Supermassive black holes in scalar field galaxy halos

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Ultralight scalar fields provide an interesting alternative to WIMPS as halo dark matter. In this paper we consider the effect of embedding a supermassive black hole within such a halo, and estimate the absorption probability and the accretion rate of dark matter onto the black hole. We show that the accretion rate would be small over the lifetime of a typical halo, and hence that supermassive central black holes can coexist with scalar field halos.

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I. INTRODUCTION

The standard assumption concerning galaxy dark matter halos is that they are comprised of some weakly interacting massive particle (WIMP). However recently there has been increased interest in an alternative possibility that the dark matter halo may be comprised of some ultralight scalar field [1–11]. A large number of such scalar particles, all in their ground state, can be bound by their self-gravity; the configurations possess a core radius related to the Compton wavelength of the particles in question, and for suitable choices of parameters such halos can give a good description of observed rotation curves [4,10], and optimistically may even provide possibilities to alleviate the "cuspy core" [5,7,10,11] and "substructure" [5,8,9] problems of the standard WIMP hypothesis [12,13].

Development of this scenario is at a primitive stage compared to the WIMP hypothesis. While it is known that the linear theory evolution of perturbations matches the standard scenario, and that time-independent equilibrium configurations can broadly reproduce desired halo properties, the scenario has not been developed in a full cosmological setting where halo formation is tracked. Nevertheless, what is known so far is sufficiently intriguing that the scenario merits further study.

In this paper, we address one requirement of the scalar-field halo model, which is that such a halo must be able to survive the existence of a supermassive black hole at its center, as it is widely believed that such black holes reside within many or perhaps even all galaxy halos [14]. In the WIMP scenario, the angular momentum of the individual dark matter particles, combined with their low interaction rate, ensures that the capture cross section for halo particles by the central black hole is sufficiently small. However, the scalar-field halo regime is markedly different; the individual particles do not possess angular momentum and indeed are expected to have a Compton wavelength upwards of one parsec so that the individual particles occupy a considerable volume of space. It is important to verify that the halos are able to survive the presence of a central black hole if the scenario is to remain feasible.

This paper is constructed as follows. In Sec. II, we describe the basic steps to model a spherical scalar halo without luminous matter. The main intention is to provide a simple panorama of the modeling and its appealing properties, such as the smooth scalar profiles. In Sec. III, we use two complementary views of the interaction of a scalar halo and a black hole: the classical Newtonian picture and the semiclassical approximation. The latter will give us information about the absorption probability and the accretion rate of scalar matter onto the black hole, the main result of this paper. Finally, we discuss the main results and some points deserving further investigation.

II. SCALAR FIELD HALOS

We briefly describe a galaxy halo assuming that it is made only of scalar field matter. A description including, for instance, an exponential disk of luminous matter [1,10], would not significantly change the final results. Two similar but distinct kinds of scalar field objects have been proposed in the literature to explain galaxy halo structure: boson "stars" (comprised of a complex scalar field) [15–18] and oscillatons (made from a real scalar field) [11,19,20]. For simplicity, we restrict ourselves to the case of boson stars, though the main results can be easily extended to the case of oscillatons.

The simplest boson stars are those possessing spherical symmetry, for which the metric is written in the form

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=-Bdt^2+Adr^2+r^2(d\theta^2+\sin^2\theta d\varphi^2), \tag{1}$$

where $A(t,r)$ and $B(t,r)$ are functions to be determined self-consistently from the matter distribution. At the classical level, a complex scalar field Φ endowed with a scalar potential $V(|\Phi|)$ is described by the energy-momentum tensor

$$T_{\mu\nu}=\frac{1}{2}[\Phi_{,\mu}\Phi^*_{,\nu}+\Phi^*_{,\mu}\Phi_{,\nu}-g_{\mu\nu}(\Phi^{,\sigma}\Phi^*_{,\sigma}+V)]. \tag{2}$$

A self-gravitating boson star is found by solving the coupled Einstein-Klein-Gordon (EKG) equations

$$G_{\mu\nu}=\kappa_0 T_{\mu\nu},$$

$$\Box\Phi=\frac{dV}{d\Phi^*}, \tag{3}$$

$$\Box\Phi^* = \frac{dV}{d\Phi},$$

where $G_{\mu\nu}$ is the Einstein tensor corresponding to the metric Eq. (1), $\kappa_0 = 8\pi G$ (we are taking units such that $c = \hbar = 1$) and $\Box$ is the covariant d'Alambertian operator.

