Gravitational waves from neutron stars with large toroidal *B* fields

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We show that NS's with large toroidal *B* fields tend naturally to evolve into potent gravitational-wave (GW) emitters. The toroidal field B_t tends to distort the NS into a prolate shape, and this magnetic distortion dominates over the oblateness "frozen into" the NS crust for $B_t \gtrsim 3.4 \times 10^{12}$ G ($\nu_s/300$ Hz)^2. An *elastic* NS with frozen-in *B* field of this magnitude is clearly secularly unstable: the wobble angle between the NS's angular momentum J^i and the star's magnetic axis n_B^i grows on a dissipation time scale until J^i and n_B^i are orthogonal. This final orientation is clearly the optimal one for GW emission. The basic cause of the instability is quite general, so we conjecture that the same final state is reached for a realistic NS, with superfluid core. Assuming this, we show that for LMXB's with $B_t \sim 2 \times 10^{12} - 2 \times 10^{14}$ G, the spindown from GW's is sufficient to balance the accretion torque—supporting a suggestion by Bildsten. The spindown rates of most millisecond pulsars can also be attributed to GW emission sourced by toroidal *B* fields, and both these sources could be observed by LIGO II. While the first-year spindown of a newborn NS is most likely dominated by electromagnetic processes, reasonable values of B_t and the (external) dipolar field B_d can lead to detectable levels of GW emission, for a newborn NS in our own Galaxy.

I. INTRODUCTION

Nearly periodic gravitational waves (GW's) from distorted or wobbling neutron stars (NS's) are among the most promising sources for the large-scale gravitational-wave detectors now coming online. In this paper we show that NS's with large toroidal *B* fields tend naturally to evolve into potent GW emitters.

While most known, young pulsars have dipolar magnetic field strengths $B_d \sim 10^{12}$ G, studies of soft-gamma repeaters and anomalous x-ray pulsars suggest that a substantial fraction of NS's are born with larger magnetic fields, $B \sim 10^{14} - 10^{15}$ G [1–4]. In cases where the *B* field is initially "wound up" by differential rotation (as opposed to convective motions), one would expect the field's toroidal piece B_t to dominate the poloidal part B_p.

The toroidal field B_t tends to distort a NS into a prolate shape, and we show that this magnetic distortion dominates over the oblateness "frozen into" the NS crust for $B_t \gtrsim 3.4 \times 10^{12}(\nu_s/300$ Hz)^2 G, where ν_s is the NS spin frequency. An *elastic* NS with frozen-in *B* field of this magnitude is clearly secularly unstable: the wobble angle between the NS's angular momentum J^i and the star's magnetic axis n_B^i grow on a viscous time scale until J^i and n_B^i are orthogonal. This final orientation is the optimal one for GW emission. The basic cause of the instability seems so general that we conjecture that the same final state is reached for a realistic NS (with crust and superfluid core). In this paper we consider NS's where $B_t > B_p$, we assume that most of the results for elastic NS's can be carried over to the realistic NS case, and examine the consequences. We show that for LMXB's with $B_t \sim 2 \times 10^{12} - 2 \times 10^{14}$ G, the spindown from GW's is sufficient to balance the accretion torque—supporting a sug-

gestion by Bildsten [5]. The spindown rates of most millisecond pulsars can also be attributed to GW emission sourced by toroidal *B* fields of comparable size, $B_t \sim 10^{12} - 10^{13}$ G. For several known millisecond pulsars, the gravitational waves would be detectable by the Laser Interferemetic for Gravitational Wave Observatory (LIGO II). Finally, while the first-year spindown of a newborn NS is most likely dominated by electromagnetic (EM) processes, reasonable values of B_t and the (external) dipolar field B_d can still lead to detectable levels of GW emission, for a newborn NS in our own galaxy.

