Cosmological perturbations in a generalized gravity including tachyonic condensation

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We present unified ways of handling the cosmological perturbations in a class of gravity theory covered by a general action. This gravity includes our previous generalized $f(\phi,R)$ gravity and the gravity theory motivated by the tachyonic condensation. We present a general prescription to derive the power spectra generated from vacuum quantum fluctuations in the slow-roll inflation era. An application is made to a slow-roll inflation based on the tachyonic condensation with an exponential potential.

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I. INTRODUCTION

In our present paradigm of physical cosmology, the observed large-scale cosmic structures and the anisotropies of the cosmic microwave background (CMB) are regarded as small deviations from the spatially homogeneous and isotropic Friedmann world model [1]. In such a paradigm the structures in the large-scale limit and in the early stage of the evolution are assumed to be linear deviations from the background world model [2]. Although the observations are consistent with the perturbed Friedmann world model, these, however, do not necessarily constrain the underlying gravity theory (and the matter content) to be the Einstein one. Generalized forms of gravity appear in a variety of situations involving the quantum aspects of the gravity theory and the low energy limits of the unified theories of gravity with other fundamental forces. Thus, it is likely that the early stages of the universe were governed by the gravity more general than Einstein one.

We have been studying the cosmological perturbations in the so-called $f(\phi,R)$ gravity theory which includes diverse generalized gravity theories known in the literature as cases, [3]. In this work, motivated by the recent interests on the action based on the tachyonic condensation [4], and also by a previous study in the context of "k inflation" [5], we extend our study to a more general form of gravity presented in Eq. (1). Section III presents the classical evolutions in a unified form. Section IV presents the quantum generation process and the generated power spectra under the slow-roll assumption and others. Section V is an application a tachyonic slow-roll inflation. We set $c\equiv 1\equiv \hbar$.

II. GRAVITY

We consider an action

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}f(R,\phi,X)+L_m\right], \qquad (1)$$

where $X\equiv \frac{1}{2}\phi^{;c}\phi_{,c}$, and f is a general algebraic function of R, ϕ and X. This action includes the following gravity theories as cases. (1) A minimally coupled scalar field: $f=(1/8\pi G)R-2X-2V(\phi)$. (2) $f(\phi,R)$ gravity: $f=\tilde{f}(\phi,R)-2\omega(\phi)X-2V(\phi)$. (3) $p(\phi,X)$ gravity: $f=(1/8\pi G)R+2p(\phi,X)$. (4) Tachyonic condensation: $f=(1/8\pi G)R-2V(\phi)\sqrt{1+2X}$.

The gravitational field equation and the equation of motion become

$$G_{ab}=\frac{1}{F}\left[T^{(m)}_{ab}+\frac{1}{2}(f-FR)g_{ab}+F_{,a;b}-F^{;c}{}_{c}g_{ab}-\frac{1}{2}f_{,X}\phi_{,a}\phi_{,b}\right]\equiv 8\pi G T_{ab}, \qquad (2)$$

$$(f_{,X}\phi^{;c})_{;c}=f_{,\phi}, \qquad (3)$$

$$T_{(m)a;b}^{\ \ b}=0, \qquad (4)$$

where $F\equiv f_{,R}$. T_{ab} is the effective energy-momentum tensor, and $T^{(m)}_{ab}$ is the energy-momentum tensor of additional matters.

III. CLASSICAL PERTURBATIONS

We consider the Friedmann background with the scalar- and the tensor-type perturbations. Our metric convention follows Bardeen's [6]:

$$ds^2=-a^2(1+2\alpha)d\eta^2-2a^2\beta_{,\alpha}d\eta dx^\alpha+a^2[g^{(3)}_{\alpha\beta}(1+2\varphi)+2\gamma_{,\alpha|\beta}+2C_{\alpha\beta}]dx^\alpha dx^\beta. \qquad (5)$$

The energy-momentum tensor is decomposed as

$$T^0_0=-(\bar{\mu}+\delta\mu), \quad T^0_\alpha=-(\mu+p)v_{,\alpha}/k,$$

$$T^\alpha_\beta=(\bar{p}+\delta p)\delta^\alpha_\beta+\left(\frac{1}{k^2}\nabla^{(3)\alpha}\nabla^{(3)}_\beta+\frac{1}{3}\delta^\alpha_\beta\right)\pi^{(s)}+\pi^\alpha_\beta. \qquad (6)$$

A vertical bar $|$ and $\nabla^{(3)}_\alpha$ are the covariant derivatives based on $g^{(3)}_{\alpha\beta}$.