The EKG equations, (3), admit solutions of the form $\sqrt{\kappa_0}\Phi = \phi(r)e^{-i\omega t}$. With such an ansatz, the scalar energy-momentum tensor Eq. (2) and the metric functions $g_{\mu\nu}$ in Eq. (1) are time independent. If we now search for regular and asymptotically flat solutions, we should set the boundary conditions $\phi'(r=0)=0$, $A(r=0)=1$ and $\phi(r=\infty)=0$, $A(r=\infty)=1$, respectively. The EKG equations are then reduced to an eigenvalue problem; for each central value of the field $\phi(r=0)\equiv\phi_0$, it is necessary to determine the (eigen)values of the fundamental frequency ω and $B(r=0) \equiv B_0$ to find solutions in which the field has n nodes and satisfy the above boundary conditions.

In principle, we should also impose the boundary condition $B(r=\infty)=1$. However, the eigenvalue problem is further simplified since we can *absorb* ω into the metric function B. In this way, ω does not appear explicitly in the EKG equations and then it becomes an output value determined by $\omega/m = 1/\sqrt{B(r=\infty)}$. The normalized temporal metric coefficient is calculated via $g_{tt} = -(\omega/m)^2 B(r)$.

According to observations, the gravitational well in galaxies is quite weak, which suggests that we should seek boson star solutions in the weak-field limit. It is then appropriate to choose a quadratic scalar potential $V(|\Phi|) = m^2|\Phi|^2$ [1,2,10]. This choice is made not only for simplicity, as a quadratic potential can also be considered an approximation to more complicated ones possessing a minimum [5–9,11].

Using the dimensionless radial coordinate $x = rm$, the EKG equations become the so-called Schrödinger-Newton (SN) equations [2,10,16,17,21] in the weak-field limit: $(\phi_0, -g_{tt}-1, g_{rr}-1) \ll 1$. Thus, we need only solve the simpler set of ordinary differential equations

$$(x\phi)'' = xU\phi, \tag{4}$$

$$(xU)'' = x\phi^2, \tag{5}$$

where primes denote derivatives with respect to x. In order to clarify the meaning of function $U(x)$, we take a look at the metric coefficients in the weak-field limit,

$$-g_{tt} \simeq 1 + U(x) - U_\infty,$$

$$g_{rr} \simeq 1 + xU'(x). \tag{6}$$

Hence, the usual Newtonian potential is given by $U_N = (1/2)[U(r) - U_\infty]$, while the value of the fundamental frequency is given by $(\omega/m)^{-2} = 1 + U_\infty$, with $U(x=\infty) \equiv U_\infty$.

Despite its simplicity, the system above still has to be solved numerically, with the different solutions characterized by, for example, the central value ϕ_0. As in the relativistic case, the solution of Eqs. (4) and (5) is an eigenvalue problem; we have to find the one value $U(0) \equiv U_0$ in order to satisfy the boundary conditions stated above and to find n-node solutions of the scalar field $\phi(x)$.

To give an order of magnitude estimation of the quantities involved, the scalar halo models in the literature [1,2,9–11] consider an ultralight boson mass $m \sim 10^{-23}$ eV, whose corresponding Compton length is $\lambda_C = m^{-1} \sim 1$ pc. On the other hand, the central amplitude of the scalar field would be proportional to the gravitational well in galaxies, and then $\phi_0 \sim |U_0| \sim v^2 \sim 10^{-6}$ with v the rotational velocity of luminous matter in galaxies (in units of c).

All information of the properties of the scalar halo is contained in Eqs. (4) and (5). Of special interest are the asymptotic behaviors of the scalar and gravitational fields near the center. It is easy to show that [21]

$$\phi(x) = \phi_0[1 + (1/6)U_0x^2 + \mathcal{O}(x^4)], \tag{7}$$

$$U(x) = U_0 + (1/6)\phi_0^2 x^2 + \mathcal{O}(x^4), \tag{8}$$

and so the scalar field remains constant up to radii of the order $r \sim |U_0|^{-1/2}\lambda_C \sim 1$ kpc. Therefore, the resulting self-gravitating object has a smooth central profile up to distances much larger than the Compton length of its particles.