The idea that a large, toroidal magnetic field should drive the magnetic axis orthogonal to J^i is not new; a literature search revealed that Jones had this idea 28 years ago [6]. Jones [7–10] went on to develop a scenario in which the dipole axes of young pulsars (which he assumed have $B_t > B_p$) are *first* driven orthogonal to J^i by dissipative forces, on a time scale of $\sim 10^3$ yr, but then are slowly driven back parallel to J^i by em radiation reaction, for times $\gtrsim 10^4$ yr. That is, Jones estimated that at around age $10^3 - 10^4$ yr, dissipative forces would diminish to the point that EM backreaction would drive the dynamics. Clearly, Jones was strongly guided by the observation that most young pulsars do *not* appear to have dipole axes orthogonal to J^i (nor can they be perfectly aligned with J^i, as one would expect for $B_p > B_t$, since they pulse). Given his strong argument that dissipation should drive the magnetic axis orthogonal to J^i, and the apparent contradiction by observation, there are three likely ways out: (1) assume dissipative effects become negligible on a time scale of order the ages of known, young pulsars, (2) assume the pulsar emission region is not aligned along a principal axis of the *internal B* field, or (3) assume that the internal *B* field is either not born with, or quickly loses, any strong directionality. Jones developed a model around the first option, but found little support for it in the data.

Here we essentially resurrect the Jones argument, but concentrate on a different class of objects possessing much

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smaller external dipole fields B_d: the LMXB's and millisecond pulsars. For these objects, it seems clear that dissipation must dominate over the EM torque in determining the alignment of the NS's axes. We are agnostic on the question of why the *normal* (i.e., young, slow) pulsars are generally neither aligned nor orthogonal rotators. It is irrelevant to our argument whether option (1) or (2) above is correct; option (3) seems the least likely to us, but if option (3) is always correct it would render this paper irrelevant.

Of course, it has been noted before that magnetically distorted NS's can generate GW's; see, e.g., Bonazzola and Gourgouhlon [11] and Jones [12] and references therein. Also, Melatos and Phinney [13] recently showed how the poloidal B field on accreting NS's might build up "mountains" of accreted matter on the magnetic poles, leading to GW emission. What is new here is our observation that NS's with large toroidal B fields should naturally evolve to configurations with large GW emission, and our indentification of LMXB's and millisecond pulsars as likely "sites" for this phenomenon.

In Sec. II we derive the basic energetics and time scales. These are applied to LMXB's and millisecond pulsars in Sec. III, and to newborn NS's in Sec. IV. For our estimates, we shall always use the following fiducial NS parameters: $M = 1.4 M_\odot$, $R = 10$ km, and $I = 10^{45}$ g cm^2.

II. ENERGETICS AND TIME SCALES

Let n_B^i be the NS's magnetic axis (so that if we call n_B^i the z axis, then the toroidal field B_t points in the ϕ direction). Following Pines and Shaham [14], we can write the NS's inertia tensor I^{ij} as the sum of four pieces—a spherical piece and three quadrupolar distortions—as follows:

$$I^{ij} = I_0[e^{ij} + \epsilon_\Omega \, n_\Omega^i n_\Omega^j + \epsilon_d \, n_d^i n_d^j + \epsilon_B \, n_B^i n_B^j]. \quad (2.1)$$

Here e^{ij} is the flat, spatial 3-metric, $I_0 e^{ij}$ is the spherically symmmetric part of the inertia tensor, and the terms proportional to ϵ_Ω, ϵ_d, and ϵ_B are the quadrupolar distortions due to the star's spin, crustal shear stresses, and the magnetic field, respectively.

The centrifugal piece "follows" the instantaneous (unit-)spin vector n_Ω^i, but the unit-vectors n_d^i and n_B^i are assumed fixed in the body frame. (However on long time scales, n_d^i can be "re-set" by crustal relaxation.)

The centrifugal piece ϵ_Ω in Eq. (2.1) is approximately given by $\epsilon_\Omega \approx 0.3 (\nu_s / \text{kHz})^2$, where ν_s is the spin frequency. The term ϵ_d is due to the elastic crust's "memory" of some preferred shape. Absent the magnetic field, it is the residual oblateness the NS *would* have if it were spun down to zero frequency, without the crust breaking or otherwise relaxing. (To understand this, consider a rotating NS with relaxed crust. Removing the centrifugal force would decrease the NS's oblateness, but then shear stresses would also build up that tended to push the crust back to its relaxed, oblate shape.) For an NS with relaxed crust, ϵ_d is proportional to the centrifugal oblateness: $\epsilon_d = b \epsilon_\Omega$. The coefficient b was recently calculated by Cutler *et al.* [15], who solved the coupled hydroelastic equations describing the deformed crust

and found $b \approx 2 \times 10^{-7}$. (An older, back-of-the-envelope estimate of $b \sim 10^{-5}$ used by many authors turned out to be too high by a factor ~ 40.) Therefore we adopt the estimate

$$\epsilon_d \approx 6 \times 10^{-8} \left(\frac{\nu_s}{\text{kHz}} \right)^2. \quad (2.2)$$

