

# From QCD lattice calculations to the equation of state of quark matter

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We describe two-flavor QCD lattice data for the pressure at a nonzero temperature and vanishing chemical potential within a quasiparticle model. Relying only on thermodynamic consistency, the model is extended to nonzero chemical potential. The results agree with lattice calculations in the region of a small chemical potential.

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## I. INTRODUCTION

One of the fundamental issues which triggered, and has influenced since, heavy ion physics is the question of the phase structure and the thermodynamic properties of strongly interacting matter at energy densities above  $1 \text{ GeV}/\text{fm}^3$ . Under such conditions, exceeding the energy density in nuclei but still far away from the asymptotic regime, the coupling strength  $\alpha_s$  is large, which makes the theoretical description of the many-body problem challenging.

In the recent past the understanding of this field has become much more detailed. The phase diagram for QCD with  $n_f=2$  massless flavors, which is the case we will consider in the following, can be briefly described as follows (we refer to [1] for a recent review). At a zero quark chemical potential,  $\mu=0$ , the broken chiral symmetry of hadron matter is restored within the quark-gluon plasma, at a critical temperature  $T_c \approx 170 \text{ MeV}$ . It is thought that this second order transition persists also for nonzero  $\mu$ , thus defining a critical line, which changes to a first order transition line at the tricritical point. For small temperatures and  $\mu \gtrsim \mu_c$  one anticipates a color-superconducting phase of quark matter. The value of  $\mu_c$  is expected to be 100–200 MeV larger than the quark chemical potential  $\mu_n=307 \text{ MeV}$  in nuclear matter. Quantitative results for large  $\alpha_s$  can be obtained from first principles by lattice calculations which were, however, restricted to a nonzero temperature and  $\mu=0$  until very recently. Therefore, the described picture for  $\mu \neq 0$  is mainly based on general arguments combined with results from various models, including extrapolations of perturbative QCD.

As a phenomenological description of the thermodynamics of deconfined strongly interacting matter we proposed a quasiparticle model [2,3]. Its parameters are fixed by the lattice data at  $\mu=0$ . We then use the fact that within the model the thermodynamic potentials at zero chemical potential and  $\mu \neq 0$  are related by thermodynamic consistency. In [3] we analyzed lattice data for  $n_f=2$  flavors [4], and  $n_f=4$  [5], which were, however, still derogated by sizable lattice artifacts which have an effect on the absolute scaling of the data. We therefore introduced a constant effective number of de-

grees of freedom of the quasiparticles as an additional model parameter to obtain first qualitative estimates. Later we considered in [6] the lattice data [7], where also the physical case of 2+1 flavors was simulated. As the absolute scaling of the lattice data enters as important information, in particular near  $T_c$ , we pragmatically applied the continuum extrapolation of the data, which was proposed in [7] for  $T > 2 T_c$ , also for smaller temperatures. The results of this prescription can now be compared to new lattice data [8]. Meanwhile, there are other lattice calculations which allow us to test directly the assumptions underlying the quasiparticle model as well as, for the first time, some of its predictions for nonzero chemical potential.

We will therefore consider here the presently available lattice data for  $n_f=2$ . Based on that, we will fit and discuss the quasiparticle parameters at  $\mu=0$  in Sec. III. In Sec. IV, we will briefly summarize how to extend the model to nonzero chemical potential, and compare our findings with the results [9] from lattice simulations studying the region of small  $\mu$ . Section V concludes with the discussion of some physical implications.

## II. FINITE TEMPERATURE LATTICE DATA

The simulations [8] are performed on lattices with spatial extent  $N_\sigma=16$  and temporal sizes  $N_\tau=4$  and  $N_\tau=6$ , with an improved Wilson quark action and renormalized quark masses corresponding to fixed ratios  $m_{ps}/m_v$  of the pseudo-scalar to vector meson masses. We first consider the data for two light flavors, corresponding to  $0.6 \leq m_{ps}/m_v \leq 0.75$ . Although this is larger than the physical value, the results are almost insensitive to the ratio, which suggests that they are not too far from the chiral limit. As expected for the rather small lattice sizes, the results for  $N_\tau=4$  and 6 differ. However, we observe that normalizing the pressure data by  $p_0^{\text{cont}}/p_0^{N_\tau}$ , the ratio of the free limits in the continuum and on the lattice, improves considerably the consistency between the data sets. As a matter of fact, the normalized  $N_\tau=4$  data are in agreement with the normalized  $N_\tau=6$  data after rescaling by a constant of 1.14. This simple scaling behavior for

