Operator analysis of neutrinoless double beta decay

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We study the effective operators of the standard model fields which would yield an observable rate of neutrinoless double beta decay. We particularly focus on the possibility that neutrinoless double beta decay is dominantly induced by lepton-number-violating higher dimensional operators other than the Majorana neutrino mass. Our analysis can be applied to models in which neutrinoless double beta decay is induced either by strong dynamics or by quantum gravity effects at a fundamental scale near the TeV scale as well as the conventional models in which neutrinoless double beta decay is induced by perturbative renormalizable interactions.

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I. INTRODUCTION

The neutrinoless double beta decay ($\beta\beta_{0\nu}$) provides a very sensitive probe of lepton-number (L) violating interactions. The most commonly quoted origin of $\beta\beta_{0\nu}$ is the ee component of Majorana neutrino mass matrix in the charged lepton mass eigenbasis, which is given by

$$(m^\nu)_{ee}=\sum_i U_{ei}^2 m_i$$
$$=c_{12}^2c_{13}^2e^{i\alpha_1}m_1+s_{12}^2c_{13}^2e^{i\alpha_2}m_2+s_{13}^2e^{i\alpha_3}m_3, \quad (1)$$

where $m_i(i=1,2,3)$ are the neutrino mass eigenvalues, θ_{ij} $(i\neq j)$ and α_i denote the mixing angles and CP phases in the 3×3 neutrino mixing matrix U, and $c_{ij}=\cos\theta_{ij}$, $s_{ij}=\sin\theta_{ij}$. Recently there has been a report to claim $\beta\beta_{0\nu}$ with a half-life of $\tau_{1/2}\approx10^{25}$ yr [1]. Though the claim is still debatable [2], some implications of this observation have already been discussed in many papers [3]. If the origin of $\beta\beta_{0\nu}$ were due to $(m^\nu)_{ee}$, the data suggest

$$|(m^\nu)_{ee}|=0.1-0.6 \text{ eV}. \quad (2)$$

When combined with the information from atmospheric and solar neutrino data, this value of $(m^\nu)_{ee}$ severely constrains the possible form of neutrino mass matrix. In particular, it does not allow the neutrino mass eigenvalues in normal hierarchy. As is well known, the atmospheric neutrino data imply [4]

$$\Delta m_{\rm atm}^2\equiv|m_3^2-m_2^2|\approx3\times10^{-3} \text{ eV}^2. \quad (3)$$

As for the solar neutrino anomaly, the following four solutions are possible [5]:

$$\Delta m_{\rm sol}^2=|m_2^2-m_1^2|\sin^2 2\theta_{12},$$
$$\text{LMA:} \quad 3.2\times10^{-5} \text{ eV}^2, \quad 0.75,$$
$$\text{SMA:} \quad 5.0\times10^{-6} \text{ eV}^2, \quad 2.4\times10^{-3},$$
$$\text{LOW:} \quad 1.0\times10^{-7} \text{ eV}^2, \quad 0.96,$$
$$\text{VAC:} \quad 8.6\times10^{-10} \text{ eV}^2, \quad 0.96, \quad (4)$$

where LMA, SMA, LOW, and VAC mean the large mixing angle Mikheyev-Smirnov-Wolfenstein (MSW), small mixing angle MSW, low mass, and vacuum oscillation solutions, respectively, and the numbers in each solution represent the best-fit values. There exists also a constraint on θ_{13} from the CHOOZ reactor experiment [6]

$$\sin\theta_{13}\lesssim0.2. \quad (5)$$

If the neutrino mass eigenvalues are in normal hierarchy, i.e., $m_3\gg m_2\gg m_1$, which is one of the plausible scenarios, the above information on θ_{ij} and m_i imply

$$|(m^\nu)_{ee}|=\begin{cases}(2-5)\times10^{-3} \text{ eV} \quad (\text{LMA}),\\ 10^{-6} \text{ eV}-\text{Max}(m_1,s_{13}^2m_3) \quad (\text{SMA}),\\ 10^{-4} \text{ eV}-s_{13}^2m_3 \quad (\text{LOW}),\\ 10^{-5} \text{ eV}-s_{13}^2m_3 \quad (\text{VAC}),\end{cases} \quad (6)$$

where

$$s_{13}^2m_3=2\times10^{-3}\left(\frac{s_{13}^2}{4\times10^{-2}}\right) \text{ eV}\lesssim2\times10^{-3} \text{ eV}. \quad (7)$$

Obviously, these values of $(m^\nu)_{ee}$ are too small to induce $\beta\beta_{0\nu}$ with $\tau_{1/2}\approx10^{25}$ yr. So if the claimed $\beta\beta_{0\nu}$ turns out to be correct, we would have either (approximately) degenerate neutrino masses or the observed $\beta\beta_{0\nu}$ is *not* due to $(m^\nu)_{ee}$, but due to some other L-violating interactions. This would be true as long as $\tau_{1/2}\ll10^{29}$ yr which can be tested in future experiments [7].

The possibility that $\beta\beta_{0\nu}$ is dominantly induced by L-violating interactions *other than* $(m^\nu)_{ee}$ has been discussed before in the context of specific models [8–14], and also a brief operator analysis of $\beta\beta_{0\nu}$ has been made in Ref. [15]. In this paper, we wish to provide a more detailed operator analysis of $\beta\beta_{0\nu}$ by studying generic L-violating but baryon-

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number (B) conserving higher-dimensional operators of the standard model (SM) fields which may induce $\beta\beta_{0\nu}$. The main focus will be given to the possibility that $\tau_{1/2}(\beta\beta_{0\nu}) \approx 10^{25}$ yr though $(m^{\nu})_{ee}$ is in the range of Eq. (6). Our result can also be easily matched to the previous studies on specific models in which $\beta\beta_{0\nu}$ is induced by perturbative renormalizable interactions. It can be applied to models in which $\beta\beta_{0\nu}$ is induced by either a strong dynamics or quantum gravity effects at energy scales near the TeV scale.