Again, this estimate assumes the crust is nearly relaxed. While this might not be a good assumption for young, slow NS's, it seems safe for the fast, old NS's of interest to us here, for two reasons. First, during the LMXB phase, the entire crust is replaced by accreted matter as the NS spins up, and it seems unlikely the new crust would "remember" the preferred shape from an earlier era. Second, the alternative requires the crust to withstand average strains of order $3 \times 10^{-3} (\nu_s / 300 \, \text{Hz})^2$ for $\sim 10^7 - 10^{10}$ yr without relaxing [15], which also seems unlikely.

Next we need ϵ_B. Of course, the actual distortion produced by some toroidal field B_t depends on the precise distribution of B field (and hence currents) within the star, which is highly uncertain. So we must be satisfied with a rough estimate of ϵ_B, which we derive as follows. Let n_B^i be along the z axis. For an incompressible, constant-ρ star that is not superconducting, $\epsilon_B \equiv (I_{zz} - I_{xx})/I_{zz}$ depends only on the total energy in the toroidal field [17,18]:

$$\epsilon_B = -\frac{15}{4} E_G^{-1} \int \frac{1}{8\pi} B_t^2 \, dV, \quad (2.3)$$

where $E_G = (3/5) GM^2 / R$ is the star's gravitational binding energy. The sign of ϵ_B is negative because the toroidal magnetic field lines are like a rubber belt, pulling in the star's waist at the magnetic equator.

Now, it is generally believed that for interior temperatures $T < 10^9 - 10^{10 \, \circ} \text{K}$, the protons in the NS interior are a type II superconductor; hence the magnetic field is confined to flux tubes with field strength $B_{c1} \approx 10^{15}$ G. (For $B > B_{c1}$, the magnetic fluxoids overlap.) Then for $B < B_{c1}$, the anisotropic part of the mean magnetic stress tensor is increased over its nonsuperconducting value by a factor B_{c1}/B, and ϵ_B increases by the same factor [6,19]. (The virial-theorem-based derivation of ϵ_B in Ostriker and Gunn [18] makes this completely clear.) Putting this factor together with Eq. (2.3), but using $E_G = (3/4) GM^2 / R$ (i.e., the $n = 1$ polytrope result, which quite accurately gives the binding energy of realistic NS's), we adopt the estimate

$$\epsilon_B = \begin{cases} -1.6 \times 10^{-6} (\langle B_t \rangle / 10^{15} \, \text{G}), & B_t < B_{c1}, \\ -1.6 \times 10^{-6} (\langle B_t^2 \rangle / 10^{30} \, \text{G}), & B_t > B_{c1}, \end{cases} \quad (2.4)$$

where $\langle \cdots \rangle$ means "volume-averaged over the NS interior." Of course, the first line of Eq. (2.4) only applies to NS's older than $\sim 0.1 - 1$ yr (i.e., old and cold enough to be superconducting).

For simplicity we are assuming, here and below, that B_t is sufficiently greater than B_p that we can neglect the latter. When is this a good approximation? For an incompressible, constant-ρ, nonsuperconducting NS, with B^i of the form

$B_p \hat{z}^i + B_t \hat{\phi}^i$ in the interior and matched to a dipolar field in the exterior, the deformation is [17,18][1]

$$\epsilon_B = \frac{25R^4}{24GM^2} \left(\frac{21}{10} \langle B_p^2 \rangle - \langle B_t^2 \rangle \right). \quad (2.5)$$

For a superconducting NS, with $B_t, B_p \ll B_{c1}$, we claim this becomes

$$\epsilon_B = \frac{25R^4}{24GM^2} (2\langle B_p B_{c1} \rangle - \langle B_t B_{c1} \rangle). \quad (2.6)$$

[The small change $(21/10) \rightarrow 2$ reflects the fact that the *external* poloidal magnetic field energy becomes a negligible fraction of the total field energy in the superconducting case.] Clearly then, as long as $\langle B_t \rangle \gtrsim 4 \langle B_p \rangle$, our estimates should be fairly reliable.

