Interacting dark matter disguised as warm dark matter

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We explore some of the consequences of dark-matter–photon interactions on structure formation, focusing on the evolution of cosmological perturbations and performing both an analytical and a numerical study. We compute the cosmic microwave background anisotropies and matter power spectrum in this class of models. We find, as the main result, that when dark matter and photons are coupled, dark matter perturbations can experience a new damping regime in addition to the usual collisional Silk damping effect. Such dark matter particles (having quite large photon interactions) behave like cold dark matter or warm dark matter as far as the cosmic microwave background anisotropies or matter power spectrum are concerned, respectively. These dark-matter–photon interactions leave specific imprints at sufficiently small scales on both of these two spectra, which may allow us to put new constraints on the acceptable photon–dark-matter interactions. Under the conservative assumption that the abundance of $10^{12}M_{\odot}$ galaxies is correctly given by the cold dark matter, and without any knowledge of the abundance of smaller objects, we obtain the limit on the ratio of the dark-matter–photon cross section to the dark matter mass $\sigma_{\gamma\text{-DM}}/m_{\rm DM} \lesssim 10^{-6}\sigma_{\rm Th}/(100\ \text{GeV}) \simeq 6\times10^{-33}\ \text{cm}^2\ \text{GeV}^{-1}$.

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I. INTRODUCTION

The nature of dark matter particles remains one of the major challenges for both fundamental physics and astrophysics. Whereas cold dark matter (CDM) perfectly explains the formation of large scale structure [1] on scales greater than 1 Mpc, there seem to be various discrepancies on smaller (subgalactic) scales. Some of these come from the following.

(1) *N*-body CDM simulations, which give cuspy halos with divergent profiles toward the center [2], in potential disagreement with the galaxy rotation curves [3] and with observations from gravitational lensing [4].

(2) Bar stability in high surface brightness spiral galaxies which also demands low-density cores [5].

(3) CDM models which for years have been seen to yield an excess of small scale structures [6–8]. Numerical simulations [9] found 1–2 orders of magnitude more satellite galaxies than what is observed [10]. However, recent work [11–13] indicates that there may be no problem for the galaxy mass function after all.

(4) The formation of disk galaxy angular momentum, which is much too small in galaxy simulations [2].

Problems 3 and 4 can be solved with the usual warm dark matter (WDM) which experiences free streaming and hence suppresses power on small scales [14–16]. Such a particle physics candidate is easy to find in a minimalistic extension of the standard model, namely, a sterile neutrino [17–20]. However, collisionless WDM does not solve problems 1 and 2 (see Ref. [21]) and one is therefore forced to propose more complicated models like scenarios of nonthermal production of weakly interacting massive particles (WIMPs) [22] for instance.

On the other hand, strongly interacting dark matter (SIDM) has been suggested [23] to solve problems 1 and 2 and does so successfully provided the cross section is within the range $2\times10^{-25}\ \text{cm}^2\ \text{GeV}^{-1} \lesssim \sigma_{\rm DM\text{-}DM}/m_{\rm DM} \lesssim 10^{-23}\ \text{cm}^2\ \text{GeV}^{-1}$ [24,25] (where $m_{\rm DM}$ and $\sigma_{\rm DM\text{-}DM}$ are the dark matter mass and self-interaction cross section, respectively). Problem 3 is also partly solved in this scenario [24] but the survival of galactic halos excludes the range $6\times10^{-25}\text{cm}^2\ \text{GeV}^{-1} \lesssim \sigma_{\rm DM\text{-}DM}/m_{\rm DM} \lesssim 2\times10^{-20}\ \text{cm}^2\ \text{GeV}^{-1}$ [26]. Furthermore, since the inner regions of massive clusters are elliptical, one must have $\sigma_{\rm DM\text{-}DM}/m_{\rm DM} \lesssim 3\times10^{-26}\ \text{cm}^2\ \text{GeV}^{-1}$ [27]. One therefore concludes that the allowed cross section is slightly too small and that SIDM, on its own, cannot solve problems 1–4.

This paper is finally motivated by the recent findings [28] that either dark-matter–photon or dark-matter–neutrino interactions can transfer to dark matter the damping that the photon or neutrino fluids undergo. This process, characterized in the simplest cases by an exponential cutoff in the matter power spectrum, was referred to as "induced damping" in Ref. [28]. In particular, by requiring that the damping induced by relativistic particles does not wash out the dark matter primordial fluctuations responsible for the formation of the smallest galaxies, it was found that the ratio of the corresponding cross sections to the dark matter mass must satisfy $\sigma_{\gamma\text{-DM}}/m_{\rm DM} < 10^{-30}\ \text{cm}^2\ \text{GeV}^{-1}$ and $\sigma_{\nu\text{-DM}}/m_{\rm DM} < 10^{-34}\ \text{cm}^2\ \text{GeV}^{-1}$. It was then suggested that, at the edge

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to be satisfied, these constraints could provide an alternative scenario for warm dark matter.

However, the exact value of these cross sections as well as the shape of the resulting power spectrum depend on the details of the interactions history of the fluids. We therefore determine—in this paper—the transfer function resulting from non-negligible interactions between dark matter and photons. The case of neutrino–dark-matter interactions will be examined in a subsequent paper. The effects thereof are naturally different from those due to self-interactions. However, one should keep in mind that a realistic dark matter particle probably has interactions both with itself and with other particles.

In Sec. II, we discuss the motivations for such dark matter particles. In Sec. III we describe the effect of interacting dark matter[1] (IDM) on the early evolution of cosmological perturbations and, in Sec. IV, we give an analytical fit to the main features produced by IDM on the matter power spectrum. In the Conclusion, we discuss the main result of our work, namely that, for dark matter coupled to photons, the original fluctuations are damped as soon as they enter the horizon, because of the impeded growth due to this coupling. We also discuss a few implications of this result.

II. MOTIVATIONS

Until the recent dark matter crisis, it was very well known that weakly interacting particles with a mass greater than a few keV (such as supersymmetric particles [29–32] for instance) did not suffer from prohibitive free streaming [33] or collisional damping effects [34] and could therefore represent a very promising solution to the dark matter puzzle. In fact, the interactions of such particles are generally assumed to be so weak that they can be neglected as far as structure formation is concerned.

However, the precise order of magnitude of the dark matter interactions for which it is justified to neglect these damping effects has never been given explicitly. A hint at the answer comes from investigating both the free-streaming and collisional damping scales of dark matter primordial fluctuations, taking into account all the possible interactions. By requiring that the latter do not wash out the fluctuations responsible for the formation of the smallest primordial structures [$M_{\mathrm{struct}} \sim (10^6 - 10^9) M_\odot$], one can obtain bounds on the dark matter particle's mass and interaction rates.

Let us consider primordial fluctuations made of dark matter and ordinary species like photons, neutrinos, baryons, and electrons, etc. One can show that the largest collisional damping effects may be due to the dark matter interactions with relativistic particles (e.g., neutrinos and photons). Focusing on the dark-matter–photon interactions, an analytic calculation [28] shows that dark matter must decouple from the photons at a redshift z_{dec} greater than $\sim 10^5$ to ensure that the spectrum at scales $k \leqslant k_{\mathrm{struct}} \sim 10$–$100\ \mathrm{Mpc}^{-1}$ (corresponding to the mass M_{struct} given above) is not exponentially damped by the photon interactions with baryons and electrons. Because of the exponential nature of the collisional damping effect, this necessary condition holds whatever the amplitude of the initial fluctuation spectrum is, and whatever its past history is.