III. THE CENTRAL BLACK HOLE

The geodesics of scalar halos allow massive particles to reach the center of the halo, and in principle the accumulation of matter at the center is not prohibited.[1] Therefore, a black hole can form in the center of bosonic objects and become a threat to their existence. Such a caveat has been recognized before [23], but it is only recently that the interaction between black holes and cosmic scalar fields has begun to be studied seriously [24,25].

Our aim now is to outline the physical consequences of the interaction between black holes and the scalar halos considered above. For this, we will take the two simplest approximations at hand: the classical picture, in which the black hole is taken as a central pointlike gravitational source, and the semiclassical picture in which the scalar field lives in the curved space-time outside a black hole. As we shall see below, these approximations are complementary and can be matched into the scalar halo picture of Sec. II.

A. The classical picture

Taking into account that the Schwarzschild radius

$$r_s \equiv 2GM_{\rm bh} \simeq 9.57 \times 10^{-14}\ {\rm pc}\frac{M_{\rm bh}}{M_\odot} \tag{9}$$

of a central black hole is much smaller than any of the typical length scales present in realistic scalar halo (e.g. the op-

[1] However, it has been shown that *rotating* Newtonian boson stars could provide extra repulsive forces at the center [22], which suggests that the inclusion of rotation could avoid the excessive accumulation of matter at the center of scalar objects.

tical radius $r_{\rm opt}$ or the scalar Compton length m^{-1}; see Ref. [14] and references therein), we can deal with it within the Newtonian regime.

In the classical picture, the total gravitational potential is the superposition $U_N=(1/2)[U(r)-U_\infty-r_s/r]$, where $U(r)$ is the scalar self-gravitational well and (r_s/r) is the gravitational field of the black hole, which can be seen as a solution in vacuum. Thus, we need only modify Eq. (4) to include the gravitational influence of the black hole

$$(x\phi)''=x(U-mr_s/x)\phi, \tag{10}$$

which resembles the Schrödinger equation in a Coulomb-like potential $\sim 1/r$. We can still construct regular solutions for the scalar field and the other metric functions, but we need to change the boundary condition of the radial derivative of the scalar field at $r=0$ to

$$\phi'(0)=-\phi_0 mr_s/2. \tag{11}$$

The other boundary conditions remain the same.

In this classical picture, we notice that the black hole only affects the behavior of the field at small r, but the scalar profile is still regular. At large distances, the scalar profile is unperturbed by the presence of the central black hole. In other words, in the Newtonian regime the existence of the scalar halo is not threatened by the central gravitational source.

B. The quantum field theory picture

The approximation we now make is to consider that the scalar field lives near the horizon of the black hole in a fixed Schwarzschild background:

$$ds^2=-g(r)dt^2+\frac{dr^2}{g(r)}+r^2(d\theta^2+\sin^2\theta d\varphi^2), \tag{12}$$

where $g(r)=1-r_s/r$, and then its properties are determined by the field theory in such a curved space-time. This is reasonable since, as stated above, the self-gravitating effects of the scalar field appear only at distances of order $r\gg m^{-1}\gg r_s$.

Recalling that we are working with a quadratic scalar potential, an s-scalar wave[2] obeys the Klein-Gordon equation in metric Eq. (12):

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 g\frac{\partial\Phi}{\partial r}\right)-\frac{1}{g}\frac{\partial^2\Phi}{\partial t^2}=m^2\Phi, \tag{13}$$

with the corresponding equation for the complex conjugate field Φ^*. Equation (13) is separable in the form $\sqrt{\kappa_0}\Phi(t,r)=\phi(r)e^{-imt}$, where we have set $\omega=m$, anticipating the classical result in which the fundamental frequency does not appear explicitly. The change of variable preserves the notation of Sec. II.

At this point, it is convenient to take the Schwarzschild factor $g(r)$ itself as the independent variable. Then, the differential equation of $\phi(r)$ near the horizon is

$$g^2\phi''+g\phi'+m^2r_s^2(1-g)^{-3}\phi=0, \tag{14}$$

where prime denotes derivative with respect to g.