The organization of this paper is as follows. In Sec. II, we classify the L-violating operators of the SM fields which can trigger $\beta\beta_{0\nu}$. In Sec. III, we tabulate the contraint on those $\Delta L=2$ operators from the condition $\tau_{1/2}(\beta\beta_{0\nu}) \gtrsim 10^{25}$ yr, and also estimate $(m^{\nu})_{ee}$ which would be radiatively induced by the operators triggering $\beta\beta_{0\nu}$. In Sec. IV, we consider two specific models, i.e., a left-right symmetric model [16,11] and a model with scalar diquark and dilepton [17,13], which can give $\tau_{1/2}(\beta\beta_{0\nu}) \approx 10^{25}$ yr, while having $(m^{\nu})_{ee}$ in the range of Eq. (6). We match our results to the previous analysis on these models. Section V is the conclusion.

II. *L*-VIOLATING OPERATORS

In this section, we classify the L-violating but B-conserving operators of the SM fields which would trigger $\beta\beta_{0\nu}$. A complete analysis of $\Delta L=2$ operators which would induce a nonzero Majorana neutrino mass can be found in Ref. [15]. The $\Delta L=2\,\beta\beta_{0\nu}$ process may be induced by a double insertion of $\Delta L=1$ interactions or a single insertion of $\Delta L=2$ interaction. However with the SM fields alone, there is no way to construct a B-conserving operator with $\Delta L=1$. Since $\beta\beta_{0\nu}$ occurs at energy scales far below the weak scale, the effects of quark-flavor-changing SM interactions on $\beta\beta_{0\nu}$ are suppressed by the small Fermi constant and also the small quark-mixing angles. Also there is no renormalizable lepton-flavor-changing interaction in the SM. With these observations, we can ignore the effects of fermion-flavor violation, so limit the analysis to the $\Delta L=2$ operators containing only the first generation of quarks and leptons. We also limit our study only to the operators without spacetime derivative or gauge field.

We use a notation in which all fermions are two-component Weyl spinors, i.e., ψ is a left-handed spinor and $\bar{\psi}$ is its right-handed Hermitian conjugate. Generic fermion bilinear can be a Lorentz scalar, vector, or tensor:

$$(\psi\chi)_S=(\psi\chi), \quad (\psi\bar{\chi})_V=(\psi\sigma^{\mu}\bar{\chi}), \quad (\psi\chi)_T=(\psi\sigma^{\mu\nu}\chi),$$

where $\sigma_{\mu\nu}=(\sigma_{\mu}\bar{\sigma}_{\nu}-\sigma_{\nu}\bar{\sigma}_{\mu})/4$. Left-handed fermions relevant for $\beta\beta_{0\nu}$ are

$$\ell=(1,2)_{-1/2}, \quad e^c=(1,1)_1, \quad q=(3,2)_{1/6},$$
$$u^c=(\bar{3},1)_{-2/3}, \quad d^c=(\bar{3},1)_{1/3}, \tag{8}$$

where $SU(3)_C\times SU(2)_L\times U(1)_Y$ quantum numbers are indicated in parentheses. The SM Higgs doublet is denoted by

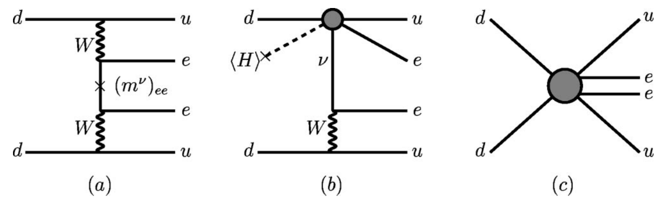

FIG. 1. Feynman diagrams for $\beta\beta_{0\nu}$. (a) corresponds to the conventional $\beta\beta_{0\nu}$ induced by $(m^{\nu})_{ee}$. (b) represents $\beta\beta_{0\nu}$ induced by the combined effects of a $d=7$, $\Delta L=2$ operator (dark blob) and the SM charged current weak interaction, while (c) represents $\beta\beta_{0\nu}$ induced by a $d=9$ or 11, $\Delta L=2$ operator.

$H=(h^+,h^0)=(1,2)_{1/2}$ and $\bar{H}$ is its Hermitian conjugate. We then have the following $\Delta L=2$ lepton bilinears:

$$(\ell^i\ell^j)_S=(\ell^j\ell^i)_S, \quad (\ell^i\ell^j)_T=-(\ell^j\ell^i)_T,$$
$$(\ell^i\overline{e^c})_V, \quad (\overline{e^c}\,\overline{e^c})_S, \tag{9}$$

where i,j are $SU(2)$-doublet indices.

There is a unique $\Delta L=2$, dimension$(d)=5$ operator [18]

$$\mathcal{L}_{d=5}=-\frac{\xi}{\Lambda}(\ell^i\ell^j)_S H^k H^l \epsilon_{ik}\epsilon_{jl}+\text{H.c.}, \tag{10}$$

where Λ denotes the mass scale of L-violating new physics which is assumed to exceed the weak scale, and the dimensionless ξ represents the strength of the couplings and/or the possible loop suppression factor involved in the mechanism to generate the above $d=5$ operator. After the electroweak symmetry breaking by $\langle H\rangle=(0,v/\sqrt{2})$ with $v=246$ GeV, it gives the ee component of the 3×3 Majorana neutrino mass matrix:

$$(m^{\nu})_{ee}=\frac{\xi v^2}{\Lambda}. \tag{11}$$

This neutrino mass is bounded to be less than 1 eV by $\beta\beta_{0\nu}$. Such a small neutrino mass can be a consequence of a very large value of Λ [19,20], e.g., $\xi\sim1$ and $\Lambda\sim10^{14}$ GeV. Alternatively, Λ can be of order TeV, but m^{ν} is small because ξ is small due to small couplings in the underlying dynamics which may be a consequence of some flavor symmetries [21], e.g., $\xi\sim10^{-11}$ and $\Lambda\sim1$ TeV. At any rate, when combined with the double insertion of the charged-current weak interaction, this Majorana neutrino mass leads to $\beta\beta_{0\nu}$ as in Fig. 1(a).

