Brane world cosmologies and statistical properties of gravitational lenses

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(Received 30 May 2002; published 29 October 2002)

Brane world cosmologies seem to provide an alternative explanation for the present accelerated stage of the Universe with no need to invoke either a cosmological constant or an exotic *quintessence* component. In this paper we investigate statistical properties of gravitational lenses for some particular scenarios based on this large scale modification of gravity. We show that a large class of such models are compatible with the current lensing data for values of the matter density parameter $\Omega_{\rm m} \leqslant 0.94(1\sigma)$. If one fixes $\Omega_{\rm m}$ to be $\simeq 0.3$, as suggested by most of the dynamical estimates of the quantity of matter in the Universe, the predicted number of lensed quasars requires a slightly open universe with a crossover distance between the 4- and 5-dimensional gravities of the order of $1.76H_o^{-1}$.

DOI: 10.1103/PhysRevD.66.083511 PACS number(s): 98.80.Es, 04.50.+h, 95.35.+d

I. INTRODUCTION

The results of observational cosmology in the past years have opened up an unprecedented opportunity to test the veracity of a number of cosmological scenarios as well as to establish a more solid connection between particle physics and cosmology. The most remarkable finding among these results comes from distance measurements of type Ia supernovae (SNe Ia) that suggest that the expansion of the Universe is speeding up, not slowing down [1]. As widely known such a result poses a crucial problem for all cold dark matter (CDM) models since their generic prediction is a decelerating universe $(q_o > 0)$, whatever the sign adopted for the curvature parameter. Indirectly, similar results have also been obtained, independent of the SNe Ia analyses, by combining the latest galaxy clustering data with CMB measurements [2].

To reconcile these observational results with theory, cosmologists have proposed more general models containing a negative-pressure dark component that would be responsible for the present accelerated stage of the Universe. Although a large number of pieces of observational evidence have consistently suggested a universe composed of $\sim 2/3$ of dark energy, the exact nature of this new component is not well understood at present. Among the several candidates for dark energy discussed in the recent literature, the simplest and most theoretically appealing possibility is the vacuum energy or cosmological constant. Despite the serious problem that arises when one considers a nonzero vacuum energy [3], models with a relic cosmological constant (ΛCDM) seem to be our best description of the observed universe, being considered as a serious candidate for standard cosmology.

On the other hand, motivated by particle physics considerations, there has been growing interest in cosmological models based on the framework of brane-induced gravity [4–7]. The general principle behind such models is that our 4-dimensional Universe would be a surface or a brane embedded into a higher dimensional bulk space-time on which gravity can propagate. In some of these scenarios, there is a certain crossover scale r_c that defines what kind of gravity an observer on the brane will observe. For distances shorter than r_c, such an observer will measure the usual 4-dimensional gravitational $1/r^2$ force whereas for distances larger than r_c the gravitational force follows the 5-dimensional $1/r^3$ behavior. In this way, gravity gets weaker at cosmic distances and, therefore, it is natural to think that such an effect has some implications on the dynamics of the Universe [8].

Several aspects of brane world cosmologies have been explored in the recent literature. For example, the issue related to the cosmological constant problem has been addressed [9] as well as evolution of cosmological perturbations in the gauge-invariant formalism [10], cosmological phase transitions [11], inflationary solutions [12], baryogenesis [13], stochastic background of gravitational waves [14], singularity, homogeneity, flatness and entropy problems [15], among others (see [16] for a discussion on the different perspectives of brane world models). From the observational viewpoint, however, the present situation is somewhat controversial. While the authors of Refs. [17,18] have shown that such models are in agreement with the most recent cosmological observations [for example, they found that a flat universe with $\Omega_{\rm m} = 0.3$ and $r_c \simeq 1.4H_o^{-1}$ is consistent with the currently SNe Ia+cosmic microwave background (CMB) data], the authors of Ref. [19] have claimed that a larger sample of SNe Ia data can also be used to rule out these models at least at the 2σ level. Recently, one of us [20] used measurements of the angular size of high-z compact radio sources to show that the best fit model for these data is a slightly closed universe with $\Omega_{\rm m} \simeq 0.06$ and a crossover radius of the order of $0.94H_o^{-1}$.

