Supersymmetry effects on the exclusive semileptonic decays $B\to\pi\tau^+\tau^-$ and $B\to\rho\tau^+\tau^-$

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Semileptonic decays of B mesons are known to be very sensitive to any new physics effects. Amongst various possibilities the transition at the quark level $b\to d\ell^+\ell^-$ is more suited than $b\to s\ell^+\ell^-$ for the purpose of studying *CP* violation. Here in this work we will discuss the effects of supersymmetry on the various experimentally measurable quantities such as the decay rate, forward backward asymmetry, various polarization asymmetries, and *CP* violation asymmetries in the exclusive channels $B\to\pi\ell^+\ell^-$ and $B\to\rho\ell^+\ell^-$. We will focus mainly on the neutral Higgs boson effects on these measurements, with a view to eliciting information about possible *CP* violating as well as non-Hermitian terms in the effective Hamiltonian.

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I. INTRODUCTION

The rare B meson decays induced by the flavor changing neutral current $b\to s(d)$ transition offer a deeper probe for the weak interaction sector of the standard model (SM) as they go through second order in weak interactions. These decays can give information regarding fundamental parameters such as the Cabibbo-Kobayashi-Maskawa (CKM) factors, leptonic decay constants, etc. These decays can also be very useful in testing the various new physics scenarios such as the two Higgs doublet [1,2] and the minimal supersymmetric standard model (MSSM) [3–7]. Among the weak decays of B mesons the leptonic and semileptonic decays are very useful because of their relative cleanness. Between these two, the exclusive decays lately have received special attention [8,9]. For semileptonic and leptonic decays such as $B\to X_{s,d}\ell^+\ell^-$, $B\to K(K^*)\ell^+\ell^-$, $B\to\pi\ell^+\ell^-$, $B\to\rho\ell^+\ell^-$, etc., the basic quark level process is $b\to s(d)\ell^+\ell^-$ [10]. The basic quark level process $b\to s(d)\ell^+\ell^-$ occurs through the intermediate t, c, or u quark. These processes can be described in terms of an effective Hamiltonian which contains information about short and long distance effects. For the quark level process $b\to s\ell^+\ell^-$ the various contributions due to intermediate t, c, and u quarks enter into the matrix elements with factors $V_{tb}V_{ts}^*$, $V_{cb}V_{cs}^*$, and $V_{ub}V_{us}^*$. Off these three factors the third one is extremely small as compared to the first two. The unitarity relationship of the CKM matrix becomes (approximately) $V_{tb}V_{ts}^*+V_{cb}V_{cs}^*\approx 0$, so that the second factor can be written effectively as the negative of the first one. The (effective) Hamiltonian for the $b\to s\ell^+\ell^-$ transition thus involves essentially only one independent CKM factor $V_{tb}V_{ts}^*$, and hence the process $b\to s\ell^+\ell^-$ is not sensitive to CKM phases within the SM [11].

For the transition $b\to d\ell^+\ell^-$ the CKM factors involved, $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$, and $V_{ub}V_{ud}^*$, are comparable in magnitude and so the cross sections for processes having the quark level process $b\to d\ell^+\ell^-$ can have significant interference terms between them, and this could open up the possibility of observing the complex CKM factors from the interference terms. In the semileptonic decays (having a lepton pair in the final state) one can discuss several other kinematical variables associated with final state leptons, such as lepton pair forward-backward (FB) asymmetry and various polarization asymmetries. Supersymmetry (SUSY) effects on the FB asymmetries in various exclusive decay modes of the B meson, such as $B\to(\pi,K)\ell^+\ell^-$ [8], $B\to(\pi,\rho)\ell^+\ell^-$ [8,9], and $B\to\ell^+\ell^-\gamma$ [12], have been extensively studied. Sometime back, as pointed out by one of us [13] for the inclusive decay mode $B\to X_d\ell^+\ell^-$ and by Krüger and Sehgal [14] for the exclusive mode $B\to(\pi,\rho)\ell^+\ell^-$, along with *CP* violation from partial width asymmetry, one can look for *CP* violation in FB asymmetry also. We will explore this possibility also.

Along with FB asymmetry associated with final state leptons one can also discuss the three polarization asymmetries (longitudinal, normal, and transverse) associated with the final state leptons in various semileptonic decays. The importance of polarization asymmetries associated with final state leptons in various inclusive and exclusive semileptonic decay modes has been extensively discussed in many works [5,12,15,16]. In this communication, we study the three possible polarization asymmetries with the idea of exploring *CP* asymmetries as well as the *non-Hermiticity* of the effective Hamiltonian[1] through their measurement.

The paper is organized as follows. In Sec. II we present the general formalism, where we write the general effective Hamiltonian and present our definitions of the FB asymmetry and polarization asymmetries. Section III is devoted to the decay mode $B\to\pi\ell^+\ell^-$ where we discuss all the kinamatical variables associated with this decay mode. In Sec. IV we will discuss the decay mode $B\to\rho\ell^+\ell^-$. Finally, we will conclude with results and discussion in Sec. V.

II. GENERAL FORMALISM

The QCD corrected effective Hamiltonian for the decay $b\to d\ell^+\ell^-$ in the general SUSY model can be written as [4]

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[1]Which arises out of the quark loops.

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$$\mathcal{H}_{\rm eff}=\frac{4G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{td}^*\left[\sum_{i=1}^{10}C_iO_i+\sum_{i=1}^{10}C_{Q_i}Q_i-\lambda_u\{C_1[O_1^u-O_1]+C_2[O_2^u-O_2]\}\right], \qquad (2.1)$$

where we have used the unitarity of the CKM matrix $V_{tb}V_{td}^*+V_{ub}V_{ud}^*\approx-V_{cb}V_{cd}^*$, and $\lambda_u=V_{ub}V_{ud}^*/V_{tb}V_{td}^*$. Here O_1 and O_2 are the current current operators, $O_3,\ldots,O_6$ are called QCD penguin operators, and O_9 and O_{10} are semileptonic electroweak penguin operators [10]. The new operators Q_i ($i=1,...,10$) arises due to NHB (neutral Higgs boson) exchange diagrams [2,4]. In addition to the short distance corrections included in the Wilson coefficients, there are some long distance effects also, associated with real $c\overline{c}$ resonances in the intermediate states. This is taken into account by using the prescription given in [17], namely, by using the Breit-Wigner form of the resonances that add to $C_9^{\rm eff}$:

$$C_9^{res}=\frac{-3\pi}{\alpha^2}\kappa_v\sum_{V=J/\psi,\psi',\ldots}\frac{M_V{\rm Br}(V\to\ell^+\ell^-)\Gamma_{total}^V}{(s-M_V^2)+i\Gamma_{total}^VM_V}, \qquad (2.2)$$

thus we are taking the final leptons to be τ so only five resonances of the $c\overline{c}$ will contribute. The phenomenological factor κ_v is taken to be 2.3 for numerical calculations [16]. In this work we use the Wolfenstein parametrization [18] of the CKM matrix with four real parameters λ, A, ρ, and η, where η is the measure of CP violation. In terms of these parameters we can write λ_u as

$$\lambda_u=\frac{p(1-\rho)-\eta^2}{(1-\rho)^2+\eta^2}-i\frac{\eta}{(1-\rho)^2+\eta^2}+O(\lambda^2). \qquad (2.3)$$

From the relevant part of the above effective Hamiltonian Eq. (2.1) we can write the QCD corrected matrix element as

$$\mathcal{M}=\frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{td}^*\Big\{-2C_7^{\rm eff}\frac{m_b}{q^2}(\overline{d}i\sigma_{\mu\nu}q^\nu P_Rb)(\overline{\ell}\gamma^\mu\ell)+C_9^{\rm eff}(\overline{d}\gamma_\mu P_Lb)(\overline{\ell}\gamma^\mu\ell)+C_{10}(\overline{d}\gamma_\mu P_Lb)(\overline{\ell}\gamma^\mu\gamma_5\ell)+C_{Q_1}(\overline{d}P_Rb)(\overline{\ell}\ell)+C_{Q_2}(\overline{d}P_Rb)(\overline{\ell}\gamma_5\ell)\Big\}, \qquad (2.4)$$

where q is the momentum transfer and $P_{L,R}=(1\mp\gamma_5)/2$; we have neglected the mass of the d quark. The Wilson coefficients $C_7^{\rm eff}$ and C_{10} are given in many works [4,7,19] and the other Wilson coefficients C_{Q_1} and C_{Q_2} are given in [3,4]. The definition of $C_9^{\rm eff}$ is given in [9,13,20].