It is rather easy to see that there is no B-conserving $d=6$, $\Delta L=2$ operator. As for $d=7$, $\Delta L=2$ operators which would trigger $\beta\beta_{0\nu}$, we have

TABLE I. Numerical values of Φ_{λ_i} obtained using the results of Ref. [22].

λ_1	λ_2	λ_3	λ_4	λ_5
6.9×10^{-10}	2.9×10^{-8}	6.9×10^{-10}	2.0×10^{-7}	1.7×10^{-13}

$$\mathcal{L}_{d=7}=\frac{1}{\Lambda^3}[\lambda_1^{S,T}\epsilon_{ik}\epsilon_{lj}(\ell^i\ell^j)_{S,T}(q^k d^c)_{S,T}H^l + \lambda_2^T\epsilon_{ij}\epsilon_{kl}(\ell^i\ell^j)_T(q^k d^c)_T H^l + \lambda_3^{S,T}\epsilon_{jl}(\ell^i\ell^j)_{S,T}(\overline{q_i}\overline{u^c})_{S,T}H^l + \lambda_4^T\epsilon_{ij}(\ell^i\ell^j)_T(\overline{q_k}\overline{u^c})_T H^k + \lambda_5(\ell^i\overline{e^c})_V(d^c\overline{u^c})_V H^j\epsilon_{ij}] + \text{H.c.}, \tag{12}$$

where again Λ is the mass scale at which the above operators are generated. After the electroweak symmetry breaking, these $d=7$ operators yield the following 4-fermion operators:

$$\frac{v}{\sqrt{2}\Lambda^3}[\lambda_1\ (\nu_e e)_S(ud^c)_S+\lambda_2\ (\nu_e e)_T(ud^c)_T + \lambda_3\ (\nu_e e)_S(\overline{du^c})_S+\lambda_4\ (\nu_e e)_T(\overline{du^c})_T + \lambda_5\ (\nu_e\overline{e^c})_V(d^c\overline{u^c})_V]+\text{H.c.}, \tag{13}$$

where

$$\lambda_1=\lambda_1^S,\quad \lambda_2=\lambda_2^T-\lambda_1^T,\quad \lambda_3=\lambda_3^S,\quad \lambda_4=\lambda_4^T-\lambda_3^T.$$

When combined with a single insertion of the standard charged current weak interaction, the above $\Delta L=2$ four-fermi interactions lead to $\beta\beta_{0\nu}$ as in Fig. 1(b).

As for the operators with $d\geqslant 8$, we are interested in the operators which can induce $\beta\beta_{0\nu}$ *without* involving the SM weak interaction. Such operators should contain two electrons, so they can be written as

$$\mathcal{L}_{d\geqslant 8}=(\ell^i\ell^j)_S\mathcal{O}^S_{ij}+(\overline{e^c}\ell^i)_V\mathcal{O}^V_i+(\overline{e^c}\overline{e^c})_S\mathcal{O}^S, \tag{14}$$

where $\mathcal{O}_I=\{\mathcal{O}^S_{ij},\mathcal{O}^V_i,\mathcal{O}^S\}$ are the operators made of the quarks and Higgs fields. In order to be $\Delta Q_{\rm em}=2$, $\mathcal{O}_I$ must contain at least 4 quarks, so we need $d\geqslant 9$. Any $\Delta Q_{\rm em}=2$ four-quark operator can be written as a product of two $\Delta Q_{\rm em}=1$ quark-antiquark bilinears. $\mathcal{O}_I=J_I J_I'$, where the quark-antiquark bilinears J_I, J_I' can be either color-singlet or color-octet. The hadronic matrix element for $\beta\beta_{0\nu}$ can be approximated as $\langle Z+2|\mathcal{O}_I|Z\rangle\propto\langle p|J_I|n\rangle\langle p|J_I'|n\rangle$ for the neutron state $|n\rangle$ and the proton state $|p\rangle$, and then O_I with color octet J_I can be ignored in the operator analysis of $\beta\beta_{0\nu}$. The most general $d=9$ operators containing two color-singlet quark-antiquark bilinears together with two electrons are given by

$$\mathcal{L}_{d=9}=\frac{1}{\Lambda^5}[\kappa_1^{S,T}\epsilon_{ik}\epsilon_{jl}(\ell^i\ell^j)_S(q^k d^c)_{S,T}(q^l d^c)_{S,T} + \kappa_2^{S,T}(\ell^i\ell^j)_S(\overline{q_i}\overline{u^c})_{S,T}(\overline{q_j}\overline{u^c})_{S,T} + \kappa_3^{S,T}\epsilon_{kj}(\ell^i\ell^j)_S(q^k d^c)_{S,T}(\overline{q_i}\overline{u^c})_{S,T} + \kappa_3^V\epsilon_{ki}(\ell^i\ell^j)_S(q^k\overline{q_j})_V(d^c\overline{u^c})_V + \kappa_4^{S,T}\epsilon_{ji}(\ell^i\overline{e^c})_V(d^c\overline{u^c})_V(q^j d^c)_{S,T} + \kappa_5^{S,T}(\ell^i\overline{e^c})_V(d^c\overline{u^c})_V(\overline{q_i}\overline{u^c})_{S,T} + \kappa_6(\overline{e^c}\overline{e^c})_S(d^c\overline{u^c})_V(d^c\overline{u^c})_V]+\text{H.c.}, \tag{15}$$

where all quark bilinears in the parentheses are color- singlet. These $d=9$ operators give the following 6-fermion operators which would trigger $\beta\beta_{0\nu}$ as in Fig. 1(c):

$$\frac{1}{\Lambda^5}[\kappa_1^{S,T}(ee)_S(ud^c)_{S,T}(ud^c)_{S,T}+\kappa_2^{S,T}(ee)_S(\overline{du^c})_{S,T}(\overline{du^c})_{S,T} + \kappa_3^{S,T}(ee)_S(ud^c)_{S,T}(\overline{du^c})_{S,T}+\kappa_3^V(ee)_S(u\overline{d})_V(\overline{u^c}d^c)_V + \kappa_4^{S,T}(e\overline{e^c})_V(d^c\overline{u^c})_V(ud^c)_{S,T} + \kappa_5^{S,T}(e\overline{e^c})_V(d^c\overline{u^c})_V(\overline{du^c})_{S,T} + \kappa_6(\overline{e^c}\overline{e^c})_S(d^c\overline{u^c})_V(d^c\overline{u^c})_V]+\text{H.c.} \tag{16}$$

