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Update on a very light CP-odd scalar in the two-Higgs-doublet model

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In a previous work we have shown that a general two-Higgs-doublet model with a very light CP-odd scalar can be compatible with electroweak precision data, such as the ρ parameter, BR($b\to s\gamma$), R_b, A_b, BR($\Upsilon\to A\gamma$), BR($\eta\to A\gamma$), and $(g-2)$ of the muon. Prompted by the recent significant change in the theoretical status of the latter observable, we comment on the consequences for this model and update the allowed parameter region. It is found that the presence of a very light scalar with a mass of 0.2 GeV is still compatible with the new theoretical prediction of the muon anomalous magnetic moment.

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I. INTRODUCTION

The possibility of a Higgs boson decaying into a pair of light CP-odd scalars was considered in Ref. [1]. Although it is very unlikely that this particle can be accommodated in the minimal supersymmetric standard model (MSSM), in light of the restrictions imposed by the current low-energy data on the parameters of this model, a very light CP-odd scalar A can still arise in some other extensions of the standard model (SM), such as the minimal composite Higgs model [2], or the next-to-minimal supersymmetric model [3]. Therefore the existence of a very light CP-odd scalar not only proves new physics but also casts the most commonly studied MSSM in doubt. Furthermore, studying the couplings of the light CP-odd scalar to the SM fermions may help discriminating models of electroweak symmetry breaking – either a weakly interacting model (e.g., the next-to-minimal supersymmetric model) or a strongly interacting model (e.g., the minimal composite Higgs model). Apart from the above implications arising from the existence of a very light CP-odd scalar, our main interest in studying this particle stems from the fact that its phenomenology is indeed rather exciting: an interesting aspect of a light A is that if its mass M_A is less than twice that of the muon m_μ, i.e., less than about 0.2 GeV, it can only decay into a pair of electrons ($A\to e^+e^-$) or photons ($A\to\gamma\gamma$). Hence the decay branching ratio BR($A\to\gamma\gamma$) can be sizable. Consequently, A can behave like a fermiophobic CP-odd scalar and predominantly decay into a photon pair, which would register in detectors of high energy collider experiments as a single photon signature when the momentum of A is much larger than its mass [1].

In a previous work [4] we performed an extensive analysis within the framework of the two Higgs doublet model (THDM) and found that a very light CP-odd scalar can still be compatible with precision data, such as the ρ parameter, BR($b\to s\gamma$), R_b, A_b, BR($\Upsilon\to A\gamma$), and the muon anomalous magnetic moment a_μ [4]. We considered different values for $\sin^2(\beta-\alpha)$ and found the constraints imposed on the remaining parameters of the model, which we summarize in Table I, where M_H (M_h) stands for the heavy (light) CP-even scalar and $M_{H^\pm}$ for the charged scalar. As for the soft breaking term μ_{12}, it is not involved in any of the above observables, so they cannot be used to constrain it. Since μ_{12} has no relevance to the present discussion (the purpose of this work is to update the bounds derived from the changes in the status of the theoretical value of the muon anomaly), we refer the reader to Ref. [4], where CERN e^+e^- collider LEP-2 data were used to set bounds on this parameter. In

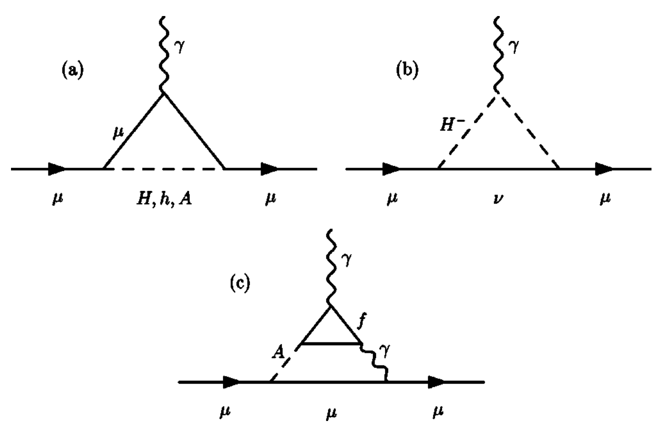

FIG. 1. Contribution from the THDM to the anomalous magnetic moment of muon: (a) neutral Higgs bosons, (b) charged Higgs boson, and (c) the leading two-loop contribution from the CP-odd scalar.

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TABLE I. Constraints from the low energy data for type-I and type-II THDMs, with $M_A = 0.2$ GeV. The old calculation of the hadronic light by light contribution to a_μ, cf. Table II, was used together with the calculation of Ref. [8] for the hadronic vacuum polarization. When $\sin^2(\beta-\alpha)=1$ there is no M_h dependence on ρ, otherwise, we assume $M_h = 110$ GeV.