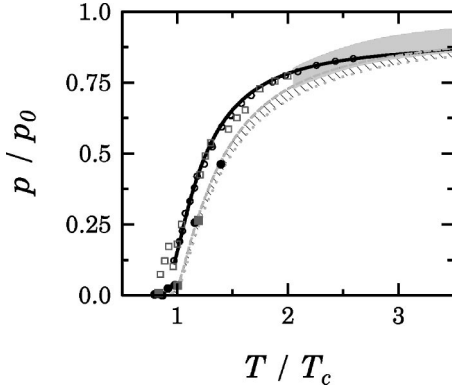


FIG. 1. Compilation of  $n_f=2$  lattice data for the pressure in units of the free pressure  $p_0$ . Shown are the scaled (see text) data [8] for light quarks corresponding to meson mass ratios of  $0.65 \leq m_{ps}/m_v \leq 0.75$  (open circles:  $N_\tau=4$ ; open squares:  $N_\tau=6$ ), and the continuum estimate [7] (gray band). The full line is the quasiparticle result. The full symbols represent the data [8] for large quark masses, with  $m_{ps}/m_v=0.95$ . For comparison, the hatched band shows the SU(3) lattice data (dotted line: [10]; dashed line: [11]) normalized to the corresponding free pressure.

large coupling is rather remarkable. Based on this observation we suggest the continuum estimate for the pressure shown in Fig. 1. We assume here that the normalized  $N_\tau=6$  data are already close to the continuum limit. This is supported by the fact that the thus interpreted data match the aforementioned continuum estimate from the staggered quark simulations [7].<sup>1</sup> Therefore, a consistent picture forms for the thermodynamics of QCD with  $n_f=2$  light flavors.

In Fig. 2, the corresponding data for the entropy are shown. It is noted that since the slope of the continuum extrapolated pressure [7] is slightly larger than that from the data [8] (see Fig. 1), the upper part of the error band is already for  $T \sim 3 T_c$  very close to the free limit. This would be in contrast to the pure gauge case, where the uncertainty due to lattice artifacts has become small, so we will assume that the lower side of this estimate is more relevant.

### III. QUASIPARTICLE MODEL

For completeness, we briefly recall here the main ideas of the quasiparticle model [2,3] of the QCD plasma.

For weak coupling  $g$ , the thermodynamic behavior of the system is dominated by its excitations with momenta  $\sim T$ . While hard collective modes (the longitudinal plasmon and the quark hole excitation) are exponentially suppressed, the transverse gluons and the quark particle excitations propagate predominantly on simple mass shells,  $\omega_i^2(k) \approx m_i^2 + k^2$  [12]. In the chiral limit the so-called asymptotic masses are given by

<sup>1</sup>In these calculations  $m_q=0.1 T$  was assumed, corresponding to  $m_{ps}/m_v=0.7$  at  $T_c$ . From the weak quark mass sensitivity observed in [8], both results should indeed be comparable.

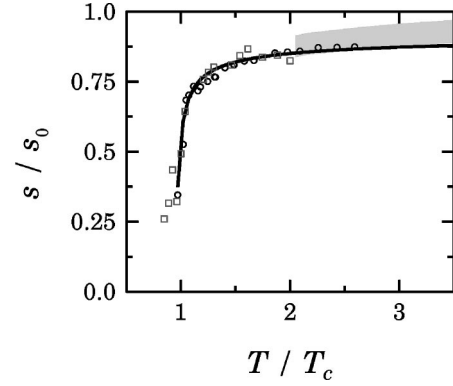


FIG. 2. The lattice data for the entropy corresponding to the data for the pressure shown in Fig. 1, and the quasiparticle fit.