For the reasons presented earlier, the comparison between any alternative cosmology and ΛCDM models is very important. In this concern, statistical properties of gravitational

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lenses may be an interesting tool because, as is well known, they provide restrictive limits on the vaccum energy contribution (see, for instance, [21]). On the other hand, in brane world models the distance to an object at a given redshift z is smaller than the distance to the same object in ΛCDM models (assuming the same value of Ω_{m}). Therefore, we expect that the constraints coming from lensing statistics will be weaker for these models than for their ΛCDM counterparts.

In this paper, we explore the implications of gravitationally lensed quasistellar objects (QSOs) for models based on the framework of the brane-induced gravity of Dvali *et al.* [5] that have been recently proposed in Refs. [6,8]. We restrict our analysis to these scenarios because, as explained in [6], due to a geometrical effect of the bulk gravity on the brane, they present a ''self-inflationary'' solution with $H \sim r_c^{-1}$ (H is the Hubble parameter), or equivalently, they undergo accelerated expansion at the late stages of the universe (in agreement with SNe Ia data). It is worth mentioning that this accelerated phase may be transitional which helps to reconcile the description of an accelerating universe with the requirements of string or M-theories [22] (see also [23] for a discussion on this topic).

This paper is organized as follows. In Sec. II we present the basic field equations and distance formulas relevant for our analysis. We then proceed to analyze the constraints from lensing statistics on these models in Sec. III. In Sec. IV our main conclusions are presented.

II. THE MODEL: BASIC EQUATIONS AND DISTANCE FORMULAS

The Friedmann equation for the kind of models we are considering is [8,17]

$$\left[\sqrt{\frac{\rho}{3M_{pl}^2}+\frac{1}{4r_c^2}}+\frac{1}{2r_c}\right]^2 = H^2+\frac{k}{R(t)^2}, \tag{1}$$

where ρ is the energy density of the cosmic fluid, $k=0,\pm 1$ is the spatial curvature, M_{pl} is the Planck mass and $r_c = M_{pl}^2/2M_5^3$ is the crossover scale defining the gravitational interaction among particles located on the brane (M_5 is the 5-dimensional reduced Planck mass). For simplicity, throughout this paper we consider that the cosmic fluid is dominated by nonrelativistic matter (see [8] for exact expressions for a general fluid component). In this case, from the above equation we find that the normalization condition is given by

$$\Omega_k + [\sqrt{\Omega_{\mathrm{r_c}}}+\sqrt{\Omega_{\mathrm{r_c}}+\Omega_{\mathrm{m}}}]^2 = 1 \tag{2}$$

where Ω_{m} and Ω_k are, respectively, the matter and curvature density parameters (defined in the usual way) and

$$\Omega_{\mathrm{r_c}} = 1/4r_c^2H_o^2, \tag{3}$$

is the density parameter associated with the crossover radius r_c. For a flat universe, the normalization condition becomes [17]

$$\Omega_{\mathrm{r_c}} = \left(\frac{1-\Omega_{\mathrm{m}}}{2}\right)^2 \quad \text{for} \quad \Omega_{\mathrm{r_c}}<1 \quad \text{and} \quad \Omega_{\mathrm{m}}<1. \tag{4}$$

In order to derive the constraints from lensing statistics in the next section we shall use the concept of angular diameter distance, $D_A(z)$. Such a quantity can be easily obtained in the following way: consider that photons are emitted by a source with coordinate $r=r_1$ at time t_1 and are received at time t_o by an observer located at coordinate $r=0$. The emitted radiation will follow null geodesics on which the dimensionless comoving coordinates θ and ϕ are constant. The comoving distance of the source is defined by

$$r_1 = \int_{t_1}^{t_o}\frac{dt}{R(t)} = \int_{R(t)}^{R_o}\frac{dR}{\dot{R}(t)R(t)}. \tag{5}$$

From Eqs. (1) and (5), it is possible to show that the comoving distance $r_1(z)$ can be written as [20]

$$r_1(z) = \frac{1}{R_oH_o|\Omega_k|^{1/2}}\mathcal{F}\left[|\Omega_k|^{1/2}\int_{x'}^{1}\frac{dx}{x^2 f(\Omega_j,x)}\right], \tag{6}$$

where the subscript o denotes present day quantities, $x' = R(t)/R_o = (1+z)^{-1}$ is a convenient integration variable and the function $\mathcal{F}(r)$ is defined by one of the following forms: $\mathcal{F}(r)=\sinh(r)$, r, and $\sin(r)$, respectively, for open, flat and closed geometries. The dimensionless function $f(\Omega_j,x)$ is given by

$$f(\Omega_j,x) = [\Omega_k x^{-2} + (\sqrt{\Omega_{\mathrm{r_c}}}+\sqrt{\Omega_{\mathrm{r_c}}+\Omega_{\mathrm{m}}x^{-3}})^2]^{1/2}, \tag{7}$$

where j stands for m, r_c and k.