Constraint	Type-I THDM	Type-II THDM
$(g-2)_\mu$	$\tan\beta > 0.4$	$\tan\beta < 2.6$
$[\tan\beta > 1]$ $b \to s\gamma$	$M_{H^+} > 100$ GeV	$M_{H^+} > 200$ GeV
$[0.5 < \tan\beta < 1]$ $b \to s\gamma$		$M_{H^+} > 200$–350 GeV
$[0.6 < \tan\beta < 1]$ R_b	$M_{H^+} > 200$–600 GeV	$M_{H^+} > 200$–600 GeV
$[\sin^2(\beta-\alpha)=1]$ $\Delta\rho$	$M_H \sim M_{H^+}$	$M_H \sim M_{H^+}$
$[\sin^2(\beta-\alpha)=0.8]$ $\Delta\rho$	$M_H \sim 1.2 M_{H^+}$	$M_H \sim 1.2 M_{H^+}$
$[\sin^2(\beta-\alpha)=0.5]$ $\Delta\rho$	$M_H \sim 1.7 M_{H^+}$	$M_H \sim 1.7 M_{H^+}$

obtaining the bounds shown in Table I we have used the lower value of 110 GeV for M_h. We recall that the LEP-2 direct search bound requires $M_h > 114.1$ GeV at the 95% C.L. [5]. However, in the presence of new physics such a bound can be substantially relaxed. As explained in Ref. [4], the reason why the LEP-2 bound ($M_h > 114.1$ GeV) does not apply to our model is because this bound is based on the SM specific value of BR($h \to b\bar{b}$) [6]. In the THDM, the new $h \to AA$ decay mode can significantly reduce the $h \to b\bar{b}$ branching ratio. This was clearly illustrated in Fig. 9 of Ref. [4] for some allowed parameter space of the model. In any case, the new decay channel ($h \to AA$) registers as a diphoton signature ($h \to \gamma\gamma$) for which LEP-2 has already set a lower bound. By taking both the AA and $b\bar{b}$ decay modes into consideration, a lower bound for $M_h > 103$ GeV can be established in our light A scenario [4].

At this point we would like to emphasize that, given the recent measurements of a_μ at Brookhaven National Laboratory (BNL) [7], the bounds on new physics effects imposed by the muon $(g-2)$ data depend largely on the theoretical value predicted by the SM for the nonperturbative hadronic contribution to a_μ. In our analysis [4], we followed a conservative approach and considered various predictions for the hadronic correction $a_\mu^{\rm had}$ [8–11], which in fact has been the source of debate recently [12–14]. For instance, the bounds shown in Table I were obtained from the calculation presented in Ref. [8], which was the one allowing the largest parameter space.

After the completion of our work, it was evident that the latest precision measurement of a_μ at BNL [7] along with some theoretical predictions for $a_\mu^{\rm had}$ disfavored the presence of a light A in the THDM. As is well known, the BNL data opened the prospect for new physics as the experimental value of a_μ appeared to be more than 2.6σ above the theory prediction based on some calculations of the hadronic vacuum polarization. At the one-loop order, a light CP-odd scalar can give a significant negative contribution to a_μ, making it harder for this type of model to be consistent with experiment. However, the two-loop calculation can yield a large correction to the one-loop result as pointed out in [15]. Although this fact seems to contradict perturbation theory, the unusual situation in which a two-loop diagram can give a contribution of similar size or even larger than that from the one-loop diagrams within a perturbative calculation was noted first by Bjorken and Weinberg when evaluating the Higgs scalar contribution to the $\mu \to e\gamma$ decay [16]. It is straightforward to see that this situation also occurs in the calculation of the Higgs scalar contribution to a_μ. The reason is that the coupling of the Higgs scalar to the muon enters twice in the one-loop diagram, whereas at the two-loop level there appears a diagram in which this coupling enters just once, together with a line where the Higgs scalar couples to a heavy fermion pair (see Fig. 1). This gives rise to an enhancement factor, due to the couplings, that compensates the suppression factor $g^2/(16\pi^2)$, due to an additional loop. It turns out that the diagram of Fig. 1 (c) gives by far the most dominant contribution at the two-loop level. Therefore we do not expect large uncertainties arising from unknown higher order terms. In our previous analysis, even when we considered the two-loop calculation for the CP-odd

TABLE II. Contributions to the anomalous magnetic moment of muon in the SM [19], prior to the discovery of a wrong sign in the pion pole correction to $a_\mu^{\rm had}$(l.b.l), which significantly changed the $a_\mu^{\rm theory}$ prediction. All values are given in units of 10^{-11}.

Contribution	SM prediction
$a_\mu^{\rm QED}$	116 584 705.7(1.8)
$a_\mu^{\rm weak}$	151(4)
$a_\mu^{\rm had}$(l.b.l)	$-79.2(15.4)$[a]
$a_\mu^{\rm had}$(h.o.)	-101 (6)

[a]This value has been found to be wrong in Ref. [17].

TABLE III. The most recent evaluations of the hadronic light by light contribution to $a_\mu^{\rm had}$(l.b.l.). These corrected values contrast with the wrong one shown in Table II.

Reference	$a_\mu^{\rm had}$(l.b.l.)$\times 10^{11}$
[17]	83(12)
[20]	89(15)
[21]	83(32)
[22]	56[a]

[a]This value accounts only for the pion pole contribution.

TABLE IV. Some of the most recent calculations of $a_\mu^{\rm had}$(h.v.p.) together with the respective theory prediction $a_\mu^{\rm theory}$ and the discrepancy Δa_μ between experiment and theory. The last column represents the bounds on any new physics contribution $a_\mu^{\rm NP}$ at the 95% C.L. All of the values are given in units of 10^{-11}.