We next consider the kinetic energy (KE) of the spinning NS as the orientation of the body changes, for fixed angular momentum J^i. While in the inertial frame the NS's angular velocity Ω^i clearly must remain approximately fixed (near the star's angular momentum J^i), Ω^i will migrate with respect to the body axes as the star precesses (and as the precession angle secularly changes). Using $J^i = I^{ij}\Omega_j$, $KE = \frac{1}{2}J^i\Omega_i$, and expanding everything to lowest order in ϵ_Ω, ϵ_d and ϵ_B, we find

$$KE = \frac{J^2}{2I_0}[1 - \epsilon_\Omega - \epsilon_d(n_d^i n_{Ji})^2 - \epsilon_B(n_B^i n_{Ji})^2], \quad (2.7)$$

where n_J^i is the unit vector along J^i.

Now consider the case where $\epsilon_B < 0$ and $|\epsilon_B| \gg \epsilon_d$, and let θ be the "wobble angle" between J^i and n_B^i. Then the θ-dependent piece of KE is $(2I_0)^{-1}J^2(-\epsilon_B)\cos^2\theta$, which is clearly minimized for $\theta = \pi/2$. [More precisely—including the effect of nonzero ϵ_d—KE is minimized for $|n_B^i n_{Ji}| = \frac{1}{2}|(\epsilon_d/\epsilon_B)\sin 2\beta|$, where β is the angle between n_B^i and n_d^i.] In addition to the kinetic energy, the wobbling NS stores potential energy (PE) in the stressed crust and the distorted fluid. One easily convinces oneself that the θ-dependent pieces of PE and KE have the same sign and same functional form ($\propto \cos^2\theta$), and that they have comparable magnitudes for NS's with relaxed crusts. Therefore, considering the total energy E = KE + PE leads to the same conclusion about energy-minimization or stability as considering KE alone.

It is worth emphasizing that the NS need not actually be prolate to be unstable; indeed, for the cases of interest here ($\nu_s > 100$ Hz), the centrifugal oblateness will be orders of magnitude larger than $|\epsilon_B|$. But what matters, for both precessional dynamics and stability, is the net prolateness or

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[1]Ostriker and Gunn [18] actually give 3, not 2.1, as the coefficient of $\langle B_p^2 \rangle$ in Eq. (2.5), but we claim they made an algebra error; the coefficient on the RHS of their Eq. (A19) should be $-7/40$, not their $-1/4$. With this correction, their coefficent 3 becomes 2.1, in agreement with Wentzel [17].

oblateness that is "frozen into" the star's body frame, not the centrifugal piece that "follows" the star's angular velocity. Therefore the condition for instability is $-\epsilon_B > \epsilon_d$, or

$$B_t \gtrsim 3.4 \times 10^{12} \text{ G} \left(\frac{\nu_s}{300 \text{ Hz}} \right)^2. \quad (2.8)$$

Assuming inequality (2.8) is satisfied, how does the instability grow? For an elastic star with frozen-in magnetic field, this is completely clear. In the body frame, Ω^i precesses around n_B^i with precession frequency $\nu_s \epsilon_B$ (neglecting a small correction due to nonzero ϵ_d). Because of dissipation, the angle between Ω^i and n_B^i steadily increases until they are nearly orthogonal. (For a review of NS precessional dynamics, see Cutler and Jones [16] and references therein.)

For a realistic NS, with superfluid core, the precessional dynamics could be more complicated. But as long as those dynamics do not reduce $-\epsilon_B$ below ϵ_d (i.e, by destroying the large-scale coherence of the internal field or building up an equally large internal poloidal piece), it seems inevitable, by simple energetics, that dissipation will tend to drive n_B^i orthogonal to J^i. In what follows, we shall assume this is true. Some support for this assumption comes from the work of Goldreich and Reisenegger [20], who show that the composition of a NS interior is stably stratified, impeding convective motions that might otherwise distort the interior B field.