As for the next higher dimensional $d=11$ operators, we are interested in the operators which can *not* be obtained by multiplying the gauge-invariant Higgs bilinear $H^i\overline{H_i}$ to $d=9$ operators in Eq. (15). Among such operators, the following ones are relevant for $\beta\beta_{0\nu}$:

TABLE II. Numerical values of Φ_{κ_i} and Φ_{η_i} obtained using the results of Ref. [22].

$\kappa^S_{1,2,3},\eta^S_{5,6,7}$	$\kappa^T_{1,2,3},\eta^T_{5,6,7}$	$\kappa_6,\eta_{1,2}$	κ_3,η_7	$\kappa^S_{4,5},\eta^S_{3,4}$	$\kappa^T_{4,5},\eta^T_{3,4}$
6.2×10^{-13}	1.4×10^{-8}	3.5×10^{-11}	5.6×10^{-10}	1.4×10^{-12}	1.4×10^{-10}

$$\mathcal{L}_{d=11}=\frac{1}{\Lambda^7}[\eta_1'\epsilon_{km}\epsilon_{ln}(\ell^i\ell^j)_S(q^k\overline{q_i})_V(q^l\overline{q_j})_V H^m H^n+\eta_1''\epsilon_{kj}\epsilon_{ln}(\ell^i\ell^j)_S(q^k\overline{q_i})_V(q^l\overline{q_m})_V H^m H^n$$
$$+\eta_1'''\epsilon_{ik}\epsilon_{jm}(\ell^i\ell^j)_S(q^k\overline{q_l})_V(q^m\overline{q_n})_V H^l H^n+\eta_2(\ell^i\ell^j)_S(d^c\overline{u^c})_V(d^c\overline{u^c})_V\overline{H_i}\overline{H_j}$$
$$+\eta'^{S,T}_3\epsilon_{jl}\epsilon_{km}(\ell^i\overline{e^c})_V(q^j\overline{q_i})_V(q^k d^c)_{S,T}H^l H^m+\eta''^{S,T}_3\epsilon_{ij}\epsilon_{ml}(\ell^i\overline{e^c})_V(q^j\overline{q_k})_V(q^l d^c)_{S,T}H^k H^m$$
$$+\eta'''^{S,T}_3\epsilon_{il}\epsilon_{mj}(\ell^i\overline{e^c})_V(q^j\overline{q_k})_V(q^l d^c)_{S,T}H^m H^k+\eta'^{S,T}_4\epsilon_{jl}(\ell^i\overline{e^c})_V(q^j\overline{q_i})_V(\overline{q_k}\overline{u^c})_{S,T}H^k H^l$$
$$+\eta''^{S,T}_4\epsilon_{ji}(\ell^i\overline{e^c})_V(q^j\overline{q_k})_V(\overline{q_l}\overline{u^c})_{S,T}H^l H^k+\eta'''^{S,T}_4\epsilon_{jl}(\ell^i\overline{e^c})_V(q^j\overline{q_k})_V(\overline{q_i}\overline{u^c})_{S,T}H^l H^k$$
$$+\eta^{S,T}_5\epsilon_{ik}\epsilon_{jl}(\overline{e^c}\overline{e^c})_S(q^i d^c)_{S,T}(q^j d^c)_{S,T}H^k H^l+\eta^{S,T}_6(\overline{e^c}\overline{e^c})_S(\overline{q_i}\overline{u^c})_{S,T}(\overline{q_j}\overline{u^c})_{S,T}H^i H^j$$
$$+\eta^{S,T}_7\epsilon_{ik}(\overline{e^c}\overline{e^c})_S(q^i d^c)_{S,T}(\overline{q_j}\overline{u^c})_{S,T}H^j H^k+\eta_7\epsilon_{ik}(\overline{e^c}\overline{e^c})_S(q^i\overline{q_j})_V(d^c\overline{u^c})_V H^j H^k]. \quad (17)$$

After the EWSB, these $d=11$ operators give the following 6-fermion operators leading to $\beta\beta_{0\nu}$ as in Fig. 1(c):

$$\frac{v^2}{2\Lambda^7}[\eta_1(ee)_S(u\overline{d})_V(u\overline{d})_V+\eta_2(ee)_S(d^c\overline{u^c})_V(d^c\overline{u^c})_V$$
$$+\eta_3^{S,T}(e\overline{e^c})_V(u\overline{d})_V(ud^c)_{S,T}+\eta_4^{S,T}(e\overline{e^c})_V(u\overline{d})_V$$
$$\times(\overline{du^c})_{S,T}+\eta_5^{S,T}(\overline{e^c}\overline{e^c})_S(ud^c)_{S,T}(ud^c)_{S,T}$$
$$+\eta_6^{S,T}(\overline{e^c}\overline{e^c})_S(\overline{du^c})_{S,T}(\overline{du^c})_{S,T}+\eta_7^{S,T}(\overline{e^c}\overline{e^c})_S$$
$$\times(ud^c)_{S,T}(\overline{du^c})_{S,T}+\eta_7(\overline{e^c}\overline{e^c})_S(u\overline{d})_V(d^c\overline{u^c})_V]+\text{H.c.}, \quad (18)$$

where

$$\eta_1=\eta_1'+\eta_1''+\eta_1''',$$
$$\eta_3^{S,T}=\eta'^{S,T}_3+\eta''^{S,T}_3+\eta'''^{S,T}_3,$$
$$\eta_4^{S,T}=\eta'^{S,T}_4+\eta''^{S,T}_4+\eta'''^{S,T}_4. \quad (19)$$

III. CONSTRAINTS FROM $\beta\beta_{0\nu}$

To determine the $\beta\beta_{0\nu}$ rate induced by the operators presented in Sec. II, one needs to compute the nuclear matrix elements of the involved multi-quark operators.[1] In this paper, we will use the results of Ref. [22] for the necessary nuclear matrix elements. We will also assume that $\beta\beta_{0\nu}$ is dominated by one of the operators in (10),(12), (15) and (17), so ignore possible interference between the contributions from different operators. The resulting $\tau_{1/2}$ have several sources of uncertainties, e.g., the renormalization group (RG) evolution effects, hadronic uncertainties in the nuclear matrix elements, and also possible interference effects, however, still it can be used to constrain L-violating interactions with a reasonable accuracy.