Reference	$a_\mu^{\rm had}$(h.v.p.)	$a_\mu^{\rm theory}$	Δa_μ	Allowed range for $a_\mu^{\rm NP}$
[26]	7011(94)	116 591 856.3(95.54)	166.7(179.53)	[−185–519]
[10]	6924(62)	116 591 769.3(64.31)	253.7(164.92)	[−70–577]
[11]	6974(105)	116 591 833.3(112.3)	189.7(188.8)	[−180–560]
[25]	7031(77)	116 591 876.3(78.88)	146.7(171.25)	[−189–482]
[24]	6952(64)	116 591 797.3(66.25)	225.7(165.81)	[−99–551]

scalar contribution to a_μ, together with the hadronic correction quoted in Ref. [7], i.e., that by Davier and Höcker [10], we found that there was no allowed parameter space (in the type-II THDM) in the $\tan\beta$ versus. M_A plane when M_A was below 3 GeV. Nevertheless, there were other SM calculations yielding a_μ close enough to the experimental value as to allow a very light A.

Since the publication of [7], there has been a lot of controversy regarding the theoretical value of the muon anomalous magnetic moment. It is evident that before claiming the presence of any new physics effect, an extensive reexamination of every contribution to a_μ is necessary [12–14]. Along these lines, a reevaluation [17] of the hadronic light by light contribution to a_μ found a sign error in earlier calculations [18] of this contribution, which has resulted in a significant change of the a_μ prediction. Once the corrected value is taken into account, the discrepancy between experiment and theory reduces down to the level of 1.6σ. In the light of this result, we believe it is worth revisiting our work and reexamining our previous bounds.

II. ALLOWED PARAMETER RANGE FOR M_A AND $\tan\beta$

The SM prediction of a_μ is composed of the following three parts [19]:

$$a_\mu^{\rm theory}=a_\mu^{\rm QED}+a_\mu^{\rm weak}+a_\mu^{\rm had}, \tag{1}$$

where the electroweak corrections have been computed to a very good accuracy: they have a combined error of the order of 5×10^{-11}, which is already about one order of magnitude smaller than the ultimate goal of the E821 experiment [7]. In contrast, the hadronic contribution $a_\mu^{\rm had}$ contains the bulk of the theoretical error ($\sim70\times10^{-11}$) and can be decomposed into three parts [19], namely the hadronic vacuum polarization contribution a_μ(h.v.p.), the hadronic light-by-light correction a_μ(l.b.l), and other hadronic higher order terms a_μ(h.o.):

$$a_\mu^{\rm had}=a_\mu^{\rm had}(\text{h.v.p.})+a_\mu^{\rm had}(\text{l.b.l})+a_\mu^{\rm had}(\text{h.o.}). \tag{2}$$

In our previous analysis we used the values shown in Table II for each contribution to $a_\mu^{\rm theory}$ [19] together with the $a_\mu^{\rm had}$(h.v.p.) predictions to be discussed below. In the months following the publication of our work, a new situation arose: the sign of the pion pole contribution to the hadronic light by light correction was found to be wrong [17]. Very interestingly, this contribution alone represents about 70% of the full $a_\mu^{\rm had}$(l.b.l.). It turns out that after correcting this mistake, the $a_\mu^{\rm had}$(l.b.l.) value gets significantly changed and even its sign gets flipped. As a result, the discrepancy between the experiment and theory reduces down to the level of 1.6σ. Subsequent publications have confirmed this finding [20–22]. In Table III we list the most recent evaluations of $a_\mu^{\rm had}$(l.b.l.). In addition, there is one more calculation that is based on chiral perturbation theory [23]:

$$a_\mu^{\rm had}(\text{l.b.l.})=(55^{+50}_{-60}+31\hat{C})\times10^{-11},$$

where $\hat{C}$ is an unknown low-energy constant that parametrizes some subdominant terms. We will not consider this result here but only mention it as an example of a calculation that is still open to debate. For the purpose of this work we will take an average of the top three results shown in Table III and study the consequences on the allowed parameter space of the THDM.

After introducing the corrected value of $a_\mu^{\rm had}$(l.b.l.), the sum of all the contributions to $a_\mu^{\rm theory}$ except $a_\mu^{\rm had}$(h.v.p.) is

$$a_\mu^{\rm theory}-a_\mu^{\rm had}(\text{h.v.p.})=116\,584\,845.3\,(17.1)\times10^{-11}, \tag{3}$$

where the errors have been composed quadratically.[1] As for the a_μ(h.v.p) term, its evaluation has also been the source of renewed interest lately.[2] As in our previous work, here we will use a conservative approach and consider some representative evaluations of a_μ(h.v.p). In the second column of Table IV we show some of the most recent results, which were compiled in [24], whereas in the third column we show the full theory prediction, which is obtained after adding up each value in the second column to Eq. (3).

As for the experimental value $a_\mu^{\rm exp}$, the data obtained during the 1999 running period combined with previous measurements give [7]

$$a_\mu^{\rm exp}=116\,592\,023\,(152)\times10^{-11}. \tag{4}$$

One thus can obtain the discrepancy between experiment and theory $\Delta a_\mu=a_\mu^{\rm exp}-a_\mu^{\rm theory}$ for each different evaluation

[1]Throughout this work we will systematically compose the errors in quadrature.