$$m_g^2 = \frac{1}{6} \left[ \left( N_c + \frac{1}{2} n_f \right) T^2 + \frac{N_c}{2\pi^2} \sum_q \mu_q^2 \right] g^2, \quad (1)$$

$$m_q^2 = \frac{N_c^2 - 1}{8N_c} \left[ T^2 + \frac{\mu_q^2}{\pi^2} \right] g^2,$$

where  $\mu_q$  denotes the quark chemical potential, and  $N_c=3$ . Interpreting the relevant excitations as quasiparticles, the thermodynamic potential is

$$p(T, \mu) = \sum_i p_i(T, \mu_i(\mu); m_i^2) - B(m_j^2), \quad (2)$$

where  $p_i = \pm d_i T \int d^3k / (2\pi)^3 \ln(1 \pm \exp\{-(\omega_i - \mu_i)/T\})$  are the contributions of the gluons (with vanishing chemical potential) and the quarks (for the antiquarks, the chemical potential differs in the sign), and  $d_g = 2(N_c^2 - 1)$  and  $d_q = 2N_c$  count the degrees of freedom. As shown in [13], thermodynamic consistency requires the derivative, with respect to the  $m_j^2$ , of the right-hand side of Eq. (2) to vanish, i.e., the contribution  $B$  is related to the  $T$  and  $\mu$  dependent masses by

$$\frac{\partial B}{\partial m_j^2} = \frac{\partial p_j(T, \mu_j; m_j^2)}{\partial m_j^2}. \quad (3)$$

This implies that the entropy and the particle densities are simply given by the sum of the individual quasiparticle contributions,

$$s_i = \frac{\partial p_i(T, \mu_i; m_i^2)}{\partial T} \Big|_{m_i^2}, \quad n_i = \frac{\partial p_i(T, \mu_i; m_i^2)}{\partial \mu_i} \Big|_{m_i^2}, \quad (4)$$

while the energy density has the form  $e = \sum_i e_i + B$ .

Expanded in the coupling  $g$ , the above approach reproduces the leading-order perturbative results. The full expressions, however, represent a thermodynamically consistent resummation of terms of all orders in  $g$ . This suggests pondering the application of the model also in the strong cou-

pling regime.<sup>2</sup> Considering first the case  $\mu=0$ , it indeed turns out that the lattice data for the entropy shown in Fig. 2 can be described by the model with the ansatz

$$\alpha_s(T, \mu=0) = \frac{12\pi}{(11N_c - 2n_f) \ln[\lambda(T - T_s)/T_c]^2} \quad (5)$$

for  $g^2/(4\pi)$ . This is the leading order perturbative result at a momentum scale determined by the temperature:  $T_c/\lambda$  is related to the QCD scale  $\Lambda$ , while  $T_s$  parametrizes the behavior in the infrared. For the parameters we obtain<sup>3</sup>

$$\lambda = 17.1, \quad T_s = 0.89 T_c. \quad (6)$$

The resulting quasiparticle masses are large; near  $T_c$  they reach several times the value of the temperature.<sup>4</sup> The existence of such heavy excitations, which we have inferred from the thermodynamic bulk properties, has meanwhile been confirmed directly by lattice calculations of the propagators [17]. Finally, since the derivative of the ‘‘bag’’ function  $B$  is related to the quasiparticle masses by Eq. (3), the model is completely defined by fixing

$$B_0 = B(T_c) = 1.1 T_c^4, \quad (7)$$

which enters the fit in Fig. 1 as the third parameter.

Since all the information about the coupling is encoded in the parameters  $T_s$  and  $\lambda$ , it is interesting to look at their flavor dependence. Comparing to the pure gauge plasma, it is recalled that in this case the pressure becomes very small close to the transition since there it has to match the pressure of the heavy glue balls in the confined phase. Similarly, the entropy is small at  $T \sim T_c$ , which implies a large coupling there. For  $n_f=2$  the scaled entropy for  $T \sim T_c$  is somewhat larger; thus close to the transition the coupling has to be smaller than for pure SU(3). However, for fixed parameters  $\lambda$  and  $T_s$ , the coupling (5) would increase with increasing number of active flavors. Therefore, a difference of the parameters for  $n_f=2$  to those for the pure gauge plasma [3],

$$\lambda^{\text{SU}(3)} = 4.9, \quad T_s^{\text{SU}(3)} = 0.73 T_c, \quad (8)$$

is not unexpected. Interestingly, the parameter  $T_s$  does not change by much compared to the case of  $n_f=2$ .