To the background order, Eq. (2) gives

$$H^2=\frac{8\pi G}{3}\mu-\frac{K}{a^2},\quad \dot{H}=-4\pi G(\mu+p)+\frac{K}{a^2},\tag{7}$$

where $H\equiv\dot{a}/a$ and an overdot denotes a time derivative based on t with $dt\equiv a d\eta$. We also have $R=6(2H^2+\dot{H}+K/a^2)$. The effective fluid quantities are

$$8\pi G\mu=\frac{1}{F}\left[\mu^{(m)}-\frac{1}{2}(f-FR)-\frac{1}{2}f_{,X}\dot{\phi}^2-3H\dot{F}\right],$$

$$8\pi Gp=\frac{1}{F}\left[p^{(m)}+\frac{1}{2}(f-FR)+\ddot{F}+2H\dot{F}\right],\tag{8}$$

where we have $X=-\frac{1}{2}\dot{\phi}^2$. To the background order Eq. (3) gives

$$\frac{1}{a^3}(a^3f_{,X}\dot{\phi})^{\cdot}+f_{,\phi}=0.\tag{9}$$

A perturbed set of equations can be derived similarly. The perturbed set of equations in Einstein gravity based on our convention in Eqs. (5) and (6) is presented in [6].[1] These equations are valid even in our gravity theory if we re-interprete the fluid quantities as the effective ones. The perturbed order effective fluid quantities can be easily read by comparing Eq. (6) with Eq. (2).

For the scalar-type perturbation we ignore the presence of additional fluid, thus $T^{(m)}_{ab}=0$. In the following we consider two general situations: (i) $F=F(\phi)$ and $K=0$, and (ii) $F=1/8\pi G$ but general K. We introduce the Field-Shepley combination [8][2]

$$\Phi\equiv\varphi_{\delta\phi}-\frac{K/a^2}{4\pi G(\mu+p)}\varphi_\chi,\tag{10}$$

where

$$\varphi_{\delta\phi}\equiv\varphi-(H/\dot{\phi})\delta\phi,\quad \varphi_\chi\equiv\varphi-H\chi,\tag{11}$$

are gauge-invariant combinations;[3] $\chi\equiv a(\beta+a\dot{\gamma})$ is a spatially gauge-invariant combination [6].

(i) In the first case, perturbed parts of Eq. (2) can be combined to give a closed form of second-order differential equation for $\varphi_{\delta\phi}$[4]

$$\frac{1}{a^3Q}(a^3Q\dot{\varphi}_{\delta\phi})^{\cdot}+c_A^2\frac{k^2}{a^2}\varphi_{\delta\phi}=0,\tag{12}$$

$$Q\equiv\frac{\frac{3\dot{F}^2}{2F}+f_{,X}X+2f_{,XX}X^2}{\left(H+\frac{\dot{F}}{2F}\right)^2}\equiv\frac{\dot{\phi}^2}{H^2}Z,$$

$$c_A^2\equiv\left(1+\frac{2f_{,XX}X^2}{\frac{3\dot{F}^2}{2F}+f_{,X}X}\right)^{-1}.\tag{13}$$

For $f_{,X}=-2\omega(\phi)$ we recover the result derived in the $f(\phi,R)$ gravity theory [7].