[2]For a summary of the most recent evaluations of $a_\mu^{\rm had}$, see Refs. [24] and [25].

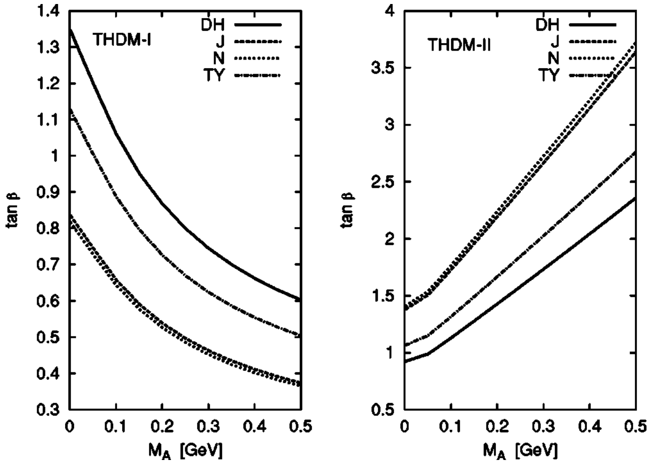

FIG. 2. The regions (above the curves for type-I and below the curves for type-II THDM) in the tan β vs M_A plane allowed by the a_μ data at the 95% C.L. Four different curves are displayed depending on whether the SM prediction is obtained from the DH, J, N, or TY calculation of $a_\mu^{\rm had}$(h.v.p.). The two-loop contribution from the light A has been used.

of a_μ(h.v.p), as shown in the fourth column of Table IV. Finally, if we assume that the discrepancy between theory and experiment is to be ascribed to new physics effects, we can obtain the bounds shown in the last column of the same table for the new physics contribution to the anomalous magnetic moment at the 95% C.L., which is denoted by $a_\mu^{\rm NP}$. Those bounds on $a_\mu^{\rm NP}$ should be compared to those used in our previous analysis, cf. Eq. (2) in Ref. [4].

Given the new bounds on $a_\mu^{\rm NP}$, we update the constraint imposed by it on the $\tan\beta - M_A$ plane within the THDM. The analytical expressions for the contribution of either a CP-even or a CP-odd scalar (Fig. 1) can be found in Appendix A of Ref. [4]. We will use the two-loop calculation for the contribution from the CP-odd scalar [15]. In order to

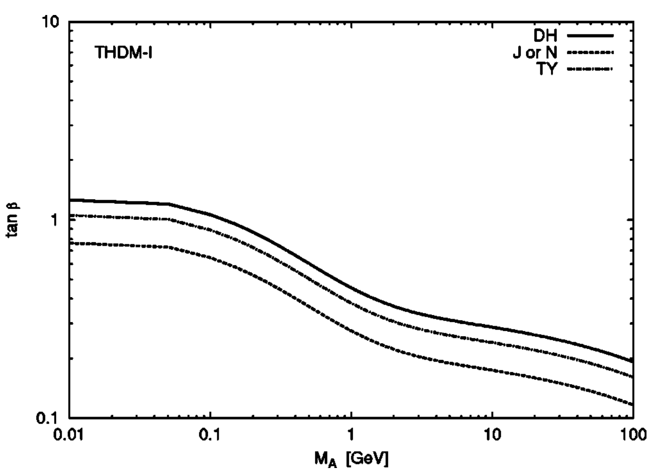

FIG. 3. The region (above the curves) in the tan β vs M_A plane of a type-I THDM allowed by the a_μ data at the 95% C.L. The allowed regions based on the DH, J, N, and TY calculations are above the curves. The two-loop contribution from the light A has been used.

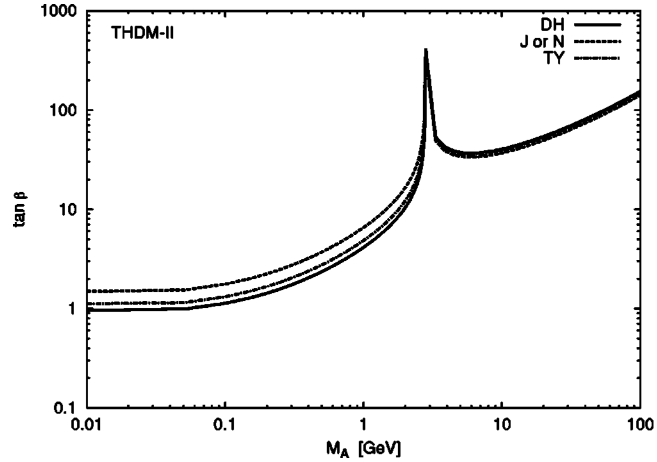

FIG. 4. The regions (below the curves) in the tan β vs M_A plane of a type-II THDM allowed by the a_μ data at the 95% C.L. The allowed regions based on the DH, J, N, and TY calculations are below the curves. The two-loop contribution from the light A has been used.