The decay rate (for any general three-body decay process $B\to P\ell^+\ell^-$) can be evaluated by doing phase space integration. On doing phase space integration we get

$$\frac{d\Gamma(B\to P\ell^+\ell^-)}{d\hat{s}dx}=\frac{m_B}{2^9\pi^3}\lambda^{1/2}(1,\hat{s},\hat{m}_p^2)\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}|\mathcal{M}|^2, \qquad (2.5)$$

where $\hat{s}=s/m_B^2$, $\hat{m}_\ell=m_\ell/m_B$, and $\hat{m}_p=m_p/m_B$ are dimensionless quantities. $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$. s is the c.m. energy of the $\ell^+\ell^-$ system, m_P is the mass of the particle labeled P, and $z=\cos\theta$ where θ is the angle between ℓ^- and the B three-momenta in the c.m. frame of $\ell^+\ell^-$. $|\mathcal{M}|^2$ is the matrix element squared of the process under consideration.

From the above expression one can get the decay rate and FB asymmetry [17]. The decay rate is simply the integration of Eq. (2.5) over the angle z. The definition of the FB asymmetry is [17]

$$A_{FB}=\frac{\int_0^1dzd\Gamma/d\hat{s}dz-\int_{-1}^0dzd\Gamma/d\hat{s}dz}{\int_0^1dzd\Gamma/d\hat{s}dz+\int_{-1}^0dzd\Gamma/d\hat{s}dz}. \qquad (2.6)$$

To define the polarization asymmetries we define the orthogonal unit vectors S in the rest frame of ℓ^- for the polarization of the lepton ℓ^- [5,16] in the longitudinal direction (L), the normal direction (N), and the transverse direction (T):

$$S_L^\mu\equiv(0,\mathbf{e}_L)=\left(0,\frac{\mathbf{p}_-}{|\mathbf{p}_-|}\right),$$

$$S_N^\mu\equiv(0,\mathbf{e}_N)=\left(0,\frac{\mathbf{q}\times\mathbf{p}_-}{|\mathbf{q}\times\mathbf{p}_-|}\right),$$

$$S_T^\mu\equiv(0,\mathbf{e}_T)=(0,\mathbf{e}_N\times\mathbf{e}_L), \qquad (2.7)$$

where $\mathbf{p}_-$ and $\mathbf{q}$ are the three-momenta of the ℓ^- and the particle P in the center-of-mass frame of the $\ell^-\ell^+$ system. Now, on boosting all three vectors given in Eq. (2.7), only the longitudinal vector will be boosted, and two (normal and transverse) will remain the same. The longitudinal vector after the boost becomes

$$S_L^\mu=\left(\frac{|\mathbf{p}_-|}{m_\ell},\frac{E_-\mathbf{p}_-}{m_\ell|\mathbf{p}_-|}\right). \qquad (2.8)$$

Now we can calculate the polarization asymmetries by using the spin projectors for ℓ^- as $1/2(1+\gamma_5\not{S})$. The lepton polarization asymmetries are defined as

$$P_x(\hat{s})\equiv\frac{d\Gamma(S_x)/d\hat{s}-d\Gamma(-S_x)/d\hat{s}}{d\Gamma(S_x)/d\hat{s}+d\Gamma(-S_x)/d\hat{s}} \qquad (2.9)$$

with $x=L,T,N$, respectively, for longitudinal, transverse, and normal polarization asymmetry.

Now ready with the terminology and definitions we will move to the calculations of the various measurable quantities that we mentioned before.

III. $B\to\pi\ell^+\ell^-$ DECAY MODE

In this section we calculate the branching ratio, FB asymmetry, and polarization asymmetries associated with the inclusive decay mode $B\to\pi\ell^+\ell^-$. Using the definition of the form factors [Eqs. (B1)–(B3)] we can write down the matrix element for the $\bar{B}\to\pi$ transition[2] as[3]

$$\mathcal{M}^{\bar{B}\to\pi}=\frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{td}^*\{A(p_B)_\mu(\bar{\ell}\gamma_\mu\ell)+B(p_B)_\mu(\bar{\ell}\gamma_\mu\gamma_5\ell)+C(\bar{\ell}\gamma_5\ell)+D(\bar{\ell}\ell)\} \tag{3.1}$$

with

$$A=C_9^{\text{eff}}F_1(q^2)-2C_7^{\text{eff}}\tilde{F}_T(q^2), \tag{3.2}$$

$$B=C_{10}F_1(q^2), \tag{3.3}$$

$$C=m_\ell C_{10}\left\{-F_1(q^2)+\frac{(m_B^2-m_\pi^2)}{q^2}\times[F_0(q^2)-F_1(q^2)]\right\}+\frac{(m_B^2-m_\pi^2)}{2m_b}\times F_0(q^2)C_{Q_2}, \tag{3.4}$$

$$D=\frac{(m_B^2-m_\pi^2)}{2m_b}F_0(q^2)C_{Q_1}. \tag{3.5}$$

From the above expression of the matrix element [Eq. (3.1)] we can get the analytical expression of the decay rate as

$$\frac{d\Gamma(\bar{B}\to\pi\ell^+\ell^-)}{d\hat{s}}=\frac{G_F^2m_B^5\alpha^2}{3\times2^9\pi^5}|V_{tb}V_{td}^*|^2\lambda^{1/2}(1,\hat{s},\hat{m}_\pi^2)\times\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\Sigma_\pi \tag{3.6}$$

with

$$\Sigma_\pi=\lambda(1,\hat{s},\hat{m}_\pi^2)\left(1+\frac{2\hat{m}_\ell^2}{\hat{s}}\right)|A|^2+\left[\lambda(1,\hat{s},\hat{m}_\pi^2)\left(1+\frac{2\hat{m}_\ell^2}{\hat{s}}\right)+24\hat{m}_\ell^2\right]|B|^2+6\frac{\hat{s}}{m_B^2}|C|^2+6\frac{\hat{s}}{m_B^2}\left(1-\frac{4\hat{m}_\ell^2}{\hat{s}}\right)|D|^2+12\frac{\hat{m}_\ell}{m_B}(1+\hat{s}-\hat{m}_\pi^2)\text{Re}(C^*B). \tag{3.7}$$

The expression for the FB asymmetry is

$$A_{FB}(\bar{B}\to\pi\ell^+\ell^-)=-6\hat{m}_\ell\lambda^{1/2}(1,\hat{s},\hat{m}_\pi^2)\times\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\frac{\text{Re}(AD^*)}{(m_B\Sigma_\pi)}. \tag{3.8}$$

As we can see from the above expression, the FB asymmetry is proportional to the new interactions, i.e., the NHB contributions. This is a point that has also been noted in some earlier work [8].

We divide this section into two subsections. In the first one we discuss the *CP* violation in $B\to\pi\ell^+\ell^-$ and in the next subsection we discuss the polarization asymmetries associated with the final state leptons.

A. *CP* violation

First we define the *CP*-violating partial width asymmetry between B and $\bar{B}$ decay. This is defined as

$$A_{CP}(\hat{s})=\frac{d\Gamma/d\hat{s}-d\bar{\Gamma}/d\hat{s}}{d\Gamma/d\hat{s}+d\bar{\Gamma}/d\hat{s}} \tag{3.9}$$

where

$$\frac{d\Gamma}{d\hat{s}}=\frac{d\Gamma(\bar{B}\to\pi\ell^+\ell^-)}{d\hat{s}},\quad \frac{d\bar{\Gamma}}{d\hat{s}}=\frac{d\Gamma(B\to\bar{\pi}\ell^+\ell^-)}{d\hat{s}}. \tag{3.10}$$

In going from Γ to $\bar{\Gamma}$ the only change we have to make is in the expression for C_9^{eff}. We define C_9^{eff} as

$$C_\alpha^{\text{eff}}=\xi_1+\lambda_u\xi_2 \tag{3.11}$$

where ξ_1, ξ_2, and λ_u are all complex. In going from $\bar{B}\to\pi\ell^+\ell^-$ to $B\to\bar{\pi}\ell^+\ell^-$, C_9^{eff} becomes

$$C_9^{\text{eff}}=\xi_1+\lambda_u^*\xi_2. \tag{3.12}$$

With this change one can get the expression for $d\bar{\Gamma}/d\hat{s}$ as

$$\frac{d\Gamma(B\to\bar{\pi}\ell^+\ell^-)}{d\hat{s}}=\frac{G_F^2m_B^5\alpha^2}{3\times2^9\pi^5}|V_{tb}V_{td}^*|^2\lambda^{1/2}(1,\hat{s},\hat{m}_\pi^2)\times\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\{\Sigma_\pi+4\,\text{Im}\,\lambda_u\Delta_\pi\} \tag{3.13}$$

with

$$\Delta_\pi=\left\{\text{Im}(\xi_1^*\xi_2)|F_1(s)|^2-2C_7^{\text{eff}}\,\text{Im}\,\xi_2F_T(s)F_1(s)\frac{m_b}{(m_B+m_\pi)}\right\}\times\lambda(1,\hat{s},\hat{m}_\pi^2)\left(1+\frac{2\hat{m}_\ell^2}{\hat{s}}\right). \tag{3.14}$$

Using Eqs. (3.6), (3.13), and (3.14) we can get the *CP*-violating partial width asymmetry as

$$A_{CP}(\hat{s})=\frac{-2\,\text{Im}\,\lambda_u\Delta_\pi}{\Sigma_\pi+2\,\text{Im}\,\lambda_u\Delta_\pi}. \tag{3.15}$$

[2] $\bar{B}$ actually is B^+.