We next turn to time scales. There are four relevant ones: τ_{GW}, τ_{EM}, τ_{ACC} and τ_{DIS}. Once n_B^i has been realigned perpendicular to J^i, the NS spins down due to GW emission on a time scale $\tau_{GW} \equiv J/(2\dot{J}_{GW})$, given by

$$1/\tau_{GW} = 5.50 \times 10^{-13} \text{ s}^{-1} \left(\frac{\epsilon_B}{10^{-7}} \right)^2 \left(\frac{\nu_s}{\text{kHz}} \right)^4. \quad (2.9)$$

We must compare this to the time scale $\tau_{EM} \equiv J/(2\dot{J}_{EM})$ on which the NS spins down due to electromagnetic processes. Define $B_d \equiv 2\mathcal{M}/R^3$, where $\mathcal{M}$ is the NS's magnetic dipole moment. (With this definition, B_d is the value of the field at the magnetic pole, on the NS surface, for an external field that is perfectly dipolar.) Then τ_{EM} is given by

$$1/\tau_{EM} = 4.88 \times 10^{-16} \text{ s}^{-1} \left(\frac{B_d}{10^9 \text{ G}} \right)^2 \left(\frac{\nu_s}{\text{kHz}} \right)^2. \quad (2.10)$$

For a magnetic dipole rotating in vacuum, the RHS of Eq. (2.10) would be multiplied by a factor $\sin^2\alpha$, where α is the angle between the spin vector and magnetic dipole direction. But Goldreich and Julian [21] showed that a realistic NS is surrounded by plasma, and as a result Eq. (2.10) is a reasonable estimate of the spindown rate even for an aligned (i.e., $\alpha = 0$) rotator.

From Eqs. (2.9), (2.10), spindown due to GW emission dominates EM spindown for

$$B_t/B_d > 6.2 \times 10^3 \left(\frac{300 \text{ Hz}}{\nu_s} \right), \quad B_t < B_{c1} \quad (2.11a)$$

$$B_t/B_d > 2.5 \times 10^2 \left(\frac{300 \text{ Hz}}{\nu_s} \right)^{1/2} \left(\frac{10^{14} \text{ G}}{B_d} \right)^{1/2}, \quad B_t > B_{c1}. \quad (2.11b)$$

Such a large ratio B_t/B_d would not be expected in a normal pulsar, but could occur in a LMXB or recycled pulsar. For these sources, accretion apparently "buries" or otherwise reduces the external dipolar field, but the toroidal field in the deep interior could be left largely intact.

The EM torque from the external dipole field causes the wobble angle θ between J^i and n_B^i to damp (or grow, for $\sin^2 \xi > 2/3$) on roughly the EM spindown time scale τ_{EM} [22]:

$$\frac{1}{\sin \theta} \frac{d \sin \theta}{dt} = -\frac{1}{2} \cos^2 \theta \left(1 - \frac{3}{2} \sin^2 \xi \right) / \tau_{EM}, \quad (2.12)$$

where ξ is the angle between the external dipole direction and the magnetic axis n_B^i of the internal B field. Similarly, gravitational radiation reaction tends to align the distortion axis n_B^i with the spin direction (independent of the sign of ϵ_B) on the GW spindown time scale τ_{GW} [16]: $\dot{\theta} = -\frac{1}{32} \theta / \tau_{GW}$ for small wobble angle θ, where here τ_{GW} is defined as the rhs of Eq. (2.9).

Next there is the time scale τ_{ACC} on which accretion can significantly change the NS's angular momentum (either magnitude or direction). We approximate $\tau_{ACC} \equiv J/2\dot{J}_{ACC}$ by [23]

$$1/\tau_{ACC} = 9.3 \times 10^{-16} \text{ s}^{-1} \left(\frac{\dot{M}}{10^{-9} M_\odot/\text{yr}} \right) \left(\frac{300 \text{ Hz}}{\nu_s} \right). \quad (2.13)$$

Lastly, we consider the dissipation time scale τ_{DIS} on which the instability acts. Define n to be the dissipation time scale τ_{DIS} divided by the wobble period, so $\tau_{DIS} \equiv nP/\epsilon_B$, where $\tau_{DIS} \sim |\epsilon_B| [J^2 \theta^2/(2I)]/\dot{E}_{DIS}$. We can rewrite this as

$$1/\tau_{DIS} = 3.0 \times 10^{-8} \text{ s} \left(\frac{10^4}{n} \right) \left(\frac{\nu_s}{300 \text{ Hz}} \right) \left(\frac{\epsilon_B}{10^{-7}} \right). \quad (2.14)$$