If $\beta\beta_{0\nu}$ is induced dominantly by $(m^\nu)_{ee}$, one finds [1]

$$\tau_{1/2}^{-1}=1.1\times10^{-13}\left(\frac{v^2}{\Lambda m_e}\right)^2|\xi|^2\ \text{yr}^{-1}=7.4\times10^{-30}\left(\frac{(m^\nu)_{ee}}{4\times10^{-3}\ \text{eV}}\right)^2\ \text{yr}^{-1}. \quad (20)$$

In the case where $\beta\beta_{0\nu}$ is dominated by one of $d\geqslant7$ operators in Sec. II, the resulting $\tau_{1/2}^{-1}$ can be written as $\tau_{1/2}^{-1}=|\epsilon|^2\Phi_\epsilon$ where ϵ contains the operator coefficient, while Φ_ϵ contains the phase space factors and nuclear matrix elements depending on the Lorentz structure of the corresponding operator. Using the results of Ref. [22], the numerical values of Φ_ϵ can be obtained as summarized in Tables I and II. For $d=7$ operators of Eq. (12) giving the 4-fermion operators (13), $\beta\beta_{0\nu}$ occurs as in Fig. 1(b). We then find the corresponding half-life time

$$\tau_{1/2}^{-1}=\frac{1}{128}\left(\frac{v^3}{\Lambda^3}\right)^2[16|\lambda_{1,3,5}|^2,|\lambda_{2,4}|^2]\Phi_{\lambda_i}\ \text{yr}^{-1}, \quad (21)$$

where the numerical values of Φ_{λ_i} are listed in Table I. The upperbound on λ's resulting from the condition $\tau_{1/2}\gtrsim10^{25}$ yr are summarized in Table III.

It is also straightforward to compute $\tau_{1/2}$ for $\beta\beta_{0\nu}$ induced by $d=9$ and $d=11$ operators of Eqs. (15) and (17). For $d=9$ operators, we find

TABLE III. Upper bounds on the coefficient of $d=7$ operators from $\tau_{1/2}\geqslant10^{25}$ yr. Here $\Lambda_{\text{TeV}}=\Lambda/\text{TeV}$.

$\lambda_1/\Lambda^3_{\text{TeV}}$	$\lambda_2/\Lambda^3_{\text{TeV}}$	$\lambda_3/\Lambda^3_{\text{TeV}}$	$\lambda_4/\Lambda^3_{\text{TeV}}$	$\lambda_5/\Lambda^3_{\text{TeV}}$
2.3×10^{-6}	1.4×10^{-6}	2.3×10^{-6}	5.3×10^{-7}	1.4×10^{-4}

[1] In fact, since the $\Delta L=2$ operators are assumed to be generated at scale Λ, one also needs to compute the renormalization group evolution of the operators over the scales from Λ to $\Lambda_{\text{QCD}}\sim1$ GeV. Taking into account the effects of such renormalization group evolution is beyond the scope of this paper, so will be ignored.

TABLE IV. Upper bounds on the coefficients of $d=9$ operators.

$\kappa^S_{1,2,3}/\Lambda^5_{\rm TeV}$	$\kappa^T_{1,2,3}/\Lambda^5_{\rm TeV}$	$\kappa^V_3/\Lambda^5_{\rm TeV}$	$\kappa^S_{4,5}/\Lambda^5_{\rm TeV}$	$\kappa^T_{4,5}/\Lambda^5_{\rm TeV}$	$\kappa_6/\Lambda^5_{\rm TeV}$
2.3×10^{-1}	6.2×10^{-3}	7.7×10^{-3}	1.5×10^{-1}	3.1×10^{-2}	3.1×10^{-2}

$$\tau^{-1}_{1/2}=\left(\frac{m_p v^4}{8\Lambda^5}\right)^2[16|\kappa^S_{1,2,3,4,5}|^2,16|\kappa^V_3|^2,4|\kappa^T_{4,5}|^2, \times|\kappa^T_{1,2,3}|^2,16|\kappa_6|^2]\Phi_{\kappa_i}\ \ \text{yr}^{-1}, \tag{22}$$

and for $d=11$ operators

$$\tau^{-1}_{1/2}=\left(\frac{m_p v^6}{16\Lambda^7}\right)^2[16|\eta_{1,2,7}|^2,16|\eta^S_{3,4,5,6,7}|^2, \times4|\eta^T_{3,4}|^2,|\eta^T_{5,6,7}|^2]\Phi_{\eta_i}\ \ \text{yr}^{-1}, \tag{23}$$

where m_p is the proton mass and the numerical values of Φ_{κ_i} and Φ_{η_i} are listed in Table II. The resulting constraints on κ's and η's for $\tau_{1/2}\gtrsim10^{25}$ yr are summarized in Tables IV and V.