[3] In writting this we have used $\ell\not{q}\ell=0$, $\bar{\ell}\not{q}\gamma_5\ell=2m_\ell\ell\gamma_5\ell$.

As argued in earlier work [13], by measuring the FB asymmetries of B and $\overline{B}$ one can observe the CP-violating phase of the CKM matrix. We will here estimate how much the predictions of the SM would change in SUSY.

In the discussion of CP violation by measuring FB asymmetry, it is important to fix the sign convention. The signs of the FB asymmetry for B and $\overline{B}$ decays are different. In fact, under strict CP conservation

$$A_{FB}(\overline{B}) = -A_{FB}(B). \tag{3.16}$$

So under CP conservation the FB asymmetries of B and $\overline{B}$ are exactly opposite.[4] So any change in Eq. (3.15) will be a measure of CP violation. We define the CP-violating parameter of FB asymmetry as

$$\delta_{FB} = A_{FB}(\overline{B} \to \pi \ell^+ \ell^-) + A_{FB}(B \to \overline{\pi} \ell^+ \ell^-). \tag{3.17}$$

We can get the expression for δ_{FB} from the expression for FB asymmetry [Eq. (3.8)] as

$$\begin{aligned}\delta_{FB} = {} & \frac{12 \hat{m}_\ell \lambda^{1/2}(1,\hat{s},\hat{m}_\pi^2)\sqrt{1-4\hat{m}_\ell^2/\hat{s}}}{m_B \Sigma_\pi (\Sigma_\pi + 4 \,\mathrm{Im}\,\lambda_u \Delta_\pi)} \\ & \times \mathrm{Im}\,\lambda_u [\Sigma_\pi F_1(s) \mathrm{Im}\,\xi_2 + \Delta_\pi [2 C_7^{\mathrm{eff}} \widetilde{F}_T(s) + F_1(s) \\ & + \mathrm{Im}\,\xi_2 \,\mathrm{Im}\,\lambda_u - F_1(s) \mathrm{Re}\,\xi_1 - F_1(s) \mathrm{Re}\,\xi_2 \,\mathrm{Re}\,\lambda_u]].\end{aligned} \tag{3.18}$$

B. Polarization asymmetries

We can also get the expression for the polarization asymmetries in the $\overline{B} \to \pi \ell^+ \ell^-$ transition using the formalism given in Sec. II. The expressions for the various polarization asymmetries of ℓ^- are

$$P_L = 0, \tag{3.19}$$

$$P_T = 0, \tag{3.20}$$

$$\begin{aligned}P_N = {} & \frac{\pi D \lambda^{1/2}(1,\hat{s},\hat{m}_\pi^2)\sqrt{\hat{s}-4\hat{m}_\ell^2}}{\Sigma_\pi} \\ & \times [\mathrm{Im}\,\xi_1 + \mathrm{Im}\,\lambda_u \,\mathrm{Re}\,\xi_2 + \mathrm{Im}\,\xi_2 \,\mathrm{Re}\,\lambda_u].\end{aligned} \tag{3.21}$$

[4]We can understand this negative sign because under CP conjugation not only does $b \leftrightarrow \overline{b}$ occur but there is a transformation in leptons also and $\ell^+ \leftrightarrow \ell^-$. Since the two leptons are emitted back to back in the c.m. frame of dileptons, the FB asymmetry defined in terms of the negatively charged lepton ℓ^- (for both B and $\overline{B}$) changes sign under CP conjugation.

IV. $B \to \rho \ell^+ \ell^-$ DECAY MODE

In this section we will calculate the possible measurables associated with the inclusive decay mode $B \to \rho \ell^+ \ell^-$. Using the definition of the form factors for the $B \to \rho$ transition given by Eqs. (C1), (C2), (C3), we can write down the matrix element as

$$\begin{aligned}\mathcal{M}^{\overline{B} \to \rho} = {} & [i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu *} p_B^\beta q^\beta A + \epsilon_\mu^* B + (\epsilon^* \cdot q)(p_B)_\mu C](\overline{\ell} \gamma^\mu) \\ & + [i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu *} p_B^\alpha q^\beta D + \epsilon_\mu^* E + (\epsilon^* \cdot q)(p_B)_\mu F] \\ & \times (\overline{\ell} \gamma^\mu \ell) + G(\epsilon^* \cdot q)(\overline{\ell}\,\ell) + H(\epsilon^* \cdot q)(\overline{\ell} \gamma_5\, \ell)\end{aligned} \tag{4.1}$$

where

$$A = 4 \frac{C_7^{\mathrm{eff}}}{s} m_b T_1(s) + C_9^{\mathrm{eff}} \frac{V(s)}{(m_B + m_\rho)}, \tag{4.2}$$

$$\begin{aligned}B = {} & -2 \frac{C_7^{\mathrm{eff}}}{s} m_b (m_B^2 - m_\rho^2) T_2(s) \\ & - \frac{1}{2}(m_B + m_\rho) A_1(s) C_9^{\mathrm{eff}},\end{aligned} \tag{4.3}$$

$$\begin{aligned}C = {} & 4 \frac{C_7^{\mathrm{eff}}}{s} m_m \left\{ T_2(s) + \frac{s}{(m_B^2 - m_\rho^2)} T_3(\hat{s}) \right\} \\ & + C_9^{\mathrm{eff}} \frac{A_2(s)}{m_B + m_\rho},\end{aligned} \tag{4.4}$$

$$D = C_{10} \frac{V(s)}{m_B + m_\rho}, \tag{4.5}$$

$$E = -\frac{1}{2}(m_B + m_\rho) A_1(s), \tag{4.6}$$

$$F = C_{10} \frac{A_2(s)}{m_B + m_\rho}, \tag{4.7}$$

$$G = -C_{Q_1} \frac{m_\rho A_0(s)}{m_b}, \tag{4.8}$$

$$\begin{aligned}H = {} & -C_{Q_2} \frac{m_\rho A_0(s)}{m_b} - C_{10} \frac{m_\ell A_2(s)}{m_B + m_\rho} \\ & + \frac{2 m_\rho m_\ell}{s} [A_3(s) - A_0(s)] C_{10}.\end{aligned} \tag{4.9}$$

From the above expression for the matrix element we can get the expression for the partial decay rate:

$$\begin{aligned}\frac{d\Gamma(\overline{B} \to \rho \ell^+ \ell^-)}{d\hat{s}} = {} & \frac{G_F^2 m_B^5 \alpha^2}{3 \times 2^{10} \pi^5} |V_{tb} V_{td}^*|^2 \lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2) \\ & \times \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \Sigma_\rho\end{aligned} \tag{4.10}$$