The bound on z_{dec} can actually be translated into a limit on the dark-matter–photon interaction rate which finally turns into a constraint on the ratio of the photon–dark-matter cross section (at the DM-γ decoupling) to the dark matter mass: $\langle \sigma_{\gamma\text{-DM}} v \rangle / (m_{\mathrm{DM}} c) < 10^{-30}\ \mathrm{cm}^2\,\mathrm{GeV}^{-1}$ (here the angular brackets denote the statistical average owing to the fact that the coupling is due to momentum transfer). Thus, the lower the cross sections are, the smaller the dark matter mass must be in order to maintain the thermal equilibrium for a very long time.

The properties of such interacting dark matter have already been discussed in Ref. [28] (and will be so in more detail in [35]). In particular, it was pointed out that if they thermally decouple at a redshift[2] close to $z \sim 10^5$ they may behave as warm dark matter particles erasing structures with a size smaller than 10–100 kpc. Many other cases of WDM were considered in these papers but that of large dark-matter–photon interactions is especially interesting because one may expect some modifications in both the cosmic microwave background (CMB) and matter power spectra.

Despite their rather strong interactions with photons, these dark matter particles may still be considered as ''dark'' particles. Their thermal decoupling indeed occurs much before the recombination epoch and they therefore keep the universe transparent to photons from the last scattering surface to nowadays (as in the standard scheme). However, one may wonder if this dark matter is able to accumulate into stars and whether or not it may affect their properties. Note that, for the cross sections mentioned above, dark matter particles—once thermalized—have a mean free path $\lambda_{\mathrm{DM}} \sim (m_{\mathrm{DM}}/1\ \mathrm{GeV})^{1/2} R_\odot$ within the sun, potentially giving rise to heat conduction if their mass is smaller than a few GeV. For $m_{\mathrm{DM}} > m_p \sim 1\ \mathrm{GeV}$ [36], on the other hand, one expects dark matter to be able to evaporate so that no dark matter particles would be left in the sun. A γ-DM cross section ~ 100 times lower (as we shall be led to consider below) would predict even less accumulation in the sun.

A lower limit on the dark matter mass can be inferred from the free-streaming constraint. As far as our specific interaction rates are concerned, the well known condition $m_{\mathrm{DM}} > 1\ \mathrm{keV}$ [37] (obtained for a weakly interacting species) has to be replaced [28] by $m_{\mathrm{DM}} > 1\ \mathrm{MeV}$. These limits, however, hold as long as the thermal decoupling of dark matter occurs before the gravitational collapse and would be disregarded if we were considering at the same time extremely strong dark matter self-interactions for instance (see the discussion in [28] for the exact conditions of validity).[3]

[1]In the following, we shall adopt the notation of interacting dark matter for convenience but the reader has to remember that only dark-matter–photon interactions are investigated in this paper.

[2]The value of the redshift corresponding to the dark-matter–photon decoupling will be determined properly in this paper.

[3]It should be stressed that in our specific model the thermal decoupling of dark matter is fixed by the dark-matter–photon decoupling.

The condition $m_{\rm DM}>1$ MeV also ensures that the number of relativistic degrees of freedom during the primordial nucleosynthesis is the usual one.[4] Hence, one does not expect any problem concerning primordial nucleosynthesis in this case.

Dark matter particles must also have an acceptable relic abundance. This requirement actually constrains the nature of dark matter. We are considering, indeed, large elastic cross sections from 10^{-33} to 10^{-27} cm^2 for a dark matter mass in the MeV–TeV range for instance (according to the previous analytical estimate [28]). If there exists any symmetry between elastic and annihilation cross sections, this may provide annihilation cross-sections much larger than the ones required for weakly interacting massive particles which are expected to be roughly of the order of 10^{-36} cm^2 (for particles having chemically decoupled at their nonrelativistic transition). However, this constraint is obtained by assuming that the number of dark matter particles is exactly equal to that of anti-dark-matter particles in the primordial universe (or assuming that dark matter particles are Majorana particles). It can be disregarded if one makes the assumption that there exists a primordial asymmetry between particles and anti particles so that large elastic photon–dark-matter cross sections may finally be relevant. Such dark matter particles would then behave like baryons but with smaller interaction rates and would probably be neutral to avoid important reionization effects. This should therefore exclude, in principle, tree-level elastic and annihilation cross sections between fundamental dark matter particles and photons.

On the other hand, it is quite interesting to note that these constraints on the elastic cross sections from structure formation potentially imply annihilation cross sections (into two photons) close to the one proposed in Ref. [38] to make strongly annihilating dark matter a possible solution to the CDM crisis.[5] This, in fact, still represents an open possibility as discussed in Ref. [39].

As mentioned previously, large elastic scattering cross sections may imply large annihilation cross sections. If the latter exceed 10^{-36} cm^2, the condition for having an acceptable relic abundance leads to requiring an initial asymmetry between dark matter particles and antiparticles. This asymmetry implies that no antiparticles should be left after the dark matter annihilation in the primordial universe (unless one imposes a very large, quite unnatural fine-tuning). Therefore, in this case, there should not be any further annihilation of dark matter particles with their antiparticles in the center of galaxies or in galaxy clusters (similarly to baryons actually). No extra x-ray emission can then be expected from such dark matter particles (still assuming they have a too large annihilation cross section). This could in fact provide an important signature. If for one reason or another dark matter is shown to annihilate in the galactic center or in cluster of galaxies, one should be able to rule out such strongly interacting particles (for masses greater than a few MeV as we will see in the following, depending on the scale of the structures considered) or at least to strongly constrain their characteristics. In any case, it would become necessary to investigate their properties more closely.

On the other hand, one could also relax the assumption of a crossing symmetry between the elastic and the annihilation cross sections. However, it is hard in this case to predict whether the annihilation cross section would be much larger than the elastic cross section or not since no realistic models exhibit such an asymmetry.

At low energy, the elastic scattering cross sections should not induce any deviations from the black body spectrum. In any case, particles annihilating before (or around) $z\sim10^{10}$ and thermally decoupling before $z\sim10^5$ leave enough time for any irregularity to be erased at the recombination epoch (because of the usual coupling between photons and baryons). At least, if some distortions exist, they should be very small.

Finally, a still important question concerns the signature of such interactions in the CMB and matter power spectra. This is precisely the aim of the present paper. We shall adopt the definitions given in [28], following which particles having a collisional damping or free-streaming length of the order of $l_{\rm struct}\propto k_{\rm struct}^{-1}$ are called warm dark matter; and particles having a collisional damping or free-streaming length much lower than $l_{\rm struct}\propto k_{\rm struct}^{-1}$ are called cold dark matter. This actually differs from the current WDM definition since we now take into account the possibility that dark matter may be "warm" not because of its mass but because of its interactions with another species (referred to as "induced damping" effects in [28]). Since the above criteria are based on the shape of the matter power spectrum (not on the form of the CMB spectrum), the signature of such interacting dark matter in the CMB anisotropies cannot be easily inferred without performing numerical calculations. We now investigate how such a kind of dark matter changes the relevant equations for determining both the CMB and the matter power spectra.

III. THE PHYSICS OF INTERACTING DARK MATTER

In this section, we first recall the main physical effects that arise when one considers coupled fluids. As a warm-up, we shall recall the main equations governing the evolution of the photon-baryon plasma before recombination (Sec. III A). We then write the modified perturbation equations for the cosmological perturbations including interacting dark matter (Sec. III B) and study the damping experienced by dark matter fluctuations (Secs. III C–III F). Finally, the most prominent observational consequences of IDM on cosmic microwave background anisotropies (Sec. III G) and on the matter power spectrum (Sec. III H) are discussed.