(ii) In the second case, perturbed parts of Eq. (2) can be combined to give[5]

$$\Phi=\frac{H^2}{4\pi G(\mu+p)a}\left(\frac{a}{H}\varphi_\chi\right)^{\cdot},\tag{14}$$

$$\dot{\Phi}=-\frac{Hc_A^2k^2}{4\pi G(\mu+p)a^2}\varphi_\chi,\tag{15}$$

where $\mu+p=-\frac{1}{2}f_{,X}\dot{\phi}^2=f_{,X}X$, and

$$c_A^2\equiv c_X^2-\frac{p_{,\phi}-c_X^2\mu_{,\phi}}{\mu+p}\frac{\dot{\phi}}{H}\frac{K}{k^2},$$

$$c_X^2\equiv\frac{p_{,X}}{\mu_{,X}}=\frac{f_{,X}}{f_{,X}+2f_{,XX}X}.\tag{16}$$

Equations (14) and (15) were derived by Garriga and Mukhanov; see Eqs. (21) and (22) in [13]. Equations (14) and (15) can be combined to give

$$\frac{1}{a^3Q}(a^3Q\dot{\Phi})^{\cdot}+c_A^2\frac{k^2}{a^2}\Phi=0,\quad Q\equiv\frac{\mu+p}{c_A^2H^2},\tag{17}$$

$$\frac{\mu+p}{H}\left[\frac{H^2}{(\mu+p)a}\left(\frac{a}{H}\varphi_\chi\right)^{\cdot}\right]^{\cdot}+c_A^2\frac{k^2}{a^2}\varphi_\chi=0.\tag{18}$$

Using

$$v\equiv z\Phi,\quad u\equiv\frac{\varphi_\chi}{\sqrt{\mu+p}},\quad z\equiv a\sqrt{Q}\equiv\frac{1}{c_A}\tilde{z},\tag{19}$$

Eqs. (17) and (18) become the well known equations [8,10]

$$v''+\left(c_A^2k^2-\frac{z''}{z}\right)v=0,\tag{20}$$

$$u''+\left(c_A^2k^2-\frac{(1/\tilde{z})''}{1/\tilde{z}}\right)u=0,\tag{21}$$

[1] See Eqs. (43)–(50) in [7]. We have $\epsilon\equiv\delta\mu$, $\pi=\delta p$, $\Psi\equiv-(a/k)(\mu+p)v$, and $\sigma=(a^2/k^2)\pi^{(s)}$.

[2] See the paragraph containing Eq. (36) in [9].

[3] $\varphi_{\delta\phi}$ is the same φ in the uniform-field gauge ($\delta\phi\equiv0$) [10]. φ_χ is the same as φ in the zero-shear gauge ($\chi\equiv0$) [11], and is the same as Φ_H which is often called the Bardeen potential [12].

[4] The procedure is exactly the same as the one used to derive Eq. (66) in [7].

[5] The procedure is exactly the same as the one used to derive Eqs. (32) and (33) in [9].

where a prime indicates a time derivative based on η. Equation (20) is valid for the first case in Eq. (12) as well.

In the large-scale limit, with $z''/z \gg c_A^2 k^2$ and $\tilde{z}(1/\tilde{z})'' \gg c_A^2 k^2$, we have exact solutions

$$\Phi = C(\mathbf{x}) - D(\mathbf{x}) \int_0^t \frac{dt}{az^2}, \tag{22}$$

$$\varphi_\chi = 4\pi G \frac{H}{a}\left[C(\mathbf{x}) \int_0^t \frac{\tilde{z}^2}{a} dt + \frac{1}{k^2} D(\mathbf{x}) \right]. \tag{23}$$

Ignoring the transient solution (which is the D-mode in expanding phases) we have a temporally conserved behavior for Φ

$$\Phi(\mathbf{x},t) = C(\mathbf{x}). \tag{24}$$

For the tensor-type perturbation, for the *general* action in Eq. (1), we have

$$\ddot{C}^\alpha_\beta + \left(3H + \frac{\dot{F}}{F}\right)\dot{C}^\alpha_\beta + \frac{k^2+2K}{a^2} C^\alpha_\beta = \frac{1}{F}\pi^{(m)\alpha}{}_\beta, \tag{25}$$

which is the same as Eq. (111) in [7] based on $f(\phi,R)$ gravity. Thus, the presence of general algebraic complication of X in Eq. (1) has *no effect* on the tensor-type perturbation. Also, Eq. (25) can be written as in Eqs. (17) and (20). In such cases we have $\Phi = C^\alpha_\beta$, $Q = F \equiv Z/(8\pi G)$, $c_A^2 = 1$, thus $z \equiv a\sqrt{F}$, and Eqs. (22) and (24) also remain valid.