The above equations (21), (22), and (23) summarizing the $\beta\beta_{0\nu}$ rate of $d=7,9,11$ operators and the resulting constraints on the operator coefficients λ's, κ's, and η's listed in Tables III, IV, V are the main results of this paper. Still one of our major concern is the possibility that $\beta\beta_{0\nu}$ is induced dominantly by one of the $d\geqslant7$ operators, not by the $d=5$ operator for $(m^\nu)_{ee}$. This would occur, for instance, if some of the λ's or κ's saturate their bounds from $\beta\beta_{0\nu}$, while ξ is small enough to give $(m^\nu)_{ee}\ll1$ eV. In fact, the condition $(m^\nu)_{ee}\ll1$ eV constrains the coefficients of $d\geqslant7$ operators also since those operators can generate $(m^\nu)_{ee}$ through loops. For instance, the $d=7$ operator with coefficients $\lambda^S_{1,3}$ in Eq. (12) generate the $d=5$ operator for $(m^\nu)_{ee}$ through the one-loop diagram of Fig. 2(a), yielding

$$\delta_\lambda\xi\sim\frac{y_{d,u}}{16\pi^2}\lambda^S_{1,3}, \tag{24}$$

where we have taken the cutoff of the loop momenta to be Λ and $y_{d,u}$ is the down(up)-quark Yukawa couplings. Other $d=7$ operators can generate ξ also, however, it involves more loops and/or more insertions of small Yukawa couplings. For instance, the operator with coefficient λ_5 in Eq. (12) generates ξ through the 2-loop diagram of Fig. 2(b), yielding

$$\delta_\lambda\xi\sim\frac{y_u y_d y_e}{(16\pi^2)^2}\lambda_5, \tag{25}$$

TABLE V. Upper bounds on the coefficients of $d=11$ operators.

$\eta_{1,2}/\Lambda^7_{\rm TeV}$	$\eta^S_{3,4}/\Lambda^7_{\rm TeV}$	$\eta^T_{3,4}/\Lambda^7_{\rm TeV}$	$\eta^S_{5,6,7}/\Lambda^7_{\rm TeV}$	$\eta^T_{5,6,7}/\Lambda^7_{\rm TeV}$	$\eta_7/\Lambda^7_{\rm TeV}$
1.0	5.1	1.0	7.6	0.2	0.3

where y_e is the electron Yukawa coupling. As a result, $\delta_\lambda\xi$ from other $d=7$ operators are negligibly small compared to $\delta_\lambda\xi$ from $\lambda^S_{1,3}$.

Similarly, the $d=9$ operators with coefficients $\kappa^S_{1,2,3}$ in Eq. (15) generate the $d=5$ operator for $(m^\nu)_{ee}$ at two-loop order [Fig. 2(c)]:

$$\delta_\kappa\xi\sim\frac{(y_d^2,y_u^2,y_d y_u)}{(16\pi^2)^2}\kappa^S_{1,2,3}, \tag{26}$$

where again the cutoff of the loop momenta is chosen to be Λ. Other $d=9$ operators and also the $d=11$ operators can generate ξ, however, the corresponding diagrams involve more loops and/or more insertions of small Yukawa couplings. For instance, the $d=9$ operator with coefficient κ_6 generates ξ through the 4-loop diagram of Fig. 2(d), yielding

$$\delta_\kappa\xi\sim\frac{y_u^2 y_d^2 y_e^2}{(16\pi^2)^4}\kappa_6, \tag{27}$$

which is absolutely negligible even when κ_6 saturates the bound from $\beta\beta_{0\nu}$.

Combining Eqs. (20), (21), and (22) with Eqs. (24) and (26), one easily finds

$$\frac{\tau^{-1}_{1/2}(\lambda^S_{1,3})}{\tau^{-1}_{1/2}(\delta_\lambda\xi)}\sim3\times10^3\left(\frac{\text{TeV}}{\Lambda}\right)^4,$$

$$\frac{\tau^{-1}_{1/2}(\kappa^S_{1,2,3})}{\tau^{-1}_{1/2}(\delta_\kappa\xi)}\sim3\times10^7\left(\frac{\text{TeV}}{\Lambda}\right)^8, \tag{28}$$

implying that if the scale Λ of L-violating interactions is about 1 TeV, it is quite possible that $\beta\beta_{0\nu}$ is dominantly induced by one of $\Delta L=2$, $d=7$, or $d=9$ operators. In particular, one of $d=7$ or $d=9$ operators can give $\tau_{1/2}(\beta\beta_{0\nu})\approx10^{25}$ yr even when $(m^\nu)_{ee}$ is in the range of Eq. (6) as suggested by the atmospheric and solar neutrino oscillation data in normal neutrino mass hierarchy.

IV. APPLICATIONS TO SOME MODELS

Our results in the previous section can be applied to various kinds of models providing L-violating interactions for $\beta\beta_{0\nu}$ and/or neutrino mass. In this section, we consider some specific models of $\beta\beta_{0\nu}$ which have been discussed in the literature [11,13] and use our results to rederive the constraints on L-violating couplings from the condition $\tau_{1/2}(\beta\beta_{0\nu})\gtrsim10^{25}$ yr.

Let us first consider a model in which $d=7$ operators can be a dominant source of $\beta\beta_{0\nu}$. An example of such model is the left-right symmetric model [16,11] with gauge group