A. Reminder of the influence of Thomson scattering on cosmological perturbations

The aim of this paper is to study the interactions between dark matter and photons which are *a priori* quite similar to

[4]Open windows for $m_{\rm DM}\lesssim1$ MeV would require a more involved scenario [35]. Together with galaxy dynamic results, this would make such a case very unlikely.

[5]However, as we will see in the next section, the constraints we will finally obtain in this paper are smaller by a factor $\sim10^{-3}$.

Thomson scattering between photons and baryons. Therefore, we shall first recall the standard case of photons coupled to baryons through Thomson scattering cross sections and collisionless dark matter. Indeed, as we are see later, some of the effects that affect the baryon fluid are also present for dark matter.

In the following sections, we introduce the Euler equation for photons, baryons, and dark matter (Sec. III A 1), and then compute the dark matter perturbation evolution (Sec. III A 2). We describe a situation in which the coupling between photons and baryons can play a significant role, and describe the usual damping phenomena which can affect the cosmological perturbations (Sec. III A 3).

1. Photon and baryon Euler equations

We consider nonrelativistic baryons coupled to photons through Thomson scattering. The corresponding Euler equations for these two fluids are

$$\dot{v}_{\mathrm{b}}=k\Phi-\mathcal{H}v_{\mathrm{b}}-R^{-1}\dot{\kappa}(v_{\mathrm{b}}-v_{\gamma}), \tag{3.1}$$

$$\dot{v}_{\gamma}=k\Phi+\frac{1}{4}k\delta_{\gamma}-\frac{1}{6}k\pi_{\gamma}-\dot{\kappa}(v_{\gamma}-v_{\mathrm{b}}), \tag{3.2}$$

where an overdot denotes a derivative with respect to the conformal time η, $\mathcal{H}$ is the conformal Hubble parameter ($\mathcal{H}\equiv\dot{a}/a$ with a being the scale factor), δ_X, v_X, and π_X represent the density contrast, the velocity divergence, the anisotropic stress of the species X, respectively (we work in Newtonian gauge), and Φ is the Bardeen potential (see, e.g., Refs. [40–42]). The Thomson scattering term between photons and baryons reads

$$\dot{\kappa}=a\sigma_{\mathrm{Th}}n_e, \tag{3.3}$$

where σ_{Th} is the standard Thomson scattering cross section, and n_e is the free electron number density. This quantity is also referred to as the differential opacity since it also gives the scattering rate of a photon by free electrons. Finally, R denotes the "baryon-to-photon ratio," that is,

$$R\equiv\frac{3}{4}\frac{\rho_{\mathrm{b}}}{\rho_{\gamma}}. \tag{3.4}$$

This factor in the baryon Euler equation ensures that the overall momentum is conserved for the two fluids.

2. The growth of dark matter perturbations

Usually, dark matter is not coupled to any species (except through gravitational interactions) so that its perturbations follow the simple equation

$$\dot{v}_{\mathrm{DM}}=-\mathcal{H}v_{\mathrm{DM}}+k\Phi. \tag{3.5}$$

It is well known that in the matter dominated epoch, the above equation implies that dark matter density perturbations grow as the scale factor: $\delta_{\mathrm{DM}}\propto a$.

In the radiation dominated epoch, it is also well known that the dark matter density contrast [as soon as the fluctuation enters into the Hubble radius (say $k>\mathcal{H}=\eta^{-1}$)] grows logarithmically [43] as

$$\delta_{\mathrm{DM}}\sim\delta_k B(k\eta), \tag{3.6}$$

where

$$B(k\eta)\sim 1-\alpha\ln(k\eta)+\beta\ln^2(k\eta), \tag{3.7}$$

with $\alpha=v_k/\delta_k\sim 0$ and $\beta=\Phi_k/\delta_k\sim 1$, where δ_k, v_k, Φ_k are the amplitudes of the corresponding quantities at $k\eta\ll 1$ (i.e., well before entering the Hubble radius; the actual values of α and β actually depend on the initial conditions given, e.g., by inflation; see, for example, [44]). This yields approximately $B(k\eta)\sim 1+\ln^2(k\eta)$.

It is particularly important to compute this quantity since the growth (3.6) will be suppressed when we consider the case where dark matter is coupled to photons when it enters into the Hubble radius.

3. Free-streaming and collisional damping

For standard cosmological perturbations, there are essentially two damping phenomena: one (which mainly concerns neutrinos) is free streaming, and the other one (which concerns photons) is collisional damping. Both of these are related to the presence of a small but nonzero anisotropic stress in Eq. (3.2). In order to compute this anisotropic stress, one must remember that the usual Euler equation is in fact part of the hierarchy of the Boltzmann equation in which one expands the angular dependence of the temperature contrast Θ in terms of multipoles, which obey the following hierarchy:

$$\dot{\Theta}_l=\frac{k}{2l+1}[l\Theta_{l-1}-(l+1)\Theta_{l+1}]+S_l-\dot{\kappa}\Theta_l. \tag{3.8}$$

This hierarchy involves source terms S_l, which include gravity, polarization, etc. [40–42]. For $l=1$, by comparing Eq. (3.8) to Eq. (3.2), we easily recover

$$\Theta_0=\delta_{\gamma}, \tag{3.9}$$

$$\Theta_1=\frac{4}{3}v_{\gamma}, \tag{3.10}$$

$$\Theta_2=\frac{1}{4}\pi_{\gamma}. \tag{3.11}$$

Photon free streaming occurs when both Thomson scattering and the source terms are negligible. In this case, the hierarchy admits a simple solution involving spherical Bessel functions:

$$\Theta_l\propto j_l(k\eta)\propto\frac{\sin(k\eta+\phi)}{k\eta},\quad k\eta\gg 1. \tag{3.12}$$

The interpretation of this behavior is that, since the mean free path of the photons is very large, they simply flow away from the overdense region toward the underdense regions.

On the contrary, collisional damping appears when photons and baryons are strongly coupled. For $l=2$, Eq. (3.8) becomes

$$\dot{\Theta}_2=\frac{k}{5}(2\Theta_1-3\Theta_3)+S_2-\dot{\kappa}\Theta_2. \tag{3.13}$$

For higher multipoles strong coupling implies that for $l>2$, $\Theta_l\propto\dot{\kappa}^{-(l-1)}$. For $l=2$, if we can neglect S_2, we have

$$\Theta_2=\frac{2}{5}\frac{k}{\dot{\kappa}}\Theta_1. \tag{3.14}$$

(If one takes polarization into account, then S_2 is of the same order of magnitude as $\dot{\kappa}\Theta_2$, and the above two equations are modified by small numerical factors.) Injecting this result into Eq. (3.2) and also taking into account the Boltzmann equation for $l=0$, one obtains a damped oscillator equation, in which the term involving π_γ acts as a damping term, which takes into account both viscosity and heat conduction. This damping is seen to be exponential and appears only when $k^2\gg\dot{\kappa}/\eta$. The interpretation of the above limit is that the damping occurs only for scales smaller than the photon diffusion length. In conclusion, the photon density perturbations follow

$$\delta_\gamma\sim\delta_k\cos(k\eta/\sqrt{3})e^{-2k^2\eta/15\dot{\kappa}}. \tag{3.15}$$

(We have taken the limit $R\to0$ here and neglected the influence of polarization; see Refs. [45,46] for more detailed calculations.)