with

$$\Sigma_\rho=\left(1+\frac{2\hat{m}_\ell^2}{\hat{s}}\right)\lambda(1,\hat{s},\hat{m}_\rho^2)\left[4m_B^2\hat{s}|A|^2+\frac{2}{m_B^2\hat{m}_\rho^2}\left(1+12\frac{\hat{m}_\rho^2\hat{s}}{\lambda(1,\hat{s},\hat{m}_\rho^2)}\right)|B|^2\right.$$
$$\left.+\frac{m_B^2}{2\hat{m}_\rho^2}\lambda(1,\hat{s},\hat{m}_\rho^2)|C|^2+\frac{2}{\hat{m}_\rho^2}(1-\hat{m}_\rho^2+\hat{s})\mathrm{Re}(B^*C)\right]+4m_B^2\lambda(1,\hat{s},\hat{m}_\rho^2)(\hat{s}-4\hat{m}_\ell^2)|D|^2$$
$$+\frac{2}{m_B^2}\frac{[2(2\hat{m}_\ell^2+\hat{s})-2(2\hat{m}_\ell^2+\hat{s})(\hat{m}_\rho^2+\hat{s})+2\hat{m}_\ell^2(\hat{m}_\rho^4-26\hat{m}_\rho^2+\hat{s}^2)+\hat{s}(\hat{m}_\rho^4+10\hat{m}_\rho^2\hat{s}+\hat{s}^2)]}{\hat{m}_\rho^2\hat{s}}|E|^2$$
$$+\frac{m_B^2}{2\hat{m}_\rho^2\hat{s}}\lambda(1,\hat{s},\hat{m}_\rho^2)\{(2m\hat{m}_\ell^2+\hat{s})[\lambda(1,\hat{s},\hat{m}_\rho^2)+2\hat{s}+2\hat{m}_\rho^2]-2[2\hat{m}_\ell^2(\hat{m}_\rho^2-5\hat{s})+\hat{s}(\hat{m}_\rho^2+\hat{s})]\}|F|^2$$
$$-3\frac{4\hat{m}_\ell^2-\hat{s}}{\hat{m}_\rho^2}\lambda(1,\hat{s},\hat{m}_\rho^2)|G|^2+3\frac{\hat{s}}{\hat{m}_\rho^2}\lambda(1,\hat{s},\hat{m}_\rho^2)|H|^2+\frac{2\lambda(1,\hat{s},\hat{m}_\rho^2)}{\hat{m}_\rho^2\hat{s}}[-2\hat{m}_\ell^2(\hat{m}_\rho^2-5\hat{s})+(2\hat{m}_\ell^2+\hat{s})$$
$$-\hat{s}(\hat{m}_\rho^2+\hat{s})]\mathrm{Re}(E^*F)+\frac{12\hat{m}_\ell}{m_B\hat{m}_\rho^2}\lambda(1,\hat{s},\hat{m}_\rho^2)\mathrm{Re}(H^*E)+\frac{2m_B\hat{m}_\ell}{\hat{m}_\rho^2}\lambda(1,\hat{s},\hat{m}_\rho^2)(1-\hat{m}_\rho^2+\hat{s})\mathrm{Re}(H^*F). \quad (4.11)$$

For the $B\to\rho$ transition we can find the FB asymmetry as

$$A_{FB}(\overline{B}\to\rho\ell^+\ell^-)=\left\{-12\lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2)\sqrt{1-\frac{4\hat{m}_\ell^3}{\hat{s}}}\hat{s}[\mathrm{Re}(A^*D)+\mathrm{Re}(A^*E)]-3\frac{\hat{m}_\ell\lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2)}{\hat{m}_\rho^2}\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\right.$$
$$\left.\times\left[2\,\mathrm{Re}(G^*B)\frac{(1-\hat{m}_\rho^2-\hat{s})}{m_B}+\mathrm{Re}(G^*C)m_B\lambda(1,\hat{s},\hat{m}_\rho^2)\right]\right\}/\Sigma_\rho. \quad (4.12)$$

We will discuss the CP violation in the $B\to\rho$ transition and the polarization asymmetries associated with the final state lepton in the next subsections.

A. *CP* violation

To find the CP-violating partial width asymmetry we require the expression for the partial width of $B\to\overline{\rho}\ell^+\ell^-$. The expression for the partial decay rate for $B\to\overline{\rho}\ell^+\ell^-$ is

$$\frac{d\Gamma(B\to\overline{\rho}\ell^+\ell^-)}{d\hat{s}}=\frac{G_F^2m_B^5\alpha^2}{3\times2^{10}\pi^5}|V_{tb}V_{td}^*|^2\lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2)\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}(\Sigma_\rho+4\,\mathrm{Im}\,\lambda_u\;\Delta_\rho) \quad (4.13)$$

with

$$\Delta_\rho=\left[\mathrm{Im}(\xi_1^*\xi^2)\left\{4\hat{s}\frac{|V(s)|^2}{1+\hat{m}_\rho^2}+(1+\hat{m}_\rho^2)\left(\frac{6\hat{s}}{\lambda(1,\hat{s},\hat{m}_\rho^2)}+\frac{1}{2\hat{m}_\rho^2}\right)|A_1(s)|^2+\frac{\lambda(1,\hat{s},\hat{m}_\rho^2)}{2\hat{m}_\rho^2(1+m_\rho)^2}|A_2(s)|^2\right.\right.$$
$$\left.-\frac{1-\hat{m}_\rho^2-\hat{s}}{\hat{m}_\rho^2}A_1(s)A_2(s)\right\}+2\frac{C_2^{\mathrm{eff}}\hat{m}_b}{\hat{s}}\mathrm{Im}(\xi_2)\left\{8\frac{T_1(s)V(s)\hat{s}}{1+\hat{m}_\rho}+2A_1(s)T_2(s)(1+\hat{m}_\rho)^2(1-\hat{m}_\rho)\right.$$
$$\times\left(6\frac{\hat{s}}{\lambda(1,\hat{s},\hat{m}_\rho^2)}+\frac{1}{2\hat{m}_\rho^2}\right)+A_2(s)\left(T_2(s)+\frac{\hat{s}}{1-\hat{m}_\rho^2}T_3(s)\right)\frac{\lambda(1,\hat{s},\hat{m}_\rho^2)}{\hat{m}_\rho^2(1+\hat{m}_\rho)}$$
$$-(1+\hat{m}_\rho)A_1(s)\left(T_2(s)+\frac{\hat{s}}{1-\hat{m}_\rho^2}T_3(s)\right)$$
$$\left.\left.\times\frac{1-\hat{m}_\rho^2-\hat{s}}{\hat{m}_\rho^2}+A_2(s)T_2(s)(1-\hat{m}_\rho)\frac{1-\hat{m}_\rho^2-\hat{s}}{\hat{m}_\rho^2}\right\}\right]\left(1+\frac{2\hat{m}_\ell^2}{\hat{s}}\right)\lambda(1,\hat{s},\hat{m}_\rho^2). \quad (4.14)$$

Plugging in the expressions for the differential decay rates of $\overline{B}\to\rho\ell^+\ell^-$ and $B\to\overline{\rho}\ell^+\ell^-$ given by Eqs. (4.10) and (4.13), respectively, we can get the expression for the partial width CP asymmetry;

$$A_{CP}(\hat{s})=\frac{-2\,\mathrm{Im}\,\lambda_u\;\Delta_\rho}{\Sigma_\rho+2\,\mathrm{Im}\,\lambda_u\;\Delta_\rho} \tag{4.15}$$

with Σ_ρ and Δ_ρ as given in Eqs. (4.11) and (4.14).

Another measure of CP violation could be the sum of the FB asymmetries of $\overline{B}\rightarrow\rho\ell^+\ell^-$ and $B\rightarrow\overline{\rho}\ell^+\ell^-$. One can calculate this by use of Eq. (4.12) for $\overline{B}\rightarrow\rho\ell^+\ell^-$, and for $B\rightarrow\overline{\rho}\ell^+\ell^-$ by making appropriate changes in the expression for C_9^{eff}. The final expression is

$$\begin{aligned}\delta_{FB}&=A_{FB}(\overline{B}\rightarrow\rho\ell^+\ell^-)+A_{FB}(B\rightarrow\overline{\rho}\ell^+\ell^-)\\&=-6\lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2)\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\,\mathrm{Im}\,\lambda_u\frac{[\mathrm{Im}\,\xi^2\Gamma_1-2\Delta_\rho\{\mathrm{Re}(C_9^{\mathrm{eff}})\Gamma_1+(2C_7^{\mathrm{eff}}\hat{m}_b/\hat{s})\Gamma_2\}]}{\Sigma_\rho(\Sigma_\rho+4\,\mathrm{Im}\,\lambda_u\Delta_\rho)}\end{aligned} \tag{4.16}$$

with Γ_1 and Γ_2 given by

$$\Gamma_1=4\hat{s}A_1(s)V(s)C_{10}-\frac{\hat{m}_\ell(1-\hat{m}_\rho^2-\hat{s})(1+\hat{m}_\rho)}{2\hat{m}_\rho\hat{m}_b}A_0(s)A_1(s)C_{Q_1}+\frac{\hat{m}_\ell\lambda(1,\hat{s},\hat{m}_\rho^2)}{\hat{m}_\rho\hat{m}_b(1+\hat{m}_\rho)}A_0(s)A_2(s)C_{Q_1}, \tag{4.17}$$

$$\begin{aligned}\Gamma_2=&\,4\hat{s}T_2(s)V(s)(1-\hat{m}_\rho)C_{10}+4\hat{s}(1+\hat{m}_\rho)A_1(s)T_1C_{10}-\frac{(1-\hat{m}_\rho^2-\hat{s})(1-\hat{m}_\rho^2)\hat{m}_\ell}{\hat{m}_\rho\hat{m}_b}A_0(s)T_2(s)C_{Q_1}\\&\times\frac{2\lambda(1,\hat{s},\hat{m}_\rho^2)\hat{m}_\ell}{\hat{m}_\rho\hat{m}_b}A_0(s)\left(T_2(s)+\frac{\hat{s}}{1-\hat{m}_\rho^2}T_3(s)\right)C_{Q_1}.\end{aligned} \tag{4.18}$$