The angular diameter distance to a light source at $r=r_1$ and $t=t_1$ and observed at $r=0$ and $t=t_o$ is defined as the ratio of the source diameter to its angular diameter, i.e.,

$$D_A = \frac{\ell}{\theta} = R(t_1)r_1. \tag{8}$$

In the general case, the angular diameter distance, $D_{LS}(z_L,z_S) = R_o r(z_L,z_S)/(1+z_S)$, between two objects, for example, a lens at z_L and a source (galaxy) at z_S, reads

$$D_{LS}(z_L,z_S) = \frac{H_o^{-1}}{(1+z_S)|\Omega_k|^{1/2}}\mathcal{F}\left[|\Omega_k|^{1/2}\int_{x_S'}^{x_L'}\frac{dx}{x^2 f(\Omega_j,x)}\right]. \tag{9}$$

III. CONSTRAINTS FROM LENSING STATISTICS

In this paper we work with a sample of 867 ($z>1$) high luminosity optical quasars which include 5 lensed quasars. These data are taken from optical lens surveys such as the Hubble Space Telescope (HST) Snapshot survey [24], the Crampton survey [25], the Yee survey [26], Surdej survey [27], the NOT Survey [28] and the FKS survey [29]. Since the lens surveys and quasar catalogs usually use V magnitudes, we transform m_V to B-band magnitude by using $B-V=0.2$ as suggested by Bahcall *et al.* [30].

The differential probability $d\tau$ of a beam having a lensing event in traversing dz_L is [32,31]

$$d\tau = F^*(1+z_L)^3\left(\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\right)^2 \frac{1}{R_0}\frac{dt}{dz_L}dz_L, \qquad (10)$$

where

$$\frac{c\,dt}{dz_L} = \frac{H_o^{-1}}{(1+z_L)f(\Omega_j, x_L)}, \qquad (11)$$

and

$$F^* = \frac{16\pi^3}{cH_0^3}\phi_* v_*^4 \Gamma\left(\alpha + \frac{4}{\gamma} + 1\right). \qquad (12)$$

D_{OL}, D_{OS} and D_{LS} are, respectively, the angular diameter distances from the observer to the lens, from the observer to the source and between the lens and the source. For simplicity we use the singular isothermal model (SIS) for the lens mass distribution. The Schechter luminosity function is adopted and lens parameters for E/SO galaxies are taken from Loveday, Peterson, Efstathiou, and Maddox [33] (LPEM), i.e., $\phi_* = 3.2\pm 0.17h^3\,10^{-3}\mathrm{Mpc}^{-3}$, $\alpha = 0.2$, $\gamma = 4$, $v_* = 205.3$ km/s and $F^* = 0.010$. It is worth mentioning that, although the recent galaxy surveys have increased considerably our knowledge of the galaxy luminosity function, they do not classify the galaxies by their morphological type [34]. In this work we restrict ourselves to the LPEM parameters because they have been derived in a highly correlated manner and they also take into account the morphological distribution of the E/S0 galaxies [35].

The differential optical depth of lensing in traversing dz_L with angular separation between ϕ and $\phi + d\phi$ is given by

$$\frac{d^2\tau}{dz_L d\phi}d\phi dz_L = F^*(1+z_L)^3\left(\frac{D_{OL}D_{LS}}{R_o D_{OS}}\right)^2 \frac{1}{R_o}\frac{dt}{dz_L} \times \frac{\gamma/2}{\Gamma\left(\alpha+1+\dfrac{4}{\gamma}\right)}\left(\frac{D_{OS}}{D_{LS}}\phi\right)^{(\gamma/2)(\alpha+1+(4/\gamma))} \times \exp\left[-\left(\frac{D_{OS}}{D_{LS}}\phi\right)^{\gamma/2}\right]\frac{d\phi}{\phi}dz_L \qquad (13)$$

where $\phi = \Delta\theta/8\pi(v_*/c)^2$, with the velocity dispersion v_* corresponding to the characteristic luminosity L_* in the Schechter luminosity function. The total optical depth is obtained by integrating $d\tau$ along the line of sight from $z=0$ (z_O) to z_S. One obtains

