# Measurements of the supersymmetric Higgs self-couplings and the reconstruction of the Higgs potential

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We address the issue of the reconstruction of the scalar potential of a two-Higgs-doublet model having in mind that of the MSSM. We first consider the general $CP$ conserving dim-4 effective potential. To fully reconstruct this potential, we show that even if all the Higgs boson masses and their couplings to the standard model particles are measured, one needs not only to measure certain trilinear Higgs self-couplings but some of the quartic couplings as well. We also advocate expressing the Higgs self-couplings in the mass basis. We show explicitly that in the so-called decoupling limit the most easily accessible Higgs self-couplings are given in terms of the Higgs boson mass while all other dependences on the parameters of the general effective potential are screened. This also helps to easily explain how, in the MSSM, the largest radiative corrections which affect these self-couplings are reabsorbed by using the corrected Higgs boson mass. We also extend our analysis to higher order operators in the effective Higgs potential. While the above screening properties do not hold, we argue that these effects must be small and may not be measured considering the foreseen poor experimental precision in the extraction of the SUSY Higgs self-couplings.

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## I. INTRODUCTION

The most important issue at the upcoming colliders is the elucidation of the mechanism of symmetry breaking and the hunt for the Higgs boson. Within the standard model (SM) there is strong indirect evidence that the latter might be light. But at the same time within the SM such a light Higgs boson poses the problem of naturalness. Supersymmetry (SUSY) solves this problem and for a large array of models predicts a light Higgs boson in accordance with the present precision data. The task of the next colliders will therefore be not so much the discovery of the (lightest) Higgs boson but a careful study of the properties of the Higgs system since this will be an ideal window on the mechanism of (super)symmetry breaking. Many state-of-the-art studies have analyzed the couplings of the lightest Higgs boson to fermions and to gauge bosons. The most useful conclusions are in the context of a next linear collider (LC),[1] for a summary see [2,3]. One can, for example, discriminate between a light Higgs boson within the SM and one within a supersymmetric model. Precision studies on other supersymmetric particles that may be produced at these colliders can nicely complement these studies. However, one key ingredient that still requires further studies and simulations concerns the important aspect of the Higgs potential. Not only because this triggers electroweak symmetry breaking but also because supersymmetry breaking is also encoded in this potential. Some studies [4–8] have addressed the issue of the measurements of some of the Higgs self-couplings within minimal SUSY. These studies have been rather purely phenomenological studies in the sense that one has, within the minimal supersymmetric model (MSSM), quantified various cross sections for double (and for some triple) Higgs boson production at a high energy collider. From these one has derived a sensitivity on some individual Higgs self-couplings by varying the strength of these couplings independently, while fixing the Higgs boson mass spectrum. We would, however, expect that a deviation in one of the Higgs self-couplings should not only affect other Higgs self-couplings but also affect the Higgs boson mass spectrum. Moreover, one would like to see how a particular deviation in the Higgs self-couplings relates to the fundamental parameters of the Higgs potential and also what order of magnitude should one expect from these deviations. This can most efficiently be addressed through an effective potential approach and would be similar to what has been applied in the measurements of the trilinear [9] or even quadrilinear [10] self-couplings of the weak vector bosons. Within the one Higgs doublet of the SM a general parametrization of the Higgs self-couplings has been given [11,12] and its effects at the colliders studied [11–13]. As for the case of SUSY where one needs two Higgs doublets such a study is missing although a leading order parametrization has been known [14]. The aim of this paper is to fill this gap. As we will see this parametrization of the scalar potential is important; not only can it embody through an effective potential many of the well-known radiative corrections [15–19] but it will also make clear the link between what can be learned about the Higgs potential by measuring the Higgs boson masses and what additional, if any, information can be gained if one can study the self-couplings. This will also show that the couplings of the charged Higgs boson to the neutral Higgs bosons can embody the same information as some of the trilinear neutral Higgs couplings. Measurements of the Higgs self-couplings involving charged Higgse bosons have, as far as we know, never been addressed although a calculation of charged Higgs pair production at the CERN Large Hadron Collider (LHC) has been made [6,20]. One conclusion though from [20] is that the trilinear Higgs self-couplings contribution is rather too small or becomes non-

[1]A Higgs factory at a muon facility gives astounding results [1]; however, many technical problems need to be solved before the design of such a facility.

 

negligible but with a quite small cross section. Our findings also help understand why the radiative corrections to the Higgs boson self-couplings of the lightest Higgs in the MSSM though substantial (as are those to the Higgs boson mass) become tiny when expressed in terms of the lightest Higgs boson mass [21]. With the five $(h,H,A,H^{\pm})$ Higgs bosons of the MSSM one has eight possible Higgs trilinear self-couplings. One would then think that, together with the measurements of the Higgs boson masses and their couplings to ordinary matter, the measurement of these trilinear self-couplings would allow a full reconstruction of the leading order dim-4 effective Lagrangian describing the Higgs potential. Even in the most optimistic scenarios where all the Higgs bosons are light, we find that a full reconstruction requires the measurements of some quartic couplings which are extremely difficult, if not impossible [22], to measure even at the linear collider. In the decoupling regime [14] where only one of the Higgs bosons is light and with properties very much resembling those of the SM, the effective parameters of the Higgs potential will be screened and thus extremely difficult to measure. One can, of course, entertain that new physics affecting the Higgs potential appears as higher dimensional operators, dim-6, in which case the above "screening" effects are not operative. However, one expects these effects to be too small to be measured considering the expected accuracy, no better than $\sim 10\%$, at which the Higgs self-couplings are to be probed (for a nice review see [2,3,22]).

## II. LEADING ORDER PARAMETRIZATION OF THE HIGGS POTENTIAL AND THE HIGGS SELF-COUPLINGS

When trying to parametrize the effects of some new physics on the properties of *familiar* particles, the effective Lagrangian is most useful. One uses all known symmetries of the model and then writes the tower of operators according to their dimensions. One expects thus that the allowed lowest dimension operators have the most impact on the low energy observables, higher order operators being suppressed due to the large mass scale needed for their parametrization. Therefore, for the minimal SUSY the lowest dimension operator is of dim-4. For the MSSM one needs two Higgs doublets $H_1$ and $H_2$ with opposite hypercharge $Y=\mp 1$, respectively. They may be written as

$$H_1=\begin{pmatrix} v_1+\frac{1}{\sqrt{2}}(h_1+i\varphi_1^0) \\ \varphi_1^- \end{pmatrix},$$

$$H_2=\begin{pmatrix} \varphi_2^+ \\ v_2+\frac{1}{\sqrt{2}}(h_2+i\varphi_2^0) \end{pmatrix}. \tag{2.1}$$

Requiring $CP$ conservation one can write the following [14]:

$$\begin{aligned} V_{eff}=&(m_1^2+\mu^2)|H_1|^2+(m_2^2+\mu^2)|H_2|^2 \\ &-[m_{12}^2(\epsilon H_1H_2)+\text{H.c.}]+\frac{1}{2}\left[\frac{1}{4}(g^2+g'^2)+\lambda_1\right] \\ &\times(|H_1|^2)^2+\frac{1}{2}\left[\frac{1}{4}(g^2+g'^2)+\lambda_2\right](|H_2|^2)^2 \\ &+\left[\frac{1}{4}(g^2-g'^2)+\lambda_3\right]|H_1|^2|H_2|^2 \\ &+\left[-\frac{1}{2}g^2+\lambda_4\right](\epsilon H_1H_2)(\epsilon H_1^*H_2^*)+\left(\frac{\lambda_5}{2}(\epsilon H_1H_2)^2\right. \\ &\left.+[\lambda_6|H_1|^2+\lambda_7|H_2|^2](\epsilon H_1H_2)+\text{H.c.}\right), \end{aligned} \tag{2.2}$$

where $\epsilon$ is the antisymmetric matrix with $\epsilon_{12}=-1$. $g$ and $g'$ are the SU(2) and U(1) gauge couplings. $\mu$ is a supersymmetry preserving mass term. The case with all $\lambda_i$ being zero corresponds to the original MSSM potential at the tree level. Moreover, exact supersymmetry imposes that $m_1^2=m_2^2=m_{12}^2=0$ so that no electroweak symmetry breaking ensues. $m_1$, $m_2$ and $m_{12}$ are thus essential for electroweak symmetry breaking and encode also supersymmetry breaking. These dimensionful quantities are soft SUSY breaking parameters. As for the $\lambda$'s, practically all analyses of the Higgs phenomenology have only viewed them as "soft" terms originating from higher order loop effects. As known these loop effects can be substantial as they are enhanced by large Yukawa couplings (corrections are quartic in the top quark mass) and have kept the MSSM alive; see [23], for example. However it may well be that models of supersymmetry breaking can provide a *direct* contribution to these parameters—for instance, through nonrenormalizable operators in the Kähler potential. Technically these contributions would be deemed *hard*. It has, however, been stressed recently [24,25] that if these parameters are related to the source of soft SUSY breaking, then they would not destabilize the scalar potential and would evade the "unnatural" quadratic divergence problem, thus leading to a viable model. In such circumstances such terms may lead to the lightest SUSY Higgs boson with a mass much in excess of 150 GeV and with "no unnaturalessness" dilemma. Therefore, the reconstruction of the Higgs potential is crucial. In a different context—supersymmetric models with a warped fifth dimension [26]—it was shown that some of the quartic couplings (apart from the usual ones of gauge origin) can even be supersymmetric and originate from a nonminimal Kähler potential. Supersymmetry breaking terms also contribute to the latter as well as to the quadratic terms. However, to obtain a satisfactory electroweak breaking in this scheme constrains the parameters such that $t_\beta=1$ is picked up (see below the definition of $t_\beta$).

Of course, the approach we follow and the results we obtain can be made to apply directly to a general two-Higgs-doublet model (2HDM); one only needs to switch the gauge couplings contributions off in the potential Eq. (2.2). This being said when we investigate precision measurements extracted from the Higgs bosons to fermions, one should stick to a model whose characteristics are close to the MSSM. The 2HDM we have in mind is the so-called type II, in the terminology of [27], where down-type quarks and leptons couple to $H_1$ and up-type quarks to $H_2$ as in the MSSM. However, in most studies of the 2HDM, $\lambda_{6,7}$ are not considered on the basis that they may induce too large flavor changing neutral current (FCNC).

