Two-photon decay of the pseudoscalars, the chiral symmetry breaking corrections

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The extrapolation of the decay amplitudes of the pseudoscalar mesons into two photons from the soft meson limit where it is obtained from the axial anomaly to the mass shell involves the contribution of the 0^- continuum. These chiral symmetry breaking corrections turn out to be large. The effects of these corrections on the calculated π^0 decay rate, on the values of the singlet-octet mixing angle, and on the ratios f_8/f_π and f_0/f_π are discussed. The implications for the transition form factors $\gamma\gamma^*\to\pi^0$, η, η' are also evaluated and confronted with the available experimental data.

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I. INTRODUCTION

Current algebra and PCAC (partial conservation of the axial-vector current) together with the Adler-Bell-Jackiw (ABJ) axial anomaly [1] express the two-photon decay rate of the pseudoscalar mesons ($P=\pi^0,\eta,\eta'$) in terms of the coupling constants f_{π^0}, f_0, and f_8 and of the singlet-octet mixing angle θ:

$$\begin{aligned}
\langle 0|A_\alpha|\pi^0(p)\rangle &= -2if_{\pi^0}p_\alpha,\\
\langle 0|A_\alpha^{(8)}|\eta(p)\rangle &= 2if_8(\cos\theta)p_\alpha,\\
\langle 0|A_\alpha^{(0)}|\eta(p)\rangle &= -2if_0(\sin\theta)p_\alpha, \qquad (1.1)\\
\langle 0|A_\alpha^{(8)}|\eta'(p)\rangle &= 2if_8(\sin\theta)p_0,\\
\langle 0|A_\alpha^{(0)}|\eta'(p)\rangle &= 2if_0(\cos\theta)p_\alpha,
\end{aligned}$$

with

$$f_{\pi^0}\simeq f_{\pi^+}=92.4\ \text{MeV},$$

and where the axial-vector currents are given in terms of the quark fields:

$$\begin{aligned}
A_\alpha &= (\bar{u}\gamma_\alpha\gamma_5 u-\bar{d}\gamma_\alpha\gamma_5 d),\\
A_\alpha^{(8)} &= \frac{1}{\sqrt{3}}(\bar{u}\gamma_\alpha\gamma_5 u+\bar{d}\gamma_\alpha\gamma_5 d-2\bar{s}\gamma_\alpha\gamma_5 s), \qquad (1.2)\\
A_\alpha^{(0)} &= \sqrt{\frac{2}{3}}(\bar{u}\gamma_\alpha\gamma_5 u+\bar{d}\gamma_\alpha\gamma_5 d+\bar{s}\gamma_\alpha\gamma_5 s).
\end{aligned}$$

Unlike f_π, f_0 and f_8 are not directly related to any physical process.

SU(3) breaking enters through singlet-octet mixing and through the deviation of the ratio f_8/f_π from unity. Another symmetry breaking effect originates from SU(3)×SU(3) breaking: The decay rate is obtained in the soft meson limit from the ABJ anomaly [1] and the extrapolation to the mass shell involves corrections $O(m_P^2)$, which are expected to be small for the π^0 but which are not necessarily so for the η and η'. These corrections to the PCAC limit, which arise from the 0^- continuum, are similar to the corrections to the Goldberger-Treiman relation [2]. Attempts to estimate these corrections have been undertaken [3–5]. In Refs. [3] and [4] only the contribution of the high-energy part of the 0^- spectrum was considered and in Ref. [5] only the low-energy part was used. These three calculations are, moreover, heavily model dependent.

It is the purpose of the present work to provide an estimate of the PCAC corrections which enter in the evaluation of the two photon decay rates of the pseudoscalars under the sole assumptions that the V-A-A vertex is given by the quark triangle graph in the deep euclidean region and that the main contribution to SU(3)×SU(3) breaking arises from the energy interval

$$1\ \text{GeV}^2\leqslant s\leqslant 2\ \text{GeV}^2$$

of the 0^- continuum.