B. Polarization asymmetries

Finally, we calculate the three polarization asymmetries, namely, longitudinal, transverse, and normal, for $\overline{B}\rightarrow\rho\ell^+\ell^-$. The longitudinal polarization asymmetry (P_L) is

$$\begin{aligned}P_L=\Bigg\{&24\,\mathrm{Re}(A^*B)(1-\hat{m}_\rho^2-\hat{s})\hat{s}\left(-1+\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\right)+4m_B^2\lambda(1,\hat{s},\hat{m}_\rho^2)\hat{s}\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\,\mathrm{Re}(A^*D)\\&+\frac{1}{\hat{m}_\rho^2}\left(3+\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\right)\{2\,\mathrm{Re}(B^*E)[1+\hat{m}_\rho^2+2\hat{m}_\rho^2\hat{s}+\hat{s}^2-2(\hat{m}_\rho^2+\hat{s})]\\&+m_B^2\,\mathrm{Re}(C^*E)[1-3(\hat{m}_\rho^2+\hat{s})-(\hat{m}_\rho^2-\hat{s})(\hat{m}_\rho^2+\hat{s})+(3\hat{m}_\rho^4+2\hat{m}_\rho^2\hat{s}+3\hat{s}^2)]\}\\&+\frac{1}{\hat{m}_\rho^2}[\mathrm{Re}(B^*F)(1-\hat{m}_\rho^2-\hat{s})+\mathrm{Re}(C^*F)m_B^2\lambda(1,\hat{s},\hat{m}_\rho^2)]\Bigg[\left(3+\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\right)[1+\hat{m}_\rho^2(\hat{m}_\rho^2-\hat{s})-2\hat{m}_\rho^2]\\&+\left(3-7\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\right)\hat{s}(\hat{m}_\rho^2-\hat{s})-8\hat{s}\sqrt{1-\frac{4\hat{m}_\ell^2}{\hat{s}}}\Bigg]\Bigg\}\Bigg/\;\Sigma_\rho.\end{aligned} \tag{4.19}$$

The normal polarization asymmetry (P_N) is

$$\begin{aligned}P_N=&\,\lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2)\sqrt{(\hat{s}-4\hat{m}_\ell^2)}\,\pi\\&\times\Bigg[2\,\mathrm{Im}(E^*F)\frac{1+\hat{m}_\rho^2-\hat{s}}{\hat{m}_\rho^2}+2\,\mathrm{Im}(A^*E+B^*D)\\&+\frac{1}{4\hat{m}_\rho^2}\{2(1-\hat{m}_\rho^2-\hat{s})\mathrm{Im}(G^*B)\\&+m_B^2\lambda(1,\hat{s},\hat{m}_\rho^2)\mathrm{Im}(G^*C)\}\Bigg].\end{aligned} \tag{4.20}$$

Finally, the transverse polarization asymmetry (P_T) is

$$\begin{aligned}P_T=&\,\lambda^{1/2}(1,\hat{s},\hat{m}_\rho^2)\sqrt{\hat{s}}\hat{m}_\ell\pi\Bigg[-4\,\mathrm{Re}(A^*B)\\&+\frac{1}{4\hat{m}_\rho^2\hat{s}}\{2[2(1-\hat{m}_\rho^2-\hat{s})\mathrm{Re}(B^*E)\\&+m_B^2\lambda(1,\hat{s},\hat{m}_\rho^2)\mathrm{Re}(C^*E)]\}\Bigg].\end{aligned} \tag{4.21}$$

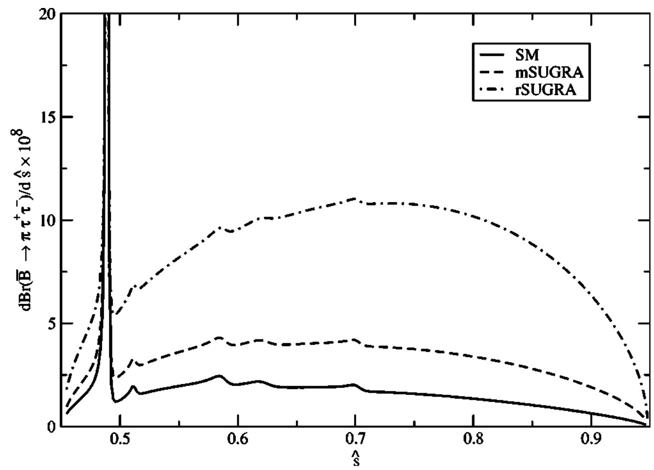

FIG. 1. Branching ratio for $\overline{B}\to\pi\tau^+\tau^-$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, $\mathrm{sgn}(\mu)$ is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

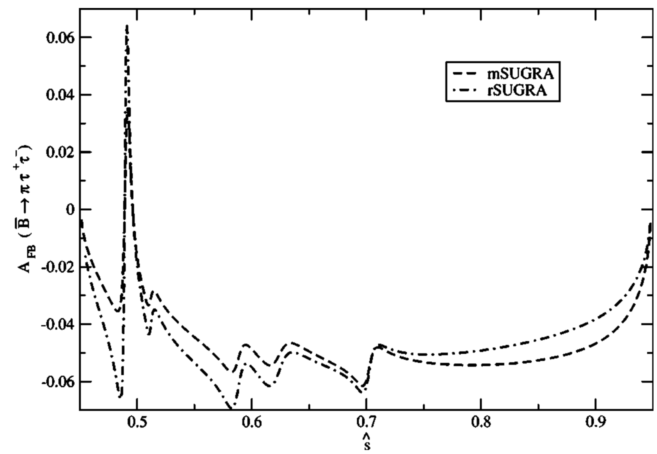

FIG. 2. FB asymmetry for $\overline{B}\to\pi\tau^+\tau^-$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, $\mathrm{sgn}(\mu)$ is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

V. RESULTS AND DISCUSSION

We have performed a numerical analysis of all the kinematical variables that we have evaluated in Secs. III and IV.

For our numerical analysis we could use the minimal supersymmetric standard model, which is the simplest (and the one having the least number of parameters) extension of the SM. Actually, the MSSM itself has a fairly large number of parameters, which makes it difficult to do phenomenology with it. We therefore resort to models that reduce the large parameter space of the MSSM to a manageable level. The models that exist include minimal supergravity (mSUGRA), no-scale, dilaton, etc., models. For our analysis we use the supergravity (SUGRA) models. The basic feature of all these models is that they assume some sort of unification of the parameters at some unifying scale. In the numerical analysis of SUGRA models the parameters we have chosen satisfy the radiative electroweak symmetry breaking condition.

In the mSUGRA model unification of all the scalar masses, all gaugino masses, and all coupling constants is assumed at grand unified theory (GUT) scale. So effectively we are left with five parameters (beyond the SM parameters) at the GUT scale. They are m (unified mass of all the scalars), M (unified mass of all the gauginos), A (unified trilinear coupling constants), $\tan\beta$ (the ratio of vacuum expectation values of the two Higgs doublets), and finally $\mathrm{sgn}(\mu)$.[5] As emphasized in many works [3,5,6] the universality of the scalar masses is not a necessary requirement and one can relax this. The only constraint for this relaxation is K^0-$\overline{K}^0$ mixing. To suppress this mixing it is sufficient to give a unified scalar mass to all the squarks but the Higgs sector can be given a different unified mass. We explore a sort of SUGRA model also, which we will call the rSUGRA model. In this model we will take the mass of the pseudoscalar Higgs boson to be another parameter. For our MSSM parameter space analysis we will take the 95% C.L. bound [21]

$$2\times10^{-4}<\mathrm{Br}(B\to X_s\gamma)<4.5\times10^{-4},$$

which is in agreement with CLEO and ALPEH results. We are primarily interested in finding the effects of NHBs and as it is emphasized in the literature that these effects become more profound when the final state leptons are τ [22]. Thus we will take the final state leptons to be τ here.

Our results are summarized in Figs. 1–12, where the spikes in the distributions are because of the charm resonances as given in Eq. (2.2). For $B\to\pi\tau^+\tau^-$, in Fig. 1 we have plotted the variation of the branching ratio with the scaled c.m. energy of the dileptons. As we can see from the graphs, the deviation from the respective SM values is fairly large for almost the whole region of invariant dilepton mass. The deviation is more profound for the rSUGRA model than for the mSUGRA model. In Fig. 2 we have plotted the FB asymmetry for the transition. As has already been noted in earlier work [8], in the SM the FB asymmetry vanishes. But if we consider SUSY then one can have a finite value of FB asymmetry. So any observation of FB asymmetry in this decay mode ($B\to\pi\tau^+\tau^-$) should be a clear signal of new physics. In Fig. 3 we have given estimates of the *CP*-violating partial width asymmetry. As expected from the result of Eq. (3.15) the new Wilson coefficients do not contribute to the numerator of the asymmetry but the denominator (which essentially is the decay rate) gets contributions from NHBs and hence the NHB effects actually lower the SM estimates of the *CP*-violating partial width asymmetry. The reduction is greater for the rSUGRA model where all the scalar masses are not unified; in this case one can take the Higgs boson mass as a parameter also.[6] The new Wilson coefficients C_{Q_1} and C_{Q_2} crucially depend on the Higgs

[5] Our sign convention for μ is such that μ enters the chargino mass matrix with positive sign.

