU(5)$ decuplets is given by $N_g(N_g-1)/2$ while the composite antiquintets by the number N_g by itself (whether they are the baryonic or meson composites), one is unavoidably led to the $SU(5)$ anomaly cancellation condition of the type

$$\frac{N_g(N_g-1)}{2}=N_g, \tag{18}$$

from which it immediately follows that $N_g=3$. Thus the above model actually predicts three full generations of composite quarks and leptons being the triplets of the chiral global family symmetry $SU(3)_h$ automatically appearing in the composite spectrum.

Proceeding as in the previous section one can easily determine the desired R charges. If we identify the composite quarks and leptons with fermionic zero modes of the baryonic composites D_1 and $\overline{Q}$ in Eqs. (16), then preonic R charges along with Eq. (9) must satisfy the following equations:

$$3q_{\overline{5}}+2q_{\overline{s}}=\frac{q_{\overline{5}}+q_5}{2}, \tag{19}$$

$$4q_5+q_s=\frac{q_{\overline{5}}+q_5}{2}. \tag{20}$$

The desired solutions are provided by Z_6-twisted boundary conditions (which is the minimal one) with R charges defined as

$$q_5=q_{\overline{5}}=\frac{1}{6},\quad q_{\overline{s}}=-\frac{1}{6},\quad q_s=\frac{1}{2}. \tag{21}$$

In the case when the composite $SU(5)$ decuplets are identified with fermionic zero modes of the baryonic composite D_1, Eqs. (16), while the composite antiquintet with the mesonic composite $\overline{Q}'$, Eqs. (17), one should replace Eq. (20) by Eq. (11). It is easy to verify that the minimal solution will be once again provided by Z_6-twisted boundary conditions but now with the following R charges:

$$q_5=q_{\overline{5}}=\frac{1}{6},\quad q_{\overline{s}}=\frac{1}{3},\quad q_s=0. \tag{22}$$

Remarkably only three generations of composite quarks and leptons emerge as massless states while all other composites are massive, thus decoupling from the low energy particle spectrum.

V. DISCUSSION AND CONCLUSION

Some questions concerning the dynamics of the composite models discussed above should certainly be elaborated further. The major ones are the following. How, are the $SU(5)$ and subsequently the electroweak symmetries broken? How are the masses for composite quarks and leptons generated? Can one naturally explain the hierarchies of masses and mixings of composite quarks and leptons? Here we will briefly outline some possible scenarios one can think of.

One can indeed use the $SU(5)$-adjoint superfield Φ to break $SU(5)$ symmetry down to the $SU(3)_C\otimes SU(2)_W\otimes U(1)_Y$ standard model gauge group . In the supersymmetric uncompactified limit there are degenerate flat vacuum directions for the scalar component of Φ. Among these vacua one can certainly find the $SU(5)$ breaking and $SU(3)_C\otimes SU(2)_W\otimes U(1)_Y$ invariant one. In such a vacuum the preons will acquire $SU(5)$ noninvariant masses but this will not affect their subsequent dynamics resulting in the formation of composite states. The degeneracy of vacuum states of course is removed when one takes into account supersymmetry breaking effects due to Scherk-Schwarz compactification. Alternatively one can break $SU(5)$ symmetry through the condensation of the scalar components of composite mesonic superfield M, Eqs. (7),(17). Similarly, to break $SU(2)_W\otimes U(1)_Y$ electroweak symmetry one can use the doublet (antidoublet) components of composite quintets (antiquintets). Since the supersymmetry is broken, one inevitably faces the gauge hierarchy problem which can be resolved by fine-tuning as in the usual nonsupersymmetric grand unified theories (GUTs). Alternatively one could imagine that the solution to the gauge hierarchy problem appears due to the strong renormalization of the electroweak Higgs boson mass which is driven to an infrared stable fixed point of the order of electroweak scale while being of the order of the GUT scale at higher energies [14]. The relatively large extra dimensions play a crucial role in this scenario by inducing fast (power-law) evolution of gauge and Yukawa couplings.