In Sec. II we give details of the calculation and the resulting constraints on the values of f_8, f_0, and the mixing angle θ are discussed. Finally an evaluation of the implications of our results on the transition form factors $\gamma^*\gamma\to P$ are presented in Sec. III and compared with the available experimental data.

II. THE CHIRAL SYMMETRY BREAKING CORRECTIONS

Consider the three-point function

$$\begin{aligned}
T_{\alpha\mu\nu}(p,q_1,q_2) &= 2\pi i\int\int dx\,dy\,\exp(-iq_1x+ipy)\\
&\quad\times\langle 0|TA_\alpha(y)V_\mu(x)V_\nu(0)|0\rangle\\
&= \frac{1}{\pi}\frac{1}{p^2-m_\pi^2}\epsilon_{\mu\nu\lambda\sigma}q_1^\lambda q_2^\sigma p_\alpha F(p^2,q_1^2,q_2^2)\\
&\quad+\cdots, \qquad (2.1)
\end{aligned}$$

where the dots represent other tensor structures and where $V_{\mu,\nu}$ denotes the electromagnetic current

$$V_\mu=\tfrac{2}{3}\bar{u}\gamma_\mu u-\tfrac{1}{3}\bar{d}\gamma_\mu d-\tfrac{1}{3}\bar{s}\gamma_\mu s. \qquad (2.2)$$

The pion pole contribution has been isolated in expression (2.1).

In the soft π limit and with both photons on the mass shell, the axial anomaly [1] yields

$$F(0,0,0)=1. \tag{2.3}$$

The rate of the decay $\pi^0\to 2\gamma$ provides a measurement of

$$F(m_\pi^2,0,0)=1+\Delta_\pi, \quad \Delta_\pi=O(m_\pi^2). \tag{2.4}$$

Equation (2.1) is used to define the off-shell symmetric amplitude

$$F(s,t)=F(s=p^2,t=q_1^2=q_2^2). \tag{2.5}$$

In the deep Euclidean region $F(s,t)=F^{\rm QCD}(s,t)$, $F^{\rm QCD}(s,t)$ includes perturbative and nonperturbative contributions

$$F^{\rm QCD}(s,t)=F_p(s,t)+F_{np}(s,t). \tag{2.6}$$

The perturbative part $F_p(s,t)$ is obtained from the quark triangle graph, the contribution of which takes a particularly simple form in the symmetric case [6]

$$F_p(s,t)=-2(s-m_\pi^2)\int\int_0^1 dx\,dy\,\frac{x\bar{x}\bar{y}^2}{[y(x\bar{x}s-t)+t]},$$
$$\bar{x}=1-x. \tag{2.7}$$

The nonperturbative part $F_{np}(s,t)$ arises from the contribution of the vacuum condensates

$$F_{\rm np}(s,t)=\frac{b_1}{t^2}+\frac{b_2}{t^3}+\frac{b_3}{st}+\cdots, \tag{2.8}$$

where

$$b_1=-\frac{\pi^3}{9}\left\langle\frac{\alpha_s G^2}{\pi}\right\rangle, \quad b_2=-\frac{64}{27}\pi^4\langle\alpha_s(qq)^2\rangle, \quad b_3=-b_1. \tag{2.9}$$

For fixed t, $F(s,t)$ is an analytic function of the complex variable s with a cut on the positive real axis that extends from $s=9m_\pi^2$ to ∞ [7].

A dispersion relation between $F(s=m_\pi^2,t)$ and $F(0,t)$ is obtained from the integral

$$\frac{1}{2\pi i}\int_c \frac{ds}{s(s-m_\pi^2)}F(s,t), \tag{2.10}$$

where c is the closed contour in the complex plane consisting of a circle of large radius R and two straight lines lying immediately above and immediately below the cut. Cauchy's theorem then yields

$$F(m_\pi^2,t)=F(0,t)+\frac{m_\pi^2}{2\pi i}\int_c \frac{ds}{s(s-m_\pi^2)}F(s,t). \tag{2.11}$$