The n factor is hard to estimate, but fortunately most of our conclusions are essentially independent of n over a huge range. Previous authors who have considered precessional damping in NS's have generally concluded that precession damps quickly. E.g., Chau and Henriksen [24] considered dissipation in the elastic crust as Ω^i (and hence the centrifugal bulge) precess through the body frame. The crust is periodically distorted by the wobbling motion, and the elastic energy stored in the crust is some fraction F of the total wobble energy, where we estimate $0.05 < F < 0.5$. (For the Earth, F is 0.11 [25].) If some fraction $1/Q$ of the elastic energy is dissipated in each wobble period, then $n \sim Q/F$. Typically $Q \sim 10^4$ for terrestrial metals, so one might estimate $n \lesssim 10^5$. Other mechanisms seem likely to lead to faster damping: the core and crust will not wobble together rigidly, and relative internal motions will lead to frictional damping, e.g., at the crust-core interface, or through interactions of

neutron superfluid vortices with electrons and nuclei in the inner crust. (E.g., Alpar and Sauls [26] estimated $n \approx 10^2 - 10^4$ for slowly rotating NS's, with the dissipation due to electron scattering off pinned superfluid vortices. However superfluid vortices certainly will *not* remain pinned in the wobble regimes of interest to us—large wobble angle and $\nu_s \gtrsim 300$ Hz so this particular estimate does not seem immediately relevant to our case; see Link and Cutler [27].) We therefore leave n as unknown parameter to be determined by observation or improved theory. The point is that, for LMXB's and millisecond pulsars, any n less than $\sim 10^9$ means the dissipation time scale is much shorter than all the others, and so should successfully drive n_B^i orthogonal to J^i.

We have collected all the necessary formulas. We now apply them to interesting cases: LMXB's, millisecond pulsars, and newborn NS's.

III. LMXB'S AND MILLISECOND PULSARS

A. Evolution scenario

The arguments and time scales above lead to the following evolution scenario for (some large fraction of) LMXB's and millisecond pulsars. The NS is born with $B_t \gtrsim 4B_p$ and $B_d \sim 10^{12} - 10^{14}$ G. At birth, n_B^i is likely nearly aligned with the NS's spin (and so is nearly aligned with n_d^i). The NS spins down electromagnetically, but much later it is recycled by accretion from a companion. Accretion reduces the exterior field B_d below $\sim 10^9$ G; however in the interior, B_t remains in the range $\sim 10^{12} - 10^{15}$ G. As the NS spins back up and its external B field decays, it reaches a state where τ_{DIS} is shorter than the other time scales (including τ_{EM}, thanks to the decay of B_d), while $|\epsilon_B| > \epsilon_d$. Then dissipation rapidly "flips" n_B^i perpendicular to J^i. Initially n_d^i flips along with n_B^i, but we expect the crust of a rapidly spinning NS will crack or relax such that its new preferred axis n_d^i is again aligned with Ω^i, but now is perpendicular to n_B^i. This final configuration is clearly a minimum of both KE and PE (for fixed J), independent of the relative sizes of ϵ_d and $|\epsilon_B|$. Thus while the NS continues to spin up, dissipation will ensure that n_d^i remains aligned with J^i, with n_B^i perpendicular to both. This is important because ϵ_d increases quadratically with spin frequency, and so (as we shall see) eventually surpasses $|\epsilon_B|$ for the fastest LMXB's and millisecond pulsars. Our point is that as long as n_d^i has already been "re-set" along J^i *before* that happens, then n_B^i will remain orthogonal to J^i. After accretion stops, the NS continues to spin down by GW emission. And because n_B^i is time-varying (rotating around J^i with frequency ν_s) while n_d^i is not, it is the magnetic distortion that sources the gravitational waves.

A potential objection to this scenario arises if one assumes that the pulsar beam is aligned with the axis n_B^i of the B field in deep interior, since then we would predict that (a large fraction of) millisecond pulsars should be orthogonal rotators, which is not observed. So for our picture to be sensible, we should assume that the B field near the NS surface

evolves (e.g., via accretion and crust-cracking) in such a way that that the pulsar beam is *not* aligned with n_B^i.