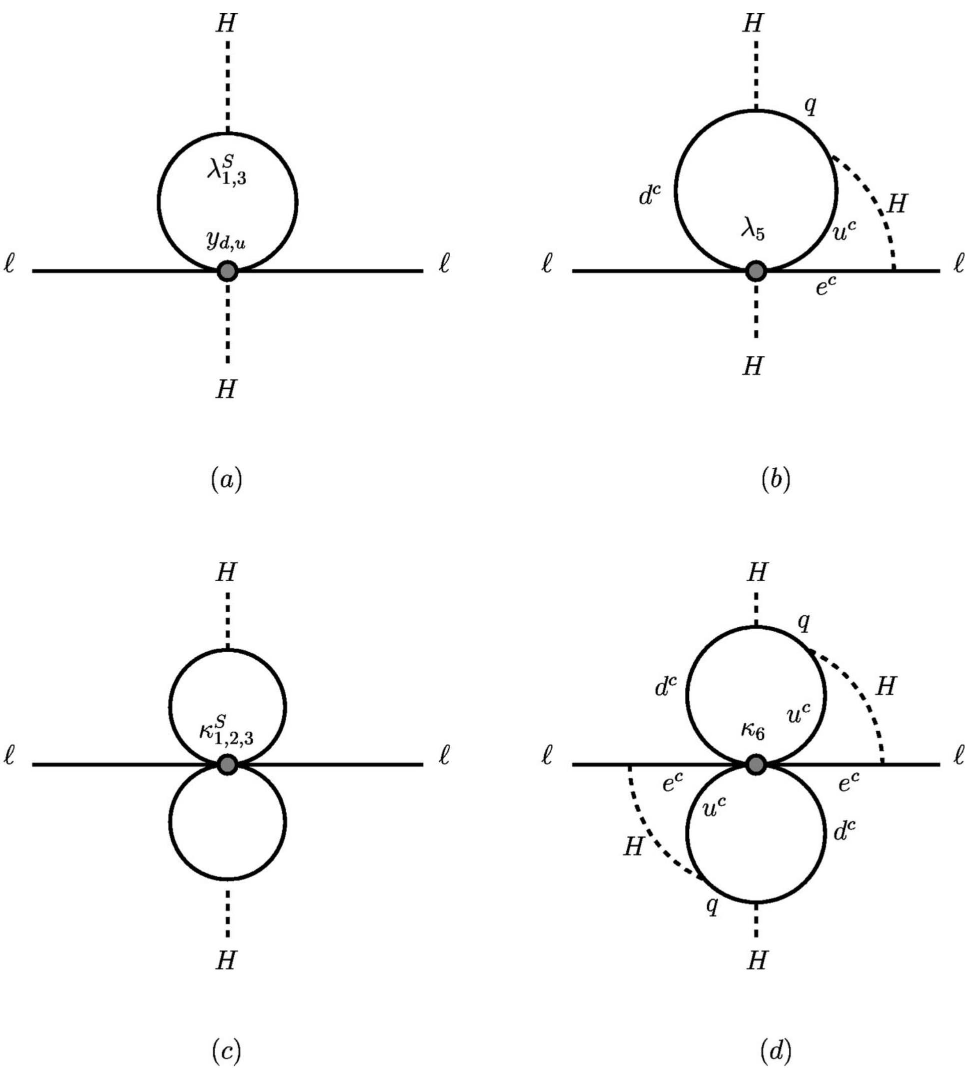

FIG. 2. Feynman diagrams for the $d=5$ operator for $(m^\nu)_{ee}$ radiatively induced by $d=7$ or $d=9$ operators.

$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$. The Higgs sector of the model contains a bidoublet ϕ and also triplets Δ_L and Δ_R whose $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ quantum numbers are given by

$$\Delta_L=(3,1)_2=\begin{pmatrix}\delta_L^+/\sqrt{2} & \delta_L^{++}\\ \delta_L^0 & -\delta_L^+/\sqrt{2}\end{pmatrix},$$

$$\Delta_R=(1,3)_2=\begin{pmatrix}\delta_R^+/\sqrt{2} & \delta_R^{++}\\ \delta_R^0 & -\delta_R^+/\sqrt{2}\end{pmatrix},$$

$$\phi=(2,2)_0=\begin{pmatrix}\phi_1^0 & \phi_1^+\\ \phi_2^- & \phi_2^0\end{pmatrix},$$

where the subscripts are the $U(1)_{B-L}$ charges. The model contains also the left- and right-handed lepton doublets

$$\ell_L=\begin{pmatrix}\nu\\ e\end{pmatrix}=(2,1)_{-1},\quad \ell_R=\begin{pmatrix}\overline{N^c}\\ \overline{e^c}\end{pmatrix}=(1,2)_{-1},$$

as well as the quark doublets

$$Q_L=\begin{pmatrix}u\\ d\end{pmatrix}=(2,1)_{1/3},\quad \overline{Q}_R=\begin{pmatrix}\overline{u^c}\\ \overline{d^c}\end{pmatrix}=(1,2)_{1/3}.$$

Yukawa interactions of the 1st generation are given by

$$\mathcal{L}_Y=h\ell_L\phi\ell_R+\tilde{h}\ell_L\sigma_2\phi^*\sigma_2\ell_R+h^Q Q_L\phi Q_R+\tilde{h}^Q Q_L\tilde{\phi}Q_R$$
$$+f\ell_L i\sigma_2\Delta_L\ell_L+f\ell_R i\sigma_2\Delta_R^\dagger\ell_R+\text{H.c.} \tag{29}$$

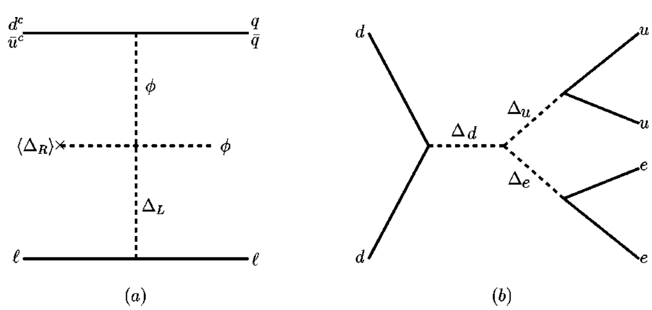

FIG. 3. (a) Feynman diagram for the $d=7$ operator with coefficients $\lambda^S_{1,3}$ in the left-right symmetric model. (b) Feynman diagram for the $d=9$ operator with coefficient κ_6 induced by the exchange of scalar diquarks and dilepton.