The vector-type perturbation of additionally present fluid(s) is described by Eq. (4) which is *not* affected by the generalized nature of the gravity theory in Eq. (1).

IV. SLOW-ROLL INFLATION

As in [14] the quantum generation process can be presented in a unified form. From Eq. (17) we can construct the perturbed action [10]

$$\delta^2 S = \frac{1}{2}\int a^3 Q\left(\dot{\Phi}^2 - c_A^2 \frac{1}{a^2}\Phi^{|\gamma}\Phi_{,\gamma}\right) dt d^3x, \tag{26}$$

which is valid for both the scalar-type and tensor-type perturbations in a unified form. The rest of the canonical quantization process is straightforward; see [14]. Under an *ansatz*

$$z''/z = n/\eta^2, \quad c_A^2 = \text{const}, \tag{27}$$

where $n = n_s$, n_t for the two perturbation types, the mode function has an exact solution in terms of the Hankel functions; see Eq. (24) in [14]. The power spectrum based on the vacuum expectation value of $\hat{\Phi}$ can be constructed as in Eq. (26) of [14], and in the large-scale limit we have[6]

$$\mathcal{P}^{1/2}_{\hat{\Phi}}|_{LS} = \frac{H}{2\pi}\frac{1}{aH|\eta|}\frac{\Gamma(\nu)}{\Gamma(3/2)}\left(\frac{k|\eta|}{2}\right)^{3/2-\nu}\frac{1}{c_A^\nu\sqrt{Q}}, \tag{28}$$

where $\nu \equiv \sqrt{n+1/4}$. We can read the spectral indices

$$n_S - 1 = 3 - \sqrt{4n_s+1}, \quad n_T = 3 - \sqrt{4n_t+1}. \tag{29}$$

We introduce the slow-roll parameters [7]

$$\epsilon_1 \equiv \frac{\dot{H}}{H^2}, \quad \epsilon_2 \equiv \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 \equiv \frac{1}{2}\frac{\dot{F}}{HF}, \quad \epsilon_4 \equiv \frac{1}{2}\frac{\dot{E}}{HE},$$

$$E \equiv F\left(\frac{3\dot{F}^2}{2\dot{\phi}^2 F} - \frac{1}{2}f_{,X} - f_{,XX}X\right). \tag{30}$$

Compared with the Einstein gravity in [16] we have two additional parameters ϵ_3 and ϵ_4 for the scalar-type perturbation which reflect the effects of additional parameters F ($\equiv f_{,R}$) and $f_{,X}$ in our generalized gravity; for the tensor-type perturbation we have only one additional parameter ϵ_3 from F. Compared with [7] the only difference occurs in our definition of E which includes the $f(\phi,R)$ gravity in [7] as a case. Using our present definition of ϵ_i's our unified analyses made in Eqs. (30)–(32) of [14] remain valid.

To the first-order in the slow-roll parameters, i.e., *assuming*

$$\dot{\epsilon}_i = 0, \quad |\epsilon_i| \ll 1, \tag{31}$$

we can derive

$$\begin{aligned}\mathcal{P}^{1/2}_{\hat{\varphi}_{\delta\phi}}\Big|_{LS} &= \frac{H}{|\dot{\phi}|}\mathcal{P}^{1/2}_{\delta\hat{\phi}_\varphi}\Big|_{LS} \\ &= \frac{H^2}{2\pi|\dot{\phi}|}\frac{1}{\sqrt{Z_s}}\{1+\epsilon_1+[\gamma_1+\ln(k|\eta|)] \\ &\quad \times(2\epsilon_1-\epsilon_2+\epsilon_3-\epsilon_4)\}c_A^{-\nu_s},\end{aligned} \tag{32}$$

$$\begin{aligned}\mathcal{P}^{1/2}_{\hat{C}^\alpha_\beta}\Big|_{LS} &= \sqrt{16\pi G}\frac{H}{2\pi}\frac{1}{\sqrt{Z_t}}\{1+\epsilon_1+[\gamma_1+\ln(k|\eta|)] \\ &\quad \times(\epsilon_1-\epsilon_3)\},\end{aligned} \tag{33}$$