B. LMXB's

The distribution of LMXB spin periods suggest that many have "hit a wall" at $\nu_s \sim 260$–590 Hz—far below the likely maximum rotation rate of 1–1.5 kHz [5,28]. Bildsten [5] proposed that they had reached an equilibrium where accretion torque was balanced by spindown from gravitational radiation (resuscitating an idea of Wagoner's [29]).

Equilibrium between accretion torque and GW spindown implies an ellipticity

$$\epsilon = 4.5 \times 10^{-8} \left(\frac{\dot{M}}{10^{-9} M_\odot/\mathrm{yr}} \right)^{1/2} \left(\frac{300 \text{ Hz}}{\nu_s} \right)^{5/2}. \quad (3.1)$$

For the LMXB's with known spin rates, the range of $\dot{M}$'s is $\sim 10^{-11}$–$2 \times 10^{-8} M_\odot/\mathrm{yr}$, with implied ϵ's in the range $\sim 3 \times 10^{-9}$–3×10^{-7} [28].

On this assumption, GW's from the brightest LMXB's (especially Sco X-1) should be detectable by LIGO II. Ushomirsky *et al.* [23] showed that crustal "mountains" can provide the required ellipticity, but only if the crustal breaking strain σ_{max} is larger than $\sim 10^{-3}$–10^{-2}. Toroidal B fields place no such requirement on the crust's breaking strain: $B_t \sim 2 \times 10^{12}$–$2 \times 10^{14}$ G provides the required ellipticity. Our picture requires that GW torque dominates EM torque, or $B_d \lesssim 5 \times 10^8$–$5 \times 10^{10}$ G, by Eq. (2.11a). There are no direct measurements of B_d in LMXB's to compare this to, but $B_d \lesssim 10^9$ G is indicated by the lack of x-ray pulses.

While for simplicity we have assumed $B_t \gg B_p$, it should be clear that the basic picture remains the same if they are comparable, so long as the total ϵ_B is negative and $|\epsilon_B| \sim 10^{-8}$–$10^{-7}$.

C. Millisecond pulsars

We turn now to millisecond radio pulsars (which we define as those with spin periods $P < 10$ ms), which are commonly assumed to be the descendants of LMXB's. Our picture suggests than many should now be spinning down by GW emission. The NS's characteristic age $\tau_c \equiv P/(2\dot{P})$ is then just τ_{GW}. We can rewrite Eq. (2.9) as

$$|\epsilon_B| = 6.0 \times 10^{-9} \left(\frac{10^{10} \text{ yr}}{\tau_c} \right) \left(\frac{P}{5 \text{ msec}} \right)^2, \quad (3.2)$$

or $B_t \sim 4 \times 10^{12}$ G for our fiducial millisecond pulsar values ($\tau_c = 10^{10}$ yr and $P = 5$ msec)—consistent with the lower end of the range we inferred for LMXB's. (A comparison of derived ϵ_B's in the LMXB and millisecond pulsar populations would be interesting, but is clearly complicated by selection effects; e.g., low-$|\epsilon_B|$ millisecond pulsars live longer, and so will dominate the observed sample.) Note that $|\epsilon_B| > \epsilon_d$ only for $P \gtrsim 4.0$ msec $(\tau_c/10^{10} \text{ yr})^{1/2}$ [from Eqs. (2.2) and (3.2)], so the oldest or fastest millisecond pulsars would not be unstable in the way we described in Sec. II. However, as explained in Sec. III A, that probably does not matter:

their magnetic axes were flipped orthogonal to J^i while they were being spun up, and will not flip back if the crust has relaxed.