Cosmological baryon density perturbations will essentially follow the evolution of photon fluctuations as long as strong coupling is effective. At sufficiently small scales, they will therefore be damped before recombination. However, they will rapidly fall into the dark matter potential well afterward, so that any damping in the baryon fluid is rapidly "forgotten" after recombination. Of course, this occurs only because there is an "extra" fluid (dark matter) which was never coupled to photons and which can subsequently have some gravitational interactions with baryons.

We shall now investigate how these conclusions are modified by the presence of interacting dark matter.

B. Perturbation equations for IDM

Assuming IDM is always nonrelativistic during the epochs of interest, and interacts with photons only, we have the following modified Euler equations for baryons, photons, and dark matter:

$$\dot{v}_b=k\Phi-\mathcal{H}v_b-R^{-1}\dot{\kappa}(v_b-v_\gamma), \tag{3.16}$$

$$\dot{v}_\gamma=k\Phi+\frac{1}{4}k\delta_\gamma-\frac{1}{6}k\pi_\gamma-\dot{\kappa}(v_\gamma-v_b)-\dot{\mu}(v_\gamma-v_{DM}), \tag{3.17}$$

$$\dot{v}_{DM}=k\Phi-\mathcal{H}v_{DM}-S^{-1}\dot{\mu}(v_{DM}-v_\gamma), \tag{3.18}$$

where we have set

$$S\equiv\frac{3}{4}\frac{\rho_{DM}}{\rho_\gamma} \tag{3.19}$$

(note that we also have $S\propto a$ and that $1+R^{-1},1+S^{-1}\propto a^{-1}$ at early times and ~1 at late times), and where $\dot{\mu}$ represents the interaction rate between photons and dark matter. By analogy with Eq. (3.3), we write

$$\dot{\mu}\equiv a\sigma_{\gamma\text{-DM}}n_{DM}, \tag{3.20}$$

where $n_{DM}=\rho_{DM}/m_{DM}$ is the dark matter number density and $\sigma_{\gamma\text{-DM}}$ is the photon–dark-matter cross section. For simplicity, we shall assume that it is constant at low energy as for the Thomson scattering case. Since both dark matter and baryons are supposed to be nonrelativistic, both $\dot{\mu}$ and $\dot{\kappa}$ behave as a^{-2} at high redshift. Their ratio is therefore constant in this regime, and is characterized by the parameter

$$u\equiv\left[\frac{\sigma_{\gamma\text{-DM}}}{\sigma_{Th}}\right]\left[\frac{m_{DM}}{100\ \text{GeV}}\right]^{-1}. \tag{3.21}$$

This reads $\dot{\mu}=u\dot{\kappa}\Omega_{DM}/106\Omega_b$, or $\dot{\mu}\sim\frac{1}{16}u\dot{\kappa}$ with our choice of cosmological parameters. We emphasize that the above parameter u (measuring the relative size of $\dot{\mu}$ and $\dot{\kappa}$) is defined before recombination. Note that after recombination $\dot{\kappa}$ is strongly suppressed (by a factor $\sim10^{-4}$, see [37]) because of the drastic subsequent drop in the free electron density, while we assume that $\dot{\mu}$ never suffers from such a modification.

We shall now investigate the damping experienced by the dark matter perturbations because of their coupling with photons.

C. Dark-matter–photon coupling

For many cases of interest, the differential opacity $\dot{\mu}$ is large compared to the wave number k. In other words, the photon mean free path is small compared to the scale of interest. This is true, in particular, at high redshift, because $\dot{\mu}$ grows as $(1+z)^2$. In practice, one obtains a stiff system of equations, which physically means that the relative velocity between the two species is small, so that they can be considered as a single fluid. It is therefore more convenient to consider the following two quantities:

$$v_{\gamma DM}\equiv\frac{v_\gamma+Sv_{DM}}{1+S}, \tag{3.22}$$

$$w_{\gamma DM}\equiv v_\gamma-v_{DM}. \tag{3.23}$$

In term of these two variables, we have

$$v_\gamma=v_{\gamma DM}+\frac{S}{1+S}w_{\gamma DM}, \tag{3.24}$$

$$v_{\mathrm{DM}}=v_{\gamma\mathrm{DM}}-\frac{1}{1+S}w_{\gamma\mathrm{DM}}. \tag{3.25}$$

Keeping in mind that $S\propto a$, so that $\dot{S}=\mathcal{H}S$, we then have

$$\dot{v}_{\gamma\mathrm{DM}}=k\Phi+\frac{k}{1+S}\left(\frac{1}{4}\delta_\gamma-\frac{1}{6}\pi_\gamma\right)-\frac{S}{1+S}\mathcal{H}v_{\gamma\mathrm{DM}}-\frac{\dot{\kappa}}{1+S}(v_\gamma-v_\mathrm{b}), \tag{3.26}$$

$$\dot{w}_{\gamma\mathrm{DM}}=\frac{1}{4}k\delta_\gamma-\frac{1}{6}k\pi_\gamma-\dot{\kappa}(v_\gamma-v_\mathrm{b})-\frac{1+S}{S}\dot{\mu}w_{\gamma\mathrm{DM}}. \tag{3.27}$$

In order to study the evolution of these quantities, we have to consider various cases, depending on the relative values of k, $\mathcal{H}$, and $(1+S^{-1})\dot{\mu}$. For a given wavelength, these various case occur in the following chronological order.

Case 1: Large wavelength limit. This occurs when a given wavelength has not yet entered into the Hubble radius:

$$k<\mathcal{H}. \tag{3.28}$$

In this case the cosmological perturbations do not experience any significant evolution, whatever the amplitude of the scattering is. Large wavelengths eventually enter into the Hubble radius after dark matter has decoupled from photons. In this case, we switch directly to case 4 below. Otherwise, we have case 2.

Case 2: Strong coupling regime. This occurs when the scattering rate is higher than the expansion rate and the photon oscillation frequency:

$$\mathcal{H}<k<(1+S^{-1})\dot{\mu}. \tag{3.29}$$

In this case, Eq. (3.27) reduces to

$$w_{\gamma\mathrm{DM}}=\frac{1}{(1+S^{-1})}\frac{k}{\dot{\mu}}\left(\frac{1}{4}\delta_\gamma-\frac{1}{6}\pi_\gamma-\frac{\dot{\kappa}}{k}(v_\gamma-v_\mathrm{b})\right). \tag{3.30}$$

The anisotropic stress can be neglected in the above equation, whereas in Eq. (3.26) one has

$$\pi_\gamma=\frac{8}{5}\frac{k}{\dot{\kappa}+\dot{\mu}}v_\gamma. \tag{3.31}$$

This ensures that the bulk velocities of the two fluids are almost identical, so that $v_{\gamma\mathrm{DM}}$ can be replaced by v_γ in Eq. (3.26). Microphysics plays a significant role in the evolution of the cosmological perturbations. Thus, although small [see Eq. (3.31)], the photon anisotropic stress cannot be neglected. In particular, this is the regime in which the photon fluctuations—with their Silk damping—are fully transferred to the dark matter.

Case 3: Weak coupling regime. This represents the intermediate case, where

$$\mathcal{H}<(1+S^{-1})\dot{\mu}<k. \tag{3.32}$$

This regime always occurs between the strong coupling regime and decoupling since the interaction rate must drop below k before reaching $\mathcal{H}$. This regime is really effective when it lasts several expansion times, and gives rise to new, unexpected effects.