This brings us to the issue concerning the order of magnitude for the various $\lambda_i$ in a general supersymmetric context. For the conventional MSSM, one effectively gets a Yukawa-enhanced contribution, starting at one loop, which affects all seven paramaters [23]. The largest contribution for moderate $\tan\beta$ stems from $\lambda_2$ and is of order 0.1. In [24] where "natural hard" terms are discussed, some orders of magnitude for the $\lambda_i$ are given based on how and at what scale SUSY breaking is transmitted. Values as high as 1.0 could be entertained. Such values ($<4\pi$) are still perturbative. In the warped fifth dimension model [26] it is interesting to note that all $\lambda_i$ but $\lambda_5$ get a SUSY conserving contributions in addition to some SUSY breaking contribution, while $\lambda_5$ is of purely SUSY breaking origin. All these contribution disappear in the limit of an infinitely large effective fundamental scale (scale in lieu of $M_{\rm Planck}$), which is directly related to the warp factor. The effective couplings can be large enough, in fact so large that the authors estimate a lightest Higgs boson mass of order 700 GeV as a possibility while the couplings are still perturbative up to the cutoff scale. Note, however, that the cutoff scale is identified with the "low" fundamental scale as compared to the usual Planck scale that is usually used to set an upper bound on the quartic coupling and hence the mass of the Higgs boson. Moreover, it is argued that the model is safe as regards FCNC [26].

Some bounds, though not so strong, exist. In all its generality, if we allow some combinations of couplings within the range $-1<\lambda_i<1$, independently of $\tan\beta$, this can lead to too low values of the Higgs boson masses (even negative-squared masses and problems with vacuum stability can occur). Furthermore, to constrain the parameter space some authors have imposed vacuum stability of the potential and perturbativity of the couplings up to high scales [28] as well as tree-level unitarity constraints of the elastic scattering of the Higgs bosons [29] in the 2HDM. Limits from $\Delta\rho$ [30] can also be quite useful, although they are model dependent if, for instance, the top squark contribution to $\Delta\rho$ plays a role. Nonetheless, all of these requirements still leave a large parameter space that can be quite drastically reduced once the Higgs boson masses and their couplings are directly measured.

We will therefore follow a general approach based on the potential (2.2), assuming the quadratic terms ($m_{1,2,12}$) to satisfy the usual conditions for a stable minimum with nonvanishing vacuum expectation values. The minimization of the potential and the absence of tadpoles imposes the following constraint on the "soft" SUSY parameters $m_1, m_2$:

$$m_1^2=-m_{12}^2 t_\beta-\mu^2-M_Z^2 c_{2\beta}/2+v^2(-\lambda_1 c_\beta^2-\lambda_3 s_\beta^2-\lambda_4 s_\beta^2 -\lambda_5 s_\beta^2+3\lambda_6 c_\beta s_\beta+\lambda_7 s_\beta^3/c_\beta), \qquad (2.3)$$

$$m_2^2=-m_{12}^2/t_\beta-\mu^2+M_Z^2 c_{2\beta}/2+v^2(-\lambda_2 s_\beta^2-\lambda_3 c_\beta^2-\lambda_4 c_\beta^2 -\lambda_5 c_\beta^2+\lambda_6 c_\beta^3/s_\beta+3\lambda_7 c_\beta s_\beta), \qquad (2.4)$$

where $v^2=v_1^2+v_2^2=2M_W^2/g^2$, $t_\beta=\tan\beta=v_2/v_1$, $s_\beta=\sin\beta$, and so on.

Then, the parameter $m_{12}^2$ can be fixed if we choose $M_A$, the Higgs pseudoscalar mass, as an independent variable:

$$m_{12}^2=-c_\beta s_\beta[M_A^2+v^2(2\lambda_5-\lambda_6/t_\beta-\lambda_7 t_\beta)]. \qquad (2.5)$$

### A. Higgs boson masses

At this stage, beside $M_A$ and $t_\beta$, there are seven independent parameters. Luckily some of these enter the expressions of the Higgs boson masses and the couplings to fermions and vector bosons.

The mass of the charged Higgs boson reads as

$$M_{H^\pm}^2=M_A^2+M_W^2-v^2(\lambda_4-\lambda_5). \qquad (2.6)$$

This already shows that a measurement of $M_{H^\pm}$ and $M_A$ can put a limit on the combination $(\lambda_4-\lambda_5)$. A dedicated study addressing this particular issue at the LHC to differentiate between the MSSM and a general 2HDM has very recently appeared [31].

The two $CP$-even Higgs states are determined by the mixing angle $\alpha$. $h$ and $H$ will denote the $CP$-even Higgs scalars. Introducing

$$\mathcal{N}=M_A^2 s_{2\beta}+M_Z^2 s_{2\beta}-2v^2[s_{2\beta}(\lambda_3+\lambda_4)-2c_\beta^2\lambda_6-2s_\beta^2\lambda_7], \qquad (2.7)$$

$$\mathcal{D}=M_A^2 c_{2\beta}-M_Z^2 c_{2\beta}-2v^2[\lambda_1 c_\beta^2-\lambda_2 s_\beta^2-\lambda_5 c_{2\beta} -(\lambda_6-\lambda_7)s_{2\beta}], \qquad (2.8)$$

the masses of the $CP$-even Higgs scalars, $M_h^2$ and $M_H^2$, and the mixing angle $\alpha$ are defined through

$$\tan 2\alpha=\frac{\mathcal{N}}{\mathcal{D}}=\frac{M_A^2 s_{2\beta}+M_Z^2 s_{2\beta}-2v^2[s_{2\beta}(\lambda_3+\lambda_4)-2c_\beta^2\lambda_6-2s_\beta^2\lambda_7]}{M_A^2 c_{2\beta}-M_Z^2 c_{2\beta}-2v^2[\lambda_1 c_\beta^2-\lambda_2 s_\beta^2-\lambda_5 c_{2\beta}-(\lambda_6-\lambda_7)s_{2\beta}]},$$

$$\sin 2\alpha=-\frac{\mathcal{N}}{\sqrt{\mathcal{D}^2+\mathcal{N}^2}}, \qquad (2.9)$$

$$M_H^2 = M_Z^2 c_{\alpha+\beta}^2 + M_A^2 s_{\alpha-\beta}^2 + 2v^2[\lambda_1 c_\alpha^2 c_\beta^2 + \lambda_2 s_\alpha^2 s_\beta^2 + 2(\lambda_3+\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + \lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2) - 2s_{\alpha+\beta}(\lambda_6 c_\alpha c_\beta + \lambda_7 s_\alpha s_\beta)], \tag{2.10}$$

$$M_h^2 = M_Z^2 s_{\alpha+\beta}^2 + M_A^2 c_{\alpha-\beta}^2 + 2v^2[\lambda_1 s_\alpha^2 c_\beta^2 + \lambda_2 c_\alpha^2 s_\beta^2 - 2(\lambda_3+\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + \lambda_5(c_\alpha^2 c_\beta^2 + s_\alpha^2 s_\beta^2) + 2c_{\alpha+\beta}(\lambda_6 s_\alpha c_\beta - \lambda_7 c_\alpha s_\beta)]. \tag{2.11}$$

With $-\pi/2 \leqslant \alpha \leqslant \pi/2$, $h$ ($H$) defines the lightest (heaviest) $CP$-even Higgs boson mass. The decoupling limit [32] is usually defined as $M_A \gg M_Z$, we will extend this to mean $M_A \gg v$ [with the $\lambda_{1-7}$ never exceeding $\mathcal{O}(1)$]. Having the decoupling limit in mind it is very instructive and useful to express the dependence in the mixing angle $\alpha$ through $c_{\beta-\alpha}$ and $s_{\beta-\alpha}$ since these two quantities are a direct measure of the couplings of $h$ (and $H$) to vector bosons and to fermions. In units of the SM Higgs couplings, the couplings to vector bosons are

$$g_{hVV,HVV} = s_{\beta-\alpha}, c_{\beta-\alpha},$$

while, for instance,

$$g_{hb\bar{b}} = -s_\alpha / c_\beta = s_{\beta-\alpha} - t_\beta c_{\beta-\alpha}. \tag{2.12}$$

Therefore, especially if $t_\beta$ has been identified from other SUSY processes, these combinations could be easily extracted once the light Higgs boson has been produced at the linear collider, and after allowing for some (important) QCD and in some cases model-dependent vertex corrections. The couplings of $H$ can be easily translated from those of $h$ by the substitution $h \to H$, $M_h \to M_H$, $s_{\beta-\alpha} \to c_{\beta-\alpha}$, $c_{\beta-\alpha} \to -s_{\beta-\alpha}$. Moreover, $c_{\beta-\alpha}$ is a very good measure of decoupling since in this limit $c_{\beta-\alpha} \sim 1/M_A^2$. To wit,

$$M_A^2 c_{\beta-\alpha} s_{\beta-\alpha} \sim M_A^2 c_{\beta-\alpha} \to s_{2\beta} c_{2\beta} \left\{ M_Z^2 - v^2 \left( \lambda_3 + \lambda_4 + \lambda_5 + (\lambda_6 - \lambda_7) t_{2\beta} - \lambda_1 \frac{c_\beta^2}{c_{2\beta}} + \lambda_2 \frac{s_\beta^2}{c_{2\beta}} - \frac{\lambda_6}{t_\beta} - \lambda_7 t_\beta \right) \right\}. \tag{2.13}$$

It is important to keep in mind for later reference that in the decoupling limit we have

$$c_{\beta-\alpha}, (M_A^2 - M_H^2) \to \mathcal{O}(1/M_A^2). \tag{2.14}$$

In the decoupling limit, the lightest Higgs boson mass is written as

$$M_h^2 \to M_Z^2 c_{2\beta}^2 + 2v^2 \{ \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + s_{2\beta}[(\lambda_3+\lambda_4+\lambda_5) s_\beta c_\beta - (\lambda_6+\lambda_7) - (\lambda_6-\lambda_7) c_{2\beta}] \}. \tag{2.15}$$

These simple considerations already show that if some of the $\lambda_{1-7}$ are not too tiny, one should observe their effects by measuring the Higgs boson masses and also the Higgs couplings to ordinary fermions. This MSSM generalization as pointed out in [24] can allow for a lightest Higgs boson mass in excess of 150 GeV, say, independently of $t_\beta$.