$$\tau(z_S) = \frac{F^*}{30}[D_{OS}(1+z_L)]^3 R_o^3. \qquad (14)$$

Figure 1 shows the normalized optical depth as a function of the source redshift (z_S) for $\Omega_{\rm m}=0.3$ and values of $\Omega_{\rm r_c} = 0.1$, 0.2 and 0.3. For comparison, the standard prediction ($\Omega_{\rm r_c}=0$) is also displayed. Note that, at higher redshifts ($z > 2.5$), an increase in $\Omega_{\rm r_c}$ at fixed $\Omega_{\rm m}$ tends to reduce the optical depth for lensing. For example, at $z_S = 3.0$ the value of τ/F^* for $\Omega_{\rm r_c}=0.1$ is down from the standard value by a factor of ~ 1.10. This decrease of the optical depth as the value of $\Omega_{\rm r_c}$ is increased (at a fixed z_S and $\Omega_{\rm m}$) occurs because, at high redshift, say $z>2.5$, the distance between two redshifts (e.g., z_O and z_S) is smaller for higher values of $\Omega_{\rm r_c}$.

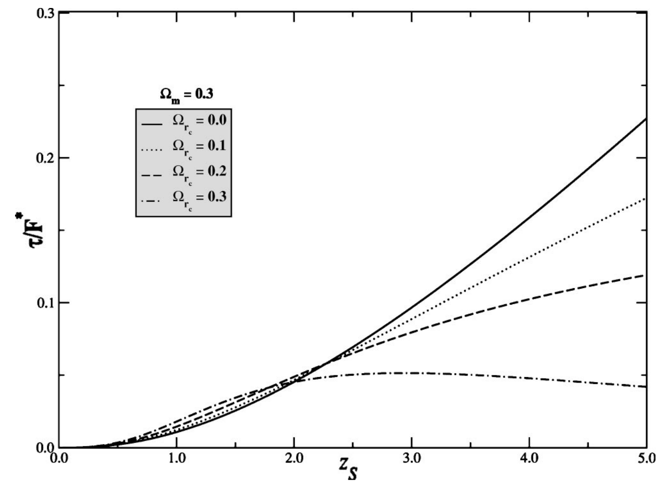

FIG. 1. The normalized optical depth (τ/F^*) as a function of the source redshift (z_S) for some selected values of $\Omega_{\rm r_c}$. In all curves, the value of the matter density parameter has been fixed ($\Omega_{\rm m}=0.3$).

In order to obtain the correct lensing probability we have made two corrections to the optical depth, namely, magnification bias and selection function. Magnification bias, $\mathbf{B}(m,z)$, is considered in order to take into account the increase in the apparent brightness of a quasar due to lensing which, in turn, increases the expected number of lenses in flux limited sample. The bias factor for a quasar at redshift z with apparent magnitude m is given by [31,21]

$$\mathbf{B}(m,z) = M_0^2 B(m,z,M_0,M_2), \qquad (15)$$

where

$$B(m,z,M_1,M_2) = 2\left(\frac{dN_Q}{dm}\right)^{-1}\int_{M_1}^{M_2}\frac{dM\,dN_Q}{M^3\,dm} \times [m+2.5\log(M),z]. \qquad (16)$$

In the above equation $[dN_Q(m,z)/dm]$ is the measure of number of quasars with magnitudes in the interval $(m, m+dm)$ at redshift z. Since we are modeling the lens by a SIS profile, $M_0 = 2$, we adopt $M_2 = 10^4$ in the numerical computation.

We use Kochanek's ''best model'' [21] for the quasar luminosity function:

$$\frac{dN_Q}{dm}(m,z) \propto (10^{-a(m-\bar{m})} + 10^{-b(m-\bar{m})})^{-1}, \qquad (17)$$

where

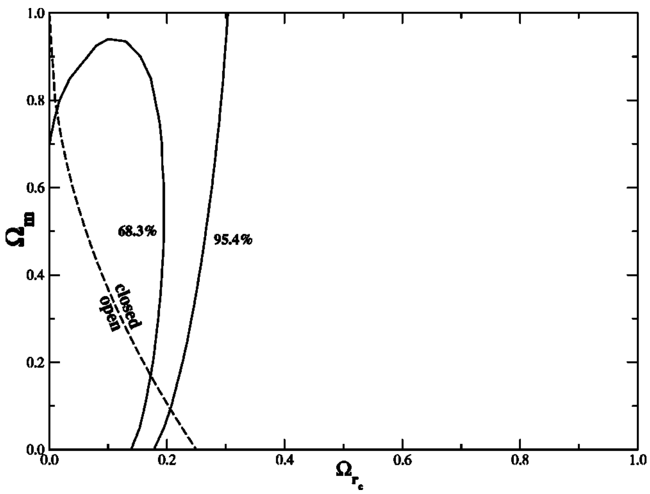

FIG. 2. Confidence regions in the plane $\Omega_{\mathrm{m}}-\Omega_{r_c}$ arising from lensing statistics. Solid lines indicate contours of constant likelihood at 68% and 95.4%.