where $\gamma_1 \equiv \gamma_E + \ln 2 - 2 = -0.7296\ldots$, with γ_E the Euler constant. We have

$$Z_s = \frac{E/F}{(1+\epsilon_3)^2}, \quad Z_t = 8\pi G F, \tag{34}$$

where Z's become unity in Einstein gravity. Thus, besides ϵ_1, the scalar-type perturbation is affected by ϵ_2, ϵ_3 and ϵ_4 (thus, $f_{,\phi}$, F and $f_{,X}$), whereas the tensor-type perturbation is affected by ϵ_3 (thus, F) only. The spectral indices of the scalar and tensor-type perturbations in Eq. (29) become

$$n_S - 1 = 2(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4), \quad n_T = 2(\epsilon_1 - \epsilon_3). \tag{35}$$

[6]For $\nu = 0$ we have an additional 2 $\ln(c_A k|\eta|)$ factor. For the gravitational we should consider additional $\sqrt{2}$ factor [15].

For the scale independent Harrison-Zel'dovich ($n_S-1\simeq 0 \simeq n_T$) spectra [17] the CMB quadrupole anisotropy becomes

$$\langle a_2^2\rangle = \langle a_2^2\rangle_S + \langle a_2^2\rangle_T = \frac{\pi}{75}\mathcal{P}_{\varphi_{\delta\phi}} + 7.74\frac{1}{5}\frac{3}{32}\mathcal{P}_{C_{\alpha\beta}}, \quad (36)$$

which is valid for $K=0=\Lambda$. The four-year Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) data give $\langle a_2^2\rangle \simeq 1.1\times 10^{-10}$, [18]. From Eqs. (36), (32) and (33) the ratio between two types of perturbations $r_2 \equiv \langle a_2^2\rangle_T/\langle a_2^2\rangle_S$ becomes

$$\begin{aligned} r_2 &= 13.8\times 4\pi G\frac{\dot\phi^2}{H^2}\left|\frac{Z_s}{Z_t}\right| c_A^{2\nu_s} \\ &= 13.8\frac{1}{(1+\epsilon_3)^2}\left|(\epsilon_1-\epsilon_3)(1+\epsilon_3)+\frac{\dot\epsilon_3}{H}\right| c_A^{2-n_S} \\ &\simeq 13.8|\epsilon_1-\epsilon_3| c_A \\ &\simeq 6.92|n_T| c_A, \end{aligned} \quad (37)$$

where in the last two steps we used the slow-roll conditions in Eq. (31). In the limit of Einstein gravity we have $r_2 = -13.8\epsilon_1 = -6.92 n_T$ which is independent of V and is known as a consistency relation. The c_A factor difference from the Einstein gravity for $p(\phi,X)$ gravity was noticed in [13]. For the $f(\phi,R)$ gravity we have $c_A^2=1$.

V. TACHYONIC CONDENSATION

The recently popular tachyonic condensation is a case of our gravity with a form $f=(1/8\pi G)R - 2V\sqrt{1+2X}$: if based on the string theory, we should regard the field in this action as being written in the unit where the string theory is relevant. We have

$$Q = \frac{\dot\phi^2}{H^2}\frac{V}{(1-\dot\phi^2)^{3/2}}, \quad c_A^2 = 1-\dot\phi^2. \quad (38)$$

Equations (20) and (21) in this case were derived in Eq. (17) of [19] and in Eq. (44) of [20], respectively. We have $\epsilon_3 = 0$ and $E=(V/8\pi G)(1-\dot\phi^2)^{-3/2}$.

Assuming a set of *slow-roll* conditions $\ddot\phi \ll 3H\dot\phi$ and $\dot\phi^2 \ll 1$, and under an *ansatz* $V\equiv V_0 e^{-\alpha\phi}$ [21], from Eqs. (7)–(9) for $K=0$ we have [22]

$$\phi = -\frac{2}{\alpha}\ln\left(C - \frac{\sqrt{3}\alpha^2 M_{pl}}{6\sqrt{V_0}}t\right),$$

$$a \propto e^{(C\sqrt{V_0}/\sqrt{3}M_{pl})t - (\alpha^2/12)t^2}, \quad (39)$$

where $M_{pl}^2 \equiv 1/(8\pi G)$. If we set $t_i=0$, we have $C = e^{-(\alpha/2)\phi_i}$ and $V_i = V_0 C^2$. For $t\simeq t_i$ we have an accelerated expansion stage. In such a situation we have the slow-roll conditions in Eq. (31) are well met, with the result

$$\epsilon_1 = -\epsilon_2 = \epsilon_4 = -\frac{\alpha^2 M_{pl}^2}{2V_i}, \quad \epsilon_3 = 0. \quad (40)$$