satisfy the bounds shown in Table I, we are assuming that the remaining four Higgs scalars are much heavier than the CP-odd scalar A, so their contribution to a_μ turns out to be negligibly small as compared to that coming from the latter. Also, we are considering that $\sin^2(\beta-\alpha)=1$. The reason why we make this choice is because in our scenario with a very light CP-odd scalar the most convenient way to meet the constraint imposed by the ρ parameter is to have M_H and M_{H^+} nearly degenerate and $\sin^2(\beta-\alpha)$ close to 1 [4]. For comparison purposes, we will analyze the bounds arisen from the theoretical predictions based on the Davier-Höcker (DH) [10], Jegerlehner (J) [11], Narison (N) [25], and de Troconiz-Yndurain (TY) [24] calculations of $a_\mu^{\rm had}$(h.v.p.), which are the most representative and recent ones. We would like to note that, as observed through Figs. 2–6, the bounds from the J [11] and N [25] calculations are almost indistinguishable.

In Figs. 2–4 we show the allowed regions in the $\tan\beta - M_A$ plane for both types of THDMs. In Fig. 2, which

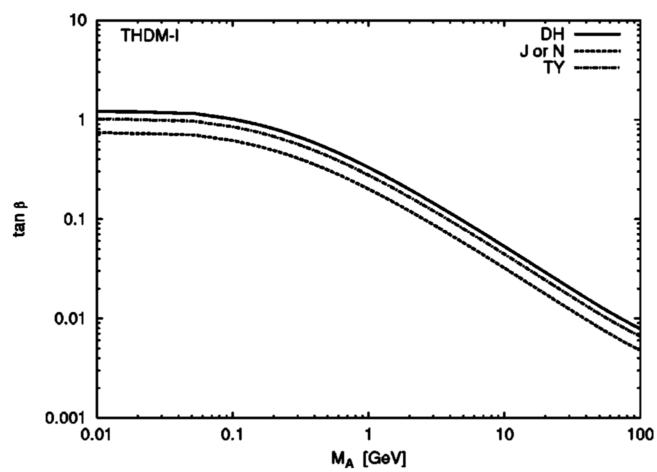

FIG. 5. Same as Fig. 3, but only the one-loop contribution from the light A is considered.

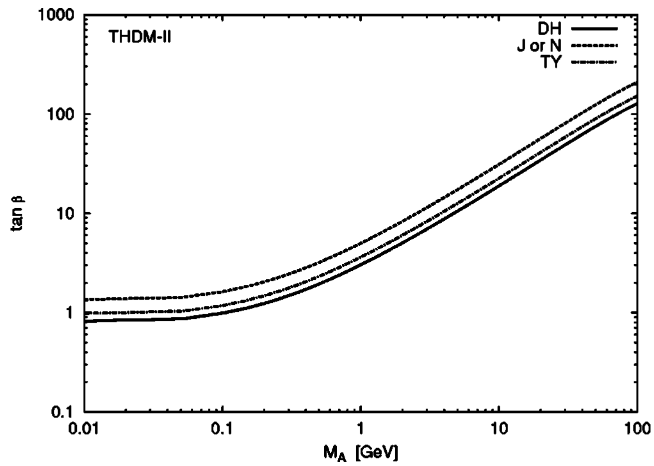

FIG. 6. Same as Fig. 4, but only the one-loop contribution from the light A is considered.

shows the low M_A regime, it can be seen clearly that even if one considers the DH calculation of a_μ^{had}, which is the one with the smallest error, there is still the possibility of having a CP-odd scalar with a mass of the order of 0.2 GeV in the type-II THDM as long as $\tan\beta<1.43$, whereas for a type-I THDM $\tan\beta$ has to be greater than 0.87. This is a very significant change with respect to the results obtained when using the old (uncorrected) value of a_μ^{theory}. In that case, the DH calculation did not allow for a light CP-odd scalar in either type of THDM, though other calculations did allow such a possibility.

As stated above, so far our results have been derived from the two-loop contribution from the CP-odd scalar to a_μ^{NP}. It is also interesting to repeat the above analysis using only the one-loop calculation for a_μ^{NP}. Its result is depicted in Figs. 5 and 6. The old (uncorrected) theory prediction based on the DH calculation required any new physics contribution to a_μ to be positive. However, the one-loop contribution from a light CP-odd scalar is always negative. Therefore the old SM theory prediction for a_μ combined with the THDM one-loop correction strongly disfavored the existence of a very light CP-odd scalar. This is to be contrasted with the conclusion drawn from the corrected value of a_μ^{theory}. In that case, there is indeed an allowed region of $\tan\beta$ when $M_A \sim 0.2$ GeV, though this region is smaller than the one allowed by the two-loop calculation of a_μ^{NP} (cf. Figs. 3 and 5, and Figs. 4 and 6). As shown in Fig. 4, there is an interesting feature in the $\tan\beta$ versus M_A plane of a type-II THDM when M_A is around 2.6 GeV. It is because for $M_A \sim 2.6$ GeV, the two-loop contribution from a light CP-odd scalar becomes as large as the respective one-loop contribution but with an opposite sign, so the total effect cancels.

TABLE V. Constraints on $\tan\beta$ from the muon $(g-2)$ data for type-I and type-II THDM, with $M_A=0.2$ GeV, based on various SM theory predictions of a_μ^{had}(h.v.p). The two-loop contribution for the CP-odd scalar has been used.