$$\overline{m}=\begin{cases} m_o+(z-1) & \text{for } z<1, \\ m_o & \text{for } 1<z\leqslant 3, \\ m_o-0.7(z-3) & \text{for } z>3, \end{cases} \tag{18}$$

and we assume $a=1.07\pm0.07$, $b=0.27\pm0.07$ and $m_o=18.92\pm0.16$ at B magnitude [21].

The magnitude corrected probability, p_i, for a given quasar i at z_i and apparent magnitude m_i to be lensed is

$$p_i=\tau(z_i)\mathbf{B}(m_i,z_i). \tag{19}$$

Due to selection effects the survey can only detect lenses with magnifications larger than a certain magnitude M_f. It can be shown that the corrected lensing probability and image separation distribution function for a single source at redshift z_S are [21,36]

$$p_i'(m,z)=p_i\int \frac{d(\Delta\theta)p_c(\Delta\theta)B(m,z,M_f(\Delta\theta),M_2)}{B(m,z,M_0,M_2)} \tag{20}$$

and

$$p_{ci}'=p_{ci}(\Delta\theta)\frac{p_i}{p_i'}\frac{B(m,z,M_f(\Delta\theta),M_2)}{B(m,z,M_0,M_2)}, \tag{21}$$

where

$$p_c(\Delta\theta)=\frac{1}{\tau(z_S)}\int_0^{z_S}\frac{d^2\tau}{dz_L d(\Delta\theta)}dz_L \tag{22}$$

and

$$M_f=M_0(f+1)/(f-1) \quad \text{with} \quad f=10^{0.4\,\Delta m(\theta)}. \tag{23}$$

Equation (21) defines the configuration probability, i.e., the probability that the lensed quasar i is lensed with the observed image separation. To obtain selection function corrected probabilities, we follow [21] and divide our sample into two parts, namely, the ground based surveys and the HST survey.

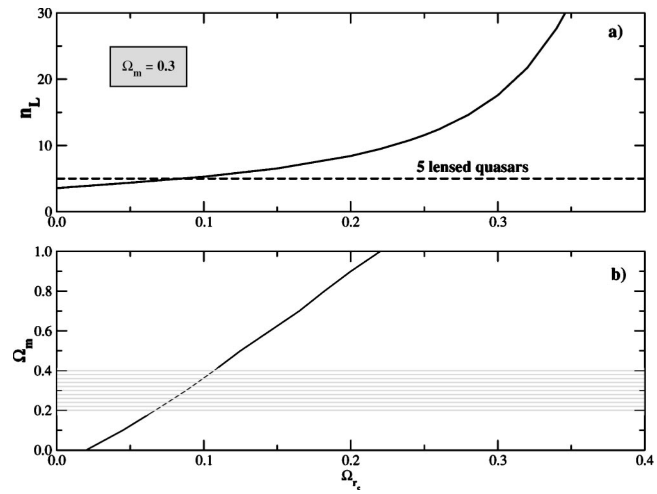

FIG. 3. (a) Predicted number of lensed quasars as a function of Ω_{r_c} for a fixed value of the matter density parameter ($\Omega_{\mathrm{m}}=0.3$) and image separation $\Delta\theta\leqslant 4$. (b) Contour for five lensed quasars in the parametric space $\Omega_{\mathrm{m}}-\Omega_{r_c}$. The shadowed horizontal region corresponds to the observed range $\Omega_{\mathrm{m}}=0.3\pm0.1$ [38].

In order to constrain the parameters Ω_{m} and Ω_{r_c} we perform a maximum-likelihood analysis with the likelihood function given by [21] (see also [37] for an alternative analysis based on the Fisher matrix approach)

$$\mathcal{L}=\prod_{i=1}^{N_U}(1-p_i')\prod_{k=1}^{N_L}p_k'\,p_{ck}', \tag{24}$$

where N_L is the number of multiple-imaged lensed quasars, N_U is the number of unlensed quasars, and p_k' and p_{ck}^{i} are the probability of quasar k to be lensed and the configuration probability defined, respectively, by Eqs. (20) and (21).