This regime, indeed, is new: for baryons coupled to photons, it is not relevant. Thomson scattering gives rise to the same succession of events: $k<\mathcal{H}$, $\mathcal{H}<k<(1+R^{-1})\dot{\kappa}$. However, in this case, for the relevant wavelengths, recombination occurs in the strong coupling regime. Thus, within a negligible fraction of time, one switches to $(1+R^{-1})\dot{\kappa}<\mathcal{H}<k$, therefore skipping the weak coupling regime. Moreover, the damping due to the weak coupling regime is due to an averaging of the photon fluctuations, which are transmitted to the dark matter, over many oscillations. This requires in the baryon case $k\delta\tau\gg 1$, where $\delta\tau$ is the thickness of the last scattering surface. In this regime, many other sources of damping are present, compared to which weak coupling effects are negligible. The weak coupling regime for baryons does not occur for waves of horizon size such as the ones considered for dark matter.

Case 4: No coupling. This occurs when the scattering rate is negligible with respect to the expansion rate:

$$(1+S^{-1})\dot{\mu}<\mathcal{H}<k. \tag{3.33}$$

In this case, it is safe to neglect all the terms involving DM-γ scattering in the above equations (3.17), (3.18) and the two fluids evolve independently from each other.

Clearly, the interesting cases to be discussed are cases 2 and 3.

D. Dark-matter–photon decoupling before recombination

The (more interesting) case we will focus on in detail is when dark matter is coupled to photons and the photon interactions are dominated by the Thomson scattering process (e.g., when the photon–dark-matter interactions decouple before recombination). We will also assume that we are before the radiation to matter transition. In this case, the dark matter perturbations will be *driven* by the photon perturbations, which follow their usual behavior. As we suppose dark matter decouples from photons before recombination, we have $u<2$. This implies that the photon coupling is stronger with baryons than with dark matter. This condition is valid for any realistic nonzero dark-matter–photon interactions. Depending on the amplitude of the photon–dark-matter coupling (and, hence, on the epoch where they decouple) several different effects are to be expected. We are now going to discuss cases 2 and 3 defined above (strong and weak coupling).

For simplicity, we have considered only the case of a radiation dominated universe, with $R,S\ll 1$. This will be sufficient to explain the important results: the only cases where this does not hold are not relevant, because realistic models predict that dark matter must decouple from the photons before equality [28]. A more complete classification may be found in Ref. [35]. The extension to the matter dominated case, if desired, is straightforward anyway. Our numerical results are of course given without such a restriction.

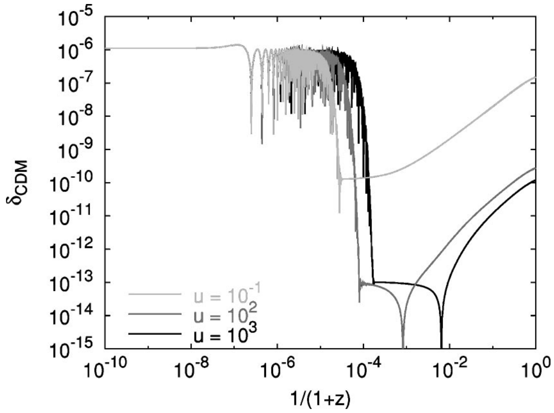

FIG. 1. Evolution of dark matter density perturbation as a function of the redshift for various (*large*) interaction rates with photons, all leading to collisional damping. In this plot, as in the others, we consider a dark matter model with a cosmological constant, with $h=0.65$ (i.e., $H_0=100h$ km s^{-1} Mpc^{-1}), $\Omega_\Lambda=0.7$, $\Omega_{\rm mat}=0.3$, $\Omega_b h^2=0.019$, and a scalar perturbation index $n_S=1$. We have considered the mode $k=40$ Mpc^{-1}. At $z>10^7$, the mode is outside the Hubble radius and the perturbation is frozen. The perturbation first experiences undamped oscillations (strong coupling regime) and then exponentially damped oscillations, corresponding to the collisional damping regime. The γ-DM cross sections are parametrized by the quantity $u \equiv [\sigma_{\gamma\text{-DM}}/\sigma_{\rm Th}][m_{\rm DM}/100\ \text{GeV}]^{-1}$.

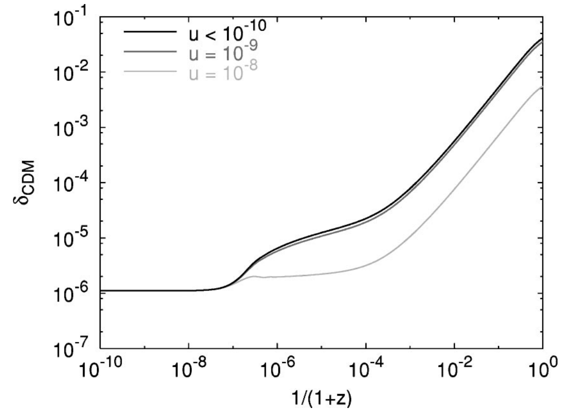

FIG. 2. Evolution of CDM density perturbation as a function of the redshift for various (small) interaction rates with photons. As expected, for sufficiently small cross sections, almost no effect is noticeable. For slightly larger cross sections, the coupling between dark matter and photons stops just after the mode has entered into the Hubble radius, preventing the perturbation from experiencing the small growth in the radiation dominated era.

1. Strong coupling regime ($\mathcal{H}<k<S^{-1}\dot{\mu}$)

In this case, we have $\delta_{\rm DM}\sim\delta_\gamma$, with the photon fluctuations given by their usual expression (3.15).

a. Late decoupling. Collisional damping in the dark matter fluid occurs when the photons are subject to damping, which is due—as usual—to the presence of the baryons, when $k^2 \gg \dot{\kappa}/\eta$. This as previously implies many oscillations after the mode k enters into the Hubble radius before damping is to occur (Fig. 1). Of course, as one can see on Fig. 1, the damping phenomenon starts later, but lasts longer and is more important as the γ-DM cross-section increases. Note also (this will be important later) that the epoch at which the collisional damping stops is difficult to compute. In the case considered here, it occurs when Φ in Eqs. (3.16)–(3.18), although quite small, is no longer negligible compared to the strongly damped density contrasts. In other words, the damping stops when gravity becomes the dominant term in the Euler equation.

b. Early decoupling. As opposed to the above case, for much smaller cross section, there still can be coupling between dark matter and photons, but without collisional damping at the scales of interest. This occurs for cross sections sufficiently small so that the decoupling occurs soon after the mode enters into the Hubble radius, in which case only the small logarithmic growth (3.6) of dark matter perturbations during the radiation dominated epoch can be suppressed. Some examples of this are shown on Fig. 2. Note that the suppression of the logarithmic growth during the radiation era is enough to reduce the power spectrum of the dark matter fluctuations by one order of magnitude, which is already a large effect.

2. A new damping regime: The weak coupling regime ($\mathcal{H}<S^{-1}\dot{\mu}<k$)

When one considers cross sections intermediate to those considered in the above paragraphs, a new phenomenon can occur. For this, several conditions must be satisfied.

(1) The coupling between dark matter and photons is sufficiently small so that we have $v_{\rm DM} \not\sim v_\gamma$.

(2) The coupling between dark matter and photons is sufficiently large so that the velocity of dark matter perturbation is "driven" by that in the photon perturbations: this means that they experience oscillation at the same frequency and therefore $\dot{v}_{\rm DM}\sim(k/\sqrt{3})v_{\rm DM}$.

(3) Gravity must be negligible in the dark matter Euler equation.