Theory prediction	Type-I THDM	Type-II THDM
DH [10]	$\tan\beta>0.87$	$\tan\beta<1.43$
J [11]	$\tan\beta>0.54$	$\tan\beta<2.19$
N [25]	$\tan\beta>0.53$	$\tan\beta<2.24$
TY [24]	$\tan\beta>0.73$	$\tan\beta<1.67$

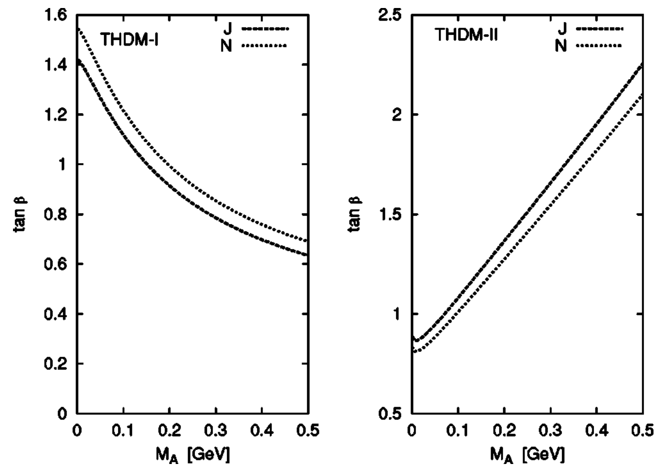

FIG. 7. Same as Fig. 2, but with the latest experimental data from the muon $(g-2)$ collaboration [31]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.

Bounds on $\tan\beta$ from meson decays

For completeness we now turn to analyze the bounds obtained on THDMs with a very light CP-odd scalar from meson decays. A very light Higgs scalar (CP-odd or CP-even) can be a decay product of some hadrons, like the η and Υ mesons. For the latter, a measured upper bound to the $X+\gamma$ decay channel has been set [27] that can be used to constrain the $A\bar{b}b$ coupling. Denote the Yukawa coupling of

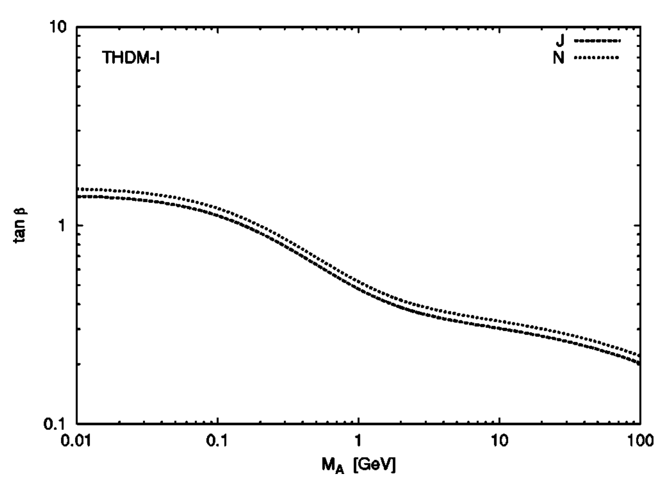

FIG. 8. Same as Fig. 3, but with the latest experimental data from the muon $(g-2)$ collaboration [31]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.

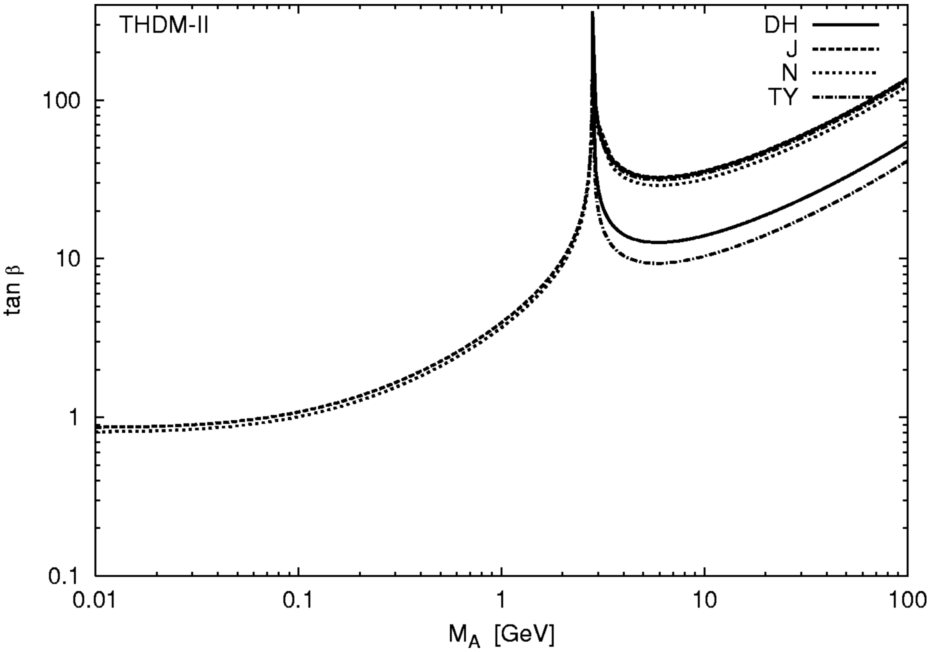

FIG. 9. Same as Fig. 4, but with the latest experimental data from the muon $(g-2)$ collaboration [31]. The region allowed by the DH [10] and TY [24] calculations is bounded by the respective lines.