The same mechanism could explain the observed hierarchies of quark-lepton masses and mixings along the lines discussed in [15]. These scenarios can be actually operative in the case of composite quarks and leptons as well. However, following a more traditional way, one can think that the hierarchy of quark-lepton masses and mixings is related to spontaneous breaking of the global chiral $SU(3)_h$ horizontal symmetry appearing in our model together with the three quark-lepton generations predicted. This is as far as one can presently think of the main benefit of the above consideration. Actually the chiral horizontal symmetry $SU(3)_h$ is known [16] to work successfully in both the quark and lepton sectors and can readily be extended to composite quarks and leptons as well. It would be interesting to gauge this symmetry within the preon model. However, a straightforward gauging of the chiral horizontal symmetry typically leads to $SU(3)_h$ triangle anomalies in the effective 4D theory. One way to overcome this problem is to introduce some extra massless states which properly cancel these anomalies in a traditional way. Another and perhaps a more interesting possibility is to cancel 4D anomalies by the Callan-Harvey anomaly inflow mechanism [17] assuming the presence of a 4D hypersurface (3-brane) in the 5D bulk space-time where the composite quarks and leptons are localized.

From a purely phenomenological point of view it is certainly interesting to study whether the compositeness scale as well as the compactification one can be lowered down to energies accessible for the future high energy colliders. Of course these and related issues deserve a more detailed investigation.

It would be also interesting to study various extensions of the simple models presented here. One can consider different gauge groups and more extra dimensions as well. Particularly one can study the possibility to unify the $SU(5)$ symmetry with the gauged horizontal $SU(3)_h$ and/or hypercolor $SU(N)_{HC}$ symmetries within a single gauge group (for earlier attempts see, e.g., [18]). It is certainly interesting to investigate the dynamical emergence of gauge symmetries themselves with the composite gauge bosons within the approach undertaken in this paper. And finally from a more fundamental point of view it could be illuminating to study string theories where string excitations are identified with preons rather than physical quarks and leptons (for an earlier discussion see [19]).

To conclude we have proposed a new approach towards quark and lepton compositeness within higher dimensional unified theories where thanks to a proper Scherk-Schwarz compactification the composite quarks and leptons turn out to be massless in four dimensions while all unwanted states (residing in the bulk) are massive. The prototype models discussed here are rather simple and economical so that we think this approach will help to construct largely realistic composite models of quarks and leptons in the not too distant future.

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[1] Particle Data Group, D. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[2] *Recent Developments in Gauge Theories*, Proceedings of the NATO Advanced Study Institute, Series B: Physics, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[3] For a review and references of earlier works see, e.g., L. Lyons, Prog. Part. Nucl. Phys. **10**, 227 (1983).

[4] For earlier supersymmetric composite models see the review in R.R. Volkas and G.C. Joshi, Phys. Rep. **159**, 303 (1988), and references therein.

[5] M.J. Strassler, Phys. Lett. B **376**, 119 (1996); A.E. Nelson and M.J. Strassler, Phys. Rev. D **56**, 4226 (1997); M.A. Luty and R.N. Mohapatra, Phys. Lett. B **396**, 161 (1997).

[6] N. Seiberg, Phys. Rev. D **49**, 6857 (1994); Nucl. Phys. **B435**, 129 (1995); for a review see K. Intriligator and N. Seiberg, Nucl. Phys. B (Proc. Suppl.) **45BC**, 1 (1996).

[7] J. Scherk and J.H. Schwarz, Phys. Lett. **82B**, 60 (1979).

[8] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); A. Pomarol and M. Quirós, *ibid.* **438**, 255 (1998); I. Antoniadis, S. Dimopoulos, A. Pomarol, and M. Quirós, Nucl. Phys. **B544**, 503 (1999); R. Barbieri, L.J. Hall, and Y. Nomura, Phys. Rev. D **63**, 105007 (2001).