[6] Here we have taken the Higgs pseudoscalar mass to be a parameter and all the rest of the Higgs boson masses can be evaluated in terms of this.

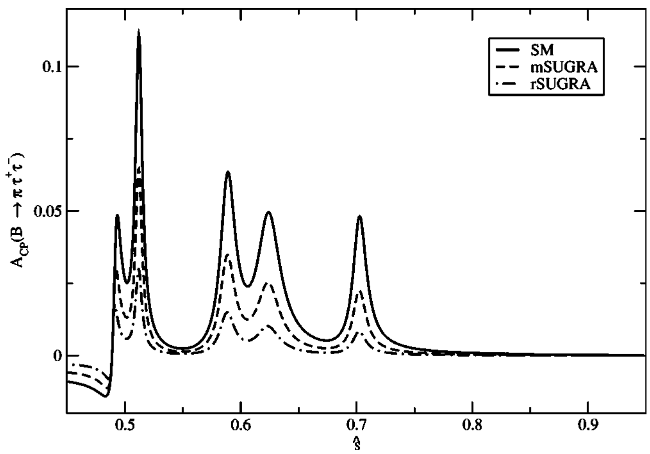

FIG. 3. CP-violating asymmetry (A_{CP}) in $\overline{B}\to\pi\tau^+\tau^-$ and $B\to\overline{\pi}\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

boson mass: if it is low their value is high. In the rSUGRA model one can have a lower Higgs boson mass and hence a high value of the new Wilson coefficients and a high partial decay rate, which effectively reduces the CP-violating partial width asymmetry. But there exists another measure of CP violation, the sum of the FB asymmetries of $\overline{B}\to\pi\ell^+\ell^-$ and $B\to\overline{\pi}\ell^+\ell^-$. This is a type of measurement which can be done in an environment having equal numbers of B and $\overline{B}$ pairs, and as argued earlier [13] it does not require any tagging. The important point here is that FB asymmetry in this decay is zero in the SM and hence the sum of the FB asymmetries of $\overline{B}\to\pi\tau^+\tau^-$ and $B\to\overline{\pi}\tau^+\tau^-$ is also zero. So the parameter δ_{FB} [which we introduced in Eq. (3.17)] is zero. But if we consider SUSY then this parameter can have a finite value. In fact, as we have shown in Fig. 4 δ_{FB} is greater for the rSUGRA model than for the mSUGRA model

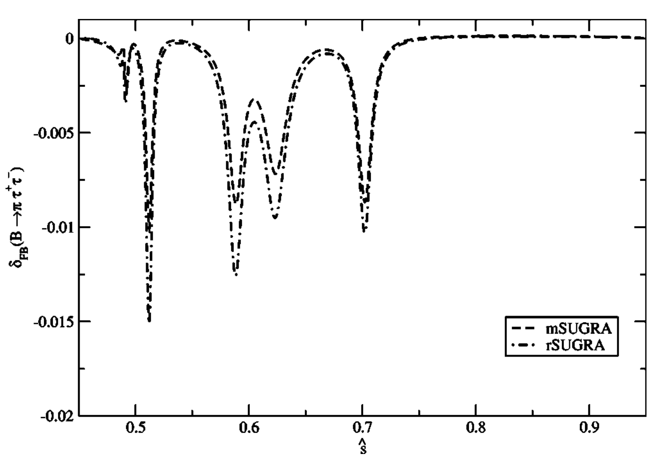

FIG. 4. CP-violating asymmetry (δ_{CP}) in $\overline{B}\to\pi\tau^+\tau^-$ and $B\to\overline{\pi}\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

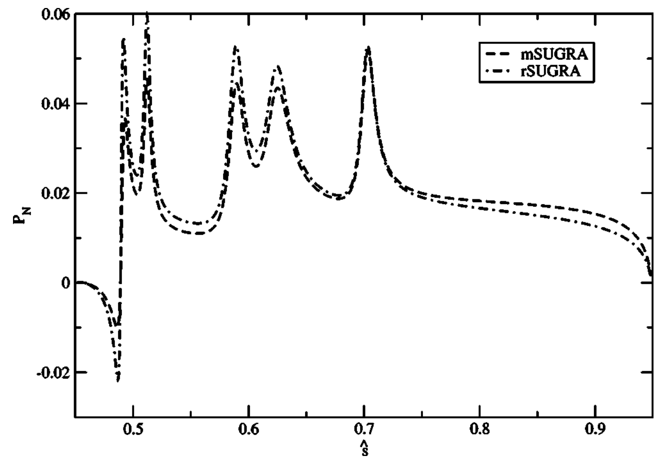

FIG. 5. Normal polarization asymmetry (P_N) in $\overline{B}\to\pi\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

(which is contrary to the partial width CP asymmetry). So this quantity could also turn out to be an important probe for new physics. In Fig. 5 we have plotted the normal polarization asymmetry of the final state lepton ℓ^-. As we can see from the expression of Eq. (3.21) the value of P_N is zero in the SM. So the observation of nonzero P_N could also be interpreted as a signal of some new physics.[7] Rest two polarization asymmetries the longitudinal (P_L) and transverse (P_T) vanishes with or without NHBs.

In Fig. 6 we have plotted the branching ratio of $\overline{B}\to\rho\tau^+\tau^-$ with a scaled invariant mass of dileptons. As we can see there is a fairly large deviation from the SM value. In Fig. 7 we have plotted the variation of FB asymmetry with $\hat{s}$; again one can observe the variation of mSUGRA and rSUGRA results from the SM values. Both the partial decay rate and FB asymmetry increase as compared to the SM values in both mSUGRA and rSUGRA models. In Fig. 8 we have plotted the CP-violating partial width asymmetry. As we can see, here the predictions of the mSUGRA and SUGRA models decrease as compared to the SM value. The reason is the same as explained for the $B\to\pi\tau^+\tau^-$ transition. But again here if we look at CP violation through the FB asymmetry (Fig. 9) we have the same effect as in $B\to\pi\tau^+\tau^-$; the SUGRA models have larger values than in the SM. Here the SM values of FB asymmetry as well as δ_{FB} are not zero, but still the SUGRA models give an enhancement of more than one order of magnitude (i.e., about a factor of 10) for almost the whole region of the invariant dileptonic mass. In Figs. 10, 11, 12 we have plotted the longitudinal (P_L), normal (P_N), and transverse (P_T) polarizations, respectively. All three show variation from the SM values but the general trend is that all these polarization asymmetries

[7] P_N is a T-odd observable because of the non-Hermiticity of the effective Hamiltonian, associated with the real $c\overline{c}$ intermediate states, so it cannot be taken as a measure of CP violation.

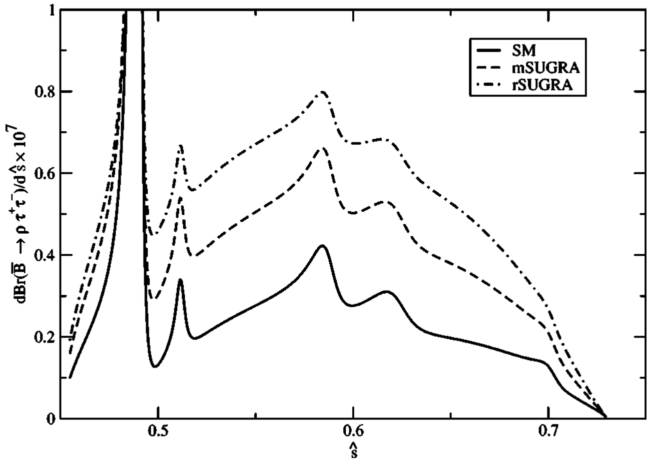

FIG. 6. Branching ratio for $\overline{B}\to\rho\tau^+\tau^-$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