It is easy to see that this can occur in the above defined *weak coupling regime*. Then, Eq. (3.18) reduces to

$$\dot{v}_{\rm DM} \simeq S^{-1}\dot{\mu} v_\gamma \propto k v_{\rm DM}. \tag{3.34}$$

This implies

$$\delta_{\rm DM} \sim \frac{S^{-1}\dot{\mu}}{\mathcal{H}} \frac{\sin(k\eta/\sqrt{3})}{k\eta/\sqrt{3}} e^{-2k^2\eta/15\dot{\mu}}, \quad k\eta \gg 1. \tag{3.35}$$

Now, in the radiation dominated era, as long as there is no collisional damping between photons and baryons, v_γ experiences undamped oscillations. This means that *the dark matter fluctuations are damped as $S^{-1}\dot{\mu}$ as a function of time*, that is, as a^{-3}.

Except for a time-dependent normalization factor (which reduces to unity at the DM-γ decoupling), this form is very similar to the damping due to the free streaming of a relativistic fluid. Obviously, in the present case, the fluids are coupled and far from free streaming. Both the photon and dark matter mean free paths are still small; therefore we are

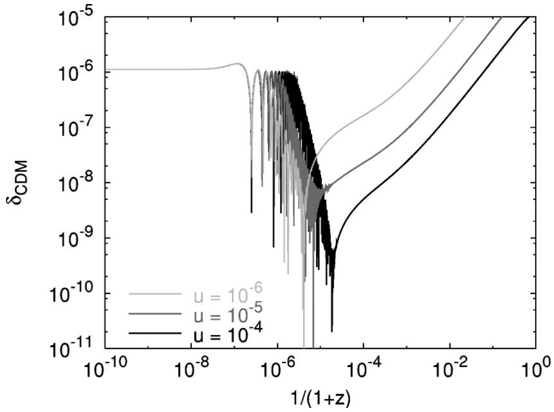

FIG. 3. Evolution of CDM density perturbation as a function of the redshift for various (intermediate) interaction rates with photons. As explained in the text, the dark matter perturbations experience a power law decay due to their weak coupling with photons. This is the main effect we can expect to have at small scales for acceptable cross sections.

well in the collisional regime. However, the coupling rate between dark matter and photons $S^{-1}\dot{\mu}$ is much smaller than the photon oscillation frequency $k/\sqrt{3}$. The slow reaction of the dark matter to the photon oscillations then mixes modes with different phases, as does the free-streaming process which collects particles of different origin. Of course, should v_γ be damped or affected by another fluid for one reason or another, then the CDM fluctuations would also feel it. For example, collisional damping in the photon-baryon fluid may already be at work in the weak coupling regime, or may also be effective after weak coupling occurs and thus less apparent. This is in fact what we can barely see on Fig. 3 for the highest cross section where the damping obviously increases soon before decoupling.

To be fully developed, the weak coupling regime requires $k \gg \mathcal{H}$. For $k \geqslant \mathcal{H}$, which will be a case of importance below, it is not well separated from the strong coupling regime. Due to the rapid variations in time of $S^{-1}\dot{\mu}$ compared to $\cos(k\eta/\sqrt{3})$ near $k\eta \sim 1$, Eq. (3.37) still holds during the first few oscillations. The dark matter fluctuations are thus given by Eq. (3.37) for $k \sim \mathcal{H}$, that is $k\eta \sim 1$, and go over to Eq. (3.35) for $k\eta \gg 1$.

E. Dark-matter–photon decoupling after recombination

In this nonstandard scheme, the photon–dark-matter interactions decouple after the recombination epoch. This is therefore the case where the usual coupling between photons and baryons is vanishingly small after recombination as opposed to the coupling between photons and dark matter. With our choice of cosmological parameters, this case occurs for $u > 2$. This unrealistic example is given here for pedagogical purpose only. Clearly, collisional damping in this case is larger than the Silk damping, implying that this scenario is of no cosmological relevance.

As dark matter and photons are more tightly coupled than photons and baryons, the damping of the fluctuations is there due to the interaction of the photons with the dark matter, not with the baryons. In this case the expression (3.31) for π_γ becomes

$$\pi_\gamma \sim \frac{k}{\dot{\mu}} \frac{8}{5} v_\gamma. \tag{3.36}$$

Hence, after recombination (and also before recombination in case $\dot{\mu} > \dot{\kappa}$, that is $u > 16$), we have

$$\delta_{\rm DM} \sim \delta_\gamma \sim \delta_k \cos(k\eta/\sqrt{3}) e^{-2k^2\eta/15\dot{\mu}}. \tag{3.37}$$

Note that even in this case, the damping starts only after many oscillations as one must require $k^2 \gg \dot{\mu}/\eta$, which may not yet hold when the mode enters into the Hubble radius. Figure 1 shows some examples of dark matter and photons experiencing collisional damping due to γ-DM interactions.

F. Dark matter damping factor

We may now evaluate the total damping at the decoupling of the dark matter with the photons. Provided the fluctuations are not too much damped (this is indeed the only relevant case: it is of no interest to evaluate very accurately fluctuations which are negligible), it is given by Eqs. (3.15) or (3.35) to which the exponential collisional damping is added. The latter is to be taken at the time where the dark matter decouples from the photons, namely, $\eta = \eta_{\rm dec}$, the solution of $S^{-1}\dot{\mu} \sim \mathcal{H}$:

$$T_{\rm IDM} \sim \begin{cases} \dfrac{\cos(k\eta_{\rm dec}/\sqrt{3})}{B(k\eta_{\rm dec})} e^{-2k^2\eta_{\rm dec}/15\dot{\kappa}}, & k\eta_{\rm dec} \sim 1, \quad (3.38) \\[2ex] \dfrac{\sin(k\eta_{\rm dec}/\sqrt{3})}{B(k\eta_{\rm dec})k\eta_{\rm dec}/\sqrt{3}} e^{-2k^2\eta_{\rm dec}/15\dot{\kappa}}, & k\eta_{\rm dec} \gg 1. \quad (3.39) \end{cases}$$

The preexponential oscillating factor is the reduction due to the strong and weak coupling to the photons, and $B(k\eta_{\rm dec})$ is the (logarithmic) reduction due to the impeded growth of the dark matter fluctuations compared to the noninteracting case. These factors come into play as soon as $k\eta_{\rm dec}$ is larger than unity, that is, for the modes which just enter the Hubble radius at the DM-γ decoupling. The exponential factor is the damping due to viscous effects. It enters into play for modes which are of the size of the length traveled by the collisional photons at the time of the DM-γ decoupling. Due to the strong γ-e interaction, this length is in the present case much smaller than the Hubble radius.