Figure 2 shows contours of constant likelihood (68% and 95.4%) in the parameter space $\Omega_{r_c}-\Omega_{\mathrm{m}}$. The maximum value of the likelihood function is located at $\Omega_{\mathrm{m}}=0$ and $\Omega_{r_c}=0.03$. At the 1σ level, our analysis requires $\Omega_{\mathrm{m}}\leqslant 0.94$ and $\Omega_{r_c}\leqslant 0.19$. Such a result means that a large class of these particular scenarios of brane world cosmology studied here are compatible with the current gravitational lensing data at this confidence level. In Fig. 3(a) the expected number of lensed quasars, $n_L=\Sigma p_i'$ (the summation is over a given quasar sample), is displayed as a function of Ω_{r_c} with the matter density parameter fixed at $\Omega_{\mathrm{m}}=0.3$ (as indicated by clustering estimates [38]). The horizontal dashed line indicates $n_L=5$, that is the number of lensed quasars in our sample. By this analysis, one finds $\Omega_{r_c}\simeq 0.08$, a value that is very close to that obtained by Deffayet *et al.* [17] ($\Omega_{r_c}=0.12$) using SNe Ia and CMB data and also with the same fixed value for the matter density parameter. In Fig. 3(b) we show the contour for five lensed quasars in the parametric space $\Omega_{\mathrm{m}}-\Omega_{r_c}$. The shadowed horizontal region corresponds to the observed range $\Omega_{\mathrm{m}}=0.3\pm0.1$ [38]. We observe that the higher the value of Ω_{m} the higher the contribution of Ω_{r_c} that is required to fit these data.

At this point it is interesting to estimate the value of r_c (the crossover distance between 4-dimensional and 5-dimensional gravities) from our estimates of Ω_{r_c}. In this case, an elementary combination of our best fit ($\Omega_{r_c}=0.03$) with Eq. (3) provides

$$r_c \simeq 2.8 H_o^{-1}, \tag{25}$$

while at the 1σ level ($\Omega_{r_c}\leqslant 0.19$), we have

$$r_c \geqslant 1.14 H_o^{-1}. \tag{26}$$

The former value is considerably larger than that found in Refs. [17,20], i.e., $r_c\simeq 1.4H_o^{-1}$ and $r_c\simeq 0.94H_o^{-1}$ in analyses involving SNe Ia + CMB and angular size of high-z sources data, respectively. However, it is worth mentioning that the estimate of r_c obtained in Ref. [17] refers to a flat model in which the value of the matter density parameter was fixed in $\Omega_{\mathrm{m}}=0.3$. As we have seen, by fixing this value for Ω_{m}, the predicted number of lensing quasars (Fig. 3a) requires $\Omega_{r_c}\simeq 0.08$ which, in turn, implies $r_c\simeq 1.76H_o^{-1}$.

IV. CONCLUSION

The recent observational evidences for a presently accelerated stage of the Universe have stimulated renewed interest for alternative cosmologies. In general, such models contain an unknown negative-pressure dark component that explains the SNe Ia results and reconciles the inflationary flatness prediction ($\Omega_{\mathrm{T}}=1$) with the dynamical estimates of the quantity of matter in the Universe ($\Omega_{\mathrm{m}}\simeq 0.3\pm0.1$). In this paper we have focused our attention on another dark energy candidate, one arising from gravitational *leakage* into extra dimensions [6,8]. We have shown that some particular scenarios based on this large scale modification of gravity are in agreement with the current gravitational lensing data for values of $\Omega_{\mathrm{m}}\leqslant 0.93$ (1σ). If one fixes Ω_{m} to be $\simeq 0.3$, the predicted number of lensed quasars requires $\Omega_{r_c}\simeq 0.08$. This is a slightly open universe with a crossover radius of the order of $r_c\simeq 1.76H_o^{-1}$.

ACKNOWLEDGMENTS

The authors are very grateful to Lee Homer, Carlos J. A. P. Martins and C. Deffayet for helpful discussions and a critical reading of the manuscript. J.S.A. is supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brasil) and CNPq (62.0053/01-1-PADCT III/Milenio).

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