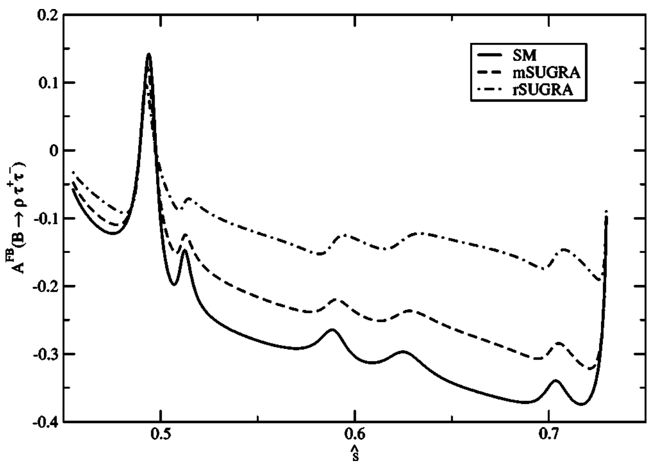

FIG. 7. FB asymmetry for $\overline{B}\to\rho\tau^+\tau^-$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

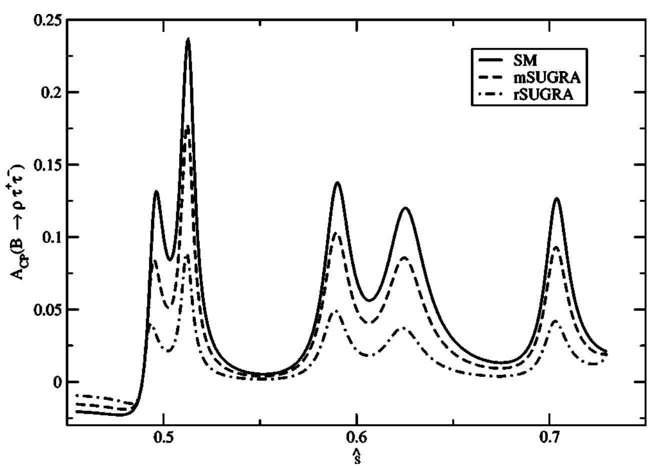

FIG. 8. *CP*-violating asymmetry (A_{CP}) in $\overline{B}\to\rho\tau^+\tau^-$ and $B\to\overline{\rho}\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

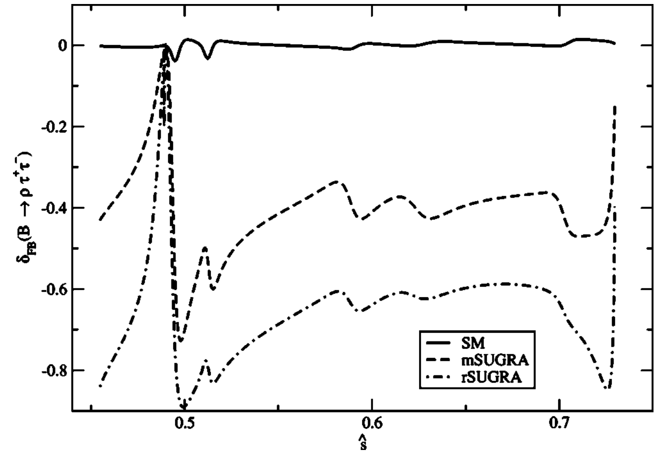

FIG. 9. *CP*-violating asymmetry (δ_{CP}) in $\overline{B}\to\rho\tau^+\tau^-$ and $B\to\overline{\rho}\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

tend to decrease in SUGRA models as compared to SM values for almost the whole region of invariant mass. For all the plots we have taken the variation of all the kinematical variables with the dileptonic invariant mass because the variation with respect to the invariant mass has more information than the result which we get after integrating over the invariant mass, and also the variation of kinematical variables with respect to the invariant mass is in principle accessible experimentally. In future B factories (like the Tevatron and LHCB) more than 10^{11} $b\overline{b}$ pairs are expected to be produced [21]; this is many orders more than the projected yield at the e^+e^-B factories, so these processes can be observed and some of the measurable quantities in these processes can be estimated. These processes ($B\to\pi\ell^+\ell^-$ and $B\to\rho\ell^+\ell^-$) are useful because they are relatively clean (both theoreti-

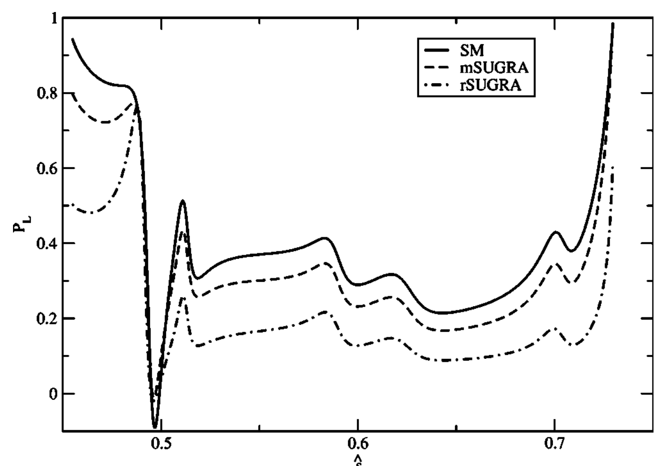

FIG. 10. Longitudinal polarization asymmetry (P_L) in $\overline{B}\to\rho\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

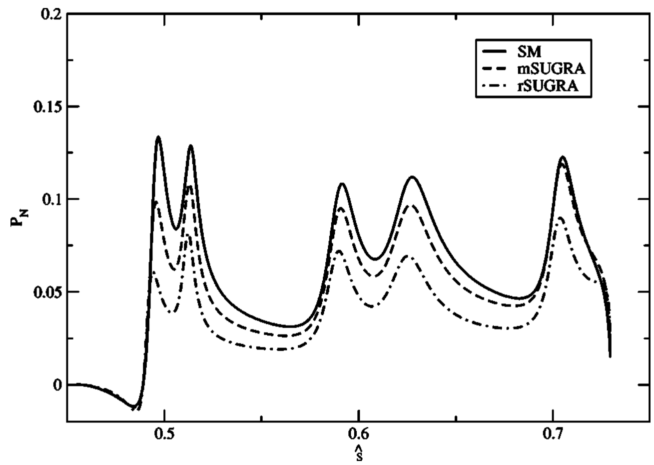

FIG. 11. Normal polarization asymmetry (P_N) in $\overline{B}\to\rho\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

cally and experimentally). Also if there is no new source of *CP* violation (except the CKM phase) then dileptonic decays $B\to\pi\ell^+\ell^-$ and $B\to\rho\ell^+\ell^-$ should be the first where *CP* violation can be observed. As we can see from Tables I and II, the kinematical variables of $B\to\rho\tau^+\tau^-$ look more promising because of their magnitude. But in $B\to\pi\tau^+\tau^-$ there are some distributions, like FB asymmetry, δ_{FB}, and P_N, which vanish in the SM but have finite (although small) values with SUSY. This point has already been noted about FB asymmetry in much other earlier work [8]. The same phenomena occur for δ_{FB} and P_N.

Finally, our conclusions regarding SUSY effects over a wide range of SUSY parameters can be summarized as follows.

(1) *Branching ratios*. The branching ratios for both $B\to\pi\tau^+\tau^-$ and $B\to\rho\tau^+\tau^-$ show large deviations from the corresponding SM values for almost the whole region of the invariant mass.

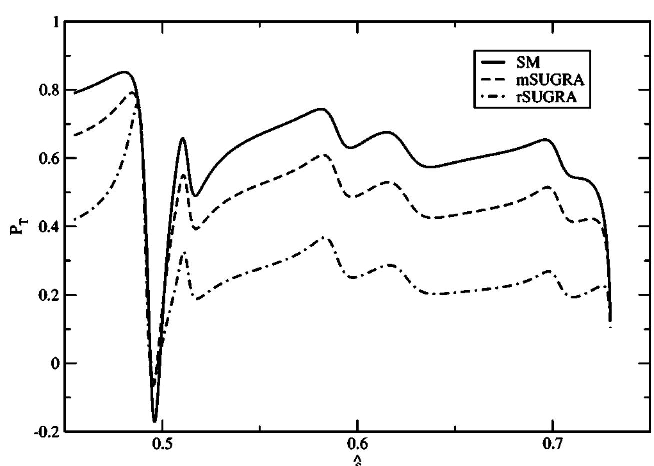

FIG. 12. Transverse polarization asymmetry (P_T) in $\overline{B}\to\rho\tau^+\tau^-$. The Wolfenstein parameters we have chosen are $\rho=-0.07$, $\eta=0.34$. Other parameters are for mSUGRA $m=200$, $M=450$, $A=0$, $\tan\beta=35$, sgn(μ) is taken to be positive. For rSUGRA $m_A=306$. All masses are in GeV.

TABLE I. Integrated kinematical variables for $B\to\pi\tau^+\tau^-$. The parameters for mSUGRA and rSUGRA are the same as given in Figs. 1–5.