<sup>2</sup>A formal reason supporting this attempt is the stationarity of the thermodynamic potential with respect to variation of the self-energies around the physical value; see [14] and the references given there. Moreover, there are heuristic arguments that resummation improved leading order results might be more appropriate at large coupling than high order perturbative results [15].

<sup>3</sup>For the fit we considered only the normalized data [8]. The result then reproduces the extrapolated data [7] on the lower side of the estimated error band; see the remark at the end of the last section.

<sup>4</sup>In an alternative approach, instead of attributing the deviations from the free limit at smaller temperatures to the mass of the quasiparticles, a variable number of degrees of freedom is proposed in [16].

#### IV. NONZERO CHEMICAL POTENTIAL

The quasiparticle model as applied in the previous section can be generalized to nonzero quark chemical potential  $\mu_q = \mu$ . The quasiparticle masses now depend also on  $\mu$ —explicitly by the dimensionful coefficients of the coupling in Eq. (1), and implicitly by the coupling itself. As shown in [3], Maxwell’s relation,  $\partial s/\partial \mu = \partial n/\partial T$ , directly implies a partial differential equation for  $\alpha_s(T, \mu)$ . It is of first order and linear in the derivatives of the coupling (but nonlinear in  $\alpha_s$ ),

$$c_T \frac{\partial \alpha_s}{\partial T} + c_\mu \frac{\partial \alpha_s}{\partial \mu} = C, \quad (9)$$

where the coefficients  $c_T$ ,  $c_\mu$  and  $C$  depend on  $T$ ,  $\mu$  and  $\alpha_s$ . It can easily be solved by reduction to a system of coupled ordinary differential equations,

$$\frac{dT(s)}{ds} = c_T, \quad \frac{d\mu(s)}{ds} = c_\mu, \quad \frac{d\alpha_s(s)}{ds} = C, \quad (10)$$

which determines the so-called characteristic curves  $T(s)$ ,  $\mu(s)$ , and the evolution of  $\alpha_s$  along such a curve, given an initial value.

With regard to the underlying physics it is worth pointing out some properties of the flow equation (9). The coefficients are combinations of products of a derivative of the quasiparticle entropy or density with respect to the quasiparticle mass, and a derivative of the quasiparticle mass with respect to  $T$ ,  $\mu$  or  $\alpha_s$ . Writing down the explicit expressions, it is easy to see that the flow equation is elliptic. In particular, one finds

$$c_T(T, \mu=0) = 0, \quad c_\mu(T=0, \mu) = 0. \quad (11)$$

The coefficient  $c_\mu$ , e.g., vanishes because not only the entropy goes to zero as  $T \rightarrow 0$ , but also its derivative with respect to the mass. Therefore, the characteristics are perpendicular to both the  $T$  and the  $\mu$  axes. This guarantees that specifying the coupling on some interval on the  $T$  axis sets up a valid initial condition problem. From the temperature dependence of the effective coupling as obtained from the lattice data at  $\mu=0$ , e.g. in the physically motivated parametrization (5), we can therefore determine numerically the coupling from Eq. (9), and hence the equation of state, in other parts of the  $\mu T$  plane.

It is instructive to consider the asymptotic limit,  $\alpha_s \rightarrow 0$ , of Eq. (9), where the coefficient  $C$  vanishes. Then the coupling is constant along the characteristics, which become ellipses in the variables  $T^2$  and  $\mu^2$ , leading to the mapping

$$T \rightarrow \left( \frac{9n_f}{4N_c + 5n_f} \right)^{1/4} \frac{\mu}{\pi}. \quad (12)$$

This holds approximately also for larger coupling, see Fig. 3, so the lattice data at  $\mu=0$  are mapped in elliptic strips into the  $\mu T$  plane. On the other hand, an ansatz analog to Eq. (5) to parametrize  $\alpha_s(T=0, \mu)$  is quantitatively less satisfactory