The integral in the equation above consists of two parts: an integral of the discontinuity of $F(s,t)$ over the cut, which provides the main contribution, and an integral over the circle of radius R where $F^{\rm QCD}(s,t)$ provides a good approximation to $F(s,t)$ except possibly in the vicinity of the positive real axis. Little is known about the integrand over the cut, which consists of the contribution of the axial-vector (1^+) and pseudoscalar (0^-) intermediate states. The contribution of the low-energy region (the three pion states) is suppressed by loop factors and amounts to little [8,9]. We expect the major part of the contribution to originate from the $a_1(1260)$ and $\pi'(1300)$ bumps, i.e., from the range $1\ {\rm GeV}^2\leqslant s\leqslant 2\ {\rm GeV}^2$. In order to eliminate this contribution, we shall consider, instead of integral (2.10), the following modified integral:

$$\frac{1}{2\pi i}\int_c \frac{ds}{(s-m_\pi^2)}\left(\frac{1}{s}-a_0-a_1 s\right)F(s,t). \tag{2.12}$$

In the equation above, the coefficients a_0 and a_1 will be chosen so as to annihilate the integrand at $m_1^2=m_{a1}^2=1.56\ {\rm GeV}^2$ and at $m_2^2=m_{\pi'}^2=1.70\ {\rm GeV}^2$, i.e.,

$$a_0=\frac{1}{m_1^2}+\frac{1}{m_2^2}, \quad a_1=-\frac{1}{m_1^2 m_2^2}. \tag{2.13}$$

This choice reduces the integrand to only a few percent of its initial value over the interval $1\ {\rm GeV}^2\leqslant s\leqslant 2\ {\rm GeV}^2$. It will also considerably reduce the contribution to the integral over the circle near the positive real axis. The contribution of the continuum has thus been drastically reduced and we neglect it. Cauchy's theorem now yields

$$F(m_\pi^2,t)\approx F(0,t)+a_0 m_\pi^2 F(m_\pi^2,t)+a_1 m_\pi^4 F(m_\pi^2,t) + \frac{1}{2\pi i}\oint \frac{ds}{(s-m_\pi^2)}\left(\frac{1}{s}-a_0-a_1 s\right)F(s,t). \tag{2.14}$$

The integral is now carried over the circle of radius R.

In order to obtain $F(s,0)$ we proceed in a similar fashion. $F(s,t)$ is an analytic function of the complex variable t except for a cut on the positive real axis extending from $t=4m_\pi^2$ to ∞. In the low-t region $F(s,t)$ is dominated by the ρ-ω double and single poles

$$F(s,t)=\frac{c_1(s)}{(t-m_\rho^2)^2}+\frac{c_2(s)}{(t-m_\rho^2)}+\cdots. \tag{2.15}$$

Consider now the integral

$$\frac{1}{2\pi i}\int_{c'} dt\,\frac{(t-m_\rho^2)^2}{t}F(s,t), \tag{2.16}$$

where c' is a contour similar to c in the complex t plane. The double and single vector meson poles have been removed, and for an appropriate choice of R' the major contribution to the integral (2.16) comes from the integral over the circle so that

$$F(s,0)\approx\frac{1}{m_\rho^4}\frac{1}{2\pi i}\oint_{R'}dt\frac{(t-m_\rho^2)^2}{t}F(s,t). \quad (2.17)$$

It follows from Eqs. (2.14) and (2.17) that

$$\begin{aligned}F(m_\pi^2,0)(1-a_0m_\pi^2-a_1m_\pi^4)&=F(0,0)+\frac{m_\pi^2}{(2\pi i)^2}\oint\oint\frac{ds\,dt}{t(s-m_\pi^2)}(t-m_p^2)^2\\&\times\left(\frac{1}{s}-a_0-a_1s\right)F^{\mathrm{QCD}}(s,t). \quad (2.18)\end{aligned}$$