Variable	SM	mSUGRA	rSUGRA
$d\Gamma/d\hat{s}\ \times\ 10^8$	2.6	3.43	5.04
$A_{FB}\ \times\ 10$	0	−0.224	−0.228
$A_{CP}\ \times\ 10^2$	0.51	0.2	0.1
$\delta_{FB}\ \times\ 10^3$	0	−0.6	−0.9
$P_N\ \times\ 10^2$	0	0.96	0.99

(2) *FB asymmetry*. The FB asymmetry for $B\to\pi\ell^+\ell^-$ vanishes within the SM. A nonvanishing FB asymmetry for $B\to\pi\ell^+\ell^-$ clearly gives indications of some new physics. The FB asymmetry for $B\to\rho\ell^+\ell^-$ shows significant increase in the mSUGRA and rSUGRA models as compared to SM values.

(3) *CP violating partial width asymmetry* (A_{CP}). The predictions of the mSUGRA and rSUGRA models are to reduce this asymmetry for both $B\to\pi\tau^+\tau^-$ and $B\to\rho\tau^+\tau^-$ as compared to SM values.

(4) *CP violation from FB asymmetry* (δ_{FB}). This observable vanishes in the SM for $B\to\pi\tau^+\tau^-$. A nonvanishing value of this clearly indicates new physics effects. For $B\to\rho\tau^+\tau^-$ the SM prediction is very low; both rSUGRA and mSUGRA models can give enhancement of over one order of magnitude for almost the whole region of the scaled dilepton invariant mass.

(5) *Polarization asymmetries*. For $B\to\pi\tau^+\tau^-$ all three polarization asymmetries (longitudinal, normal, and transverse) vanish in the SM. If we include NHB effects, although the longitudinal and transverse polarizations still remain zero, the normal polarization asymmetry becomes nonzero. So observation of the normal polarization asymmetry can still be regarded as evidence for new physics. For $B\to\rho\ell^+\ell^-$ all three polarization asymmetries decrease, with respect to their corresponding SM values, on switching on the NHB effects.

The observation of the decay modes $B\to\pi\ell^+\ell^-$ and $B\to\rho\ell^+\ell^-$ can be expected to be a very useful tool in the

TABLE II. Integrated kinematical variables for $B\to\rho\tau^+\tau^-$. The parameters for mSUGRA and rSUGRA are the same as given in Figs. 6–12.

Variable	SM	mSUGRA	rSUGRA
$d\Gamma/d\overline{s}\times10^8$	3.9	4.4	5.0
$A_{FB}\times10$	−0.72	−0.60	−0.32
$A_{CP}\times10$	0.13	0.09	0.04
δ_{FB}	0.0003	−0.11	−0.18
P_L	0.109	0.0924	0.055
$P_N\times10$	0.16	0.14	0.1
P_T	0.17	0.14	0.07

search for new physics effects, as well as for the measurement of the *CP*-violating parameters of the CKM matrix.

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APPENDIX A: INPUT PARAMETERS

$$m_u = m_d = 10 \text{ MeV},$$

$$m_b = 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_t = 176 \text{ GeV},$$

$$m_B = 5.26 \text{ GeV}, \quad m_\pi = 0.135 \text{ GeV},$$

$$m_\rho = 0.768 \text{ GeV},$$

$$|V_{tb}V_{td}^*| = 0.011, \quad \alpha = \frac{1}{129},$$

$$G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2},$$

$$m_\tau = 1.77 \text{ GeV}, \quad \tau_B = 1.54 \times 10^{-12}\ s,$$

$$\text{Wolfenstein parameters} \quad \rho = -0.07, \quad \eta = 0.34.$$

APPENDIX B: FORM FACTORS FOR $B \to \pi$ TRANSITION

We use the form factors given by Coleangelo *et al.* [23]:

$$\langle \pi(p_\pi)|\bar{d}\gamma_\mu P_{L,R} b|B(p_B)\rangle = \frac{1}{2}\left\{(2p_B - q)_\mu F_1(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} \times q_\mu [F_0(q^2) - F_1(q^2)]\right\}, \tag{B1}$$

$$\langle \pi(p_\pi)|\bar{d} i \sigma_{\mu\nu} q^\nu P_{L,R} b|B(p_B)\rangle = \frac{1}{2}\{(2p_B - q)_\mu - (m_B^2 - m_\pi^2) q_\mu\} \frac{F_T(q^2)}{m_B + m_\pi}. \tag{B2}$$

To get the matrix element for the scalar current we multiply Eq. (B1) by q_μ, giving

$$\langle \pi(p_\pi)|\bar{d} P_R b|B(p_B)\rangle = \frac{1}{2m_b}(m_B^2 - m_\pi^2) F_0(q^2) \tag{B3}$$

where we have neglected the mass of the d quark.

The definitions of the form factors F_0, F_1, and F_T are[8] (q^2 is in units of GeV^2)

$$F_0(q^2) = \frac{F_0(0)}{1 - q^2/7^2},$$

$$F_1(q^2) = \frac{F_1(0)}{1 - q^2/5.3^2},$$

$$F_T(q^2) = \frac{F_T(0)}{(1 - q^2/7^2)(1 - q^2/5.3^2)},$$

$$\tilde{F}_T(q^2) = \frac{F_T(q^2)}{(m_B + m_\pi)} m_b \tag{B4}$$

with $F_0(0) = 0$, $F_1(0) = 0.25$, and $F_T(0) = -0.14$.

APPENDIX C: FORM FACTORS FOR $B \to \rho$ TRANSITION

For the $B \to \rho$ transition we use the form factors given by Coleangelo *et al.* [23]:

$$\begin{aligned}\langle \rho(p_\rho)|\bar{d}\gamma_\mu P_L b|\bar{B}(p_B)\rangle &= i\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu *} p_B^\alpha q^\beta \frac{V(q^2)}{m_B + m_\rho} - \frac{1}{2}\left\{\epsilon_\mu (m_B + m_\rho) A_1(q^2)\right. \\ &\quad - (\epsilon^* \cdot q)(2p_B - q)_\mu \frac{A_2(q^2)}{m_B + m_\rho} - \frac{2m_\rho}{q^2}(\epsilon^* \cdot q) \\ &\quad \left.\times [A_3(q^2) - A_0(q^2)]\right\},\end{aligned} \tag{C1}$$

$$\begin{aligned}\langle \rho(p_\rho)|\bar{d} i \sigma_{\mu\nu} q^\nu P_{R,L} b|\bar{B}(p_B)\rangle &= -2i\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu *} p_B^\alpha q^\beta T_1(q^2) \pm [\epsilon_\mu^* (m_B^2 - m_\rho^2) \\ &\quad - (\epsilon^* \cdot q)(2p_B - q)_\mu] T_2(q^2) \pm (\epsilon^* \cdot q) \\ &\quad \times \left[q_\mu - \frac{q^2}{m_B^2 - m_\rho^2}(2p_B - q)_\mu\right] T_3(q^2)\end{aligned} \tag{C2}$$

where A_3 can be written in terms of A_1 and A_2, i.e.,

$$A_3(q^2) = \frac{m_B + m_\rho}{2m_\rho} A_1(q^2) - \frac{m_B - m_\rho}{2m_\rho} A_2(q^2). \tag{C3}$$

In the above equations ϵ is the polarization vector of ρ and $q = p_B - p_\rho$ is the momentum transfer.

To get the matrix element for the scalar (or pseudoscalar) current we multiply both sides of Eq. (C1) by q^μ. On simplifying we get

$$\langle \rho(p_\rho)|\bar{d} P_R b|B(p_B)\rangle = -\frac{m_\rho}{m_b}(\epsilon^* \cdot q) A_0(q^2). \tag{C4}$$

The definition of the form factors is[9] (q^2 is in units of GeV^2)

[8] The three F_0, F_1, and F_T are not independent. F_T can be related to F_0 and F_1 by the equation of motion and the relationship turns out to be $F_T = (m_B + m_\pi) m_b (F_0 - F_1)/q^2$.

[9] Here also T_3 can be related to A_3 and A_0 by the equation of motion and the relationship is $T_3 = m_B m_b (A_3 - A_0)/q^2$.

$$V(q^2)=\frac{V(0)}{q^2/5^2},$$

$$A_1(q^2)=A_1(0)(1-0.023\ \ q^2),$$

$$A_2(q^2)=A_2(0)(1+0.034\ \ q^2),$$

$$A_0(q^2)=\frac{A_3(0)}{1-q^2/4.8^2},$$

$$T_1(q^2)=\frac{T_1(0)}{1-q^2/5.3^2},$$

$$T_2(q^2)=T_2(0)(1-0.02\ \ q^2),$$

$$T_3(q^2)=T_3(0)(1+0.005\ \ q^2) \tag{C5}$$

with $V(0)=0.47$, $A_1(0)=0.37$, $A_2(0)=0.4$, $A_0(0)=0.3$, $T_1(0)=0.19$, $T_2(0)=0.19$, and $T_3(0)=-0.7$.

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