Given a millisecond pulsar at distance D, spinning down by GW emission, it is straightforward to compute the detectability of the GW signal $h(t)$. The rms S/N (averaged over source direction and polarization) is

$$S/N = 1.1 \left(\frac{1 \text{ kpc}}{D} \right) \left(\frac{T_o}{1 \text{ yr}} \right)^{1/2} \left(\frac{10^{10} \text{ yr}}{\tau_c} \right)^{1/2} \left(\frac{2 \times 10^{-24}}{S_h^{1/2}(f)} \right), \quad (3.3)$$

where $f = 2\nu_s$ is the gravity-wave frequency, $S_h(f)$ is the (single-sided) noise spectral density, and T_0 is the observation time. The broad-band LIGO-II noise curve has a broad minimum at $f_{gw} \sim 400$ Hz, with $S_h^{1/2}(f) \approx 2 \times 10^{-24}$ there [31]. Using this fiducial value for $S_h^{1/2}(f)$, ones finds there are at least 4 known millisecond pulsars with $S/N > 2.7$ in a 1 yr observation. PSR J0437-4715 ($f_{gw} = 347$ Hz, $\tau_c = 0.6 \times 10^{10}$ yr, and $D = 0.18$ kpc [32]) gives the highest S/N ratio: $S/N = 7.9$. Also, PSR J1744-1134 ($f_{gw} = 491$ Hz, $\tau_c = 0.72 \times 10^{10}$ yr, $D = 0.36$ kpc [34]) yields $S/N = 3.6$; PSR J1024-0719 ($f_{gw} = 387$ Hz, $\tau_c = 2.7 \times 10^{10}$ yr, $D = 0.20$ kpc [32]) yields $S/N = 3.3$; and PSR J1012+5307 ($f_{gw} = 381$ Hz, $\tau_c = 0.6 \times 10^{10}$ yr, $D = 0.52$ kpc [33]) gives $S/N = 2.7$. Factor ~ 2.5 improvements in these SNR's could be obtained by operating the detector in narrow-band, signal-recycling mode.

Note that since the GW signal is almost completely known from radio observations (i.e., up to polarization, amplitude and overall phase), $S/N \gtrsim 2.7$ is enough for confident detection.

IV. NEWBORN PULSARS

As another application of our basic idea, consider the spindown GW's from a supernova in our own galaxy ($D \sim 10$ kpc). (The supernova rate in our galaxy is $\sim 1/40$ yr, so this author expects to be around for the next one.) It does not seem outlandish to posit field strengths $B_d \sim 10^{14}$ G and $B_t \sim 10^{15}$ G. Now EM radiation dominates the spindown torque, but there is significant GW emission as well.

The matched-filtering $(S/N)^2$ for a NS spinning down *solely* due $l = m = 2$ GW's is [30]

$$(S/N)^2 = \frac{2G}{5\pi D^2 c^3} \int_{f_{min}}^{f_{max}} \frac{dJ/df}{f \, S_h(f)} df. \quad (4.1)$$

For our case, we just need to multiply the numerator in Eq. (4.1) by $\dot{J}_{GW}/\dot{J}_{EM} = \tau_{EM}/\tau_{GW} \propto f^2$. Approximating the high-frequency part of the broad-band, LIGO-II noise spectrum by $S_h(f) = 2.1 \times 10^{-47}$ s $(f/10^3 \text{ Hz})^2$ [31], we find

$$S/N = 11.7 \left(\frac{10 \text{ kpc}}{D} \right) \left(\frac{\epsilon_B}{10^{-6}} \right) \left(\frac{10^{14} \text{ G}}{B_d} \right) \left[\ln \left(\frac{f_{max}}{f_{min}} \right) \right]^{1/2}. \tag{4.2}$$

Here we can take f_{min} to be the frequency (~ 500 Hz) where the actual LIGO-II broad-band noise curve "flattens out."

E.g., consider a NS with $B_t = 2 \times 10^{15}$ G and $B_d = 10^{14}$ G, which spins down from $\nu_s \sim 1$ kHz to $\nu_s = 250$ Hz in ~ 10 days (and to $\nu_s \approx 80$ Hz after one year). In order for n_B^i to "flip" before the NS spins down substantially, we need $\tau_{DIS} < \tau_{EM}$, or $n \lesssim 1.3 \times 10^3$, by Eqs. (2.10) and (2.14). Assuming n *is* this small, and taking

$[\ln(f_{max}/f_{min})]^{1/2} \approx 1$, we find $S/N = 75$ for $D = 10$ kpc, which would likely be detectable even in a semi-blind search—i.e., one where neutrino detectors and other channels give the collapse time and a rough position, but where there is no radio signal to give us the rotational phase as a function of time.

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