Parameters in the Higgs potential can be chosen to yield the Higgs vacuum expectation values $\langle \delta^0_{L,R}\rangle = v_{L,R}$ and $\langle \phi \rangle = \mathrm{diag}(\kappa,\kappa')$ with the scale hierarchy $v_R \gg \kappa \gg \kappa' \gg v_L$. Then the fermion masses of the model are given by $m_e \approx \tilde{h}\kappa$, $m^u \approx h^Q \kappa$, $m^d \approx \tilde{h}^Q \kappa$ for $\kappa \sim 180$ GeV, and the neutrino mass

$$(m^\nu)_{ee} \approx \frac{(h\kappa)^2 - 4f^2 v_L v_R}{2 f v_R}. \tag{30}$$

The model can generate also the $d=7$ operators of Eq. (12) (with coefficients $\lambda^S_{1,3}$) through the diagram of Fig. 3(a), yielding

$$\frac{\lambda^S_1}{\Lambda^3} \approx \frac{\gamma v_R f}{m^2_{\Delta_L}}\left(\frac{h^Q}{m^2_{\phi_1}} - \frac{\tilde{h}^Q}{m^2_{\phi_2}}\right),$$

$$\frac{\lambda^S_3}{\Lambda^3} \approx \frac{\gamma v_R f}{m^2_{\Delta_L}}\left(\frac{\tilde{h}^Q}{m^2_{\phi_1}} - \frac{h^Q}{m^2_{\phi_2}}\right), \tag{31}$$

where γ is the coefficient of the term $\mathrm{tr}(\Delta_L^\dagger \phi \Delta_R \phi^\dagger)$ in the Higgs potential. Then there exists a parameter range of the model in which $\beta\beta_{0\nu}$ is dominated by these $d=7$ operators. For instance, if $f v_R \sim m_{\Delta_L} \sim 10^5$ GeV, $m_\phi \sim 2\times 10^2$ GeV, and $\gamma \sim 10^{-1}$, the resulting $\lambda^S_{1,3}/\Lambda^3 \sim 10^{-6}/(\mathrm{TeV})^3$ saturates the bound listed in Table III, so lead to $\beta\beta_{0\nu}$ with $\tau_{1/2} \sim 10^{25}$ yr. Though not very natural, still the parameters of the model can be tuned to yield $(m^\nu)_{ee} = \mathcal{O}(10^{-3})$ eV, while keeping $\lambda^S_{1,3}/\Lambda^3 \sim 10^{-6}/(\mathrm{TeV})^3$. So the model can accomodate $\tau_{1/2}(\beta\beta_{0\nu}) \sim 10^{25}$ yr together with the atmospheric and solar neutrino oscillation data in the hierarchical neutrino mass scenario.

As an example of the model in which $d=9$ operators of Eq. (15) can be a dominant source of $\beta\beta_{0\nu}$, let us consider a model containing scalar diquarks and a scalar dilepton [17,13] with the following $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers:

$$\Delta_u = (6,1)_{4/3}, \quad \Delta_d = (6,1)_{-2/3}, \quad \Delta_e = (1,1)_{-2}. \tag{32}$$

The Yukawa couplings of the model are assumed to include

$$h_u \Delta_u u^c u^c + h_d \Delta_d d^c d^c + h_e \Delta_e e^c e^c \tag{33}$$

and also the Higgs potential contains

$$\mu \Delta_u \Delta_d^\dagger \Delta_e \tag{34}$$

which breaks L conservation.

With the above interactions, a $d=9$ operator for $\beta\beta_{0\nu}$ is generated at tree level as depicted in Fig. 3(b). The resulting operator corresponds to the κ_6 term of Eq. (15):

$$\frac{\kappa_6}{\Lambda^5}(\overline{e^c}\,\overline{e^c})_S (d^c \overline{u^c})_V (d^c \overline{u^c})_V, \tag{35}$$

where

$$\frac{\kappa_6}{\Lambda^5} = \frac{\mu h_u h_d h_e}{8 m^2_{\Delta_u} m^2_{\Delta_d} m^2_{\Delta_e}}. \tag{36}$$

If $h_{u,d,e} \sim 1$, $m_{\Delta_{u,d,e}} \sim 1$ TeV, and $\mu \sim 250$ GeV, the resulting $\kappa_6/\Lambda^5 \sim 3\times 10^{-2}/(\mathrm{TeV})^5$ saturates the bound in Table IV, so the model leads to $\beta\beta_{0\nu}$ with $\tau_{1/2} \sim 10^{25}$ yr. On the other hand, $(m^\nu)_{ee}$ induced by the L-violating interaction (34) is 4-loop suppressed and involves 6 powers of small Yukawa couplings as in Fig. 2(d):

$$(m^\nu)_{ee} \sim \frac{y_u^2 y_d^2 y_e^2}{(16\pi^2)^4} \frac{h_u h_d h_e \mu \langle H \rangle^2}{m^2_\Delta}. \tag{37}$$

This $(m^\nu)_{ee}$ is absolutely negligible, $(m^\nu)_{ee} \sim 10^{-30}$ eV, even when κ_6/Λ^5 saturates its bound. So one needs additional L-violating interactions to generate neutrino masses which would explain the atmospheric and solar neutrino oscillation data in the hierarchical neutrino mass scenario.

V. CONCLUSION

Motivated by the recent claim of observing $\beta\beta_{0\nu}$ with $\tau_{1/2} \approx 10^{25}$ yr, we studied the effective $\Delta L = 2$ operators of the SM fields which would generate $\beta\beta_{0\nu}$. We classified such operators up to mass dimension $d=11$, and find the upper bound on each operator coefficient resulting from the condition $\tau_{1/2} \gtrsim 10^{25}$ yr. Our results are summarized in Tables III, IV, V. We also examined the possibility that $d=7$ or 9 operators are a dominant source of $\beta\beta_{0\nu}$ in the context of generic operator analysis, particularly the possibility that $\tau_{1/2}(\beta\beta_{0\nu}) \sim 10^{25}$ yr, while $(m^\nu)_{ee} = \mathcal{O}(10^{-3})$ eV as suggested by the atmospheric and solar neutrino oscillation data in hierarchical neutrino mass scenario. As we have demonstrated in Sec. IV, our results can be easily matched to the previous analysis on specific models in which $\beta\beta_{0\nu}$ is induced by perturbative renormalizable interactions. They can be also applied to models in which $\beta\beta_{0\nu}$ is induced by either a strong dynamics or quantum gravity effects at scales near the TeV scale.

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