The ingoing solution of Eq. (14) is given, around $g=0(r=r_s)$, in the series form (found using the computer algebra package MAPLE, www.maplesoft.com)

$$\Phi(v,r)=\Phi^{(0)}(v,r)\left[1-(mr_s)^2\times\sum_{n=1}^{\infty}(P_n+imr_sQ_n)g^n\right], \tag{15}$$

where

$$\Phi^{(0)}(v,r)=e^{-im[v-r-r_s\ln(r/r_s)]}. \tag{16}$$

Here $v=t+r_*$ is the advanced time coordinate defined via the usual Kruskal coordinate $r_*=r+r_s\ln(r/r_s-1)$ and we have used the relationship

$$g^{r_s}(r)=e^{r_*-r-r_s\ln(r/r_s)}. \tag{17}$$

The coefficients P_n, Q_n in Eq. (15) have the complicated form

$$P_1=\frac{3}{1+4m^2r_s^2},$$

$$P_2=\frac{3}{2^2}\frac{2+5m^2r_s^2+6m^4r_s^4}{(1+m^2r_s^2)(1+4m^2r_s^2)},$$

$$P_3=\frac{1}{2^23^2}\frac{40+110m^2r_s^2+151m^4r_s^4+36m^6r_s^6}{(1+(4/9)m^2r_s^2)(1+m^2r_s^2)(1+4m^2r_s^2)},$$

$$\ldots=\ldots$$

$$Q_1=\frac{6}{1+4m^2r_s^2},$$

$$Q_2=\frac{3}{2^2}\frac{2-m^2r_s^2}{(1+m^2r_s^2)(1+4m^2r_s^2)},$$

[2]This is the scalar wave with lowest angular momentum $l=0$, and hence also the lowest energy. This condition is satisfied for the scalar halos considered so far, which are supposed to form a (ground state) Bose condensate. The results could be also applied for the case of cosmological scalar fields at late times.

$$Q_3=\frac{1}{2^2 3^3}\frac{80-266m^2r_s^2-319m^4r_s^4-108m^6r_s^6}{(1+(4/9)m^2r_s^2)(1+m^2r_s^2)(1+4m^2r_s^2)},$$

$$\ldots=\ldots.$$

In the particular case in which $mr_s \ll 1$, we can approximate P_n, Q_n by their leading terms. Then, we find the approximate expressions

$$P_n \simeq \frac{1}{2n^2}(n+1)(n+2) \simeq \frac{n}{2}Q_n. \tag{18}$$

With these approximate formulas, the sums in Eq. (15) can be written in terms of known functions, which indicates that the series diverges for $g\to 1(r\to\infty)$.

However, we find that for distances $m^{-1}>r\gg r_s$ [for which we can neglect $(r_s/r)^2$ and higher-order terms], the radial equation for the scalar field becomes

$$\phi''+\frac{2}{r}\phi'+m^2r_s\frac{\phi}{r}=0, \tag{19}$$

where primes now denote derivatives with respect to r. The new solutions are of the form

$$\phi(r)=r^{-1/2}[CJ_1+DY_1](2\sqrt{m^2r_sr}), \tag{20}$$

where J and Y are the Bessel functions of the first and second kind, and C and D are arbitrary constants.

The overlap region between the two solutions Eqs. (15) and (20) is $m^{-1}\gg r\gg r_s$. As we said above, using the approximate formulas (18), we can estimate the sum of the series in Eq. (15). For example, if $r=10^3r_s$,

$$\Phi\simeq\Phi^{(0)}(v,10^3r_s)[1-(mr_s)^2(512+14imr_s)]. \tag{21}$$

The factor $(mr_s)^2$ highly suppresses the contribution of the series in Eq. (15), so that we can safely approximate the radial part of the latter in this region as

$$\phi(r)\simeq 1-imr_s^2/r. \tag{22}$$

On the other hand, for distances $r\ll m^{-1}$, Eq. (20) reduces to

$$\phi(r)\simeq(m^2r_s)^{1/2}C\left[1-\frac{m^2r_sr}{2}+\cdots\right]-\frac{(m^2r_s)^{-1/2}D}{\pi r}. \tag{23}$$

Notice that we have included a first-order term in Eq. (23), just to show that the next-to-order correction is simply the Coulomb-like one, which coincides with the classical picture above Eq. (11).