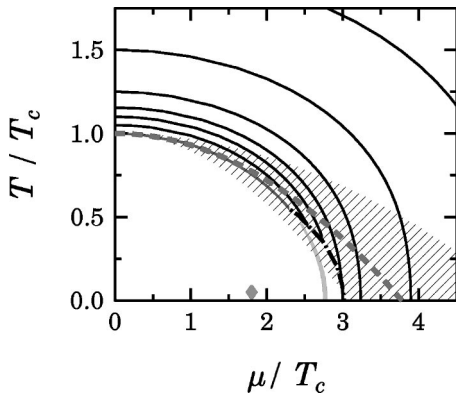


FIG. 3. Represented by the full lines are the characteristics of the flow equation (9). The characteristic through  $T_c$  coincides for small  $\mu$  with the critical line (dashed, with a hatched error band) obtained in the lattice calculation [9]. In the region under the dash-dotted line the resulting quasiparticle pressure is negative—a transition to another phase has to happen somewhere outside. Therefore, the narrow gray region under the  $p=0$  line, where the solution of the flow equation is not unique, is physically irrelevant. Indicated by the symbol (assuming, for the scaling,  $T_c=170$  MeV) is the chemical potential  $\mu_n$  in nuclear matter.

than in the case  $\mu=0$ . A closer look at the characteristics emanating from the interval  $[T_c, 1.06T_c]$  reveals that they intersect in a narrow half-crescent region, which indicates that there the solution of the flow equation is not unique. This, however, is only an ostensible ambiguity. It so happens that the extrapolation of the pressure becomes negative in a larger region; see Figs. 3 and 4. This implies that a transition to another phase, at a certain positive pressure, happens already outside this region, so the encountered ambiguity of the flow equation is of no physical relevance.<sup>5</sup>

At this point we emphasize again that this extrapolation of the quasiparticle model relies only on the requirement of thermodynamic consistency. Of course, it implicitly assumes also that the quasiparticle structure does not change, i.e., that deconfined quarks and gluons are the relevant degrees of freedom. For small enough  $\mu$  and temperatures above (or near, as  $\mu$  gets larger)  $T_c$  this is a justified assumption. However, the quasiparticle structure will change in the hadronic phase, when both  $T$  and  $\mu$  are small, as well as for sufficiently cold and dense systems where the color-superconducting phase is expected. Although the present quasiparticle model cannot make any statements about these phases, it is interesting to observe that it “anticipates” the existence of another phase only from the lattice input at  $T > T_c$  and  $\mu=0$ . An interpretation of the apparent similarity

<sup>5</sup>We remark that the region where the solution of the flow equation is not unique is determined only by  $\alpha_s(\mu=0, T)$ , i.e., by the parameters  $\lambda$  and  $T_s$  fitted from the entropy, whereas the  $p=0$  line depends also on  $p(\mu=0, T_c)$  and thus on the third parameter  $B_0$ . Therefore, the fact that the potential ambiguity is irrelevant is based in a nontrivial way on the underlying lattice data for the equation of state at  $\mu=0$ .

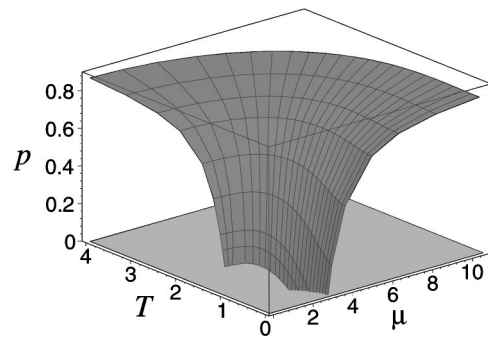


FIG. 4. The pressure scaled by the free pressure  $p_0(T, \mu)$ ;  $T$  and  $\mu$  are in units of  $T_c$ . The pressure along the characteristics starting out from  $T \sim T_c$  becomes negative at small  $T$ ; see also Fig. 3. The change to a different phase has to happen already outside this region.

of the line of vanishing pressure in Fig. 3 with the expected transition line from the hadron to the superconducting quark matter phase, see Ref. [1], remains, of course, a speculation.