Thus, Eq. (35) gives

$$n_S - 1 = 4\epsilon_1, \quad n_T = 2\epsilon_1, \quad (41)$$

and Eqs. (32)–(34) and (37) reduce to

$$\mathcal{P}^{1/2}_{\varphi_{\delta\phi}} \simeq \frac{H^2}{2\pi|\dot\phi|}\frac{1}{\sqrt{V}} \simeq \frac{1}{2\sqrt{3}\pi}\frac{V_i}{\alpha M_{pl}^3}, \quad (42)$$

$$\mathcal{P}^{1/2}_{C_{\alpha\beta}} \simeq \sqrt{16\pi G}\frac{H}{2\pi} \simeq \frac{1}{\sqrt{6}\pi}\frac{\sqrt{V_i}}{M_{pl}^2}, \quad (43)$$

$$r_2 = 6.92|n_T|. \quad (44)$$

Therefore, if the seed structures were generated from the vacuum quantum fluctuation under such a slow-roll phase, the final spectra show that (1) the spectra are nearly scale-invariant Harrison-Zel'dovich type, (2) the consistency relation is met, (3) the graviational wave is suppressed, and (4) the CMB quadrupole requires

$$\langle a_2^2\rangle \simeq \frac{1}{75\times 12\pi}\frac{V_i^2}{\alpha^2 M_{pl}^6} \simeq 1.1\times 10^{-10}. \quad (45)$$

We have assumed that, first, the seed fluctuations were generated during the slow-roll inflation stage supported by the tachyonic condensation, and secondly, the tachyonic gravity stage was switched successfully to an ordinary big-bang stage while the fluctuations stay in the large-scale limit (see [23] for the reheating problem); in such a case the relatively growing C-mode fluctuation in Eq. (22) survives as the same C-mode of the curvature fluctuation Φ now supported by the Einstein gravity with ordinary matter. We have derived these results directly based on the generalized form of gravity theory whereas the previous analyses [24,22] were based on known formulation in Einstein gravity by using some field redefinition.

VI. DISCUSSIONS

We have presented unified ways of handling the cosmological perturbations in a class of gravity theory covered by an action in Eq. (1). Section III presents the classical evolutions in a unified form, and Eqs. (28) and (29) show the generated seed fluctuations of the quantum origin under an assumption in Eq. (27). The rest of Sec. IV presents the general prescription to derive the power spectra generated under the slow-roll assumption, and Sec. V is an application to a tachyonic slow-roll inflation.

We note that even in the gravity with *additional* stringy correction terms

$$\xi(\phi)[c_1 R_{GB}^2 + c_2 G^{ab}\phi_{;a}\phi_{;b} + c_3\Box\phi\phi^{;a}\phi_{;a} + c_4(\phi^{;a}\phi_{;a})^2],$$

$$g(\phi)R\tilde{R}, \quad (46)$$

in the Lagrangian, where $R^2_{GB} \equiv R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2$ and $R\tilde{R} \equiv \eta^{abcd} R_{ab}{}^{ef} R_{cdef}$, we still have Eqs. (12) and (26) with more complicated Q and c_A^2 [25]. Thus, the rest of the analyses made above can be applied similarly as well [25]. Similar unified formulation also exists in the fluid context [10,14]. We also have studied the situation with $R^{ab}R_{ab}$ term in the action [26], in which case the gravity becomes a fourth-order theory.

We would like to emphasize that our gravity theory in Eq. (1) covers many of the modified gravity theories, and our assumption in Eq. (27) is satisfied by most of the expansion stages (including diverse classes of inflation scenarios available in analytic forms) considered in the literature, and we hope our slow-roll conditions in Eq. (31) cover most of the specific slow-roll conditions in the inflation theories based on specific modified gravity theories. We emphasize, however, that the classical evolutions studied in Sec. III are valid for the general cosmological situations governed by our action in Eq. (1).