The integral above is carried over the circles of large radii R and R' so we have replaced $F(s,t)$ by $F^{\mathrm{QCD}}(s,t)$ in the integrand. Inserting Eqs. (2.6)–(2.8) in Eq. (2.18) gives

$$\begin{aligned}F(m_\pi^2,0)(1-a_0.m_\pi^2-a_1m_\pi^4)&=F(0,0)+\frac{m_\pi^2}{9m_\rho^2}+\frac{m_\pi^2}{m_\rho^4}\left[\frac{R}{5}-\frac{1}{3}\left(R'-2m_\rho^2\ln\frac{R'}{R}-\frac{m_\rho^4}{R'}\right)\right]\\&-a_0\frac{m_\pi^2}{m_\rho^4}R\left(\frac{R}{10}-\frac{2}{3}m_\rho^2\right)-a_1\frac{m_\pi^2R^2}{m_\rho^4}\left(\frac{R}{15}-\frac{m_\rho^2}{3}\right)\\&-\frac{m_\pi^2}{m_\rho^4}(a_0+a_1m_\pi^2)b_1+2\frac{m_\pi^2}{m_\rho^2}a_1b_3. \quad (2.19)\end{aligned}$$

The nonperturbative contributions to Eq. (2.19) turn out to be negligible.

The decays $\eta\to2\gamma$ and $\eta'\to2\gamma$ are treated in a similar fashion. The axial-vector currents that project on the η and η' states are, respectively,

$$A_\alpha^\eta=\left(\frac{A_\alpha^{(8)}}{f_8}\cos\theta-\frac{A_\alpha^{(0)}}{f_0}\sin\theta\right),$$

$$A_\alpha^{\eta'}=\left(\frac{A_\alpha^{(8)}}{f_8}\sin\theta+\frac{A_\alpha^{(0)}}{f_0}\cos\theta\right). \quad (2.20)$$

These currents we use in the definition of the three-point function equation (2.1). The results for the two-photon decay rates of the pseudoscalars are then

$$\Gamma(\pi^0\to2\gamma)=\frac{\alpha^2m_\pi^3}{64\pi^3f_{\pi^+}^2}(1+\Delta_\pi)^2\frac{f_{\pi^+}^2}{f_{\pi^0}^2}, \quad (2.21)$$

$$\Gamma(\eta\to2\gamma)=\frac{\alpha^2m_\eta^3}{192\pi^3f_{\pi^+}^2}(1+\Delta_\eta)^2\left(\frac{\cos\theta}{F_8}-2\sqrt{2}\frac{\sin\theta}{F_0}\right)^2, \quad (2.22)$$

$$\Gamma(\eta'\to2\gamma)=\frac{\alpha^2m_{\eta'}^3}{192\pi^3f_{\pi^+}^2}(1+\Delta_{\eta'})^2\left(\frac{\sin\theta}{F_8}+2\sqrt{2}\frac{\cos\theta}{F_0}\right)^2. \quad (2.23)$$

With the notation $F_{0,8}=f_{0,8}/f_{\pi^+}$ and

$$F(m_p^2,0,0)=(1+\Delta_p)F(0,0,0) \quad (2.24)$$

Δ_π is obtained from Eq. (2.19). The same equation with the negligible nonperturbative part omitted yields Δ_η and $\Delta_{\eta'}$ when m_π is replaced by $m_{\eta,\eta'}$ and when the strange quark mass is neglected.

R', the duality radius in the ρ-meson channel, is usually taken to be $R'\approx1.5\ \mathrm{GeV}^2$ in the literature [9]. The value of R should be large enough to include the contribution of the pseudoscalar excitations but not too large to invalidate the approximations at hand. Moreover, Δ_P should be stable against small variations in R. For the parameters m_1 and m_2 we take

$$m_1^2=m_{a_1(1260)}^2=1.56\ \mathrm{GeV}^2,\quad m_2^2=m_{\pi(1300)}^2=1.70\ \mathrm{GeV}^2 \quad (2.25)$$

as discussed previously.