[9] G. Dvali and M. Shifman, Nucl. Phys. **B504**, 127 (1997); M. Chaichian, A.B. Kobakhidze, and M. Tsulaia, Phys. Lett. B **505**, 222 (2001).

[10] Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000); **105**, 691 (2001); G. Altarelli and F. Feruglio, Phys. Lett. B **511**, 257 (2001); A.B. Kobakhidze, *ibid.* **514**, 131 (2001); hep-ph/0108049; L. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001); Z. Berezhiani, I. Gogoladze, and A. Kobakhidze, Phys. Lett. B **522**, 107 (2001); T. Kawamoto and Y. Kawamura, hep-ph/0106163; A. Hebecker, and J. March-Russell, hep-ph/0107004; R. Barbieri, L.J. Hall, and Y. Nomura, Nucl. Phys. **B625**, 128 (2002); J.A. Bagger, F. Feruglio, and F. Zwirner, Phys. Rev. Lett. **88**, 101601 (2002); T. Li, Phys. Lett. B **520**, 377 (2001); Nucl. Phys. **B629**, (2001); L.J. Hall, H. Murayama, and Y. Nomura, hep-th/0107245; N. Maru, Phys. Lett. B **522**, 117 (2001); T. Asaka, W. Buchmuller, and L. Covi, *ibid.* **523**, 199 (2001); L. Hall, Y. Nomura, T. Okui, and D. Smith, Phys. Rev. D **65**, 035008 (2002).

[11] C. Csaki, G.D. Kribs, and J. Terning, Phys. Rev. D **65**, 015004 (2002); H.-C. Cheng, K.T. Matchev, and J. Wang, Phys. Lett. B **52**, 308 (2001).

[12] R.N. Mohapatra and A. Pèrez-Lorenzana, Phys. Lett. B **468**, 195 (1999); D. Chang and R.N. Mohapatra, Phys. Rev. Lett. **87**, 211601 (2001).

[13] N. Arkani-Hamed, T. Gregoire, and J. Wacker, J. High Energy Phys. **03**, 055 (2002); D. Marti and A. Pomarol, Phys. Rev. D **64**, 105025 (2001).

[14] M. Chaichian and A.B. Kobakhidze, Phys. Lett. B **478**, 299 (2000); A.B. Kobakhidze, hep-th/0012191.

[15] K.R. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998); Nucl. Phys. **B537**, 47 (1999); S.A. Abel and S.F. King, Phys. Rev. D **59**, 095010 (1999); A.B. Kobakhidze,

Phys. At. Nucl. **64**, 941 (2001); M. Bando, T. Kobayashi, T. Noguchi, and K. Yoshioka, Phys. Lett. B **480**, 187 (2000).

[16] J.L. Chkareuli, JETP Lett. **32**, 671 (1980); Z.G. Berezhiani and J.L. Chkareuli, Yad. Fiz. **37**, 1043 (1983); F. Wilczek, Report No. NSF-ITP-83-08, 1983; Z.G. Berezhiani, Phys. Lett. **129B**, 99 (1983); J.C. Wu, Phys. Rev. D **36**, 1514 (1987); Z. Berezhiani, Phys. Lett. B **417**, 287 (1998); J.L. Chkareuli, C.D. Froggatt, and H.B. Nielsen (in preparation).

[17] C.G. Callan and J.A. Harvey, Nucl. Phys. **B250**, 427 (1985).

[18] J.L. Chkareuli, JETP Lett. **32**, 671 (1980); **36**, 493 (1983); I. Montvay, Phys. Lett. **95B**, 227 (1980); J.E. Kim and H.S. Song, Phys. Rev. D **23**, 2102 (1981); M. Chaichian, Yu. N. Kolmakov, and N.F. Nelipa, Report No. HU-TFT-82-15, 1982; Z. Phys. C **43**, 381 (1989).

[19] J.C. Pati, M. Cvetic, and H.S. Sharatchandra, Phys. Rev. Lett. **58**, 851 (1987).