G. CMB anisotropies

When computing the CMB anisotropies, one must take into account the modification to the photon Boltzmann hierarchy induced by the dark matter interactions. This is due to the fact that both free electrons and dark matter particles are responsible for photon scattering. For the scalar part of the multipoles Θ_l of the distribution function, one now has

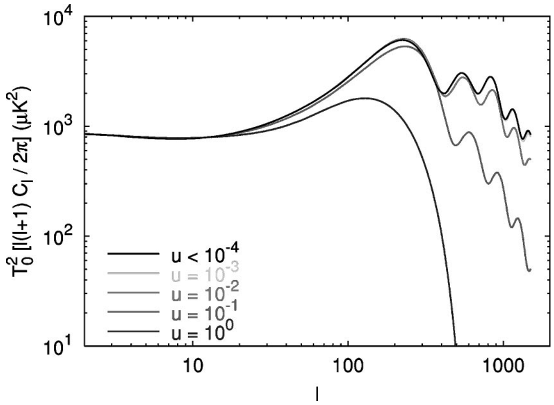

FIG. 4. Influence of interacting dark matter on the CMB anisotropy spectrum as a function of the dark-matter–photon cross section. The most spectacular effect which occurs at sufficiently large $\sigma_{\gamma\text{-DM}}$ is the apparent damping due to the large width of the last scattering surface, with some additional collisional damping due to photon–dark-matter interactions, and a slight shift of the Doppler peaks due to the decreases of the sound speed in the dark-matter–baryon–photon "plasma."

$$\dot{\Theta}_l=\frac{k}{2l+1}[l\Theta_{l-1}-(l+1)\Theta_{l+1}] +S_l-(\dot{\kappa}+\dot{\mu})\Theta_l, \tag{3.40}$$

where S_l is the usual source term which also involves an extra term involving the dark matter velocity, as photons can scatter on both baryons and dark matter. Obviously, the differential opacity is now $\dot{\kappa}+\dot{\mu}$, which can in some cases be different from the usual term $\dot{\kappa}$. In this case, recombination and the subsequent drop in the free electron density do not necessarily imply photon decoupling because of their interactions with dark matter. Therefore, large interactions between dark matter and photons can significantly delay the epoch of the photon last scattering. The net effect will be to enlarge the width of the last scattering surface, and hence to increase the damping of the observed CMB anisotropies. Of course, the effect is qualitatively similar to late reionization. The presence of several Doppler peaks as detected by the most recent experiments [47–49] therefore puts a firm upper limit on the dark-matter–photon cross section: its contribution to the differential opacity must be small at $z\sim1000$. Some examples are represented on Fig. 4. We therefore have by eye the constraint

$$\frac{\sigma_{\gamma\text{-DM}}}{m_{\rm DM}}\lesssim10^{-3}\frac{\sigma_{\rm Th}}{100\ \ \mathrm{GeV}}. \tag{3.41}$$

Actually, this is roughly speaking the constraint that one must have in order not to significantly increase the opacity *just after* recombination.

In addition, some other effects are in principle observable on the CMB anisotropy spectrum.

First, one expects that there will be collisional damping on small scales, which will also produce an exponential cutoff in the spectrum (this will be discussed later). For realistic cross sections, which could damp perturbations below 100 kpc$\sim10^{-3}$100 Mpc, this occurs roughly at angular scales $\sim10^3$ smaller that the first Doppler peak, i.e., around $l\sim10^5$. This is of course far too small to be observable even in the far future. For higher cross sections, the effect can be similar to that of large width of the last scattering surface. Distinguishing between the two is not easy, but it happens that the latter is dominant in our model.

Second, the sound speed is modified by the presence of dark matter. The sound speed is now

$$c_{\rm s}=\frac{1}{\sqrt{3}}\frac{1}{(1+R+S)^{1/2}}, \tag{3.42}$$

instead of $[3(1+R)]^{-1/2}$ in the case of strong coupling. This means that if dark matter is still coupled to photons at last scattering, the acoustic oscillation will have a lower frequency and the Doppler peak structure will be shifted to higher multipoles. This is what we can see on Fig. 4 for $\sigma_{\gamma\text{-DM}}=10^{-1},10^{-2}\sigma_{\rm Th}$. For lower cross sections, dark matter has already decoupled at $z\sim1000$, and for higher cross sections, the exponential damping also significantly shifts the peak positions in the other direction. It seems that one cannot easily shift the peak position in that way without also modifying the photon last scattering history. This is due to the fact that there is *a priori* no reason for a violent drop in $\dot{\mu}$ around $z\sim1000$ as is the case for $\dot{\kappa}$ (see also [50]).

It happens that these effects are unobservable in realistic models given the much stronger constraints that arise from the matter power spectrum.

H. Matter power spectrum

When studying the influence of IDM on the matter power spectrum, one can expect to observe four different regimes.

For small k (large wavelengths), the perturbations enter the Hubble radius rather late, when $k=\mathcal{H}\gg S^{-1}\dot{\mu}$. This occurs after the $\dot{\mu}$ terms of Eqs. (3.17),(3.18) have already become negligible. So the mode enters into the Hubble radius decoupled. For these modes, there is no difference from the usual case where there is no coupling between dark matter and photons.

For larger k (smaller wavelengths), the mode enters the Hubble radius when $k=\mathcal{H}\ll S^{-1}\dot{\mu}$, when the dark matter is coupled to the photons. This results in a reduction in the dark matter fluctuation amplitude compared to the noninteracting case as soon as the mode is within the Hubble radius. If the interaction of the dark matter with the photons is not too strong, the DM-γ decoupling occurs before collisional damping is sizable. This corresponds to the *weak coupling regime*. The spectrum then shows a characteristic behavior due to this weak coupling, namely, a series of damped oscillations of slope $k^{n_{\rm S}-6}$.

For even larger k (smaller scales), the perturbation enters the Hubble radius even earlier, still when $k=\mathcal{H}\ll S^{-1}\dot{\mu}$, but also when k is sufficiently large so that at a later time $k^2/\dot{\kappa}\gg\mathcal{H}$ before the dark-matter–photon decoupling. Then, the perturbations experience collisional damping: dark matter,

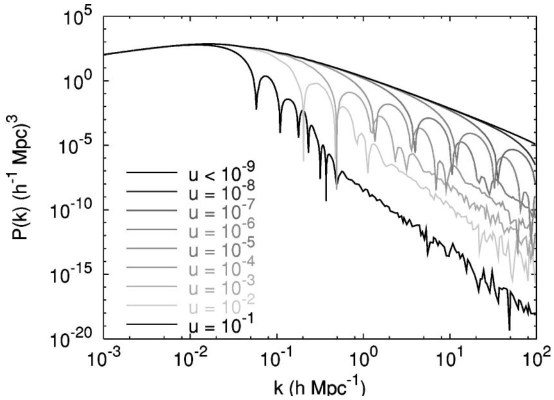

FIG. 5. Influence of interacting dark matter on the matter power spectrum. As explained in the text, the deviation from a CDM power spectrum exhibits several regimes, the strong coupling regime, the collisional damping regime, and the neutrino regime. The cosmological parameters here are the same as for Fig. 4. The wiggles at very small scales for large cross section are due to some unimportant numerical accuracy problems.

photon, and/or baryon perturbations are exponentially damped. This translates into an exponential cutoff in the matter power spectrum. This occurs for larger γ-DM cross sections or smaller scales than the previous regime. We call this the *collisional regime*. So there is a range of interaction rates and scales where only the previous regime provides the damping.

Finally, at very small scales, a new behavior appears: when the dark-matter–baryon–photon perturbations are enormously damped, the only significant perturbations that survive are those of (relativistic) neutrinos (this could be called the *neutrino regime*). These of course have also experienced significant damping because of free streaming. The latter, however, is much less effective than the collisional damping. The neutrino fluctuations can therefore eventually dominate. Through gravity, they regenerate dark matter fluctuations which, although quite small, are much larger than they would be under the sole action of collisional damping. For relativistic species, the damping of the density perturbations varies only as $(k\eta)^{-1}$ (η being the conformal time), which explains the k^{n_s-6} slope at high k on Fig 5. This undoubtedly occurs when the amplitudes are extremely small, and anyway yields a contribution which for most applications is negligible compared to the one at larger scales. Note that such a behavior can also be seen in a model when all the (noninteracting) dark matter is made of one or two massive neutrino species.

These four regimes can easily be seen on Fig. 5 for various γ-DM cross sections.