### B. Higgs self-couplings

Some of the expressions below, for the Higgs trilinear couplings, have been given elsewhere [6,14]. For completeness we list all the couplings. Introducing

$$g_h = \frac{2M_W}{g} = \sqrt{2} v, \tag{2.16}$$

we have

$$g_{hhh} = 3 g_h \lambda_{hhh},$$

$$\lambda_{hhh} = -\frac{e^2}{s_{2W}^2} s_{\beta+\alpha} c_{2\alpha} + \lambda_1 c_\beta s_\alpha^3 - \lambda_2 c_\alpha^3 s_\beta + \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5) s_{2\alpha} c_{\beta+\alpha} + \lambda_6 s_\alpha^2 \times (c_{\beta+\alpha} + 2c_\alpha c_\beta) + \lambda_7 c_\alpha^2 (c_{\beta+\alpha} - 2 s_\alpha s_\beta), \tag{2.17}$$

$$g_{Hhh} = -3 g_h \lambda_{Hhh},$$

$$\lambda_{Hhh} = \frac{e^2}{s_{2W}^2}\left( s_{\beta+\alpha} s_{2\alpha} - \frac{1}{3} c_{\beta-\alpha} \right) + \lambda_1 c_\beta s_\alpha^2 c_\alpha + \lambda_2 s_\beta c_\alpha^2 s_\alpha - \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5)\left( s_{2\alpha} s_{\beta+\alpha} - \frac{2}{3} c_{\beta-\alpha} \right) + \lambda_6 s_\alpha (c_\beta c_{2\alpha} + c_\alpha c_{\beta+\alpha}) - \lambda_7 c_\alpha (s_\beta c_{2\alpha} + s_\alpha c_{\beta+\alpha}), \tag{2.18}$$

$$g_{hHH} = 3 g_h \lambda_{hHH},$$

$$\lambda_{hHH} = \frac{e^2}{s_{2W}^2}\left( c_{2\alpha} s_{\beta+\alpha} - \frac{2}{3} s_{\beta-\alpha} \right) + c_\alpha s_\alpha (\lambda_1 c_\alpha c_\beta - \lambda_2 s_\alpha s_\beta) - \frac{(\lambda_3+\lambda_4+\lambda_5)}{2}\left( s_{\beta+\alpha} c_{2\alpha} - \frac{s_{\beta-\alpha}}{3} \right) + \lambda_6 c_\alpha (c_\beta c_{2\alpha} - s_\alpha s_{\beta+\alpha}) + \lambda_7 s_\alpha (c_{2\alpha} s_\beta + c_\alpha s_{\beta+\alpha}),$$

$$g_{hAA} = g_h \lambda_{hAA},$$

$$\lambda_{hAA}=-\frac{e^2}{s_{2W}^2}s_{\beta+\alpha}c_{2\beta}+\lambda_1 c_\beta s_\beta^2 s_\alpha-\lambda_2 s_\beta c_\beta^2 c_\alpha -(\lambda_3+\lambda_4+\lambda_5)(s_\beta^3 c_\alpha - s_\alpha c_\beta^3)+2\lambda_5 s_{\beta-\alpha} +\lambda_6 s_\beta(c_{2\beta}s_\alpha+c_\beta s_{\beta+\alpha}) +\lambda_7 c_\beta(c_{2\beta}c_\alpha - s_\beta s_{\beta+\alpha}), \quad (2.19)$$

$$g_{hH^+H^-}=g_h\lambda_{hH^+H^-},$$

$$\lambda_{hH^+H^-}=\lambda_{hAA}-\frac{e^2}{2s_W^2}s_{\beta-\alpha}+(\lambda_4-\lambda_5)s_{\beta-\alpha}, \quad (2.20)$$

$$g_{HHH}=-3g_h\lambda_{HHH},$$

$$\lambda_{HHH}=\frac{e^2}{s_{2W}^2}c_{2\alpha}c_{\beta+\alpha}+c_\alpha^3 c_\beta\lambda_1+s_\alpha^3 s_\beta\lambda_2 +\frac{(\lambda_3+\lambda_4+\lambda_5)}{2}s_{2\alpha}s_{\beta+\alpha}-\lambda_6 c_\alpha^2(s_\beta c_\alpha+3s_\alpha c_\beta) -\lambda_7 s_\alpha^2(c_\beta s_\alpha+3c_\alpha s_\beta),$$

$$g_{HAA}=-g_h\lambda_{HAA},$$

$$\lambda_{HAA}=-\frac{e^2}{s_{2W}^2}c_{\beta+\alpha}c_{2\beta}+\lambda_1 c_\beta s_\beta^2 c_\alpha+\lambda_2 s_\beta c_\beta^2 s_\alpha +(\lambda_3+\lambda_4+\lambda_5)(s_\beta^3 s_\alpha+c_\beta^3 c_\alpha)-2\lambda_5 c_{\beta-\alpha} +\lambda_6 s_\beta(c_\beta c_{\beta+\alpha}+c_\alpha c_{2\beta})-\lambda_7 c_\beta(s_\beta c_{\beta+\alpha} +s_\alpha c_{2\beta}), \quad (2.21)$$

$$g_{HH^+H^-}=-g_h\lambda_{HH^+H^-},$$

$$\lambda_{HH^+H^-}=\lambda_{HAA}+\frac{e^2}{2s_W^2}c_{\beta-\alpha}+(\lambda_5-\lambda_4)c_{\beta-\alpha}. \quad (2.22)$$

Written this way the expressions are not very telling; moreover, it is clear that some of the couplings must be related since after having measured the masses, there remains only three independent $\lambda$ while there are eight Higgs trilinear couplings.

## III. EXPRESSING THE SELF-COUPLINGS IN THE MASS BASIS

The writing of the Higgs self-couplings in terms of the fundamental parameters $\lambda_i$ is not the most judicious. The reason is that all these parameters, though in different combinations, already appear in the expression for the Higgs boson masses. Therefore the Higgs boson masses already constrain the Higgs potential, even if partially. Since the Higgs boson masses (or at least the lightest Higgs boson) will be measured first, the trilinear Higgs self-couplings being accessed only through double-Higgs-boson production whose cross section is small, one should ask how much more do we learn from the measurement of the trilinear couplings once we have measured the masses. For example, it is quite likely that the heavy Higgs bosons are too heavy so that one only has access to the lightest $CP$-even Higgs boson. In this situation one would have an extremely precise determination of its mass (either at the LHC or a linear collider) and also a very precise determination of its couplings to fermions and (gauge bosons) and hence of the angle $\alpha$ (or the combination $\alpha-\beta$) at a linear collider. One should therefore use this information—namely, trade these precisely measured physical quantities for two of the parameters $\lambda_i$ and then reexpress the self-coupling $h/H\,hh$, in terms of these physical parameters. Of course the choice of the $\lambda_i$ is not unique; however, the parametrization of the self-couplings in terms of masses will be more transparent and would have the advantage of including information on some previously measured quantities. We therefore propose to use the physical basis, using as input all the Higgs boson masses and the mixing angle $\alpha$.[2] For the latter, beside $t_\beta$, one can use the quantities $s_{\beta-\alpha}$ (and $c_{\beta-\alpha}$) which can be directly extracted from the Higgs couplings to fermions or the vector bosons. For instance a good measurement of the production cross section of $h$ at $e^+e^-$ furnishes $s_{\beta-\alpha}$. The angle $\beta$ may be measured in some purely non-Higgs processes or if one has access to some heavy Higgs bosons also through a study of their couplings to matter.[3] Thus apart from $M_A$, one can trade $M_h, M_H, M_{H^\pm}, \alpha$ for, for example, $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ through

$$\lambda_4=(M_A^2+M_W^2-M_{H^\pm}^2)/v^2+\lambda_5, \quad (3.1)$$

$$\lambda_1=\frac{M_h^2 s_\alpha^2+M_H^2 c_\alpha^2-M_Z^2 c_\beta^2-M_A^2 s_\beta^2}{2v_1^2} -\lambda_5 t_\beta^2+2\lambda_6 t_\beta, \quad (3.2)$$

$$\lambda_2=\frac{M_h^2 c_\alpha^2+M_H^2 s_\alpha^2-M_Z^2 s_\beta^2-M_A^2 c_\beta^2}{2v_2^2} -\lambda_5/t_\beta^2+2\lambda_7/t_\beta, \quad (3.3)$$

$$\lambda_3=\frac{(M_H^2-M_h^2)c_\alpha s_\alpha+(M_A^2+M_Z^2)c_\beta s_\beta}{2v_1 v_2} -\lambda_4+\lambda_6/t_\beta+\lambda_7 t_\beta. \quad (3.4)$$

With this choice the different Higgs self-couplings are given in terms of the Higgs boson masses and the remaining

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[2] In another context and in a more restricted two-Higgs-doublet model, using the ''corrected'' masses to express the self-couplings has been advocated in [29,33].

[3] Of course some of these couplings receive genuine vertex corrections that are *not* encoded in the correction to the angle $\alpha$; these genuine vertex flavor-dependent corrections should be, whenever possible, subtracted before one attempts to extract $s_{\beta-\alpha}$, for example.

$\lambda_{5,6,7}$. This choice of basis seems to be the most natural, since the structures that affect $\lambda_{1-4}$ appear already at the ''tree level'' and thus should be the most affected by radiative corrections. This is so in the MSSM where the largest corrections at moderate $t_\beta$ is in $\lambda_2$; see [23], for example. Let us first define the combinations

$$\lambda_a = \lambda_5 s_{2\beta} - \frac{\lambda_6+\lambda_7}{2} - \frac{\lambda_6-\lambda_7}{2} c_{2\beta}$$

$$(\text{note that } \ m_{12}^2 = -M_A^2 s_\beta c_\beta - v^2\lambda_a),$$

$$\lambda_b = \lambda_5 c_{2\beta} + \frac{\lambda_6-\lambda_7}{2} s_{2\beta}. \tag{3.5}$$

After some algebra, where we try as much as possible to express the $\alpha$ dependence through $\alpha-\beta$, we find

$$\lambda_{hhh} = -\frac{1}{2v^2} s_{\beta-\alpha} M_h^2 + \frac{c_{\beta-\alpha}^2}{s_\beta c_\beta}\Bigg( s_{\beta-\alpha}\lambda_a + c_{\beta-\alpha}\lambda_b + \frac{(M_A^2-M_h^2)}{2v^2}(s_{2\beta}s_{\beta-\alpha}+c_{2\beta}c_{\beta-\alpha})\Bigg). \tag{3.6}$$

This shows that in the decoupling limit the $hhh$ coupling is completely determined from the measurement of the Higgs boson mass $M_h$ and the coupling to $WWh/ZZh$, $s_{\beta-\alpha}$, both of which should be determined quite precisely from $e^+e^- \to Zh$ at the LC. The $\lambda_a$ and more so $\lambda_b$ are totally screened. The result in the decoupling limit is no surprise as it reproduces exactly the SM result. In this regime one essentially have only one Higgs doublet, and if one restricts oneself to dim-4 operators, one reproduces the SM exactly.

$$\lambda_{Hhh} = c_{\beta-\alpha}\Bigg\{ \frac{4\lambda_a}{3s_{2\beta}}\left(1-\frac{3}{2}c_{\beta-\alpha}^2\right) + \frac{2\lambda_b}{s_{2\beta}} c_{\beta-\alpha}s_{\beta-\alpha} + \frac{1}{6v^2}\Bigg( (3M_H^2-2M_h^2) + 4(M_A^2-M_H^2) + \frac{c_{\beta-\alpha}(s_{2\beta}c_{\beta-\alpha}-c_{2\beta}s_{\beta-\alpha})}{s_\beta c_\beta} \times (2M_h^2+M_H^2-3M_A^2)\Bigg)\Bigg\}. \tag{3.7}$$

In the decoupling limit,

$$\lambda_{Hhh} = \frac{c_{\beta-\alpha}}{2v^2} M_H^2. \tag{3.8}$$

Since this coupling could most easily be extracted from $H \to hh$, $M_H$ would have (together with $c_{\beta-\alpha}$) already been measured; thus the new physics effects are again screened.