There is, however, a related question which we can address with the quasiparticle model without knowing details about the other phases, just based on the fact that for nonzero chemical potential the transition from the deconfined to the confined phase occurs at the critical line  $T_c(\mu)$ . The critical line is expected to be perpendicular to the  $T$  axis, which has been confirmed in a recent lattice calculation [9] where also its curvature at  $\mu=0$  has been calculated,<sup>6</sup>  $T_c d^2 T_c(\mu)/d\mu^2|_{\mu=0} \approx -0.14$ . Within the quasiparticle model it is natural to relate, at least for small  $\mu$ , the critical line to the characteristic through  $T_c(\mu=0)$ , which, as shown above, is also perpendicular to the  $T$  axis. For small  $\mu$  where only the quadratic terms are relevant (practically even for  $\mu$  as large as  $2T_c$ ), we indeed find the  $T_c$  characteristic in a striking agreement with the critical line from [9]; see Fig. 3. Another argument supporting the above interpretation of the  $T_c$  characteristic comes from considering the case where the quark flavors have opposite chemical potentials,  $\mu_u = -\mu_d = \tilde{\mu}$ . With this isovector chemical potential the fermion determinant is positive definite, and standard Monte Carlo techniques can be applied to study this system on the lattice [18]. The lattice result [9] obtained for the curvature of the critical line in that case agrees with the value quoted above for the isoscalar potential  $\mu$ . Within the quasiparticle model, the equality of these two numbers is immediately evident.

In Ref. [9] it was furthermore mentioned that the quadratic behavior, with the same curvature as at  $\mu=0$ , of the critical line is not likely to extrapolate down to small transition temperatures since  $T_c(\mu)$  would then vanish only at  $\mu_c \sim 650$  MeV. Phenomenologically, however,  $\mu_c$  is expected to be not very much larger, say at most by 200 MeV, than the quark chemical potential  $\mu_n = 307$  MeV in nuclear

<sup>6</sup>In passing we note the amusing fact that the result agrees with the value from the bag model assuming free massless pions for the hadronic phase.

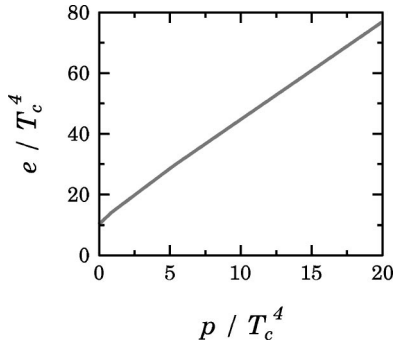


FIG. 5. The estimate for the equation of state of quark matter at  $T=0$ .

matter. In the quasiparticle model, from the chemical potential where the extrapolated pressure vanishes at  $T=0$ , we estimate  $\mu_c \approx 3 T_c \sim 500$  MeV.

This value is in the expected ballpark, which encourages us to consider the extrapolation of the model down to smaller temperatures. Although for  $T \rightarrow 0$  quark matter will be in the superconducting phase, it is still possible to give an estimate of its equation of state in that region from the quasiparticle model. The quark pairing influences thermodynamic bulk properties at the order of  $(\Delta \mu)^2$ , with the gap energy  $\Delta$  being at most 100 MeV [1]. This has little effect on the energy density  $e = \sum_i e_i + B$  as both the quasiparticle contributions and the function  $B$  are parametrically of the order  $\mathcal{O}(\mu^4)$ . For the pressure, on the other hand, the pairing effects become comparable to our expression  $p = \sum_i p_i - B$  only when the latter becomes small. Since the pressure of the thermodynamically favored superconducting phase is larger than that of the plasma phase, the relation  $e(p)$  as shown in Fig. 5 is therefore an upper estimate of the equation of state of cold quark matter. For  $p \geq 5 T_c^4$ , we obtain  $e(p) \approx 13 T_c^4 + 3.2 p$ , where the slope is mainly determined by the fact that the pressure at  $T=0$  essentially scales as  $\mu^4$ . For smaller pressure, the slope is only slightly larger, and the energy at  $p=0$  is approximately<sup>7</sup>  $11 T_c^4$ . Assuming  $T_c \approx 170$  MeV, this translates into an energy density of  $1 \text{ GeV}/\text{fm}^3$ . Bearing in mind that this is an upper estimate, and comparing to the bag model equation of state,  $e^{\text{bag}}(p) = 4\bar{B} + 3p$ , this result is still considerably larger than estimates with commonly assumed values of the bag constant  $\bar{B}$ .