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[1] A. Friedmann, Z. Phys. **10**, 377 (1922); translated in *Cosmological Constants, Papers in Modern Cosmology*, edited by J. Bernstein and F. Feinberg (Columbia University Press, New York, 1986), p. 49.
[2] E.M. Lifshitz, J. Phys. (Moscow) **10**, 116 (1946).
[3] J. Hwang, Class. Quantum Grav. **7**, 1613 (1990); J. Hwang and H. Noh, Phys. Rev. D **65**, 023512 (2002).
[4] A. Mazumdar, S. Panda, and A. Pérez-Lorenzana, Nucl. Phys. **B614**, 101 (2001); G.W. Gibbons, Phys. Lett. B **537**, 1 (2002); A. Feinstein, Phys. Rev. D **66**, 063511 (2002); T. Padmanabhan, *ibid.* **66**, 021301(R) (2002); D. Choudhury, D. Ghoshal, C.P. Jatkar, and S. Panda, hep-th/0204204.
[5] C. Armendáriz-Picón, T. Damour, and V.F. Mukhanov, Phys. Lett. B **458**, 209 (1999).
[6] J. M. Bardeen, in *Cosmology and Particle Physics*, edited by L. Fang and A. Zee (Gordon and Breach, London, 1988), p. 1; J. Hwang, Astrophys. J. **375**, 443 (1991).
[7] J. Hwang and H. Noh, Phys. Rev. D **54**, 1460 (1996).
[8] G.B. Field and L.C. Shepley, Astrophys. Space Sci. **1**, 309 (1968); G.V. Chibisov and V.F. Mukhanov, Mon. Not. R. Astron. Soc. **200**, 535 (1982); J. Hwang and E.T. Vishniac, Astrophys. J. **353**, 1 (1990); V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[9] J. Hwang and H. Noh, Class. Quantum Grav. **19**, 527 (2002).
[10] V.N. Lukash, Sov. Phys. JETP **52**, 807 (1980); JETP Lett. **31**, 596 (1980); V.F. Mukhanov, *ibid.* **41**, 493 (1985); Sov. Phys. JETP **67**, 1297 (1988).
[11] E.R. Harrison, Rev. Mod. Phys. **39**, 862 (1967).
[12] J.M. Bardeen, Phys. Rev. D **22**, 1882 (1980).
[13] J. Garriga and V.F. Mukhanov, Phys. Lett. B **458**, 219 (1999).
[14] H. Noh and J. Hwang, Phys. Lett. B **515**, 231 (2001).
[15] J. Hwang, Class. Quantum Grav. **15**, 1401 (1998).
[16] E.D. Stewart and D.H. Lyth, Phys. Lett. B **302**, 171 (1993).
[17] E.R. Harrison, Phys. Rev. D **1**, 2726 (1970); Ya.B. Zeldovich, Mon. Not. R. Astron. Soc. **160**, 1 (1972).
[18] C.L. Bennett *et al.*, Astrophys. J. Lett. **464**, L1 (1996).
[19] A. Frolov, L. Kofman, and A. Starobinsky, hep-th/0204187.
[20] G. Shiu and I. Wasserman, Phys. Lett. B **541**, 6 (2002).
[21] A. Sen, hep-th/0204143.
[22] M. Sami, P. Chingangbam, and T. Qureshi, Phys. Rev. D **66**, 043530 (2002).
[23] L. Kofman and A. Linde, J. High Energy Phys. **07**, 004 (2002).
[24] M. Fairbairn and M.H.G. Tytgat, hep-th/0204070.
[25] J. Hwang and H. Noh, Phys. Rev. D **61**, 043511 (2000); K. Choi, J. Hwang, and K.W. Hwang, *ibid.* **61**, 084026 (2000); C. Cartier, J. Hwang, and E. Copeland, *ibid.* **64**, 103504 (2001).
[26] H. Noh and J. Hwang, Phys. Rev. D **55**, 5222 (1997); **59**, 047501 (1999).