The remaining couplings involve at least two heavy Higgs bosons an thus would be more difficult to measure. Nonetheless, let us express them in terms of masses. All the couplings will be written as

$$g_{H_iH_jH_k} = \frac{g_h}{s_{2\beta}} \lambda'_{H_iH_jH_k}, \tag{3.9}$$

with

$$\lambda'_{hAA} = 2s_{\beta-\alpha}\lambda_a + 2c_{\beta-\alpha}\lambda_b + \frac{1}{2v^2}[c_{\beta-\alpha}c_{2\beta} \times (2M_A^2-2M_h^2) - M_h^2 s_{2\beta}s_{\beta-\alpha}], \tag{3.10}$$

$$\lambda'_{hHH} = s_{\beta-\alpha}\Bigg\{ \frac{1}{2v^2}\{[M_h^2+2(M_H^2-M_A^2)]s_{2\alpha} - 2M_A^2 c_{\beta-\alpha}s_{\beta+\alpha}\} + 2(\lambda_a(1-3c_{\beta-\alpha}^2) + 3c_{\beta-\alpha}s_{\beta-\alpha}\lambda_b)\Bigg\}, \tag{3.11}$$

$$\lambda'_{hH^+H^-} = -2\Bigg\{ \frac{1}{2v^2}[(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)M_h^2 - (M_A^2-M_{H^\pm}^2)c_{\beta+\alpha} - M_{H^\pm}^2 c_{2\beta}c_{\beta-\alpha}] - (s_{\beta-\alpha}\lambda_a + c_{\beta-\alpha}\lambda_b)\Bigg\}, \tag{3.12}$$

$$\lambda'_{HHH} = -6\Bigg\{ \frac{1}{2v^2}\{s_{\beta+\alpha}[(M_H^2-M_A^2)+c_{\beta-\alpha}^2 M_A^2] - s_\alpha c_\alpha c_{\beta-\alpha}M_H^2\} + s_{\beta-\alpha}^2(s_{\beta-\alpha}\lambda_b - c_{\beta-\alpha}\lambda_a)\Bigg\}. \tag{3.13}$$

Note once more that these couplings, like other $H$ couplings to fermions, sfermions, and gauge bosons, can be derived from those of $h$. For example we do verify that $g_{HHH} = g_{hhh}(h\to H,\ M_h \to M_H,\ s_{\beta-\alpha}\to c_{\beta-\alpha},\ c_{\beta-\alpha}\to -s_{\beta-\alpha})$,

$$\lambda'_{HAA} = -2\Bigg\{ \frac{1}{2v^2}[(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)(M_H^2-M_A^2) + c_{\beta-\alpha}c_\beta s_\beta M_A^2] + s_{\beta-\alpha}\lambda_b - c_{\beta-\alpha}\lambda_a\Bigg\}, \tag{3.14}$$

$$\lambda'_{HH^+H^-} = -2\Bigg\{ \frac{1}{2v^2}[s_{\beta+\alpha}(M_H^2-M_A^2) + c_{\beta-\alpha}c_\beta s_\beta(2M_{H^\pm}^2-M_H^2)] + s_{\beta-\alpha}\lambda_b - c_{\beta-\alpha}\lambda_a\Bigg\}. \tag{3.15}$$

In the physical basis and especially by explicitly displaying the dependence in the mixing angle $\alpha$ through the combination $c_{\beta-\alpha}, s_{\beta-\alpha}$ shows that, for couplings involving at least two heavy Higgs bosons,

(i) the dependence in the parameters $\lambda_i$ only appears through the combination $\lambda_a$ or $\lambda_b$. Thus one combination is not accessed in the trilinear couplings and

(ii) in the decoupling limit ($M_h \ll M_A, M_H, M_A$), it is only $\lambda_a$ which is accessible in the self-couplings involving two heavy Higgs bosons ($hAA$, $hHH$, and $hH^+H^-$), while in the self-couplings with heavy Higgs bosons only ($HHH$,$HAA$,$HH^+H^-$) only $\lambda_b$ may be accessed.

Note that the heavy Higgs boson masses $M_H, M_A, M_{H^\pm}$ enter the formulas for the couplings only through combinations involving mass differences or $M_A^2 c_{\beta-\alpha}$ so that in effect the self-couplings do not grow with the heavy Higgs boson mass as occurs in the SM.

### A. Quartic Higgs couplings

To access the remaining combination we need to consider the quartic couplings. We will write the quartic couplings directly in the physical basis.

Defining

$$\lambda_c = 2c_{2\beta}\lambda_b - \lambda_5 = \lambda_5 c_{4\beta} + \frac{\lambda_6 - \lambda_7}{2} s_{4\beta}, \tag{3.16}$$

$$\begin{aligned} g_{hhhh} = &-\frac{3M_h^2}{2v^2}(1-c_{\beta-\alpha}^2) - \frac{3}{2v^2 s_\beta^2 c_\beta^2} c_{\beta-\alpha}^2 \{ c_{\beta+\alpha}(s_{\beta-\alpha}s_{2\beta} \\ &+ c_{\beta+\alpha}c_{\beta-\alpha}^2) M_h^2 + s_\alpha^2 c_\alpha^2 M_H^2 - c_{\beta+\alpha}^2 M_A^2 \} \\ &+ \frac{3}{s_\beta^2 c_\beta^2} c_{\beta-\alpha}^2 \{ s_{2\beta}\lambda_a + 2 s_{2\beta} c_{\beta-\alpha} s_{\beta-\alpha}\lambda_b + c_{\beta-\alpha}^2 \lambda_c \}. \end{aligned} \tag{3.17}$$

We see again that in the decoupling region one is not sensitive to any of the extra couplings, as expected since we recover the SM result with only the dim-4 operator. Let us now give the formulas for some of the other quartic couplings to show that some of the novel couplings are not screened in all of the quartic couplings. We will only show the dependence in the extra parameters and will not give the full dependence in terms of masses as otherwise the formulas may be too lengthy:

$$\begin{aligned} \lambda_{hhhH} \to c_{\beta-\alpha}\{ & s_{\beta-\alpha}s_{2\beta}\lambda_a + c_{\beta-\alpha}(3s_{2\beta}\lambda_b + 2c_{\beta-\alpha}s_{\beta-\alpha}\lambda_c \\ & - 4c_{\beta-\alpha}^2 s_{2\beta}\lambda_b)\}. \end{aligned} \tag{3.18}$$

Again, all the anomalous couplings are screened:

$$\begin{aligned} \lambda_{hhHH} \to s_{2\beta}\lambda_a + 6s_{\beta-\alpha}c_{\beta-\alpha}( & s_{2\beta}\lambda_b + c_{\beta-\alpha}s_{\beta-\alpha}\lambda_c \\ & - 2c_{\beta-\alpha}^2 s_{2\beta}\lambda_b). \end{aligned} \tag{3.19}$$

In $hhHH$ only $\lambda_a$ may be accessible:

$$\begin{aligned} \lambda_{hHHH} \to s_{\beta-\alpha}\{ s_{\beta-\alpha}s_{2\beta}\lambda_b + c_{\beta-\alpha}( & s_{2\beta}\lambda_a + 2s_{\beta-\alpha}^2\lambda_c \\ & - 4c_{\beta-\alpha}s_{\beta-\alpha}s_{2\beta}\lambda_b)\}. \end{aligned} \tag{3.20}$$

In $hHHH$ only $\lambda_b$ may be accessible:

$$\lambda_{HHHH} \to s_{\beta-\alpha}^2 (s_{\beta-\alpha}^2\lambda_c + s_{2\beta}\lambda_a - 2c_{\beta-\alpha}s_{\beta-\alpha}s_{2\beta}\lambda_b). \tag{3.21}$$

In $HHHH$ both $\lambda_a$ and $\lambda_c$ may be accessible but not $\lambda_b$.

### B. Using another parametrization

The screening property is a general result which does not depend on which independent parameters we keep beside the physical masses. Had we used another set of independent parameters beside the masses, the same phenomenon would have occurred and only two independent combinations out of the three parameters would enter the expression of the trilinear couplings. Indeed with $\lambda_3, \lambda_5, \lambda_6$ as extra parameters, the role of $\lambda_{a,b,c}$ is played by $\lambda'_{a,b,c}$, such that

$$\begin{aligned} \lambda_a &\to \lambda'_a = -s_\beta c_\beta(\lambda_3 - \lambda_5), \\ \lambda_b &\to \lambda'_b = -\left( \frac{\lambda_3+\lambda_5}{2} + c_{2\beta}\frac{\lambda_3-\lambda_5}{2} \right) \\ &\quad + \frac{c_\beta}{s_\beta}\lambda_6, \\ \lambda_c &\to \lambda'_c = 2\lambda'_b c_{2\beta} - \lambda_5. \end{aligned} \tag{3.22}$$

For instance,

$$\begin{aligned} \lambda_{hhh} = &-\frac{1}{2v^2} s_{\beta-\alpha} M_h^2 + c_{\beta-\alpha}^2 \frac{1}{c_\beta s_\beta}(s_{\beta-\alpha}\lambda'_a + c_{\beta-\alpha}\lambda'_b) + \frac{c_{\beta-\alpha}^2}{2v^2 c_\beta s_\beta^2}( s_{\beta-\alpha}c_\beta s_\beta^2 (M_A^2 - M_H^2 - M_h^2 - M_Z^2 c_{2W} + 2M_{H^\pm}^2) \\ &- c_{\beta-\alpha}\{ s_\beta [ s_\beta^2 (M_A^2 - M_H^2) + c_\beta^2 (2M_H^2 + M_Z^2 c_{2W} - 2M_{H^\pm}^2 - M_h^2)] + c_{\beta-\alpha}(M_H^2 - M_h^2) \\ &\times [ s_{\beta-\alpha}c_\beta(1-4s_\beta^2) - c_{\beta-\alpha}s_\beta(3-4s_\beta^2)]\}). \end{aligned} \tag{3.23}$$

Compared to the previous parametrization, this looks rather more complicated as it involves the charged Higgs boson mass as well as the heavy $H$ beside the pseudoscalar Higgs boson mass. Nonetheless, all these masses are screened.