Coming back to the region of the phase space where the quasiparticle model is well grounded, we finally address the question of the behavior of the pressure and the energy density along the critical line near  $\mu=0$ . In the lattice simulations [9] both quantities have been found to be constant

<sup>7</sup>This value renders more precisely the rough estimate [6], which was about 40% larger. Based on the pragmatic extension of the continuum extrapolation of the lattice data [7] shown in Fig. 1 near  $T_c$ , the fit led to a similar value for  $T_s$ , but to  $\lambda \approx 11$ . This demonstrates that details of the underlying lattice data are important for quantitative predictions at  $\mu \neq 0$  but, on the other hand, that the estimates are rather robust.

within the numerical errors. This is compatible with our result for small  $\mu$ ,

$$p(T_c(\mu), \mu) - p(T_c(0), 0) \approx -0.02 \mu^2 T_c^2. \quad (13)$$

The corresponding change of the energy density is about three times larger. These results differ notably from the estimate from the bag model which, although the critical line has a similar shape for small  $\mu$ , would yield coefficients larger by a factor of four.

## V. CONCLUSIONS

Within our quasiparticle model [2,3] we analyze recent  $n_f=2$  QCD lattice calculations [8] of the equation of state at nonzero temperature and  $\mu=0$ , and then extend the quasiparticle model to nonzero baryon density. The resulting elliptic flow equation for the coupling relates the thermodynamic potential along the characteristic curves in the  $\mu T$  plane. We argue that the characteristic line through  $T_c(\mu=0)$  is related to the critical line in the phase diagram. This is confirmed by comparing our results for the curvature of the critical line at  $\mu=0$ , and the variation of the equation of state along it, with recent lattice simulations [9] exploring the region of small  $\mu$ .

We give an estimate for the equation of state of cold quark matter. Energy density and pressure are almost linearly related, as in the bag model, however with parameters obtained from the lattice data at  $\mu=0$ . The relevant physical scale is given by the transition temperature  $T_c$ , and the parameter corresponding to the bag constant turns out to be large compared to conventional estimates,  $\geq 250 \text{ MeV}^4$ .

We have restricted ourselves to the case  $n_f=2$ , for which the lattice data for the equation of state at  $\mu=0$  appear to be established best. However, we expect similar results for other numbers of flavors since the pronounced decrease of the ratio  $p/p_0$  as  $T$  approaches  $T_c$ , which indicates a large coupling strength, seems to be generic. This universality is then echoed at nonzero  $\mu$  because for all  $n_f$  the flow equation behaves for strong coupling similarly as in the perturbative limit, where  $\alpha_s$  is constant along the elliptic-like characteristics. Indeed, the shape of the phase boundary calculated in [19] for the physically relevant case  $n_f=2+1$ , although now being a crossover near  $T_c$ , is very similar to the shape for  $n_f=2$ . With the same reasoning, we remark that our estimates are robust with respect to remaining uncertainties of the underlying lattice data. Indeed, the equation of state at  $\mu \neq 0$  is not very sensitive to the precise values of the model parameters as long as they reasonably describe the gross features of the equation of state at  $\mu=0$ . Therefore, the large energy density at small pressure seems to be a general feature of the equation of state.

As shown in [3,6], this would allow for pure quark stars with masses  $\leq 1 M_\odot$  and radii  $\leq 10$  km. Similar small and light quark stars have also been obtained within other approaches, cf. [20]. Such objects are of interest in the ongoing discussion of the data of the quark star candidate RXJ1856.5-3754 [21]. It should be emphasized, however, that the outermost layers of such pure quark stars are meta-

stable with respect to hadronic matter with a larger pressure at  $\mu \sim \mu_c$ . The details of the star structure depend sensitively on the hadronic equation of state [22]. However, as discussed in [23], a stable branch of hybrid stars with a dense quark core and a thin hadronic mantle could indeed be possible.

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