In the η and η' channels two pseudoscalar excitations, $\eta(1295)$ and $\eta(1440)$, as well as two axial vectors, $f_1(1285)$ and $f_1(1420)$, dominate the 0^- and 1^+ continua and couple to both η and η' mesons with unknown strengths. Because $\eta(1295)$ and $f_1(1285)$, on the one hand, and $\eta(1440)$ and $f_1(1420)$, on the other hand, are practically degenerate in mass, it suffices to take

$$m_1^2=1.66\ \mathrm{GeV}^2,\quad m_2^2=2.04\ \mathrm{GeV}^2 \quad (2.26)$$

for both η and η'.

With these choices expression (2.19) passes through a maximum for $R\simeq2\ \mathrm{GeV}^2$ in the π channel and $R\simeq2.25\ \mathrm{GeV}^2$ in the η, η' channels, values which we adopt because of the stability criteria stated above. Numerically, then, we find

$$\Delta_\pi\simeq0.047,\quad \Delta_\eta\simeq0.77,\quad \Delta_{\eta'}\simeq6.0. \quad (2.27)$$

The relative theoretical error on the numbers above is estimated to be of the order of $\alpha_s(2.5\ \mathrm{GeV}^2)/\pi=12\%$.

The chiral symmetry breaking corrections are thus seen to be quite large. For π^0 the theoretical value of the width is increased from $\Gamma_{\pi^0\to2\gamma}=7.74$ eV to

$$\Gamma_{\pi^0\to2\gamma}=(8.25\pm0.09)\ \mathrm{eV} \quad (2.28)$$

if we take $f_{\pi^+}/f_{\pi^0}=1$. Another correction to the chiral limit could originate from π^0, η, and η' mixing. It has been estimated in chiral perturbation theory [11] and would increase the decay rate by another factor of 1.045 to

$$\Gamma_{\pi^0\to2\gamma}=(8.60\pm0.10)\ \mathrm{eV}. \quad (2.29)$$

The current world average experimental value is

$$\Gamma_{\pi^0\to2\gamma}=(7.75\pm0.60)\ \mathrm{eV}. \quad (2.30)$$

The large dispersion of the experimental results, however, suggests that the quoted error is underestimated [11]. The outcome of the PRIMEX experiment at Jefferson Laboratory [13], which aims at a precision of 1.5% in the measurement of the π^0 decay rate, is thus eagerly expected to clarify the situation.