## IV. DIM-4 OPERATORS AND INDEPENDENT PARAMETERS

To easily understand our finding about the screening and number of independent parameters in the trilinear and quadrilinear couplings, note that the trilinear and quadrilinear couplings originate from the quartic terms $\lambda_i$ only, Eq. (2.2), whereas the mass terms get an additional contribution from the bilinear terms $m_1, m_2, m_{12}$ in Eq. (2.2). Take, for instance, the case of neutral couplings. The quartic self-couplings emerge as combinations of five independent terms in the original fields (before diagonalization) of the form

$$h_1^4, h_2^4, h_1^3 h_2, h_1^2 h_2^2, h_1 h_2^3. \tag{4.1}$$

In terms of the physical scalar fields $h, H$,

$$h_1 = -h s_\alpha + H c_\alpha, \quad h_2 = h c_\alpha + H s_\alpha, \tag{4.2}$$

which we more judiciously write as

$$\begin{aligned} h_1 &= c_\beta (s_{\beta-\alpha} h + c_{\beta-\alpha} H) + s_\beta (H s_{\beta-\alpha} - h c_{\beta-\alpha}) \\ &= c_\beta h_- + s_\beta h_+ , \\ h_2 &= s_\beta h_- - c_\beta h_+ . \end{aligned} \tag{4.3}$$

However, keep in mind that $h_1$ and $h_2$ in Eq. (2.1) always appear in the form

$$\begin{aligned} v_1 + \frac{h_1}{\sqrt{2}} &= v_1 \left[ \left( 1 + \frac{h_-}{v'} \right) + t_\beta \frac{h_+}{v'} \right], \\ v_2 + \frac{h_2}{\sqrt{2}} &= v_2 \left[ \left( 1 + \frac{h_-}{v'} \right) - t_\beta^{-1} \frac{h_+}{v'} \right], \end{aligned} \tag{4.4}$$

with $v' = \sqrt{2} v$ [$= g_h$, Eq. (2.16)], which helps write the doublets, Eq. (2.1), as

$$\begin{aligned} H_1 &= v_1 \begin{pmatrix} 1 + [(h_- - iG^0) + t_\beta (h_+ + iA)]/v' \\ (-G^- + t_\beta H^-)/v \end{pmatrix}, \\ H_2 &= v_2 \begin{pmatrix} (G^+ + t_\beta^{-1} H^+)/v \\ 1 + [(h_- + iG^0) - t_\beta^{-1}(h_+ - iA)]/v' \end{pmatrix}, \end{aligned} \tag{4.5}$$

where $G^{\pm,0}$ stand for the Goldstone bosons.

Therefore in effect the quartic terms originate from a combination of the form

$$\left( 1 + \frac{h_-}{v'} \right)^2 \left[ q_1 \left( 1 + \frac{h_-}{v'} \right)^2 + q_2 \left( 1 + \frac{h_-}{v'} \right) \frac{h_+}{v'} + q_3 \frac{h_+^2}{v'^2} \right] + q_4 \left( 1 + \frac{h_-}{v'} \right) \frac{h_+^3}{v'^3} + q_5 \frac{h_+^4}{v'^4}. \tag{4.6}$$

While the bilinear terms are of the form

$$b_1 \left( 1 + \frac{h_-}{v'} \right)^2 + b_2 \left( 1 + \frac{h_-}{v'} \right) \frac{h_+}{v'} + b_3 \frac{h_+^2}{v'^2}. \tag{4.7}$$

Imposing that no tadpole remains (no linear term in $h_+, h_-$) means that $(b_1, q_1)$ and $(b_2, q_2)$ must combine such that one has

$$\begin{aligned} &4 q_1 \frac{h_-^2}{v'^2} \left( 1 + \frac{h_-}{v'} + \frac{h_-^2}{4 v'^2} \right) + 2 q_2 \frac{h_+ h_-}{v'^2} \left( 1 + \frac{3 h_-}{2 v'} + \frac{h_-^2}{2 v'^2} \right) \\ &+ \frac{h_+^2}{v'^2} \left[ b_3 + q_3 \left( 1 + \frac{h_-}{v'} \right)^2 \right] + q_4 \left( 1 + \frac{h_-}{v'} \right) \frac{h_+^3}{v'^3} + q_5 \frac{h_+^4}{v'^4}. \end{aligned} \tag{4.8}$$

Since the coefficients of $h_-^2 (q_1)$ and $h_- h_+ (q_2)$ are expressed in terms of masses, so do those of $h_-^3, h_-^4$ $(q_1)$ as well as $h_+ h_-^2, h_+ h_-^3$ $(q_2)$. This does not apply to the trilinear and quadrilinear terms involving $h_+^2$ and higher order in $h_+$. Thus for the trilinear terms there are only the two structures $h_+^2 h_-$ (with coefficient $q_3$) and $h_+^3$ (with coefficient $q_4$) that cannot be expressed solely in terms of the physical masses. The last parameter $q_5$ only appears in the quartic couplings in the form $h_+^4$. To obtain $q_1$ and $q_2$ is a straightforward matter. One only has to rewrite the $CP$-even Higgs boson masses in terms of $h_\pm$:

$$\begin{aligned} M_h^2 h^2 + M_H^2 H^2 \to h_-^2 &(M_h^2 s_{\beta-\alpha}^2 + M_H^2 c_{\beta-\alpha}^2) \\ &+ 2 h_+ h_- s_{\beta-\alpha} c_{\beta-\alpha} (M_H^2 - M_h^2) \\ &+ h_+^2 (M_H^2 s_{\beta-\alpha}^2 + M_h^2 c_{\beta-\alpha}^2), \end{aligned} \tag{4.9}$$

where $q_3$, $q_4$, and $q_5$ are directly related to the parameters $\lambda_a$, $\lambda_b$, and $\lambda_c$. The construct of Eq. (4.8) shows that in fact once we get the quartic couplings one also derives the coefficients of the various trilinear couplings. For example, take the quartic couplings as they appear in the original potential in terms of the fields $h_1$ and $h_2$:

$$\begin{aligned} Q_h = &\frac{\lambda_1}{2} h_1^4 + \frac{\lambda_2}{2} h_2^4 + (\lambda_3 + \lambda_4 + \lambda_5) h_1^2 h_2^2 - 2 \lambda_6 h_1^3 h_2 \\ &- 2 \lambda_7 h_2^3 h_1 . \end{aligned} \tag{4.10}$$

When expressed in terms of $h_-$ and $h_+$ and after moving to the mass basis with, for example, $\lambda_{5,6,7}$ as extra parameters we immediately get the following dependence of the various quartic and trilinear terms:

$$(h_+^2+A^2+2H^+H^-)\Bigg[2(\lambda_a h_- - \lambda_b h_+)+\frac{1}{v'}\{\lambda_a(h_-^2+G^{0^2}+2G^+G^-)-2\lambda_b[h_-h_+-AG^0-(H^+G^-+H^-G^+)]+\tilde{\lambda}_c(h_+^2+A^2+2H^+H^-)\}\Bigg], \tag{4.11}$$

with[4]

$$\tilde{\lambda}_c=\frac{c_{2\beta}^2}{s_{2\beta}}\lambda_5-\frac{\lambda_6+\lambda_7}{2}+\frac{\lambda_6-\lambda_7}{2}c_{2\beta}, \quad \tilde{\lambda}_c-\lambda_a=\frac{1}{s_{2\beta}}\lambda_c. \tag{4.12}$$

In fact with this parametrization one has that $q_3=\lambda_a$, $q_4=\lambda_b$ and $q_5=\tilde{\lambda}_c$. One can move to another parametrization, i.e., choosing a different set of extra parameters, through Eqs. (3.1)–(3.4). We see that by *completing* the $h_-,h_+$ dependence of $H_{1,2}$ we even get the full trilinear and quadrilinear couplings involving the pseudoscalar, charged, and Goldstone boson couplings. The completion is obtained by identifying the different $t_\beta$ dependences (namely, $1,t_\beta,t_\beta^2$) in the modulus of $H_1$, for instance. It is not hard to see that by reexpressing $h_-$ and $h_+$ in terms of $h$ and $H$, we recover all our results. Moreover, this writing immediately shows that trilinear scalar couplings involving Goldstone bosons can all be expressed most simply in terms of the physical Higgs boson masses only. The requirement for the absence of tadpoles is a crucial one and explains most of our findings when restricting ourselves to dim-4 operators.

## V. RADIATIVE CORRECTIONS

Our results can also be exploited for easily expressing the radiative corrections to the trilinear (and for that matter quadrilinear) couplings of the SUSY Higgs bosons and explain some of the properties pointed out in the literature. Three-point one-loop radiative corrections for the neutral Higgs system in the MSSM have been calculated [18] within the effective potential approximation.[5] The diagrammatic one-loop radiative corrections to both the trilinear $\lambda_{hhh}$ and quadrilinear $\lambda_{hhhh}$ lightest Higgs self-couplings have been reexamined recently in [21]. For the case of no mixing in the top squark sector it is shown analytically [21] that the bulk of the corrections in the couplings are absorbed by using the corrected Higgs boson mass while the same is demonstrated numerically for the case of large mixing. One-loop radiative corrections for $\lambda_{hAA}$ are also considered in [15,16], for $\lambda_{HAA}$ in [15], and for $\lambda_{Hhh}$ in [17]. Here also the corrections are found to be large before reexpressing the results in terms of the corrected masses.