$A\bar{b}b$ to be $k_d m_b/v$, with $k_d=\tan\beta$ $(\cot\beta)$ in the type-II (type-I) model. Then, the data of the meson decay $\Upsilon\to\gamma+X$ require $k_d<1$. (We refer the reader to Refs. [4,3] for a detailed discussion.)

As shown in Ref. [28], there is another decay process that can strongly constrain $\tan\beta$, namely $\eta\to\pi S$, where S is a very light CP-even scalar. Those results can be translated into the case of a CP-odd scalar. In particular, the experimental upper limit

$$\mathrm{BR}(\eta\to\pi^0 e^+ e^-)\leqslant 5\times 10^{-5} \tag{5}$$

can be used to obtain the following constraint on a THDM CP-odd scalar with mass M_A lying in the range $2m_e\leqslant M_A\leqslant 2m_\mu$:

$$(k_d-k_u)^2\lambda\left(1,\frac{m_\pi^2}{m_\eta^2},\frac{m_A^2}{m_\eta^2}\right)\leqslant 1.5, \tag{6}$$

where k_u is $\cot\beta$ for either type-I or type-II THDM and k_d has been defined above. The function λ is given by $\lambda^2(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$. From here we can conclude that $\cot\beta\geqslant 0.65$ for type-I THDM and $0.55\leqslant\tan\beta\leqslant 1.8$ for type-II THDM. We thus can confirm that

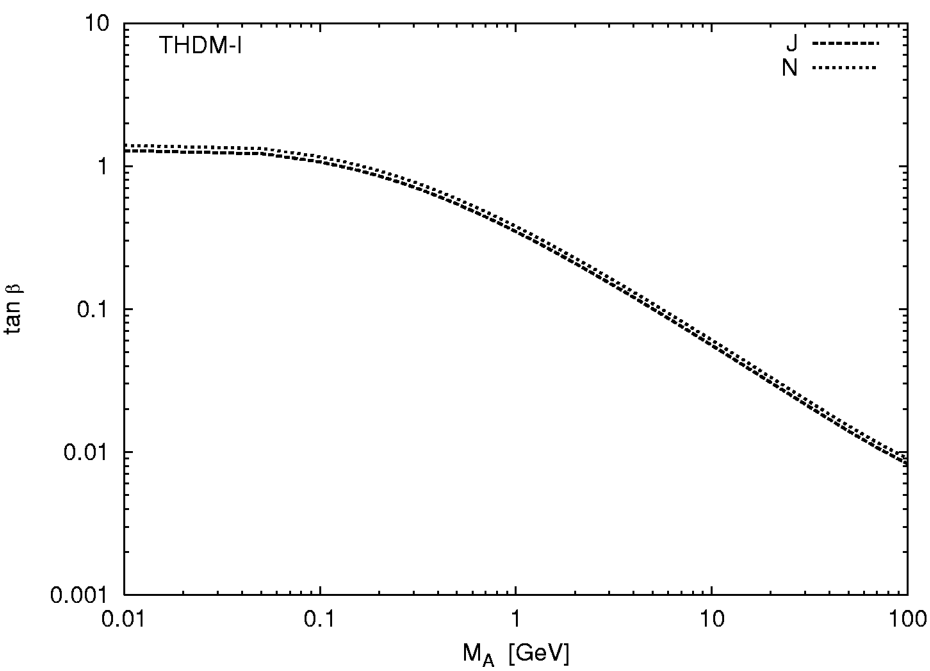

FIG. 10. Same as Fig. 5, but with the latest experimental data from the muon $(g-2)$ collaboration [31]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.