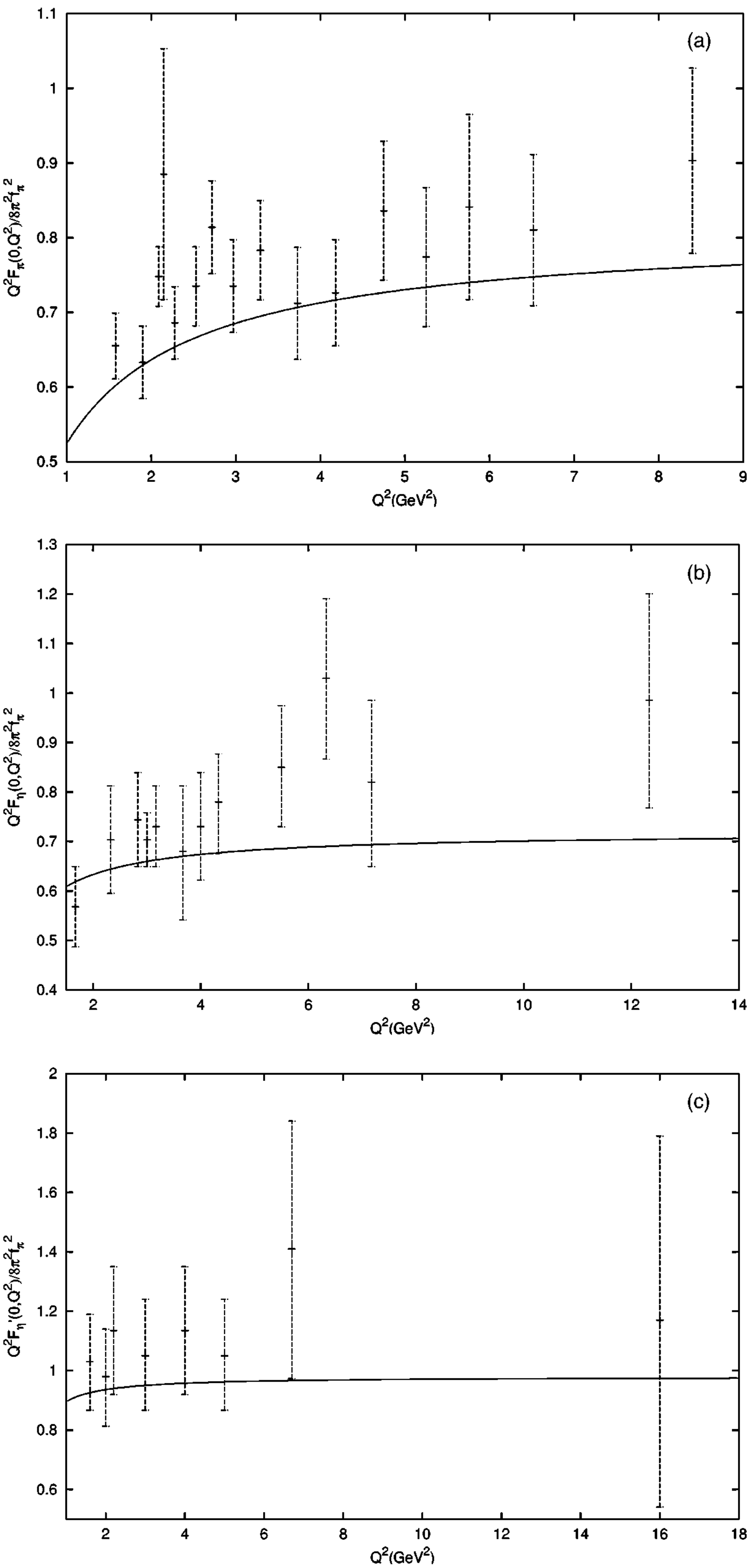

FIG. 1. The form factors of the transitions (a) $\gamma\gamma^* \rightarrow \pi^0$, (b) $\gamma\gamma^* \rightarrow \eta$, (c) $\gamma\gamma^* \rightarrow \eta'$ as obtained from Eq. (3.2). The vertical lines represent the data as taken from Ref. [12].

In order to extract information from our results on the η and η', more input has to be added. An analysis of the η, η' meson mass matrix together with chiral perturbation theory [10] yields for the mixing angle $\theta = -19.5°$. This value and the experimentally measured decay rates, when inserted in Eqs. (2.22), (2.23), and (2.27), imply

$$F_8 = 1.29 \pm 0.09, \quad F_0 = 4.70 \pm 0.45. \tag{2.31}$$

Our result for F_8 is consistent with that obtained from chiral perturbation theory, $F_8=1.25$ [10]. The value obtained for F_0 is quite larger than those appearing in the litterature. It is worth noting that a pure gluon component present in η, η' would yield the value appearing in Eq. (2.31) as an upper limit for F_0 [4].

It is unfortunate that F_8 and F_0 are not directly linked to any physically measurable quantity. Data exist, however, on the transitions $\gamma\gamma^*\to P$ [12]. We shall examine in the next section the implications of our results on the corresponding form factors.

III. FORM FACTORS OF THE TRANSITIONS $\gamma\gamma^*\to\pi^0$, η, η'

The form factors of the transitions $\gamma\gamma^*\to\pi$, η, η' were addressed before in particular by Amettler *et al.* [14] for low values of the euclidean momentum of the virtual photon Q^2 using various models as well as by Jakob, Kroll, and Raulfs [15], who deal with the problem at large Q^2 using the elaborate techniques of the hard scattering approach combined with constraints originating from the decay width of the transitions η, $\eta'\to 2\gamma$. These authors, however, neglect the chiral symmetry breaking corrections that constitute the central part of the present work. The method we have developed is now used to tackle the issue at large Q^2. In the deep Euclidean region we have [6]