Reference [23] gives analytical approximations (including two-loop leading-logarithmic corrections) for the effective quartic couplings $\lambda_i$,[6] using a renormalization-group-improved leading-logarithmic approximation. We can adapt their formulas to the one-loop case with large top squark masses so that we can compare with the direct calculation of the vertices performed in [5,18]. For instance, we find the leading one-loop contributions in the limit where the SUSY breaking term $M_S^2$, defined below, is (much) larger than the top quark mass (which is consistent with our approach of keeping only the dimension-4 operators):

$$\Delta\lambda_{hhh}=-\frac{g^4m_t^4}{64\pi^2M_W^4}\frac{c_\alpha^3}{s_\beta^3}\left\{6\log(M_S^2/m_t^2)+3\frac{f_t(c_t+f_t)}{M_S^2}-\frac{c_tf_t^3}{2M_S^4}\right\},$$

$$\Delta\lambda_{Hhh}=\frac{g^4m_t^4}{64\pi^2M_W^4}\frac{s_\alpha c_\alpha^2}{s_\beta^3}\left\{6\log(M_S^2/m_t^2)+\frac{c_t(e_t+2f_t)+f_t(f_t+2e_t)}{M_S^2}-\frac{c_te_tf_t^2}{2M_S^4}\right\},$$

$$c_t=A_t+\mu/t_\beta \quad e_t=A_t+\mu/t_\alpha \quad f_t=A_t-\mu t_\alpha$$

$$m^2_{\tilde{t}_1,\tilde{t}_2}=M_S^2\pm m_tc_t. \tag{5.1}$$

These shifts correct the tree-level expression of Eqs. (2.17) and Eq. (2.18), respectively. In the limit $\mu\to 0$, all the $\lambda_i$ in the MSSM vanish but $\lambda_2$:

$$\lambda_2^{\tilde{t}_1,\tilde{t}_2}=\frac{3}{32\pi^2}\frac{g^4m_t^4}{s_\beta^4M_W^4}\left\{\log(M_S^2/m_t^2)+\frac{A_t^2}{M_S^2}(1-A_t^2/12M_S^4)\right\}$$
$$\sim 0.15 \text{ for } A_t=M_S=1 \text{ TeV and } t_\beta=10, \tag{5.2}$$

while, in the same approximation as Eq. (5.1),

$$M_h^2=M_Z^2s_{\beta+\alpha}^2+M_A^2c_{\beta-\alpha}^2+\Delta M_h^2,$$

$$\Delta M_h^2=\frac{3}{8\pi^2}\frac{g^2m_t^4}{M_W^2}\left(\log(M_S^2/m_t^2)+\frac{f_tc_t}{M_S^2}-\frac{c_t^2f_t^2}{12M_S^4}\right)\times\left(1+\frac{c_{\beta-\alpha}}{s_\beta^2}(s_{2\beta}s_{\beta-\alpha}+c_{2\beta}c_{\beta-\alpha})\right), \tag{5.3}$$

[4]The reason $\lambda_c$ appears instead of $\tilde{\lambda}_c$ depends on how we organize the decoupling and is due to the rewriting of the terms in $s_{\beta-\alpha}^2$ as $1-c_{\beta-\alpha}^2$.

[5]See also [5] where expressions are given for some couplings assuming equal soft masses for the top squark masses. Note, however, that there is a misprint in Eq. (2.4) of [5] where in the last term of that equation one should read $A(A+\mu\cot\beta)$ instead of $\mu(A+\mu\cot\beta)$.

[6]Compared to our notation we should make $\lambda_{6,7}\to-\lambda_{6,7}$ in the expressions of [23]. Moreover, our sign convention for $\mu$ is the opposite of [23] but the same as in [5]. See [34] for a full definition of our conventions.

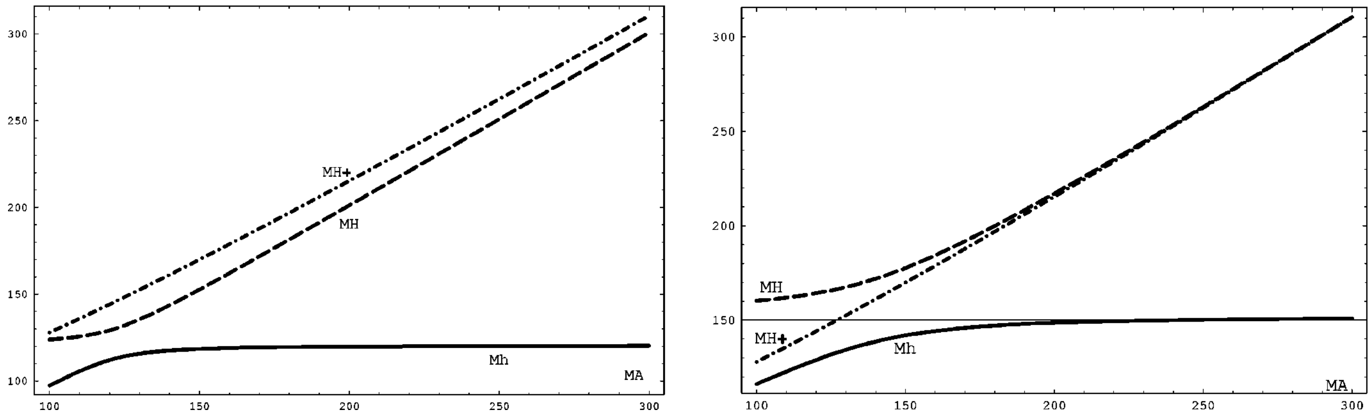


FIG. 1. Higgs boson mass spectrum without "hard" terms but with $A_t = 1000$ GeV, $M_S = 800$ GeV, and $\mu = -300$ GeV, and $\tan\beta = 10$ (left) and with the inclusion of additional terms with $\lambda_{1-5}^{\text{new}} = -\lambda_{6,7}^{\text{new}} = 0.1$ (right). All masses are in GeV.

so that one recovers the decoupling property and the fact that the bulk of the radiative corrections are reabsorbed by using the corrected Higgs boson mass,

$$\Delta\lambda_{hhh} = -\frac{\Delta M_h^2}{2v^2}\left(s_{\beta-\alpha} + \frac{c_{\beta-\alpha}}{t_\beta}\right) + \frac{3m_t^4}{16\pi^2 v^4}\frac{\mu f_t}{M_S^2} \times\left(1 - \frac{f_t c_t}{6M_S^2}\right)\frac{c_\alpha^2}{s_\beta^4} c_{\beta-\alpha}. \tag{5.4}$$

In a phenomenological analysis of the extraction of the Higgs self-couplings, one could add the contribution of the top and bottom squarks at the two-loop level through effective couplings $\lambda_i$ from the renormalization-group-improved results of [23] to which one could include new physics contributions to the $\lambda_i$.

Note that contrary to what we have presented in the previous sections, we have shown the "corrections" to the Higgs self-couplings due to radiative corrections (or presence of $\lambda$ terms) as shifts compared to the tree-level MSSM. We have done so in order to compare with the existing literature [5,18,21], which takes into account effects at one loop only. Although this shows that the bulk of the corrections is absorbed in terms of the Higgs boson mass, the notion of shifts here is somehow misleading especially that some of the correction is contained in the "corrected" mixing angle $\alpha$.

## VI. PHENOMENOLOGY AND RECONSTRUCTION OF THE HIGGS POTENTIAL AT FUTURE COLLIDERS

As we have seen, the measurement of the entire set of the dim-4 operators which is necessary to reconstruct the Higgs potential in SUSY (and 2HDM) requires that one crosses the thresholds for the production of three Higgs bosons, which is not an easy task especially that the cross sections will get tinier and tinier as the Higgs multiplicity increases. As we have seen also, a precise measurement of the Higgs boson masses and their couplings to ordinary matter is an important ingredient in the reconstruction of this potential. The LHC can thus give a first hint of the parameters $\lambda_i$. For instance, imagine that the LHC discovers some SUSY particles and identifies them as such but that one discovers also that the lightest Higgs boson has a mass in excess of 150 GeV. This would point to a scalar potential with "hard" $\lambda$ terms. We could probably even set a rough bound on their possible values. A LC with enough energy to produce some of the Higgs bosons and good luminosity to probe their couplings would constitute a nice complementary machine though. Although double-Higgs-boson production at the LHC [6–8,20,35] may not be so negligible, extracting the trilinear Higgs self-couplings will prove a challenge [8]. Therefore for the rest of this section we will only briefly outline what might be measured from the self-couplings of the Higgs boson at different stages of the LC. However, before doing so, let us illustrate what the mass measurements alone can bring and how the spectrum can be drastically affected by different forms of the potential. As an illustration we stick to $\tan\beta = 10$ and consider the situation where the $\lambda$ receive corrections from the top squark sector with the parameters $A_t = 1000$ GeV, $M_S = 800$ GeV, and $\mu = -300$ GeV. We will compare the situation where no "hard" terms are added with a situation with $\lambda_{1-5} = -\lambda_{6,7} = 0.1$, i.e., of the order of $\lambda_2^{\tilde{t}_1\tilde{t}_2}$. The mass spectrum of the Higgs system for this choice of parameters is shown in Fig. 1.

One striking feature is that $M_h$ can be substantially heavier than what it is in the usual MSSM, while the mass ordering between $M_{H^\pm}$ and $M_H$ is certainly another distinguishing feature for this particular choice of parameters.

The rate of $h$ production at $e^+e^-$, weighted by $s_{\beta-\alpha}^2$, can also provide a helpful hint and additional constraint. However, decoupling, although slightly delayed by the presence of the new $\lambda_i$, occurs rather fast in this variable as shown in Fig. 2. Having measured $\tan\beta$ greatly helps as the figure illustrates.

A full analysis from the measurements of the masses and the couplings to fermions and vector bosons is left to a forthcoming detailed analysis [36].