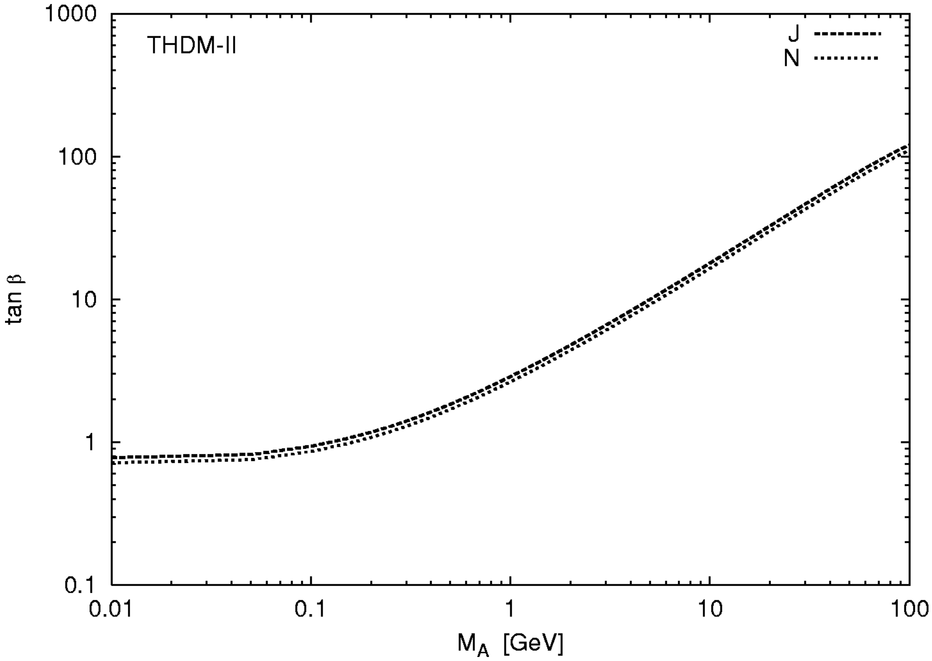

FIG. 11. Same as Fig. 6, but with the latest experimental data from the muon $(g-2)$ collaboration considered [31]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.

the hadron decay data together with the muon $(g-2)$ measurement require $\tan\beta$ to be of order 1 if there exists a very light pseudoscalar with a mass smaller than $2m_\mu$.

III. OVERALL DESCRIPTION OF THE GENERAL THDM WITH A LIGHT A

Once the allowed parameter range for $\tan\beta$ and M_A has been updated, there remains five other parameters to consider: the CP-even neutral Higgs mixing angle α, the soft breaking term μ_{12}, and the three other Higgs masses: M_h, M_H, and M_{H^+}. Since we already know that $\tan\beta$ has to be of order 1 we can address the status of the charged Higgs mass M_{H^+} independently of the other parameters. It turns out that both the $b\to s\gamma$ and the R_b data require H^+ to be considerably heavy [4,29]:

$$M_{H^+}\gtrsim 350\ \text{GeV}. \tag{7}$$

Such a high lower bound for the H^+ mass affects the allowed values of the mixing angle α. In Ref. [4] we show that the ρ parameter requires M_H and M_{H^+} to be very correlated depending on the value of $\sin^2(\beta-\alpha)$. In fact, if a very light CP-odd scalar is to be allowed, the easiest way to satisfy the bound imposed by $\rho\sim 1$ is to have M_H and M_{H^+} degenerate and $\sin^2(\beta-\alpha)=1$. With this choice, M_h is not restricted since it does not contribute to the ρ parameter. As we consider values of $\sin^2(\beta-\alpha)$ smaller than 1, it turns out that ρ is very sensitive to the masses of H and H^+. For instance, if $\sin^2(\beta-\alpha)=0.5$, M_H must be at least of the order of 500 GeV [4]. Generally speaking, our conclusion on the bounds on a very light CP-odd scalar in the THDM does not change significantly for $0.5<\sin^2(\beta-\alpha)<1$ as long as the other Higgs bosons in the model are heavy enough. For a very small value of $\sin^2(\beta-\alpha)$, much less than 0.5, the ρ-parameter data would have required the mass of H to be at the TeV order.

IV. CONCLUSION

In conclusion, with the recent correction to the SM prediction of a_μ, the current muon $(g-2)$ data, together with other precision data (cf. Table I), still allow a light (M_A $\sim$0.2 GeV) CP-odd scalar boson in the THDM. Because of this new development in the SM theory calculation of the muon $(g-2)$, the allowed range of $\tan\beta$ in the type-I or type-II THDM is modified, and our result is summarized in Table V. It is interesting to note that the phenomenology at high energy colliders predicted by the THDM with a light CP-odd Higgs boson is dramatically different from that predicted by the usual THDM in which the mass of the CP-odd scalar is at the weak scale. A detailed discussion on this point can be found in Ref. [4]. In particular, various potential discovery modes were studied in there: it was found that the Fermilab Tevatron, the CERN large hadron collider (LHC), and the planned e^+e^- linear collider (LC) have a great potential to either detect or exclude a very light A in the THDM.

Finally, we note that while a light CP-odd scalar in THDM is still compatible with all the precision data, it has been shown recently in Ref. [30] that a light CP-odd scalar in the MSSM will violate the constraint derived from the $Zb\bar{b}$ coupling. This is because in the MSSM, the masses of the five Higgs bosons are related by the mass relations required by supersymmetry. Hence, with a light CP-odd scalar, the mass of the other Higgs bosons cannot be arbitrary large, and it is difficult to yield the decoupling limit when calculating low energy observables.

Note added. During the review process of this manuscript, the muon $(g-2)$ collaboration announced a new result based on data collected in the year 2000 [31], in which the experimental uncertainty has been reduced to one-half that of the previous measurement while the central value of $a_\mu^{\rm exp}$ remains about the same. [The new data yields $a_\mu^{\rm exp} = 11\,659\,204(7)(5)\times 10^{-10}(0.7\ {\rm ppm})$.] According to the latest experimental data, we have updated Figs. 2–6 in this paper to Figs. 7–11. The new data suggests that a very light CP-odd scalar is not allowed in the type-I or type-II THDM based on the SM calculation done by DH [10] and TY [24]. However, based on the N [25] and J [11] calculations, a very light CP-odd scalar is still possible though the allowed parameter space of the THDM has been tightly constrained.

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