IV. REDUCTION OF SMALL SCALE POWER

The reduction of power on small scales for various dark matter candidates can be described by a transfer function T_X defined such that

$$P_X = T_X^2 \cdot P_{\rm CDM}, \tag{4.1}$$

where $P_{\rm CDM}$ is the corresponding cold dark matter power. For noninteracting WDM particles, this damping due to free streaming is traditionally set, by convention, at masses $M < 10^{12} M_\odot$, and is described in Ref. [51] by an exponential cutoff $T_{\rm WDM} = \exp[-kR_f/2 - (kR_f)^2/2]$, where the comoving free-streaming scale R_f is given in terms of the dark matter mass [15], $R_f = 0.2(\Omega_{\rm WDM} h^2)^{1/3} (m_{\rm WDM}/1\ \text{keV})^{-4/3}$ Mpc. A somewhat more accurate result is found from a Boltzmann code calculation, giving [52]

$$T_{\rm WDM} = [1 + (\alpha k)^{2\nu}]^{-5/\nu}, \tag{4.2}$$

where

$$\alpha = 0.048\ \text{Mpc}\ (\text{keV}/m_{\rm DM})^{1.15} \times (\Omega_{\rm DM}/0.4)^{0.15} (h/0.65)^{1.3} (1.5/g_{\rm DM})^{0.29}, \tag{4.3}$$

with $\nu = 1.2$, and $g_{\rm DM} = 1.5$ for a neutrinolike dark matter candidate. For specific sterile neutrino WDM candidates this form is somewhat changed [53].

From Fig. 5, it is clear that IDM can provide an initial reduction of small scale power similar to what WDM gives. A good fit to the transfer function

$$P_{\rm IDM} = T_{\rm IDM}^2(k) \cdot P_{\rm CDM}, \tag{4.4}$$

is (at least near $u = [\sigma_{\rm DM}/\sigma_{\rm Th}][m_{\rm DM}/100\ \text{GeV}]^{-1} \sim 10^{-6}$)

$$T_{\rm IDM} = [1 + (\alpha k)^{2\nu}]^{-5/\nu}, \tag{4.5}$$

where again $\nu = 1.2$ and

$$\alpha = 0.073\ \text{Mpc}\ (u/10^{-6})^{0.48}. \tag{4.6}$$

On smaller scales this expression naturally breaks down because of the presence of oscillations in the spectrum, but we are mostly interested in the scale at which IDM begins to produce significant deviations from the standard CDM case. It is justified since the second maximum is already down by at least an order of magnitude.[6]

A comparison of Eqs. (4.2) and (4.5) reveals that a heavy particle (e.g., $m = 100$ GeV) with scattering cross section with photons of approximately $\sigma_{\gamma\text{-DM}} = 10^{-6} \sigma_{\rm Th}$, can provide the same reduction of small scale power as a conventional WDM particle with mass $m = 1$ keV. We thus see explicitly how IDM can disguise itself as WDM.

On the other hand, one immediately gets constraints on the allowed scattering cross section for the following reason. In order to reproduce the observed properties of the Lyman-α forest in quasar spectra one gets a bound on the free-

[6]This procedure amounts to fitting the *first downward oscillation*, i.e., the moment $k\eta_{\rm dec} \sim 1$ in Eq. (3.38). Indeed, we see that α is nearly proportional to $\eta_{\rm dec}$. Our cosmological parameters imply $\eta_{\rm dec} = 0.35$ Mpc $(u/10^{-6})^{0.5}$, that is, $\alpha k \sim k\eta_{\rm dec}/5$. Undoubtedly, the fitting form (4.5) just reproduces the fall-off of $\cos(k\eta_{\rm dec}/\sqrt{3})$ damped by the factor $B(k\eta_{\rm dec})$, Eqs. (3.7),(3.38). In the case of free streaming, for a damped sinusoidal function, quite a similar fall-off is present, but for totally different reasons. This nevertheless explains the similarity of the fitting forms.

streaming scale [54], corresponding to a WDM mass of approximately 0.75 keV. Furthermore, by extending the Press-Schechter formalism to include WDM (see, e.g., [14], where the problems this raises are discussed), one can study galaxy formation with varying WDM mass. Combined with the existence of a supermassive black hole at $z=5.8$ one finds [55] a lower bound on the WDM mass of approximately 0.75 keV. These results apply equally well to IDM, because of the damping of small scale power, and the results of Refs. [54,55] translate into the bound

$$\frac{\sigma_{\gamma\text{-DM}}}{m_{\mathrm{DM}}} \lesssim 10^{-6} \frac{\sigma_{\mathrm{Th}}}{100\ \mathrm{GeV}} \simeq 6\times10^{-33}\ \ \mathrm{cm}^2\,\mathrm{GeV}^{-1}. \tag{4.7}$$

This bound is stronger than the one, $u<5\times10^{-4}$ [28], obtained by considering collisional damping alone. The reason is the reduction in the amplitude of the dark matter fluctuations, coupled to the photons before collisional damping sets in. As already mentioned this bound is also stronger than the constraint $u<10^{-3}$ arising from CMB anisotropies.

This bound is obtained under the conservative assumption that the abundance of $10^{12}M_\odot$ galaxies is correctly given by CDM, and requires no knowledge of the number of smaller objects. Should we take the more incisive point of view that CDM yields the observed abundance of $10^{9}M_\odot$ objects, then the above bound is lowered by somewhat more than an order of magnitude.

V. CONCLUSION

We have considered the effect of dark-matter–photon interactions on the evolution of primordial dark matter fluctuations. Rather than growing when they enter the horizon, the fluctuations stay of constant amplitude, as do the photon fluctuations. This impeded growth appears as a damping compared to the original amplitude of the fluctuations. Moreover, fluctuations on scales much below the size of the horizon are seen to couple to the photons at a rate much lower than the rate at which they oscillate. This is an additional, new, damping process we have called weak coupling. As a result, horizon-size dark matter fluctuations are seen to be damped if coupled to photons. The usual (exponential) damping sets in for much smaller scales. We hence have obtained a new constraint on the allowed cross sections. This bound (5.1) is two orders of magnitude stronger than the necessary condition obtained by considering the exponential damping of dark matter fluctuations induced by the photon interactions [28]. This new bound reads

$$\sigma_{\gamma\text{-DM}}/m_{\mathrm{DM}} \lesssim 10^{-32}\ \ \mathrm{cm}^2\,\mathrm{GeV}^{-1}. \tag{5.1}$$

The maximum allowed value reduces the matter power spectrum in a way corresponding to a conventional warm dark matter particle with a mass of about 1 keV but leaves the cosmic microwave background anisotropies undisturbed. Using recent bounds on the scale of reduction of the matter power spectrum thus allows us to put new bounds on the allowed photon–dark-matter cross section.

This corresponds to a universe which is well transparent to photons: the free mean path of a photon due to the interactions with dark matter in a halo core of mass density $0.02M_\odot$ pc^{-3} is of order 6×10^4 Gpc, and the optical thickness toward the (usual) last scattering surface is below 10^{-5}. The value (5.1) nevertheless remains quite large compared to the theoretical estimates usually encountered for weakly interacting particles, although there are no compelling reasons to exclude it. Anyway, this leaves open new possibilities as far as the nature of dark matter is concerned.

The lower bound (5.1) implies that dark matter decouples from the photons before the collisional Silk damping is at work, leaving an oscillating, power law, damped matter power spectrum. This new damping regime bears some similarities to the free-streaming case, although here the dark matter and photon fluids are undoubtedly coupled. Such dark matter particles therefore appear to be good warm dark matter candidates, with features in the matter power spectrum different from the conventional WDM at very small scales.

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