Matching Eq. (23) onto Eq. (22) in the overlap region, we find

$$\frac{D}{C}=i\pi(mr_s)^3, \tag{24}$$

which gives the absorption probability of an $l=0$ spherical wave as [26]

$$\Gamma=1-\left|\frac{1+\frac{D}{C}e^{i\pi/2}}{1+\frac{D}{C}e^{-i\pi/2}}\right|^2\simeq 4\pi(mr_s)^3, \tag{25}$$

where we have again assumed that $mr_s\ll 1$. The interpretation of Γ is that it gives the fraction of the ingoing wave, and hence the fraction of the incoming particles, which is absorbed by the black hole.

The last result indicates that for typical values $mr_s \sim 10^{-7}$, we have $\Gamma\sim 10^{-20}$ which implies that the absorption of the scalar field is negligible and that, from the semiclassical point of view too, a central black hole and a scalar halo can be put together. Equation (25) coincides with previous calculations, which also indicate that the absorption probability of higher l modes is further suppressed by a factor of the order $(mr_s)^{2l}$ [27].

Summarizing, we can say that the solutions of the scalar halo are given by Eqs. (10) and (5) for $r\geq m^{-1}$, by Eq. (20) for $m^{-1}>r\gg r_s$ and by Eq. (15) for $r\sim r_s$, with the absorption probability Eq. (25) calculated in the overlap region $r_s \ll r<m^{-1}$. Formally speaking, the three different solutions are well matched to each other if we multiply Eqs. (15) and (20) by the central amplitude ϕ_0 calculated for the scalar halo in Eqs. (10) and (5). Since the latter is just an overall factor, the absorption probability Eq. (25) remains the same.

Observe that the second solution in Eq. (20) could have been obtained within the classical picture in Eq. (10), but it was not taken into account because it diverges at the origin and our purpose was to construct regular solutions. But, as we have seen in this section, this second solution contains the information of the interaction between the black hole and the scalar field, since it is through it that we obtained a non-null absorption probability.

Another important issue that can be calculated is the accretion rate of the scalar field into the black hole by using the formula Eq. (3.1) in Ref. [24]. The scalar energy-momentum tensor should be written in the new variables (v,r), and then we obtain for the flux of Killing energy across the horizon

$$dM/dt=4\pi r_s^2\times T_{vv}(v,r_s)=(2G)^{-1}(\phi_0 mr_s)^2, \tag{26}$$

in which we have included the overall factor ϕ_0. Using typical numbers $\phi_0 mr_s\sim 10^{-13}$, the accretion rate is quite small, $dM/dt\simeq 10^{-14}M_\odot \mathrm{y}^{-1}$, a result that is consistent with the small absorption probability given by Eq. (25).

IV. CONCLUSIONS

We have analyzed the impact of a central supermassive black hole on galactic halos comprised of ultralight scalar particles. From simple physical grounds, we should expect that the accretion rate of a scalar halo onto a black hole is small, since the boson particles cannot ''fit'' into the horizon due to their large Compton length. In addition, the absorption probability should be proportional to the ratio of the effective 'area' of the two objects, and hence proportional to $(mr_s)^2$ as happens for massless scalar fields [26].

We found that the absorption probability is decreased by an extra factor mr_s, which assures the coexistence of the bosonic halo and the central black hole. For this, we showed how to construct consistent and regular solutions on different scales. In addition, the accretion rate of scalar matter onto the black hole is so small that the matter absorbed by the black hole is much less than a solar mass in the whole lifetime of the Universe. On the other hand, this result would indicate that the current observed accretion in galaxy black holes would not be due to matter provided by a scalar halo.

We have only investigated the equilibrium state of a relaxed scalar halo and a central black hole, and it would be interesting to have a more dynamical view studying the formation (simultaneously or not) of the two objects. This would require the evolution of the full Einstein equations, which is well beyond the scope of this paper.

A related issue is the interaction of primordial black holes with cosmic scalar fields, prior to the gravitational collapse of density perturbations, as recently outlined in Refs. [24,25]. For a cosmic scalar field endowed with a quadratic potential, the accretion rate would also be given by formula Eq. (26), and then the field would have the oscillatory behavior Eq. (15) near the black hole horizon. That is, the mass of the bosonic field still prevents a strong interaction between black holes and cosmic scalar fields. Other kinds of scalar potentials would lead to more interesting pictures [25].

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