As for the Higgs self-couplings the presence of $\lambda_i$ can have a drastic effect as shown in Fig. 3 especially for small $M_A$; see in particular the swing in $g_{HHH}$. In this region one expects though that all Higgs boson masses would have been

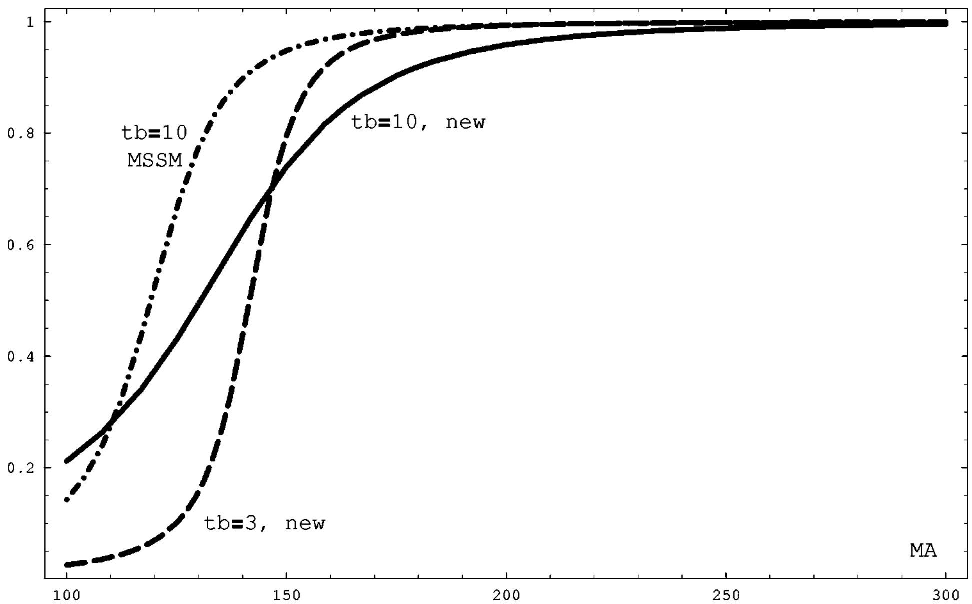


FIG. 2. $s^2_{\beta-\alpha}$ as a function of $M_A$. MSSM refers to $\lambda_i=0$ with $A_t=1000$ GeV, $M_S=800$ GeV, and $\mu=-300$ GeV while "new" has $\lambda^{\rm new}_{1-5}=-\lambda^{\rm new}_{6,7}=0.1$. tb stands for tan $\beta$.

measured and thus a good constraint on parameter space will have been provided by the masses. As soon as we enter the decoupling region, the largest coupling is $g_{hhh}$ which reaches its SM value. The other couplings remain unfortunately rather small, although some have values larger than their corresponding SM value. However, as we have seen the bulk of these deviations are due to the rather large deviations in the Higgs boson masses. In this respect let us note that Fig. 3 seems to indicate that the $HHH$ coupling can get rather large for small $M_A$. However, observe that we have plotted a reduced coupling in units of the SM coupling $\tilde{g}^{\rm SM}_{hhh}=-3M_h^2/2v^2$. The reason the reduced coupling attains a value larger than 1 is due to the larger mass of $H$ and that we are in a region of non decoupling; see Fig. 2. In this region $H$ is more standard like than $h$, as far as its couplings to gauge bosons are concerned. Had we used $\hat{g}^{\rm SM}_{hhh}=-3M_H^2/2v^2$ as a unit, the reduced coupling would be below 1.

Let us now review briefly how an $e^+e^-$ machine working at successive thresholds for Higgs boson production can attempt to unravel the Higgs potential.

### A. Stage 1

Imagine a situation where no heavy Higgs boson has been produced at a first stage of a linear collider at 500 GeV or the LHC; we would then be in the decoupling limit. The only trilinear couplings which may be accessed are $hhh$ and $Hhh$ through $e^+e^-\to Zhh$ (fusion channels are not efficient at these energies and Higgs boson masses). However, there is no sensitivity to $Hhh$. Indeed the amplitude for $e^+e^-\to Zhh$, in the unitary gauge, can be written as

$$\mathcal{M}_{Zhh}=a_h\lambda_{hhh}s_{\beta-\alpha}+a_H\lambda_{Hhh}c_{\beta-\alpha}+R_a, \qquad (6.1)$$

where $R$ stands for other contributions not containing the trilinear Higgs couplings. We have seen that $\lambda_{Hhh}$ is screened by a factor $c_{\beta-\alpha}$ [Eq. (3.7)], it is further screened by another such factor when we consider its contribution to this cross section.

At this stage the best would be to reconstruct as precisely as possible $M_h$ and the couplings of $h$ to fermions and the vector bosons. This will help give a bound on the $\lambda_i$. If one makes some model-dependent assumptions on the $\lambda_i$ (imposing some discrete or global symmetries), this can be used to extract some information on $M_A$. If an independent measurement of tan $\beta$ is missing at the time of the measurements of the Higgs boson properties, this will complicate the analyses.

If, on the other hand, the mixing angle is such that $c_{\beta-\alpha}$ is not too small and $M_A$ is not too large, $e^+e^-\to ZH$ may be accessible. Then the coupling $\lambda_{Hhh}$ could be reached directly through $H\to hh$. This may still turn out not to be too helpful, since we have seen that the $\lambda_i$ are still screened in this coupling, even though the screening in this situation could be mild. Moreover, $H$ decays into other particles ($t\bar{t}$ or $b\bar{b}$, . . . ) and superparticles (charginos and neutralinos) may still be dominant so that $Br(H\to hh)$ will be poorly determined. Let us remark at this point that most of the nice analyses of the SUSY Higgs self-couplings [3,22] that have been performed were done solely in the context of the minimal supersymmetric model, with no additional "hard" terms in the potential, and have relied heavily on the extremely good precision of the measurement of the dominant branching ratio into $b\bar{b}$. In case $h$ is heavier than 150 GeV these analyses need to be extended.

### B. Stage 2

For a machine with higher energies where $H$ and $A$ and thus most probably $H^\pm$ have been discovered, the first thresholds for double-Higgs-boson production (after that of $Zhh$) may be

$$e^+e^-\to ZhH, \nu_e\bar{\nu}_eHh$$

with

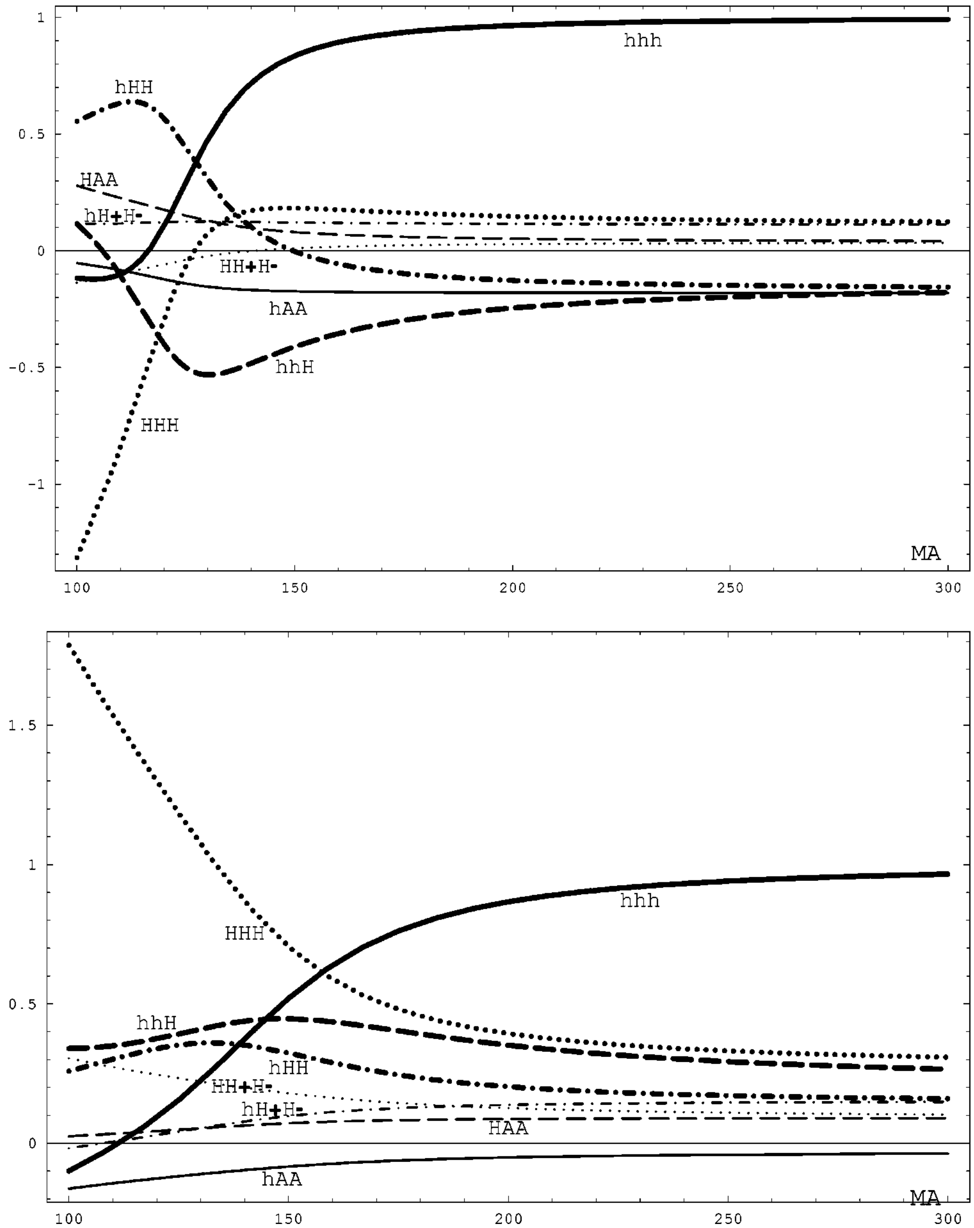


FIG. 3. $g_{H^iH^jH^k}/\tilde{g}^{\rm SM}_{hhh}$ with $\tilde{g}^{\rm SM}_{hhh}=-3M_h^2/2v^2$. The SM Higgs boson mass is identified with $M_h$ (and thus varies with $M_A$). The curve in the first panel is the usual MSSM where the one in the second panel is defined with the same parameters as in Fig. 1.

$$\mathcal{M}_{Hh}=b_h\lambda_{Hhh}s_{\beta-\alpha}+b_H\lambda_{HHh}c_{\beta-\alpha}+R_b,$$

$$e^+e^-\to hhA$$

with

$$\mathcal{M}_{Ahh}=c_h\lambda_{hhh}c_{\beta-\alpha}+c_H\lambda_{Hhh}s_{\beta-\alpha}+c_A\lambda_{hAA}c_{\beta-\alpha}+R_c. \quad (6.2)$$

Again, unfortunately these two reactions will not be very sensitive to deviations in the trilinear couplings if one takes into account the screening effect in $\lambda_{Hhh}$. Fusion processes could also be exploited at this stage and the next, but they exhibit a similar behavior to the annihilation processes as far the extraction of the parameters is concerned.