$$F_p(s,Q^2,t)=-2(s-m_P^2)\int_0^1 dx\, x\overline{x} \times\int_0^1 dy\,\frac{y^2}{[y(x\overline{x}s+xQ^2+\overline{x}q^2)-xq^2+\overline{x}t]}, \tag{3.1}$$

where $t=q_1^2$, $Q^2=-q_2^2$, and $s=p^2$ as before. In order to be able to compare with experiment, we need to evaluate $F(m_P^2,Q^2,0)$. The method used in the preceding section, modified to take into account the fact that we have now only a single pole in the vector meson channel, now yields

$$F(m_P^2,Q^2,0)=-\frac{1}{m_\rho^2}\frac{1}{(2\pi i)}\oint\oint ds\,dt\,\frac{t-m_\rho^2}{t}\left(\frac{1}{s-m_P^2}-a_0-a_1 s\right)F^{\mathrm{QCD}}(s,Q^2,t), \tag{3.2}$$

an expression valid for large Q^2. Inserting expression (3.1) in the equation above, it takes some straightforward algebra in the complex plane to evaluate $F(m_p^2,Q^2,0)$. The deep inelastic limit $Q^2\to\infty$ is readily obtained:

$$Q^2F_{\pi^0}(m_\pi^2,Q^2,0)\to\frac{\left[\frac{1}{3}R^3-(m_1^2+m_2^2)\frac{1}{2}R^2+m_1^2m_2^2R\right]}{(m_1^2-m_\pi^2)(m_2^2-m_\pi^2)}. \tag{3.3}$$

The corresponding expressions for the η and η' have to be multiplied by the factors $(1/\sqrt{3})[(\cos\theta)/F_8-(2\sqrt{2}\sin\theta)/F_0]$ and $(1/\sqrt{3})[(\sin\theta)/F_8-(2\sqrt{2}\cos\theta)/F_0]$ respectively. Numerically then

$$\frac{Q^2F_{\pi^0}(m_\pi^2,Q^2,0)}{8.\pi^2f_\pi^2}\to 0.81, \tag{3.4}$$

$$\frac{Q^2F_\eta(m_\eta^2,Q^2,0)}{8.\pi^2.f_\pi^2}\to(1.34.\pm0.12)\frac{1}{\sqrt{3}}\left(\frac{\cos\theta}{F_8}-\frac{2\sqrt{2}\sin\theta}{F_0}\right), \tag{3.5}$$

$$\frac{Q^2F_{\eta'}(m_{\eta'}^2,Q^2,0)}{8\pi^2f_\pi^2}\to(3.72.\pm0.54)\frac{1}{\sqrt{3}}\left(\frac{\sin\theta}{F_8}+\frac{2\sqrt{2}\cos\theta}{F_0}\right). \tag{3.6}$$

From the experimental widths and the values of θ, F_0, F_8 given by Eq. (2.31) we get

$$\frac{Q^2F_\eta(m_\eta^2,Q^2,0)}{8\pi^2f_\pi^2}\to 0.72\pm0.07, \tag{3.7}$$

$$\frac{Q^2F_{\eta'}(m_{\eta'}^2,Q^2,0)}{8\pi^2f_\pi^2}\to 0.98\pm0.14. \tag{3.8}$$

The outputs of Eqs. (3.4), (3.7), and (3.8) together with the experimental data [12] are shown in Fig. 1.

IV. CONCLUSION

The chiral symmetry breaking corrections to the decays $\pi^0,\eta,\eta'\to 2\gamma$ have been estimated in a model-independent way and found to be quite large. This is to be contrasted in particular with the results obtained from chiral perturbation theory [10,16] to undertake the extrapolation to the mass shell where a vanishing total contribution of the π and K loops to Δ_P was obtained. In the case of $\pi^0\to 2\gamma$ the theoretical value is considerably increased and this lends a great significance to the upcoming PRIMEX experiment [13]. In addition, the form factors of the transitions $\gamma^*\gamma\to\pi^0,\eta,\eta'$ were evaluated and shown to compare favorably with the available experimental data.

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