### C. Stage 3

With higher energies one produces two heavy Higgs bosons in association with a light Higgs boson or a $Z$:

$$e^+e^-\to ZHH, \nu_e\overline{\nu_e}HH$$

with

$$\mathcal{M}_{HH}=d_h\lambda_{HHh}s_{\beta-\alpha}+d_H\lambda_{HHH}c_{\beta-\alpha}+R_d,$$

$$e^+e^-\to ZAA, \nu_e\overline{\nu_e}AA$$

with

$$\mathcal{M}_{ZAA}=e_h\lambda_{hAA}s_{\beta-\alpha}+e_H\lambda_{HAA}s_{\beta-\alpha}+R_e,$$

$$e^+e^- \to hHA$$

with

$$\mathcal{M}_{AHh} = (f_h\lambda_{HHh} + f_H\lambda_{hAA})s_{\beta-\alpha} + (f'_h\lambda_{Hhh} + f'_H\lambda_{HAA})c_{\beta-\alpha} + R_f$$

$$e^+e^- \to H^+H^-h$$

with

$$\mathcal{M}_{H^+H^-h} = g_h\lambda_{H^+H^-h} + R_g\,,$$

$$e^+e^- \to ZH^+H^-, \nu_e\overline{\nu}_eH^+H^-$$

with

$$\mathcal{M}_{XH^+H^-} = h_h\lambda_{hH^+H^-}s_{\beta-\alpha} + h_H\lambda_{H^+H^-}c_{\beta-\alpha} + R_h\,. \quad (6.3)$$

As can be seen all of these reactions will be used to determine $\lambda_a$ ($\lambda_b$ will still be screened). Let us give some idea about the order of magnitude of the cross sections to show that things can get really tough. As a reference take all extra contributions to the $\lambda_i$ to be vanishing with SUSY parameters as those considered in the introduction of this section: $A_t = 1000$ GeV, $M_S = 800$ GeV, $\mu = -300$ GeV, and $\tan\beta = 10$ and take $M_A = 300$ GeV. The third stage could be taken as $\sqrt{s} = 1.2$ TeV. We find that $ZHH$ and $ZAA$ are about $2.3\times 10^{-2}$ fb, while the other processes listed in this stage are two orders of magnitude below. Before taking into account signatures and efficiencies this can amount to about only 25 events a year based on a luminosity of 1 ab$^{-1}$.

### D. Stage 4

At even higher energies, production of three heavy Higgs bosons could in principle allow one to determine $\lambda_b$. The processes at our disposal will be

$$\lambda_{hAA} e^+e^- \to AAA$$

with

$$\mathcal{M}_{AAA} = i_h, c_{\beta-\alpha} + i_H\lambda_{HAA}s_{\beta-\alpha}\,,$$

$$e^+e^- \to HHA$$

with

$$\mathcal{M}_{HHA} = j_h\lambda_{hHH}c_{\beta-\alpha} + j_H\lambda_{HHH}s_{\beta-\alpha} + j_A\lambda_{HAA}s_{\beta-\alpha} + R_j\,,$$

$$e^+e^- \to H^+H^-H$$

with

$$\mathcal{M}_{H^+H^-H} = k_H\lambda_{H^+H^-H} + R_k\,,$$

$$e^+e^- \to H^+H^-A$$

with

$$\mathcal{M}_{H^+H^-A} = l_hc_{\beta-\alpha}\lambda_{H^+H^-h} + l_Hs_{\beta-\alpha}\lambda_{H^+H^-H}\,. \quad (6.4)$$

Cross sections here are very small here. For instance, for the set of parameters considered above and with $\sqrt{s} = 2$ TeV, $AAA$ production is about $1.4\times 10^{-7}$ fb. Although a full study allowing a much larger parameter range (including $M_A$) is in order, it seems that a $\lambda_b$ measurement would be out of reach.

### E. Stage 5

As we have seen earlier [see Eq. (4.11)] the effect of the third combination of parameters $\lambda_c$ can only be observed in processes involving a vertex with four Higgs bosons. The first threshold where such a vertex contributes is a $Zhhh$ final state, which we could have classified in stage 2 (with a $ZHh$ final state). However, even for a SM Higgs boson $Zhhh$ or $\nu_e\overline{\nu}_e hhh$ at a 10 TeV LC with a luminosity as high as $10^{35}$ cm$^{-2}$ s$^{-1}$ yields only about five events per year [22]. Thus the prospect for a useful measurement looks grim especially that in $hhhh$ the $\lambda_c$ effect is screened as $c^4_{\beta-\alpha}$. Quartic couplings where this contribution is not screened involve any combination of the heavy Higgs bosons ($H^\pm,H,A$). Triple-$H$ production $ZHHH$ is not operative since it is triggered by $ZH$ production while quadruple production of the heavy Higgs boson is too tiny to be exploited. Thus a full reconstruction may prove to be impossible if the full set $\lambda_{1-7}$ is present.

## VII. EFFECTS FROM HIGHER ORDER OPERATORS

Up to now we have only discussed the effects of the dim-4 operators. Higher order operators are doomed to contribute less significantly, as their effects are explicitly screened by a high scale. We will illustrate this case by considering only three new operators and restrict ourselves to a few Higgs self-couplings to make the point. We consider

$$V_{eff} \to V_{eff} + \frac{1}{\Lambda^2}\{\tilde{\kappa}_1(H_1H_1^*)^3 + \tilde{\kappa}_2(H_2H_2^*)^3 + \tilde{\kappa}_3(H_1H_1^*)^2(H_2H_2^*)\}. \quad (7.1)$$

For notational ease we will use

$$\kappa_i = \frac{v^2}{\Lambda^2}\tilde{\kappa}_i\,, \quad (7.2)$$

with $\Lambda$ the scale of new physics. We find

$$\lambda^\kappa_{hhh} = -\frac{e^2}{s^2_{2W}}s_{\beta+\alpha}c_{2\alpha} + (\lambda_1 - 6\kappa_1c^2_\beta - 2\kappa_3s^2_\beta)c_\beta s^3_\alpha - (\lambda_2 - 6\kappa_2s^2_\beta)c^3_\alpha s_\beta + \frac{1}{2}[(\lambda_3+\lambda_4 - 2\kappa_3c^2_\beta) + \lambda_5] \times s_{2\alpha}c_{\beta+\alpha} + \lambda_6s^2_\alpha(c_{\beta+\alpha} + 2c_\alpha c_\beta)$$

$$+\lambda_7 c_\alpha^2(c_{\beta+\alpha}-2s_\alpha s_\beta)-4\kappa_1 c_\beta^3 s_\alpha^3$$
$$+4\kappa_2 s_\beta^3 c_\alpha^3+4s_\beta c_\beta^2 c_\alpha s_\alpha^2\kappa_3. \quad (7.3)$$

Note that we have split the effect of the new contributions in two parts. The first [second line of Eq. (7.3)] can be viewed as a shift in $\lambda_{1,2,3}$ while the other [last line in Eq. (7.3)] can be considered as a genuine new contribution beyond the effects of the dim-4 operators. The shifts mean that the combinations $(\lambda_1-6\kappa_1 c_\beta^2-2\kappa_3 s_\beta^2)$, $(\lambda_2-6\kappa_2 s_\beta^2)$, and $(\lambda_3-2\kappa_3 c_\beta^2)$ replace $\lambda_{1,2,3}$, respectively, in the definition of $\alpha$, $m_{h,H}$ in Eqs. (2.9)–(2.11). Again, this means that even in the absence of any dim-4 operator, the dim-6 operators as defined above will also affect the Higgs boson masses and couplings to fermions and vector bosons. Moving to the mass basis, keeping as extra parameters $\lambda_{5,6,7}$ and $\kappa_{1,2,3}$, we get

$$\lambda_{hhh}^\kappa=\lambda_{hhh}-4\kappa_1 c_\beta^3 s_\alpha^3+4\kappa_2 s_\beta^3 c_\alpha^3+4s_\beta c_\beta^2 c_\alpha s_\alpha^2\kappa_3 \quad (7.4)$$

and

$$\lambda_{Hhh}^\kappa=\lambda_{Hhh}-4\bigg[c_\beta^3 s_\alpha^2 c_\alpha\kappa_1+s_\beta^3 c_\alpha^2 s_\alpha\kappa_2 -\left(\frac{2}{3}-s_\alpha^2\right)c_\beta^2 s_\beta s_\alpha\kappa_3\bigg], \quad (7.5)$$

where $\lambda_{H/hhh}$ are given in Eqs. (3.6), (3.7).

We see that the higher order operators are not further reduced by the decoupling factor $c_{\beta-\alpha}$ and that all $\kappa_i$ contribute to all the self-couplings, unlike $\lambda_i$ where we are only left with a combination of two couplings. This means that if one ideally has measured all masses and couplings to ordinary fermions and quite precisely all trilinear Higgs self-couplings, one could tell whether higher order operators are contributing. However, considering the foreseen precision in the extraction of the Higgs self-couplings and the expected small contribution of the higher order terms, this would seem to be overly optimistic.

## VIII. CONCLUSION

A dedicated study of double-Higgs-boson production at a high luminosity LC [3,22] within the SM has shown that it is very difficult to extract the Higgs self-couplings with a precision better than 20% in the first stage of a LC improving to slightly better than 10% [22] at a multi-TeV LC facility, even in the most favorable case of a Higgs boson light enough to decay into $b\bar{b}$. In the decoupling limit, the lightest SUSY Higgs boson will have properties very similar to that of the SM and thus we would also get a precision on its self-coupling with a very similar precision. Unfortunately we have shown that in this limit once we have measured the mass of the Higgs boson (which will be known at better than the per-mil level) and its couplings to ordinary SM particles (with a precision of a few percent), the precision attained in the self-couplings will not be sufficient to reveal new physics. Indeed effects from ''anomalous'' operators affecting the Higgs potential have a direct impact on the Higgs boson mass and the couplings to fermions. When these are taken into account additional effects in the self-couplings are screened either by mixing angles (dim-4 operators) or large scales. Even if we are not in the decoupling regime and even if we restrict oneself to the leading dim-4 operators, we have shown that measurements of all possible trilinear self-couplings would not allow us to reconstruct the most general lowest dimension Higgs potential. To achieve this one needs to measure some of the quartic couplings. However, an analysis that would take into account the measurements of the Higgs boson masses and their couplings to ordinary particles should give some useful constraints. An analysis along these lines completed with the extraction of some of the trilinear self-couplings at different stages of the LC is under way [36].

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